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## RESEARCH ARTICLE

# Evaluating the Connectivity Reliability of the Seaborne Transportation Network Using Uncertainty Theory

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## Abstract

The shipping network serves as the backbone of international trade. However, due to unstable geopolitical conditions, unexpected events have occurred frequently in recent years that have impacted the global shipping supply chain, and this has placed enormous pressure and had adverse effects on seaborne transportation. To measure the reliability of the global shipping network under unexpected events, we abstract the global shipping network into a small world network using uncertainty theory. A method is proposed for selecting (i) the optimal reliable path and (ii) the optimal reliable path with a maximum measurement under different confidence intervals. Our approach is then applied to evaluate the connectivity reliability of China's bulk commodity shipping network, which includes 46 key strait and canal nodes. The results show that the reliability of China's bulk commodity shipping network is 0.8, and the optimal seaborne transportation path is given with different confidence intervals after an unexpected event occurs. Our findings provide managerial insights for selecting the optimal transportation path after an emergency. The results also suggest that the import and export of goods from relevant regions should be reduced, or the routes in these regions should be optimised, to minimise losses and guarantee national strategic security.

**Keywords:** Seaborne transportation, Shipping network, Uncertainty theory, Connectivity reliability, Optimal reliable path selection

## 1. Introduction

Seaborne transportation has been one of the main modes of delivery for international trade, due to its high efficiency, low cost, high loading capacity and low cargo losses [1,2]. However, in recent years, the international situation has become more volatile [3], especially in the Middle East and Africa, and this has put enormous pressure on and adversely affected seaborne transportation [4,5]. As

shown in Table 1, the Suez Canal container ship *Ever Given* ran aground and caused a canal blockage in March 2021; the Russia–Ukraine conflict broke out in February 2022 and continued to escalate, causing regional instability; the Sudan armed conflict caused a crisis in the supply of Arabic gum in April 2023; the water level of the Panama Canal dropped to a historic low due to low rainfall over the watershed area in July 2023, thus limiting the accessibility of shipping; and in October 2023, the

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Table 1. Relationships between the uncertainty or volatility of seaborne transportation and emergency events.

Emergency event	Time	Location	Effect
Grounding accident of the container ship <i>Ever Given</i>	March 2021	Middle East	Caused blockage of the Suez Canal
Russia–Ukraine conflict	February 2022	Europe	Obstructed liquefied gas and grain shipping
Armed conflict in Sudan	April 2023	Africa	Caused a supply crisis of Arabic gum
Low water levels in Panama Canal	July 2023	Latin America	Caused blockage of the Panama Canal
The Israeli–Palestinian conflict	October 2023	Middle East	Prevented ships from passing through the Suez Canal

Israeli–Palestinian conflict escalated again, which imposed huge losses on and serious challenges to the global shipping industry.

The seaborne transportation network is the main carrier of global resources. If nodes in this network are affected by unexpected events or fail, this may lead to threats to the trade of many countries, and may even directly or indirectly affect economic development and strategic security [6–8]. Hence, the connectivity reliability of the seaborne transportation network is emerging as one of the most important aspects of shipping research [9].

Tran *et al.* [10] proposed the concept of connectivity reliability for transportation networks, while Kim *et al.* [11] extended connectivity to the entire network, based on the work by Tran *et al.* [10], using an approximation and resolution approach. Wakabayashi *et al.* [12] assessed the connectivity reliability of the shipping network and simplified the computation presented by Boolean *et al.* Stern *et al.* [13] and Liu *et al.* [14] investigated the connectivity reliability of shipping networks via Monte Carlo simulations.

Some scholars have also introduced novel interdisciplinary methods. For instance, Wu *et al.* [15] calculated the connectivity reliability of the transportation network by representing the network using complex network theory; Reggiani *et al.* [16] used connectivity and accessibility to understand the loss and return in transportation systems; Edrissi *et al.* [17] introduced the concept of network reliability to identify critical sections of a road after an unexpected event; Wu *et al.* [15] considered the concept of flux entropy and proposed innovative metrics; and Song *et al.* [18] minimised the transportation risk, cost and time by modelling the path. In order to comply with the European Union Emissions Trading System (EU ETS), Sun *et al.* [19], Sun *et al.* [20] and Zhang *et al.* [21] proposed a carbon and cost accounting model for the shipping industry that could accurately calculate the emissions and the total costs of CO<sub>2</sub>.

Zhao *et al.* [22] developed a two-step approach with two programming sub-models to address the problem of ship delays and unreliable punctuality

due to inherent uncertainties at sea and in ports. Wu *et al.* [23] and Li *et al.* [24] proposed an innovative method for identifying critical breakdown points in the global container transportation network based on changes in geospatial connectivity and network breakdown processes under deliberate circumstances. Li *et al.* [25] responded to the issue of shipping service disruption, a common operational problem faced by shipping companies, by proposing a novel mathematical model of mixed-integer programming to solve the vessel schedule recovery problem (VSRP) for shipping service. Zhao *et al.* [22] and Pei *et al.* [26] proposed a two-step approach for deploying heterogeneous ships and designing reliable schedules to solve the problem of ship delays and punctuality. Many other scholars have proposed new algorithms for shipping networks; for instance, Yi *et al.* [27] and Handley *et al.* [28] used a Markov model to judge the reliability problem of traffic networks.

From the literature review presented above, it can be seen that research on the connectivity reliability of transportation networks has mainly focused on land transportation, with urban road transportation as the main research direction, and there is scant research on transportation by water, and especially seaborne transportation. Compared to land transportation, nodes in seaborne transportation networks are more susceptible to the impact of unexpected events (such as natural disasters, terrorist attacks, national conflicts, and wars) and are more likely to fail [29]. It is therefore valuable to use uncertain networks to investigate the connectivity reliability of the seaborne transportation networks. In this study, we consider the seaborne transportation network as a small world network, and then evaluate and select the connectivity reliability of each node by introducing uncertain variables and combining them with a logically extended recursive decomposition algorithm. The findings are expected to provide policy and managerial insights with the aim of optimising the seaborne transportation network and improving its reliability.

The remainder of this paper is structured as follows. Section 2 abstracts the global shipping

network into a small world network using uncertainty theory. Section 3 introduces a methodology for evaluating the reliability and connectivity of the seaborne transportation network and selecting the optimal route under uncertain conditions. Section 4 presents a case study, and Section 5 summarises our most important conclusions.

## 2. Problem description

A complex network is mainly composed of two parts, a set of points  $V(G)$  and a set of edges  $E(G)$ , i.e.,  $G = (V, E)$ . The edges  $E(G)$  denote the points of  $V(G)$ . Assuming that  $G$  is an undirected network, the number of fixed points  $N = |V|$  and the number of edges  $\varepsilon = |E|$ , where  $(i, j)$  and  $(j, i)$  correspond to each other one by one. At this point, the ports, ships, and routes are abstracted into an undirected network (see Table 2).

The composition of the seaborne transportation network is mainly composed of canals, straits and

ports, which form a complex network known as a small-world network [30]. The set of network nodes is  $V$  and the set of segments is  $E$ , both of which constitute the seaborne transportation network  $G = (V, E)$ . The details are shown in Fig. 1.

Fig. 1 illustrates the global commodity shipping network. It can be seen that the main importer of bulk commodities are China, Japan, and South Korea in Asia, followed by the Americas (North America) and Europe (Western Europe). The main exporting countries are in the Middle East, Africa, Oceania, the Americas (South America), Europe (Eastern Europe), and Asia (South Asia and South-east Asia). The major straits and canals around the world are shown, including the Taiwan Strait, the Bering Strait, the Panama Canal, the Straits of Bauk, the Suez Canal, the Straits of Malacca, the Straits of Mozambique, the Straits of Gibraltar, and others.

