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RESEARCH ARTICLE

Genetic Algorithm for Solving a Just-In-time Inventory Model With Imperfect Rework Implemented in a Serial Multi-echelon System

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Abstract

As global industrial competition intensifies, enterprises can achieve substantial competitive advantages in the supply chain management environment by promptly meeting customer demands and efficiently reducing both supply and demand costs. This paper proposes an inventory model for supply chain optimization that considers uncertain delivery lead times and defective products. Solving the model requires solving a nonlinear mixed-integer problem, which traditionally requires considerable time. Solutions to nondeterministic polynomial-time hard problems with high complexity and difficulty are often obtained using heuristic algorithms. Among these algorithms, genetic algorithms have high efficiency and quality. Therefore, we employed a genetic algorithm to solve the proposed model.

Keywords: Genetic algorithm, Multi-echelon inventory model, Quality unreliability, Uncertain delivery lead time

1. Introduction

To gain a competitive advantage, businesses must focus on two key objectives: reducing total costs and meeting customer demands in a timely manner. For suppliers and buyers, optimizing inventory quantities alone may be insufficient to optimize the entire supply chain. A logistic inventory approach could thus help suppliers and buyers determine optimal order quantities and shipping strategies.

The just-in-time (JIT) inventory system is a strategy that emphasizes continuous improvement (known as “kaizen”), and it thus achieves zero inventory by frequently producing and delivering small lots and eliminates all waste from a company's operations. The JIT system comprises a manufacturing component and a purchasing component. JIT buyers must ascertain whether products obtained from a manufacturer meet their quality standards, and manufacturers aim to minimize inventory levels to reduce holding costs.

If suppliers and buyers are geographically distant, delivery lead times can be highly variable. Such uncertain lead times may cause suppliers to increase their inventory levels or lead to shortages of goods and failures to promptly meet customer demands; both outcomes are undesirable in the JIT system. To resolve this situation, this paper proposes a time buffer policy as a form of emergency borrowing. If buyers identify defective products during inspections, they can return such products to the supplier for rework. However, if a defective product cannot be repaired (imperfect rework), the total number of received products would be insufficient. Thus, in a multi-echelon logistics inventory

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model, buyers must consider both uncertain delivery lead times and imperfect rework.

The aforementioned inventory problem can be construed as an integer nonlinear programming problem. Such problems have large solution spaces and are therefore difficult to solve through an exhaustive search. Hence, a heuristic algorithm such as a genetic algorithm (GA) is more suitable for obtaining the optimal solution or a near-optimal solution within a reasonable computational time.

2. Literature review

To provide context for the methods in this paper, this section presents the relevant literature on the JIT system, multi-echelon inventory models, delivery lead time uncertainties, unreliable quality, imperfect rework, and a GA.

2.1. JIT inventory model

In conventional inventory management systems, buyers and suppliers independently determine their respective economic lot sizes [1]. Such inventory management systems may not lead to optimal strategies for the entire supply chain. To address this problem, integrated multi-echelon inventory approaches have been used to determine the optimal order quantities and shipping policies [2].

Implementing JIT manufacturing and purchasing can be effective for increasing competitiveness. The JIT system involves frequently buying products in small lot sizes to eliminate all waste in a company's operations, with the ultimate goal of achieving zero inventory [3,4]. However, in a global supply chain, delivery times between geographically distant buyers and suppliers may be uncertain. This uncertainty can lead to product shortages and disrupt coordinated activities at different levels of a synchronized supply chain [5]. Therefore, the JIT system may be difficult to implement. Three policies could address this problem: an inventory buffer policy, time buffer policy, or urgent borrowing policy. Nevertheless, if the buffer size is not sufficient, the inventory and time buffer policies may not completely eliminate shortages. Chiu [5], demonstrated that in a JIT environment, the problem of uncertain delivery lead times can be solved by implementing emergency borrowing policies and time buffers. Several studies have attempted to address the challenges of delivery lead time uncertainty and unreliable quality by using a serial multi-echelon integrated JIT inventory (SMEIJI) model (SMEIJI problem).