Uncertain events can cause instability at several nodes within a region or country, such as the recent Houthi terrorist attacks in Yemen, the drop in the water level of the Panama Canal, and the Russia–Ukraine war, meaning that we are unable to determine the state of connectivity of a node, and whether it is safe or not. We therefore need to use uncertain variables to represent the relevant historical data, and use these data to simulate the distribution of the probability of node failure after the occurrence of unexpected events [31]. The uncertainty theory of shipping network uncertainty factor method mainly refers to Yeh *et al.* [32].

Table 2. Notation used to abstract seaborne transportation networks into undirected networks.

Element	Meaning
Nodes	Ports involved in transportation
Edges	Routes between ports
Neighbour nodes	Ports with direct shipping connections to the port
Degree of a node	Number of direct routes between the port and other ports, where each node interacts only with its neighbours

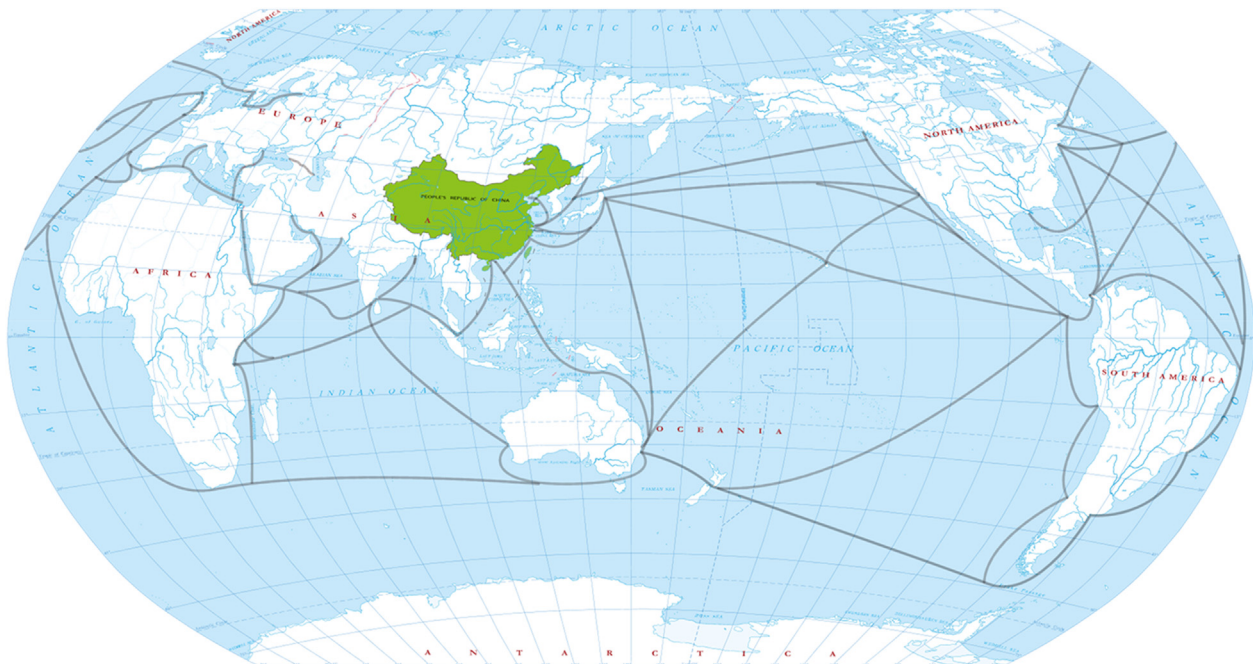


Fig. 1. The global seaborne shipping network.

### 3. Methodology

#### 3.1. Connectivity reliability assessment of the seaborne transportation network

To describe the connectivity reliability of each node after the occurrence of a contingency, we introduce the uncertain variables  $\xi = (\xi_1, \xi_2, \dots, \xi_n)$  ( $0 < \xi_1 \leq 1$ ) [33] to form an uncertain seaborne transportation network  $G = (V, E, \xi)$ . We assume that each node is in a failed state, but the state of each segment is normal. Each node segment then has an uncertainty distribution, i.e.  $\Phi_1, \Phi_2, \dots, \Phi_n$ , but these are reliable, and each node is independent of the others. To obtain the reliability of uncertain seaborne transportation network connectivity, recursive decomposition is carried out after the occurrence of a contingency.

##### 3.1.1. Recursive decomposition algorithms

The reliability of transportation network connectivity refers to the probability of connectivity from the origin to the destination path  $OD$  (origin-destination). If the network path has connectivity problems or a lack of connectivity, it will be assumed that this path also has no connectivity. The transportation network operation state function is as follows, where  $G$  is the seaborne transportation network:

$$\Phi(G) = \begin{cases} 1, & \text{if network is normal} \\ 0, & \text{if network is disabled} \end{cases} \quad (1)$$

According to Equation (1), the following expression can be obtained for each set of paths expressed by the Boolean function:

$$\Phi(G) = f\left(\bigcup_{k=1}^K S_k^n\right) \quad (2)$$

In Equation (2), the Boolean function  $f(\cdot)$  is used to represent a true event as a value of one and a false event as zero.  $S_k$  is the contingency, where there are  $K$  paths to the endpoint, and the  $k$ th path is direct. These are called the set of paths.

For  $S_0 = s_1s_2\dots s_i\dots s_{n_0}$  consisting of  $n$  network nodes,  $S_0$  is the smallest path,  $s_i$  is the one which can be connected to the  $i$ th node, and  $i = 1, 2, \dots, n_0$ . The following equation is derived from the fact that  $\Phi(G)$  in Equation (2) can be divided into complementary events  $S_0$  and  $\bar{S}_0$ , with absorptivity in the Boolean function and the disjoint sum equation:

$$\phi(G) = [f(S_0) + f(\bar{S}_0)] \Phi(G) \quad (3)$$

From De Morgan's law and the disjoint sum equation, we can obtain:

$$\bar{S}_0 = \bar{s}_1 + s_1\bar{s}_2 + \dots + s_1s_2\dots s_i\dots \bar{s}_{n_0} \quad (4)$$

Substituting Equation (4) into Equation (3), we obtain:

$$\begin{aligned} \Phi(G) &= [f(S_0) + f(\bar{S}_0)] \Phi(G) = f(S_0) + [f(\bar{s}_1) + f(s_1\bar{s}_2) \\ &+ \dots + f(s_1s_2\dots s_i\dots \bar{s}_{n_0})] \Phi(G) = f(S_0) + [f(\bar{s}_1)\Phi(G_1) \\ &+ f(s_1\bar{s}_2)\Phi(G_2) + \dots + f(s_1s_2\dots s_i\dots \bar{s}_{n_0})\Phi(G_{n_0})] \end{aligned} \quad (5)$$

where  $\bar{s}_i$  is the  $i$ th failed node, and  $G_i$  is the sub-network in the network structure with the  $i$ th node removed, where  $i = 1, 2, \dots, n_0$ . To obtain the disjoint minimal paths, it is necessary to use recursive decomposition again, to an acceptable level of accuracy, according to Equation (5). At this point, the seaborne transportation network reliability  $P(\phi(G) = 1)$  is identified at the minimum path of all disjunctions, and is calculated as follows:

$$R = P(\phi(G) = 1) = \sum_{j=1}^M P(L_j) \quad (6)$$

where  $M$  is the minimum number of all paths that are not intersected,  $P$  is the contingency rate,  $R$  is the connectivity reliability, and  $L_j$  is the minimum path that is not intersected.

Compared with traditional algorithms, recursive decomposition algorithms have the following advantages. (i) In recursive decomposition algorithms, small paths are usually replaced by short paths, which can reduce the problem of route search blindness and enable more suitable paths to be found. (ii) In the process of decomposing the shortest path, when considering the actual situation, a larger path unit decomposition will reduce the operation time and quantity, thereby improving the calculation speed. (iii) When searching for the optimal path in the seaborne transportation network, we can consider the correlation of each path and select the optimal path through gradual replacement, rather than finding the optimal path at once. A recursive algorithm is therefore more in line with the current seaborne transportation network in terms of choosing the optimal path, which is conducive to calculating the reliability of connectivity. However, recursive algorithms still have limitations: for example, when the minimum path is replaced, the search area of the seaborne transportation network may be limited, and the optimal path may be ignored. Moreover, recursive decomposition algorithms are based on path sets, which can limit the operation time. Research is therefore needed to develop novel algorithms that do not rely

on the minimum path of the seaborne transportation network.

3.1.2. Logical extensions of recursive decomposition algorithms

In this research, we apply a recursive decomposition algorithm to study the shipping network; this problem is different from the usual one involving network nodes, where a start point is typically connected to an endpoint. In this research, we investigate a network with multiple origins and multiple endpoints according to the need for seaborne transportation network connectivity and the need to connect the logical function representation, in the form of intersection or concatenation, where at least one origin corresponds to multiple endpoints or multiple origins correspond to a single endpoint. We then evaluate the reliability of the connectivity of the seaborne transportation network containing multiple OD s, and then finally carry out a recursive decomposition algorithm for the logical extension [11].

(1) Extensions to decomposition algorithms in intersection sets

Assuming that the seaborne transportation network has  $N$  OD pairs, the reliability of network connectivity can be expressed as the intersection of each OD with connectivity probability. The seaborne transportation network connectivity function is then:

$$\phi(G_p) = f\left(\bigcap_{n=1}^N \bigcup_{k=1}^{K_n} S_k^n\right) \tag{7}$$

where  $G_p$  is the global seaborne transportation network connecting all OD pairs,  $K_n$  is the number of paths the  $n$ th OD pair with  $K_n$  having connectivity is the  $k$ th path, and the number of pairs of the  $n$ th OD is  $S_k^n$ . A homogeneous decomposition of Equation (7) using Equations (3) and (4) yields the following equation:

$$\begin{aligned} \Phi(G_p) &= [f(S_{j_0}) + f(\bar{S}_{j_0})] \Phi(G_p) = f(S_{j_0}) + [f(\bar{s}_{j_1}) \\ &+ f(s_{j_1}\bar{s}_{j_2}) + \dots + f(s_{j_1}s_{j_2}\dots\bar{s}_{j_{m_0}})] \Phi(G_p) = f(S_{j_0}) \\ &+ [f(\bar{s}_{j_1}) \Phi(G_{p_1}) + f(s_{j_1}\bar{s}_{j_2}) \Phi(G_{p_2}) + \dots \\ &+ f(s_{j_1}s_{j_2}\dots s_{j_i}\dots\bar{s}_{j_{m_0}}) \Phi(G_{p_{m_0}})] \end{aligned} \tag{8}$$

$$f(S_{j_0}) \Phi(G_p) = f(S_{j_0})$$

Where  $S_{j_0}$  is the set of paths consisting of each minimum path OD pair of nodes, and  $S_{ji}$  denotes

the joint set of paths that connects the  $i$ th node of the set of paths  $S_{j_0}$ .  $\bar{s}_{ji}$  is a complement that indicates a failed node. Each  $G_{pi}$  obtains a sub-network of network  $G$  after the failure of the  $i$ th node. We continue to decompose these networks using Equation (8) until the minimum path identification is reached or an acceptable level of identification-based network reliability is obtained and there exists at least one pair of disconnected ODs.

(2) Extension of the decomposition algorithm for concatenation

We assume that the seaborne transportation network has  $N$  OD pairs and the reliability of network connectivity can be expressed as the concatenation set of individual ODs and has a probability of connectivity. Based on the above, the seaborne transportation network connection function is as follows:

$$\Phi(G_s) = f\left(\bigcup_{n=1}^N \bigcup_{k=1}^{K_n} S_k^n\right) \tag{9}$$

where  $G_s$  is at least one connected OD pair in the global seaborne transportation network,  $K_n$  denotes the number of paths for the  $n$ th OD pair, and  $S_k^n$  denotes the number of pairs at the  $n$ th OD, where the path connected by  $K_n$  is the  $k$ th path. Satisfy  $f(S_{u0})\Phi(G_s) = f(S_{u0})$  to find the path set  $S_{u0}$ , and decompose Equation (9) using the above method to obtain the following equation:

$$\begin{aligned} \Phi(G_s) &= [f(S_{u0}) + f(\bar{S}_{u0})] \Phi(G_s) = f(S_{u0}) + [f(\bar{s}_{u1}) \\ &+ f(s_{u1}\bar{s}_{u2}) + \dots + f(s_{u1}s_{u2}\dots s_{u_i}\dots\bar{s}_{u_{m0}})] \Phi(G_s) \\ &= f(S_{u0}) + [f(\bar{s}_{u1}) \Phi(G_{s1}) + f(s_{u1}\bar{s}_{u2}) \Phi(G_{s2}) + \dots \\ &+ f(s_{u1}s_{u2}\dots s_{u_i}\dots\bar{s}_{u_{m0}}) \Phi(G_{s_{m0}})] \end{aligned} \tag{10}$$

where  $s_{u0}$  is any of the  $N$  OD pairs of the smallest path in the network. The  $i$ th connected node is  $s_{ui}$ , and  $\bar{s}_{ui}$  is the complementary set denoting the failed node. In Equation (10),  $G_{si}$  is the sub-network of network  $G$  obtained from the  $i$ th node after failure, and Equation (10) is repeated for further decomposition.

3.1.3. Recursive decomposition algorithm-based assessment of seaborne transportation network connectivity reliability

The connectivity reliability of the shipping network is calculated using Equation (6), and the recursive decomposition algorithm based on intersection or concatenation with logical extension is needed to obtain the least disjoint OD paths. The

path  $L_j$  is obtained by calculating each path, and consists of a number of nodes  $n$ . The connectivity reliability is  $P(L_j) = \prod_{i=1}^n \xi_i$ , where  $\xi_i$  is the  $i$ th node uncertainty variable connectivity reliability. The expected value is the size of the mean of the uncertain variable. The expected value of  $\xi$  according to the uncertainty theory is its reliability, and whether the expectation has an uncertain distribution  $\Phi$ ,  $\xi$  expectation exists in the Equation (11):

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha) d\alpha \tag{11}$$

To calculate the reliability of path  $L_j$ , we need to calculate the  $i$ th node of the connectivity reliability  $E[\xi_i]$ , as follows:

$$E[\xi_i] = \int_0^1 \Phi_i^{-1}(\alpha) d\alpha \tag{12}$$

The inverse uncertainty distribution of the distribution of the uncertain variable  $\xi$  is  $\Phi^{-1}$  and  $\alpha \in (0, 1)$ , with the following confidence levels:

$$\phi(x) \begin{cases} 0 & \text{if } x \leq a \\ (x - a)/2(b - a) & \text{if } a \leq x \leq b \\ b(x + c - 2b)/2(c - b) & \text{if } b \leq x \leq c \\ 1 & \text{if } x \geq c \end{cases} \tag{13}$$

$\xi \sim Z(a, b, c)$  is the uncertain variable Zigzag and the regularity of the distribution of the function, where  $a < b < c$  and is real. The inverse uncertainty distribution is as follows:

$$\phi^{-1}(\alpha) = \begin{cases} (1 - 2\alpha)a + 2\alpha b & \text{if } \alpha < 0.5 \\ 2 \setminus (1 - \alpha)b + (2\alpha - 1)c & \text{if } \alpha \geq 0.5 \end{cases} \tag{14}$$

The minimum path reliability is obtained using the recursive decomposition algorithm of the logical extension, i.e., the entire seaborne transportation network connectivity reliability can be obtained by Equation (6).