2.2. Multi-echelon inventory

Managing inventory can be challenging, particularly for businesses that have numerous products in multiple locations. If these locations are distributed across different supply chain echelons, the complexity further increases [6]. Research has demonstrated that when suppliers and buyers coordinate and integrate their inventory control strategies, the average total cost incurred by the supply chain system is lower than that obtained when suppliers and buyers plan their inventory control strategies separately [8].

A multi-echelon inventory comprises multiple stages; each stage is an echelon that includes manufacturers, retailers, suppliers, or other members in the supply chain. Moreover, a multi-echelon inventory may be serial (Fig. 1), in which each echelon has only a single member, or mixed (Fig. 2), in which each echelon has multiple members with complex interactions.

In 1977, Goyal [7] introduced an integrated inventory model for minimizing the joint total cost of a one-supplier-one-customer problem. Banerjee [9] further developed this model in 1986 by using a lot-for-lot policy to create a joint economic-lot-size model. Goyal [10] subsequently adopted Banerjee's lot-for-lot assumption to propose a more general model in 1988, which can reduce the joint total cost. In 2006, Seo [11] demonstrated an improved reorder decision policy in which shared stock information is used to control a general multi-echelon distribution system [24].

2.3. Lead time uncertainties

Delivery lead times greatly affect customer service, warehouse safety, and competitiveness and can be influenced by order preparation times, transit,

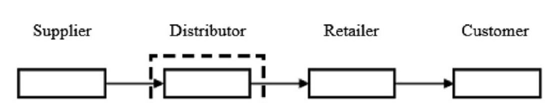

Fig. 1. Serial system of Multi-echelon inventory.

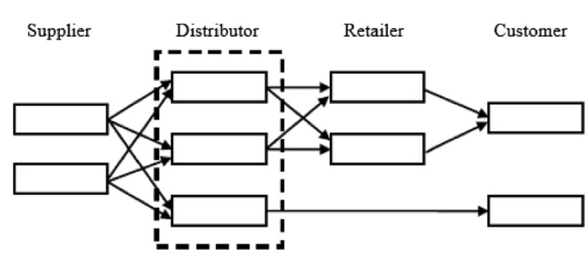

Fig. 2. Mixed system of Multi-echelon inventory.

supplier delays, and shipping delays. For geographically distant suppliers and buyers, the delivery lead time is uncertain, and this can considerably affect the optimal inventory policy. In 1987, Yano proposed a three-echelon serial inventory model to reduce delivery lead time uncertainties. In the same year, Graman and Roger [12] expanded Yano's work, creating a multi-echelon inventory model that incorporates an inventory buffer policy to handle uncertain delivery lead times. Chiu [5] further developed this concept in 2003 by introducing a multi-echelon integrated JIT inventory model that uses time buffers and emergency borrowing policies. In 2009, Kouvelis and Li stated that ensuring a timely delivery is increasingly critical for contemporary global supply chains; failure to ensure a timely delivery can result in lost sales, obsolete inventories, and high costs; the penalties for these losses can often reach billions of dollars [25].

2.4. Unreliable quality

Even in a JIT system, expecting products to have perfect quality is unrealistic. Researchers have attempted to identify the effect of defects on the production process. For example, Porteus [13] integrated defect factors into the basic economic order quantity (EOQ) model in 1986, and Lee and Rosenblatt [14] demonstrated in 1987 that the classic econometric quantity model ignores the effects of defects. Salameh and Jaber's [15] extended the traditional EOQ and economic production quantity (EPQ) models by including items with imperfect quality. In 2002, Goyal and Cardenas Barron [16] proposed a simple method for quantifying the economic losses attributable to quality defects. Huang [2] developed a model in 2004 for determining the optimal inventory policy for defective items in a JIT manufacturing environment with only one supplier and one buyer.