### 3.2. Optimal path selection for seaborne transportation networks under uncertainty

Using the recursive decomposition algorithms and according to Hsu *et al.* [34] and Yu *et al.* [35], the uncertain shipping network is obtained to be denoted as  $G = (V, E, \xi)$  after the contingency, where  $s \in V$  represents the starting point and  $t \in V$  represents the endpoint. In the seaborne transportation network discussed above, each node's reliability function after a contingency event corresponds to each path of each OD, and the connectivity of these nodes is uncertain. The aims of this

research are to find the path with maximum reliability to guarantee that transportation can be delivered in a safe and timely manner, and to solve the reliable path problem by introducing uncertainty theory to plan the optimal solution. We find the optimal reliable path  $P$  from node  $s$  to node  $t$  after the contingency and aim to maximise the objective function  $\prod_{i \in P} \xi_i$  in all paths. To minimise the problem of transformation by introducing the inverse variable of reliability  $\xi_i^{-1}$ , let  $\xi'_i$  represent  $\xi_i^{-1}$ , and  $\xi_i^{-1}$  represent uncertain risk variables at nodes that are independent of each other and have an uncertain distribution  $\Phi'_i$ . We assume that the set  $P$  contains all paths in the network between node  $s$  and  $t$ ,  $\xi_P = \{\xi'_i | i \in P\}$  denotes the path  $P$  containing uncertain risk variables, and the  $P$  risk is represented as follows:

$$f(\xi_P) = \prod_{i \in V} (1 - x_i + \xi'_i x_i) \tag{15}$$

In Equation (15), path  $P$  belongs to node  $i$  and  $x_i = 1$ , and vice versa for  $x_i = 0$ . Thus, any path risk can be expressed and uncertain as a measurable function of the space to the set of real numbers.  $f(\xi_P)$  is an uncertain variable, and its value is also uncertain. The objective function represents the problem of choosing the optimal reliable path in an uncertain network and treating it with minimisation, and its equation is as follows:

$$\min f(\xi_P) = \min \prod_{i \in V} (1 - x_i + \xi'_i x_i) \tag{16}$$

We also introduce the  $\alpha$ -optimal reliable path pessimistic value to the solution of the reliable path selection problem [21], i.e. finding the critical values of the uncertain variables, by setting the uncertainty network  $G = (V, E)$ . If  $P$  is an arbitrary path, in the interval between node  $s$  and node  $t$ , the confidence level is  $\alpha \in (0, 1)$ , and the pessimistic value of  $\alpha$  in  $\xi_P$  is inferred from the above equation as follows:

$$\xi_P^\alpha = \inf \left\{ r \mid M \left\{ f(\xi_P) \leq r \right\} \geq \alpha \right\} \tag{17}$$

The uncertainty measure  $M$  and the risk  $r$  are obtained from the above equation. If any path  $\xi_P$  obtained in  $s - t$  is the  $\alpha$ -optimal reliable path, then  $\xi_{P^*}^\alpha \leq \xi_P^\alpha$ .

The equations are obtained as follows:

$$\inf \left\{ r \mid M \left\{ f(\xi_{P^*}) \leq r \right\} \geq \alpha \right\} \leq \inf \left\{ r \mid M \left\{ f(\xi_P) \leq r \right\} \geq \alpha \right\} \tag{18}$$

By building an uncertainty network at a confidence level  $\alpha \in (0, 1)$ , the  $\alpha$ -planning model of the



optimal reliable path selection problem can be formulated to minimise the objective denoted by  $r$ . The problem can be formulated as follows:

$$\min_{P \in \rho} r \quad (19)$$

$$\text{subject to } M \left\{ \prod_{i \in P} \xi'_i \leq r \right\} \geq \alpha \quad (20)$$

Finding the path  $P$  with minimum risk with a confidence level  $\alpha$  between node  $s$  and node  $t$  is the essence of the  $\alpha$ -optimal path problem, where the risk of path  $P$  is represented by the product of the risk values of each node, that is  $f(\xi_p) = \prod_{i \in P} \xi'_i$ .

According to this theorem,  $\xi'_i$  has a regular uncertain distribution and a set  $n$  of mutually independent uncertain variables  $\xi_1, \xi_2, \dots, \xi_n$ . We assume that the function  $f(x_1, x_2, \dots, x_n)$  is mono-increasing for  $x_1, x_2, \dots, x_m$  and mono-decreasing for  $x_{m+1}, x_{m+2}, \dots, x_n$ . The uncertainty distribution is then  $\Phi_1, \Phi_2, \dots, \Phi_n$  and  $\xi = f(\xi_1, \xi_2, \dots, \xi_n)$  has an inverse distribution and an uncertain variable. The following equation is then obtained:

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \dots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \dots, \Phi_n^{-1}(1-\alpha)) \quad (21)$$

If we assume that  $\xi$  has a regular uncertain distribution, and  $f(\xi_p) = \prod_{i \in P} \xi'_i$  has an inverse distribution  $\Psi^{-1}(\alpha)$ , the following equation is obtained:

$$M\{\xi \leq t\} = M\{\xi < t\} \quad (22)$$

Equation (19) obtains an  $\alpha$  pessimistic value to minimise  $\prod_{i \in P} \xi'^{\alpha}_i$ , equivalent to minimising the objective function  $r$ , based on the fact that  $\xi'^{\alpha}_i$  is the uncertain variable  $\xi'_i$  [22], and the constraint  $M\{f(\xi_p) \leq r\} = \alpha$  is replaced by  $M\{f(\xi_p) \leq r\} \geq \alpha$  in Equation (20). Then, the solution to the problem of the reliable path is obtained as:

$$\min_{P \in \rho} \prod_{i \in P} \xi'^{\alpha}_i \quad (23)$$

For any node  $i \in V$  and any  $\alpha \in (0, 1)$ , we can obtain:

$$\xi'^{\alpha}_i = \Phi_i^{-1}(\alpha) \quad (24)$$

When a contingency occurs, the optimal path in the seaborne transportation network can be chosen according to Equations (23) and (24). At the same time, we can obtain  $\Psi'^{-1}(\alpha)$  and  $\alpha \in (0, 1)$ , by repeating the steps to obtain the uncertainty

distribution  $\Psi'(x)$  of the optimal risk value of the path  $f(\xi_p)$ .

The decision maker may set a risk criterion to find the optimal reliable path to solve an uncertain problem. The aim of this research is to select the path that is most likely to satisfy this risk criterion according to the maximum measurement.

The optimal reliable path with maximum measurement is obtained by assuming that the uncertainty network  $G = (V, E)$  and the risk of uncertain variables at each node is greater than or equal to zero. If the path between nodes  $s$  and  $t$  is  $\xi_{p^*} = \{\xi'_i | i \in P^*\}$  and any path  $P \in \rho$ , and the value of risk is predetermined to be  $r$ , then the following equation is satisfied:

$$M\{f(\xi_{p^*}) \leq r\} \geq M\{f(\xi_p) \leq r\} \quad (25)$$

The risk of each node has a regular uncertainty distribution  $\{\Phi'_i\}$  in the uncertain network  $G = (V, E)$ . The uncertainty distribution  $\Psi'$  is the risk value  $f(\xi_p)$  of the optimal path, and is independent of each other.  $P^*$  is the maximum measured optimal reliable path for a given value of risk  $r$ . The reliable path problem is obtained by solving for  $\alpha$ - [21,22].

#### 4. Case study

We now consider a case study based on China's import and export of bulk commodities. Ningbo Port, Shanghai Port, Dalian Port, Shenzhen Port, Qingdao Port, Tianjin Port, and Xiamen Port are selected as the destination ports.

##### 4.1. China's imported bulk commodity transportation

According to data from 2023, the total value of China's imported bulk cargo (raw materials) is 17.98 trillion yuan, which includes cumulative imports of crude oil of 563.994 million tons, coal and lignite of 47.297 million tons, and iron ore of 100.861 million tons. The main importing regions are the Middle East, Africa, Europe, America, Oceania, and Asia (excluding the Middle East, most of South Asia, East Asia, and Central Asia). Most of these goods are transported by sea, so only seaborne transportation is considered for the establishment of the seaborne transportation network (see Fig. 2).

The main straits and nodes through which China's seaborne transportation network for imported and exported raw material goods passes are the Taiwan Strait, the Bering Strait, the Panama Canal, the Straits of Bauk, the Suez Canal, the Straits of Malacca, the Straits of Mozambique, and the Straits of Gibraltar.



Fig. 2. China's seaborne transportation network.

The uniformity of the environment faced by many countries has led to a uniform response in the face of emergencies. Hence, the cargo import and export handling in this research are all Chinese ports, mainly Ningbo Port, Shanghai Port and Dalian Port, and the reliability of connectivity is one. Assuming

that no emergencies occur in China's ports, the set of nodes is  $V = (v_1, v_2, \dots, v_n)$ , and  $|V| = 46$ . The cargo needs to pass through most of these 46 nodes, so there is more than one OD pair. The transportation network is shown in Fig. 3, which shows the names of the nodes.

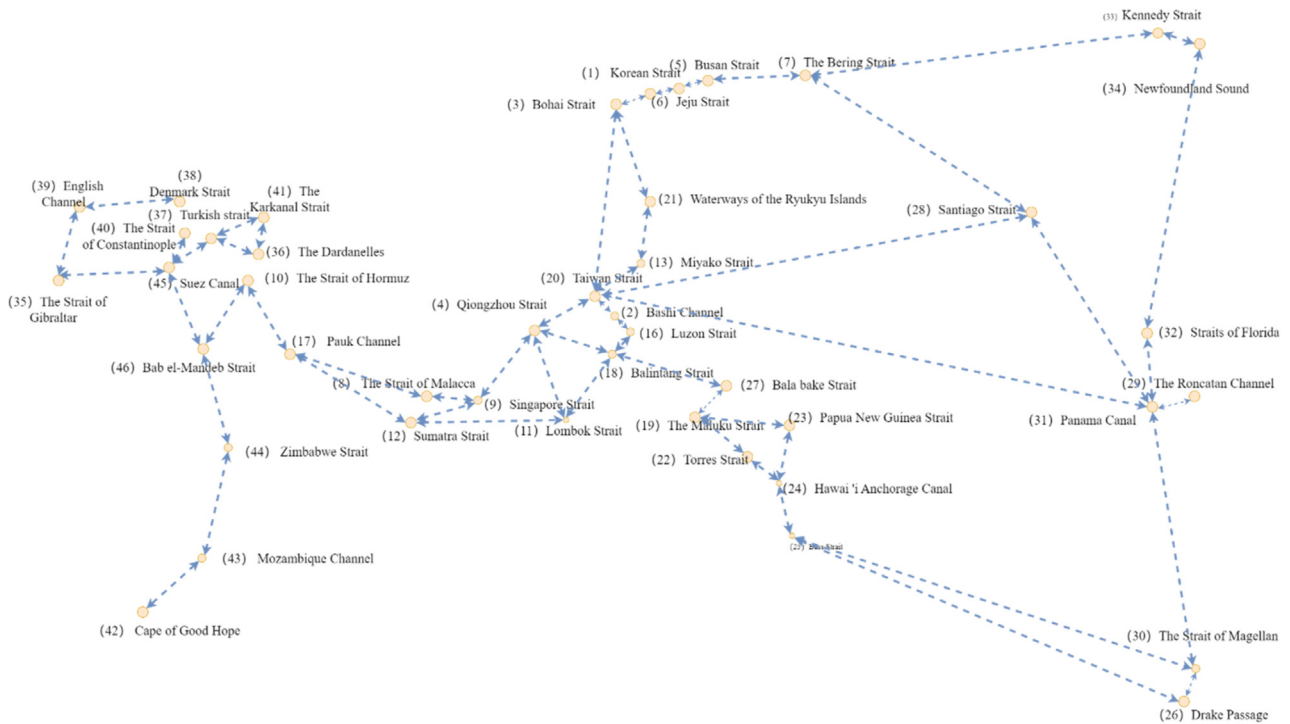


Fig. 3. Closed loop structure diagram of the seaborne transportation network.

#### 4.2. Reliability assessment of transportation network connectivity

We assume that China's import and export goods in the global seaborne transportation network are defined as multiple OD pairs that should be connected at least, but even if the network is connected to an OD can not meet the domestic demand for stable goods, there is a need for a stable route to guarantee that the goods are safe, with timely delivery. We therefore assume multiple nodes and obeys Zigzag uncertain distribution. The elaboration of each node was determined as illustrated in Fig. 3, which is reproduced from a report called “The Ease of Doing Business Ranking” published by the World Bank.

The seaborne transportation network connectivity needs to be defined according to the uncertainty reliability study of China's cargo import and export, mainly through multiple concatenation or intersection events, which can be expressed as follows:

$$\begin{aligned}
 \text{Egyztem} = & ((E1, 2, 3, 4 \rightarrow 46) \cup (E1, 2, 3, 5 \rightarrow 46) \cup (E1, 2, 3, 6 \rightarrow 46) \cup (E1, 2, 4, 5 \rightarrow 46) \cup (E1, 2, 4, 6 \rightarrow 46) \cup \\
 & (E1, 2, 5, 6 \rightarrow 46) \cup (E1, 3, 4, 5 \rightarrow 46) \cup (E1, 3, 4, 6 \rightarrow 46) \cup (E1, 3, 5, 6 \rightarrow 46) \cup (E1, 4, 5, 6 \rightarrow 46) \cup \\
 & (E2, 3, 4, 5 \rightarrow 46) \cup (E2, 3, 4, 6 \rightarrow 46) \cup (E2, 3, 5, 6 \rightarrow 46) \cup (E2, 4, 5, 6 \rightarrow 46) \cup (E3, 4, 5, 6 \rightarrow 46)) \cap \\
 & ((E7, 8 \rightarrow 46) \cup (E7, 9 \rightarrow 46) \cup (E7, 10 \rightarrow 46) \cup (E8, 9 \rightarrow 46) \cup (E8, 10 \rightarrow 46) \cup (E9, 10 \rightarrow 46)) \cap ((E11 \rightarrow 46) \\
 & \cup (E12 \rightarrow 46) \cup (E13 \rightarrow 46) \dots \cup (E45 \rightarrow 46))
 \end{aligned}
 \tag{26}$$

Using the above method, the optimal result for the reliability of all Chinese import and export cargo shipping network connectivity is obtained as 0.80. According to the logical extension of the recursive decomposition algorithm introduced here, we first calculate the minimum path  $S_{u0}$  for each concatenated set of the Chinese shipping network and connect and combine them into a set of paths, to obtain  $S_{j0}$ . We decompose this using Equation (8) to get the non-intersecting minimum path. The reliability is obtained according to Table 2 and Equations (12) and (14), and the above optimal results are obtained by accumulating Equation (6).

#### 4.3. Selection of the optimal path

China's shipping network for import and export cargoes is mainly divided into six ODs, although only one OD exists for Asia and Oceania, making the search for optimal routes meaningless. However, for other regions such as Latin America, the

Middle East and Africa, the risks are very different, and the types and numbers of nodes are also different. It is therefore important to explore reliable pathways to the endpoints in these regions.

We therefore take these six regions as virtual nodes, and the endpoints are still Ningbo Port, Shanghai Port and Dalian Port in China, as in the case study. Assuming that there are no emergencies in China's ports and the reliability is one, the risk of the other regions is uncertain. The structure of the seaborne transportation network after the introduction of virtual nodes is shown in Fig. 4. We introduce virtual nodes to explore the optimal paths in various high-risk areas after an emergency.