2.5. Imperfect rework

When a buyer receives an item, they should inspect it immediately to ensure that it is not defective before using it for manufacturing or delivering it to a subsequent buyer. If a product is defective, it must be reworked immediately. The rework process can fix only some defective products; other products are lost. In a supply chain, if losses due to defective items can be reduced at each stage, the minimum joint total cost can be obtained through the accumulated savings. Agnihothri and Kenett [17] attempted to quantify the effect of defects on multiple system performance measures for a system in which all products were inspected. Hsieh and Chiu [18] reported that although on-site rework is more expensive than in-house rework, on-site rework can be more profitable in supply chains. Liu and Yang [19] proposed the lot-size problem for a single-stage imperfect production system (i.e., a system for which failures often occur in the work process); in this system, defective items can be classified as reworkable (representing items that should be immediately reworked) and non-reworkable (representing those that should be discarded). They further developed an EPQ model with a stochastic defect rate and imperfect rework process [20]. Tiwari, Ahmed, and Sarkar [21] proposed a green production quantity model that includes random defects, service level constraints, and rework failures. Ahmadi [7]combined inspection and replacement scheduling models with other models and further included imperfect maintenance to develop a novel model of failures engendered by operating conditions. Duenyas and Nenes studied imperfect rework and inspection problems in manufacturing systems and discussed system performance optimization by adjusting rework and inspection strategies [26]. Zhu and Rabe considered a degenerate production system with either perfect or imperfect rework and proposed a control strategy for optimizing system performance [27].

2.6. Genetic algorithm

Genetic algorithm is a metaheuristic optimization algorithm based on the principles of natural selection and genetics. It first initializes a population of solutions, each represented as a chromosome comprising a set of parameters. The algorithm then evaluates the fitness of each chromosome by using a fitness function that measures how well the chromosome solves the problem. The fittest chromosomes are selected and recombined using operators such as crossover and mutation to create a new population of solutions. This process of selection, recombination, and evaluation is repeated for multiple generations until a satisfactory solution is found or a termination condition is met. Because a GA can explore a large search space and converge to a globally optimal solution, it is suitable for solving a wide range of optimization problems. Fig. 3 presents pseudocode for a GA.

Genetic algorithm has been extensively applied in recent years. Xue, Guo and Bai [22] indicated that if computational resources are limited, a GA can more effectively obtain a satisfactory result than can conventional methods. A study improved the region constraints for the initial GA population to solve the node localization problem; however, the study reported that the GA often converged prematurely,

```
Genetic Algorithm System

Choose encode method
Set parameters

Generate the initial population
While i < Max Iteration and Best fitness < Max Fitness do
    Fitness calculation
    a. Selection
    b. Crossover
    c. Mutation
end while
Decode the individual with maximum fitness

return the best solution
```

Fig. 3. Pseudo code of the genetic algorithm.

resulting in poor performance [8]. Chomatek and Duraj [23] used a multiobjective optimized GA to efficiently identify outliers and demonstrated that the algorithm could solve medical problems. Kimms proposed that a GA is efficient because its solutions are encoded in a two-dimensional matrix representation with nonbinary entries instead of being encoded as simple bit-strings. A computational study revealed that a GA could rapidly identify solutions and was competitive with a recently published tabu search approach [28]. Accordingly, this study used a GA to solve for the three decision variables in the study model.

3. Methodology

In this study, we extended the serial multi-echelon logistics inventory (SMELI) model by incorporating imperfect rework, uncertain delivery lead times, and defective products. This section outlines the multi-echelon inventory model and our methodology. Owing to the complexity of the final model, the study employed a GA to obtain solutions. The results are presented in Section 4.

3.1. Serial multi-echelon and logistic inventory model

3.1.1. Notation and assumptions

Suppose that a serial inventory system has K echelons each with one member. The members of level 1 and K are a manufacturer and buyer, respectively; the members of the other echelons act as both manufacturers and buyers. The notations used in this paper are defined as follows [24].