Using Equation (21), the individual node reliability is obtained from the inverse variable according to  $\xi'_i = 1/\xi$ ,  $\xi' = f(\xi)$ , where the inverse distribution  $\Phi'^{-1}(\alpha) = f(\Phi^{-1}(1 - \alpha))$ ,  $0 < \alpha < 1$ . The risk of the inverse distribution of uncertainty at each node can be obtained from the distribution of uncertainty at each node in Table 3.

- (1) Table 4 shows the risks of each node under  $\alpha = 0.89$ . Using Equation (23), the optimal path and risk value for  $\alpha = 0.89$  can be derived as follows: the Middle East path is  $S4 \rightarrow S11 \rightarrow S17 \rightarrow S45 \rightarrow S46$ , and the risk value is  $\Psi_{S1}^{-1}(0.89) = 3.94$ , respectively. The path for Africa is  $S3 \rightarrow S11 \rightarrow S17 \rightarrow S44 \rightarrow S43$ . with a risk value of  $\Psi_{S2}^{-1}(0.89) = 3.56$ . The path for Europe is  $S4 \rightarrow S11 \rightarrow S17 \rightarrow S45 \rightarrow S46$ , with a risk value of  $\Psi_{S3}^{-1}(0.89) = 3.20$ . The path for the Americas is  $S3 \rightarrow S6 \rightarrow S7 \rightarrow S31 \rightarrow S32$  with a risk value of  $\Psi_{S4}^{-1}(0.89) = 2.91$ . The path for the Oceania region is  $S4 \rightarrow S14 \rightarrow S2 \rightarrow S27 \rightarrow S22$  with a risk value of  $\Psi_{S5}^{-1}(0.89) = 2.84$ . The path for Asia is  $S3 \rightarrow S20 \rightarrow S18 \rightarrow S16 \rightarrow S12$  with a risk value of  $\Psi_{S6}^{-1}(0.89) = 2.68$ .

To achieve a confidence level of no less than 0.89 likelihood, it is necessary to control the value of the risk of connecting paths within a certain interval,

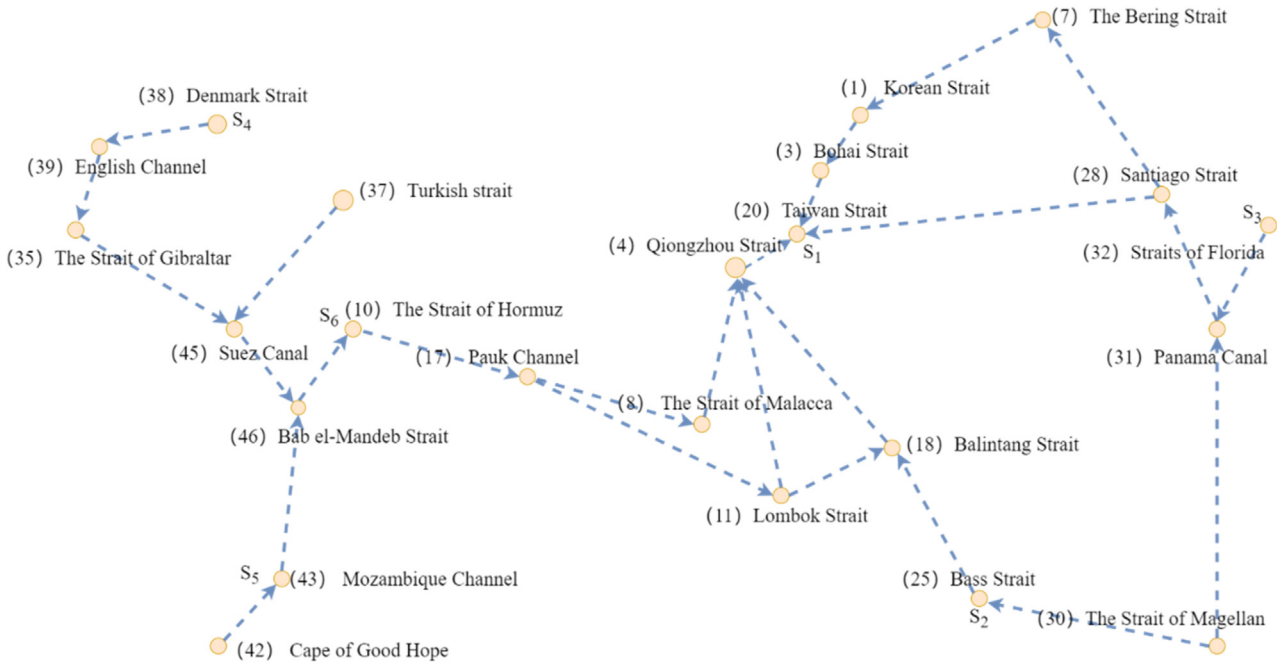


Fig. 4. Seaborne transportation network structure diagram of China's import and export goods after introducing virtual nodes. Note: At nodes 1–46, the names of the import and export source countries are used to represent the import and export strait nodes.

where the smaller the value, the better. In the Middle East and Africa, the minimum risk values are 3.94 and 3.87, respectively. Since these areas are more risky and the paths pass through fewer strait nodes, it is necessary to avoid the more risky straits. For example, at present, these include the Strait of Malacca and the Strait of Hormuz, due to the Houthi occupation of the Red Sea. It is also important to use the Suez Canal as little as possible, so we need to choose the less risky Lombok Strait, and then pass through the Taiwan Strait or the Strait of Qionghzhou, ultimately arriving at a destination such as China's ports of Ningbo, Dalian, or Shanghai. The optimal path for each region is selected based on the confidence level  $\alpha = 0.89$  and the value of the risk, as shown in Fig. 5.

In addition, although the Middle East and Africa have a higher volume of transactions and will require more routes, the risks are much higher than in other regions, especially compared to Latin America and Oceania. The numbers of imports and exports to and from the above areas is therefore increased as much as possible, to improve the reliability of transportation and guarantee national strategic security. Table 5 shows the results for different confidence levels.

From Table 5, it can be seen that the risk value of the path changes with different confidence levels. When  $\alpha = 0.29$ , the risk of choosing the optimal

route for import and export to Africa and the Middle East will also decrease. When  $\alpha = 0.59$ , the path risk of the six regions does not change significantly. It is therefore necessary to fully consider the setting of the confidence level  $\alpha$ , as this will affect the choice of the optimal route for the import and export of goods in each region after the occurrence of an emergency. It will also affect the choice of import and export regions for goods, which will directly or indirectly affect whether the shipment can be safely delivered in a timely manner to the designated area.

- (2) According to the maximum measure in Table 5, the path risk values are  $r_{S1} = 2.07, r_{S2} = 2.02, r_{S3} = 1.99, r_{S4} = 1.86, r_{S5} = 1.62, r_{S6} = 1.40$ , respectively. Based on these risk values, the uncertainty distributions are  $\Psi'_{S1}(2.07) = 0.5, \Psi'_{S2}(2.02) = 0.5, \Psi'_{S3}(1.99) = 0.5, \Psi'_{S4}(1.86) = 0.4, \Psi'_{S5}(1.62) = 0.4$ , and  $\Psi'_{S6}(1.40) = 0.4$ , respectively. The maximum measure means that the reliable path at a given risk value guarantees that the set risk of the path from the start to the end must be higher than the given path risk. By setting  $\alpha = \Psi^{(r)}$  for a given value of risk  $r$ , the results are then used to obtain the optimal path for  $\alpha$ -at different confidence levels, as shown in Table 5.
- (3) Uncertainty distribution optimal path value at risk  $f(\xi p)$  distribution.

Table 3. Uncertainty distribution reliability parameters of various.