3.1.1.1. Purchasing activity. A purchaser i executes purchase action, where $i = 1, 2, 3, \ldots K - 1$.

Decision variables:	
N_{pi}	The integer decision variable for buyer i in T_{pi} is the number of orders.
n_i	One integer decision variable in the model is the delivery time per purchase order for purchaser i.
m_{Li}	The value of m_{Li} is a real decision variable, represents the maximum delivery time that is considered acceptable without causing a shortage. $m_{Li} = \mu_{Ldi} +$ safety delivery lead time.
Parameters:	
K	The number of echelons in the chain of supply chain
T_{pi}	The purchasing time interval for a purchaser i (in years)
D_{ri}	The quantity demanded to a purchaser i (units/years)
Q_{pi}	The order lot size per purchasing order for a purchaser i; $Q_{pi} = \frac{D_{ri}T_{pi}}{N_{pi}}$ (units/order)
q_i	Quantity delivered by manufacturer i+1 to purchaser i at a time. $q_i = \frac{Q_{pi}}{n_i}$ (units/delivery),
F_{oi}	The fixed costs that purchaser i pays for each purchase ($/order)
H_{pi}	Holding cost of each purchase products for a purchaser i ($/unit/year)
F_{ei}	For buyer i , the cost of placing an emergency borrowing. ($/borrowing)
β_i	Borrowing cost of per unit per year for a purchaser i ($/unit/year)
F_{pi}	Fixed delivery cost for a purchaser i ($/delivery)
L_{di}	The delivery time from manufacturer i+1 to a buyer i. A non-negative random variable followed by a distribution: $L_{di} = \mu_{Ldi} + k\sigma_{Ldi}, E(L_{di}) = \mu_{Ldi}, \mathrm{Var}(L_{di}) = \sigma_{Ldi}$
t_i	The time interval between two consecutive deliveries to a buyer i ; $t_i = \frac{q_i}{D_{ri}} = \frac{T_{pi}}{n_i N_{pi}}$
r_i	The redelivery points to a purchaser i. A delivery will be performed by manufacturer i when stock drops to r_i ; $r_i = m_{Li}D_{ri}$
t_{si}	Screening time of q_i quantity of materials that received by purchaser i (in years)
S_{ci}	A unit screening cost for a purchaser i
R_{ri}	Repairing rate of q_i for a purchaser i
R_{ci}	The rework cost in per item of defective items for a purchaser i
t_{ri}	Rework time for defective items required by a purchaser i, $t_{ri} = \frac{\varnothing_i q_i}{\theta_i}$
$\varnothing_i$	Defective rate of a shipment that should be deducted from the items received by a purchaser i
θ_i	The times of unit item is reworked for a purchaser i

3.1.1.2. Manufacturing activity. The manufacturer i executes manufacture action, where $i = 2, 3, 4, \ldots K$. materials must be greater than or equal to the quantity required during the inspection period.

Decision variables:	
N_{mi}	The quantity of products manufactured by a manufacturer i during the period T_{mi-1} (an integer decision variable)
D_{mi}	The delivery times of per production run for a manufacturer i ; $N_{mi}D_{mi} = n_{i-1}N_{pi-1}$ (an integer decision variable)
Parameters:	
T_{mi}	The manufacturing time interval for a manufacturer i ; $T_{mi} = \frac{Q_{mi}}{P_{ri}}$ (in years)
P_{ri}	The production rate of a manufacturer i (units/years)
f_i	Conversion factor for member i ; $f_i = \frac{D_{ri}}{P_{ri}}$ (units/years)
F_{mi}	Setup cost for a manufacturer i ($/setup)
Q_{mi}	The quantity of products produced in each production run for a manufacturer i; $Q_{mi} = \frac{D_{ri}T_{pi-1}}{N_{mi}}$ (units/run)
H_{mi}	Holding cost in per produced goods and year for a manufacturer i ($/unit/year)

Other notations are introduced as necessary. Some of the assumptions are listed as follows.