Node	<i>a</i>	<i>b</i>	<i>c</i>
Korean Strait	0.54	0.60	0.68
Bashi Channel	0.72	0.78	0.83
Bohai Strait	0.74	0.80	0.88
Qiongzhou Strait	0.74	0.80	0.87
Busan Strait	0.68	0.80	0.82
Jeju Strait	0.66	0.78	0.82
Bering Strait	0.70	0.78	0.79
Strait of Malacca	0.60	0.70	0.75
Singapore Strait	0.78	0.85	0.89
Strait of Hormuz	0.77	0.78	0.88
Lombok Strait	0.70	0.75	0.85
Sumatra Strait	0.69	0.72	0.80
Miyako Strait	0.71	0.75	0.84
Belin Strait	0.68	0.78	0.79
Windward channel	0.66	0.71	0.78
Luzon Strait	0.68	0.76	0.79
Pauk Channel	0.69	0.76	0.81
Balintang Strait	0.70	0.75	0.79
Maluku Strait	0.61	0.68	0.78
Taiwan Strait	0.78	0.80	0.83
Waterways of the Ryukyu Islands	0.61	0.70	0.72
Torres Strait	0.73	0.76	0.79
Papua New Guinea Strait	0.61	0.68	0.78
Pacific canyon	0.72	0.80	0.84
Bass Strait	0.61	0.68	0.78
Drake Passage	0.72	0.76	0.82
Bala Bake	0.61	0.70	0.72
Santiago Strait	0.76	0.82	0.86
Roncatan Channel	0.60	0.72	0.75
Strait of Magellan	0.76	0.82	0.89
Panama Canal	0.75	0.82	0.85
Straits of Florida	0.69	0.81	0.86
Kennedy Strait	0.72	0.81	0.86
Newfoundland Sound	0.76	0.82	0.89
Strait of Gibraltar	0.78	0.82	0.86
Dardanelles	0.61	0.68	0.78
Turkish Strait	0.72	0.80	0.84
Denmark Strait	0.71	0.81	0.84
English Channel	0.78	0.84	0.88
Strait of Constantinople	0.72	0.78	0.85
Karkanal Strait	0.78	0.80	0.83
Cape of Good Hope	0.72	0.75	0.81
Mozambique Channel	0.78	0.82	0.89
Zimbabwe Strait	0.64	0.67	0.72
Suez Canal	0.60	0.67	0.79
Bab el-Mandeb Strait	0.67	0.76	0.80

Note: The port nodes of the country into which the goods are imported are represented by the name of that country.

To obtain  $\Psi^{-1}(\alpha)$ , we need to set different confidence levels  $\alpha$  to get the optimal path uncertainty distribution  $\Psi'$  path risk value  $f(\xi_p)$ . Fig. 6 shows the distributions of the optimal path risk value for the six regions.

From Fig. 6, the following conclusions can be drawn. For the same value of the confidence level  $\alpha$ , the optimal path choice for Asia has a lower value of risk compared to other regions, but due to the lack

Table 4. Risk values for each node

Node	$\phi_{r^{-1}}(0.89)$	Node	$\phi_{r^{-1}}(0.89)$
Korean Strait	1.26	Pacific Canyon	1.36
Bashi Channel	1.36	Bass Strait	1.43
Bohai Strait	1.46	Drake Passage	1.36
Qiongzhou Strait	1.45	Bala Baka	1.51
Busan Strait	1.41	Santiago Strait	1.32
Jeju Strait	1.38	Roncatan Channel	1.56
Bering Strait	1.39	Strait of Magellan	1.44
Strait of Malacca	1.74	Panama Canal	1.54
Singapore Strait	1.58	Straits of Florida	1.47
Strait of Hormuz	1.46	Kennedy Strait	1.45
Lombok Strait	1.38	Newfoundland Sound	1.46
Sumatra Strait	1.42	Strait of Gibraltar	1.67
Miyako Strait	1.42	Dardanelles	1.57
Belin Strait	1.52	Turkish Strait	1.55
Windward channel	1.36	Denmark Strait	1.36
Luzon Strait	1.38	English Channel	1.63
Pauk Channel	1.40	Strait of Constantinople	1.53
Balintang Strait	1.38	Karkanal Strait	1.52
Maluku Strait	1.35	Cape of Good Hope	1.70
Taiwan Strait	1.42	Mozambique Channel	1.77
Waterways of the Ryukyu Islands	1.32	Zimbabwe Strait	1.61
Torres Strait	1.38	Suez Canal	1.86
Papua New Guinea Strait	1.35	Bab el-Mandeb Strait	1.89

of import and export volume in Asia has to import and export from Africa and the Middle East. In recent years, the continuous wars in the Middle East and Africa have also directly or indirectly affected the risk value of the European region. Hence, China's imports and exports from Asia (excluding the Middle East) and Oceania are less risky; however, due to the low supply of goods, the Middle East and Africa have more sources of goods and there is a need to trade from these two regions. It is also necessary to find suitable routes and to increase the share of imports from Asia and Oceania, as well as from Latin America, to guarantee that the goods are sufficient. The value of risk for the European region increases with the value of risk for the Middle East and Africa, and as the confidence level  $\alpha$  increases, the difference in risk between the optimal paths for these two regions also increases.

Table 6 summarises prior research in this area. Based on uncertain crude oil transportation networks accurately, Liu *et al.* [14] analysed the impact of the current international situation on ship route selection, although this research focused only on the Middle East, Africa, and Europe. Wu *et al.* [23] carried out experiments and used recursive algorithms to investigate the timely arrival and delivery of sea

The most reliable routes from China to the Asian region :



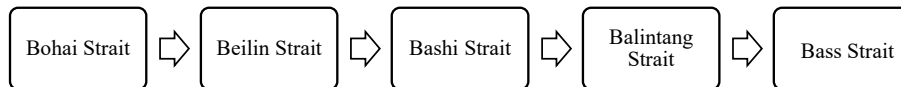
The most reliable route from China to Africa:



The most reliable route from China to Europe:



The most reliable route from China to Oceania:



The most reliable route from China to the Americas:



Fig. 5. Best routes for the bulk commodity transportation network under uncertain conditions.

transportation, but the impact of uncertain events was not considered. Based on the joint and conditional distributions of events, Prieto *et al.* [36] introduced a vine copula to construct the joint distribution of various unexpected events and evaluated the reliability of seaborne transportation network connections. However, this method was only suitable for areas with high popularity. Based

on the above research, this article considers recent ‘hot’ events, and applies recursive decomposition algorithms and uncertainty theory to determine the uncertain distribution of the most reliable path risk values for major regions around the world, thus enabling the optimal path to be selected. The results provide evidence that can aid decision-makers to make correct path choices after emergencies.

Table 5. Optimal reliable paths for different values of  $\alpha$ .

Shipping route	$\alpha$	Optimal reliable path for the value of $\alpha$	$\psi'^{-1}(\alpha), r$
Middle East	0.89	$S_4 \rightarrow S_{11} \rightarrow S_{17} \rightarrow S_{45} \rightarrow S_{46}$	3.94
Central Africa	0.89	$S_3 \rightarrow S_{11} \rightarrow S_{17} \rightarrow S_{43} \rightarrow S_{44}$	3.56
Central Europe	0.89	$S_3 \rightarrow S_6 \rightarrow S_7 \rightarrow S_{39} \rightarrow S_{35}$	3.20
Central and Latin America	0.89	$S_3 \rightarrow S_6 \rightarrow S_7 \rightarrow S_{31} \rightarrow S_{32}$	2.91
Middle ocean	0.89	$S_4 \rightarrow S_{14} \rightarrow S_2 \rightarrow S_{27} \rightarrow S_{22}$	2.81
Central Asia	0.89	$S_3 \rightarrow S_{20} \rightarrow S_{18} \rightarrow S_{16} \rightarrow S_{12}$	2.68
Middle East	0.59	$S_4 \rightarrow S_{11} \rightarrow S_{17} \rightarrow S_{45} \rightarrow S_{46}$	3.14
Central Africa	0.59	$S_3 \rightarrow S_{11} \rightarrow S_{17} \rightarrow S_{43} \rightarrow S_{44}$	2.97
Central Europe	0.59	$S_3 \rightarrow S_6 \rightarrow S_7 \rightarrow S_{39} \rightarrow S_{35}$	3.01
Central and Latin America	0.59	$S_3 \rightarrow S_6 \rightarrow S_7 \rightarrow S_{31} \rightarrow S_{32}$	3.03
Middle ocean	0.59	$S_4 \rightarrow S_{14} \rightarrow S_2 \rightarrow S_{27} \rightarrow S_{22}$	2.74
Central Asia	0.59	$S_3 \rightarrow S_{20} \rightarrow S_{18} \rightarrow S_{16} \rightarrow S_{12}$	2.30
Middle East	0.29	$S_4 \rightarrow S_{11} \rightarrow S_{17} \rightarrow S_{45} \rightarrow S_{46}$	0.97
Central Africa	0.29	$S_3 \rightarrow S_{11} \rightarrow S_{17} \rightarrow S_{43} \rightarrow S_{44}$	1.21
Central Europe	0.29	$S_3 \rightarrow S_6 \rightarrow S_7 \rightarrow S_{39} \rightarrow S_{35}$	1.30
Central and Latin America	0.29	$S_3 \rightarrow S_6 \rightarrow S_7 \rightarrow S_{31} \rightarrow S_{32}$	1.11
Middle ocean	0.29	$S_4 \rightarrow S_{14} \rightarrow S_2 \rightarrow S_{27} \rightarrow S_{22}$	0.95
Central Asia	0.29	$S_3 \rightarrow S_{20} \rightarrow S_{18} \rightarrow S_{16} \rightarrow S_{12}$	1.01

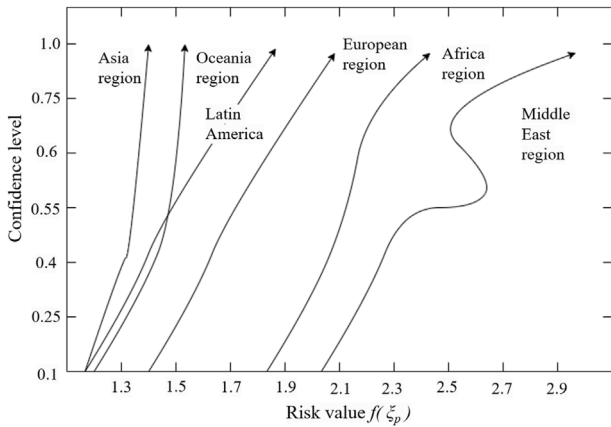


Fig. 6. Uncertainty distributions of the risk of the optimal reliable paths  $f(\xi_p)$ .

4.4. Managerial recommendations

In today's world, the international situation is becoming increasingly turbulent. For example, the recent control of the Red Sea route by Houthi armed forces has caused a series of chain reactions in the global shipping industry, leading to a huge threat to the nodes of the shipping network and seriously affecting the stability and smoothness of the international industrial and supply chains. The international community needs to work together to find solutions to restore the normal order of global shipping; only through cooperation and consultation can countries address this severe challenge and ensure the stability and prosperity of global trade. In addition, each shipping company should consider the situation in terms of network node interruption, and should formulate corresponding countermeasures for the reliability of network node connectivity, with a comprehensive consideration of each

route, to ensure the timely and safe delivery of sea freight.

The results of this research indicate that compared to other routes, the routes in Africa and the Middle East are more risky due to the recent Houthi armed conflict, unrest in Sudan, and the Israeli-Palestinian conflict. Hence, safer routes should be planned or more suitable trade areas should be sought as far as possible, to reduce dependence on resources and markets in these turbulent areas. Policies should be developed and diplomatic measures should be taken to guarantee national strategic security and reduce transportation costs and losses.

5. Conclusion

After an unexpected event occurs, key seaborne transportation network nodes experience interruptions. In this context, we aimed to provide managerial and policy insights to enable decision makers to choose the optimal seaborne transportation path. In this article, uncertainty theory was used to determine the connectivity, reliability, and optimal path of the seaborne transportation network. Through the use of recursive decomposition algorithms, the risk values and uncertainty distributions of each node were obtained, and a reliable path evaluation model was established. Our conclusions can be summarised as follows:

- (1) By using a recursive decomposition algorithm and combining uncertainty theory to identify the disjoint minimum path set of the network, the connectivity reliability of the seaborne transportation network was obtained. The reliability of China's bulk commodity shipping network was found to be 0.8.

Table 6. Research on uncertainty theory.

Reference	Methods used	Research question	Advantages	Different circumstances considered	Limitations
Liu <i>et al.</i> [14]	Logistics system analysis	On-time delivery of thermal coal by sea freight	The reliability of each link can be obtained, and targeted measures can be taken to improve the efficiency and reliability of each link	No	Fails to consider uncertainty and unexpected events
Wu <i>et al.</i> [23]	Recursive algorithms	Reliability of crude oil shipping network connection	The impact of unexpected times on transportation can be solved through recursive algorithms	Yes	Insufficient consideration of regional scope
Prieto <i>et al.</i> [36]	Vine copula	Reliability of seaborne transportation network connection	The research results can provide a basis for formulating seaborne transportation strategies	No	Only suitable for popular regions

- (2) Uncertainty theory was used to solve the problem of optimal path selection in emergencies. For example, at a confidence level value of  $\alpha = 0.89$ , the path risk value of each region varies, and the risk of the path risk value of each region varies. The optimal paths obtained in this study were as follows: the  $\alpha$ -optimal reliable path for Asia (excluding the Middle East) was the Bohai Strait - Taiwan Strait - Balintang Strait - Luzon Strait - Sumatra Strait; the  $\alpha$ -optimal reliable path for Africa was the Bohai Strait - Lombok Sela - Palk Strait - Zimbabwe Strait - Mozambique Channel; the  $\alpha$ -optimal reliable path for Latin America was the Bohai Strait - Jeju Strait - Bering Strait - Panama Canal - Florida Strait; the  $\alpha$ -optimal reliable path for Europe was the Bohai Strait - Jeju Strait - Bering Strait - English Channel - Gibraltar Strait; the  $\alpha$ -optimal reliable path for the Middle East was the Qiongzhou Strait - Lombok Strait - Palk Strait - Suez Canal - Bab el-Mandeb Strait; and the  $\alpha$ -optimal reliable path for the Oceania region was the Bohai Strait - Beilin Strait - Bashi Channel - Bahraintang Strait - Bass Strait.
- (3) When the confidence level is varied, the risk values and path selection in different regions will also vary. When  $\alpha = 0.20$ , the risk of each region will also change with the value, so the setting of the confidence level  $\alpha$  should be made based on the latest judgement after the occurrence of emergencies, to guarantee that the import and export of marine cargoes occur in a safe and timely manner.
- (4) The uncertainty distribution  $\Psi'$  can be determined from the values of  $\Psi'^{-1}(\alpha)$  obtained by iterative calculations, and can be obtained for any confidence level  $\alpha$ . The result  $\alpha = \Psi'(r)$  is obtained for a given value of risk  $r$  and can provide the decision maker with a choice of paths as well as the optimal path, which is obtained by solving for  $\alpha$ .

In response to the latest emergencies that impact the global shipping network, we used uncertainty theory to evaluate the connectivity reliability of bulk commodity shipping networks. Recursive decomposition algorithms were applied to obtain the risk value for each node in the shipping network, and uncertainty distribution methods were used to analyse the optimal transportation path in the shipping network. In future work, we will consider the impact of transportation time and capacity on reliability, to obtain improved results.

## Ethics information

Ethical approval was not required for this study as it did not involve any human or animal subjects.

## Conflicts of interest

The authors declare no conflict of interest.

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