1. Each echelon i of the supply chain inventory system has one member i, where $i = 1, 2, \ldots, K$. Each member produces a single product at each echelon.
2. Member 1 is a buyer and solely performs purchasing; member K is a manufacturer that only performs buying.
3. Each member i produces goods for member $i - 1$ in time T_{pi} and purchases materials from member $i + 1$.
4. The productivity of member $i + 1$ (total number of manufactured products) is sufficient to meet the demand for non-defective products for member i. Hence, member $i + 1$ produces more products than member i requires, but some products are defective. The rework process can be assumed to have a success rate of 40 %.
5. The delivery lead time L_{di} of member i is defined as a beta distribution with the density function $L_{dt} = f(L_{dt})$, where $0 \leq \lambda_{Li} \leq L_{di} \leq \gamma_{Li}$, $i = 1, \ldots K - 1$.
6. The time interval between two consecutive deliveries t_i for member i must be equal to or longer to γ_{Li}. This assumption is required to ensure that the problem is not unsolvable because of the lot-size problem.
7. The JIT system does not allow shortages; therefore, if a delayed delivery occurs, member i takes an emergency borrowing action. The time required to borrow items from a nearby supplier is zero.
8. For emergency borrowing, both the borrowed and returned materials are perfect items.
9. The entire serial supply chain has a finite planning period T_{pi}.
10. Each shipment of materials received by purchaser i has a defect rate of $\varnothing_i$. The number of perfect materials must be greater than or equal to the quantity required during the inspection period.
11. Inspection occurs immediately after a shipment with a screening time of t_{si} that is directly proportional to the number of received items.
12. Purchase quantities are selected with consideration of the defect and rework success rates. The rework success rate is 50 %.
13. The accumulated quantity of defective items delivered to the purchaser during production must be less than the delivery quantity; that is, $D_{mi+1} \leq \frac{1}{\varnothing_i} - 1$.

3.1.2. Model formulation

3.1.2.1. Purchasing costs for each member. Assume that all buyers use the JIT system for purchasing replenishment products. The time buffer policy can be implemented to plan delivery lead times m_{Li}, and the urgent borrowing policy can be used to handle uncertain delivery lead times. Each order lot size Q_{pi} can be subdivided into n_i small lots for frequent deliveries. During the planning period T_{pi}, the total number of deliveries for member i is $n_i \times N_{pi}$. The delivery lead time follows a probability density distribution $f(L_{dt})$, where $0 \leq \lambda_{Li} \leq L_{di} \leq \gamma_{Li}$. λ_{Li} and γ_{Li} are the lower and upper bound, respectively, of L_{di}.

The cost of purchasing activities for member i in T_{pi} includes the ordering cost, holding cost, delivery cost, transportation carrying cost, and urgent borrowing cost. However, the transportation cost is not included in the model because it is constant and does not affect any decision variables.

Delivery lead times are uncertain; hence, deliveries could be early, delayed, or on time. Buyers perform inspections immediately after receiving a shipment and deduct any defective items from the total inventory after the inspection of each batch is completed. Costs of defective items other than the inventory cost are not considered herein. Fig. 4 presents a graphical representation of the model for

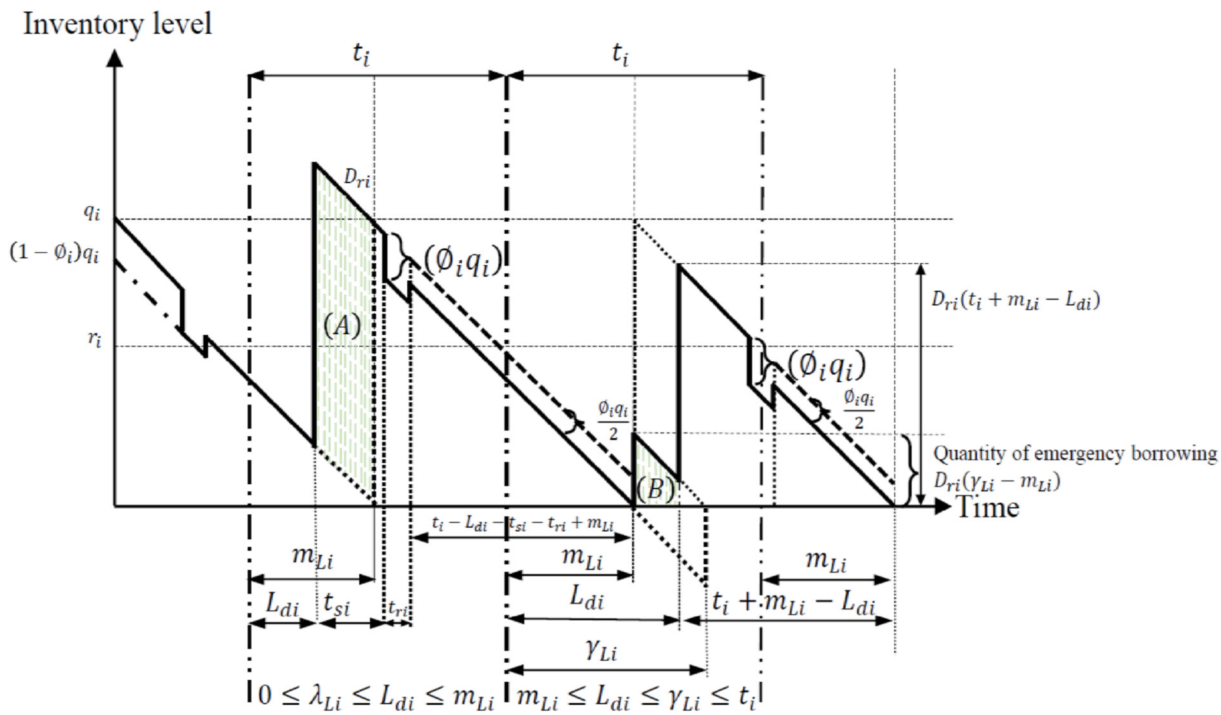

Fig. 4. Model parameters for early and delayed deliveries for a purchasing member i.

early and delayed deliveries to a purchasing member i.

For an early delivery, member i incurs unexpected holding costs. This situation is depicted in the left part of Fig. 4, where $0 \le \lambda_{Li} \le L_{di} \le m_{Li}$. In this case, the total holding cost HC_{Ai} for time t_i should be evaluated in two parts. The shadowed area A in Fig. 4 indicates the cost of holding the unexpected extra inventory and is derived as the factor q_i multiplied by the difference in the actual and expected delivery times: $q_i\ (m_{Li} - L_{di})$. The other part $\frac{\left(\frac{q_i\emptyset_i}{2}+q_i\right)t_i}{2} - q_i\emptyset_i t_{ri} - \frac{q_i\emptyset_i}{2}(t_i - L_{di} - t_{si} - t_{ri} + m_{Li})$ is the cost of holding the expected inventory. Hence,

$$HC_{Ai}=H_{pi}\left[q_i(m_{Li}-L_{di})+\frac{\left(\frac{q_i\emptyset_i}{2}+q_i\right)t_i}{2}-q_i\emptyset_i t_{ri}-\frac{q_i\emptyset_i}{2}(t_i-L_{di}-t_{si}-t_{ri}+m_{Li})\right]$$

$$=D_{ri}H_{pi}\left[t_i(m_{Li}-L_{di})+\left(\frac{t_i^2\emptyset_i}{4}+\frac{t_i^2}{2}\right)-\frac{D_{ri}(\emptyset_i t_i)^2}{\theta_i}-\frac{\emptyset_i t_i}{2}\left(t_i-L_{di}-t_{si}-\frac{\emptyset_i D_{ri}t_i}{\theta_i}+m_{Li}\right)\right] \tag{3.1}$$

Where $t_i = \frac{q_i}{D_{ri}} \Rightarrow q_i = D_{ri}t_i$, $t_{ri} = \frac{\emptyset_i q_i}{\theta_i}$.

If a delivery is delayed, member i borrows items from a nearby supplier outside the supply chain (Fig. 4, right panel; $m_{Li} \le L_{di} \le \gamma_{Li} \le t_i$). The shaded area B represents the number of items borrowed (the urgent borrowing quantity), which is $D_{ri}(\gamma_{Li} - m_{Li})$ units. The quantity of remaining items $D_{ri}(\gamma_{Li} - m_{Li})$ includes the items consumed before the delayed delivery arrives $D_{ri}(L_{di} - m_{Li})$ and the remaining consumption $D_{ri}(\gamma_{Li} - L_{di})$. When the delayed delivery arrives, member i must immediately return the urgent borrowing quantity. The factor q_i indicates the number of the items consumed before the delayed arrival in the next replenishment. Borrowed and remaining units are not inspected, and the inspection time is $\frac{q_i - D_{ri}(\gamma_{Li} - m_{Li})}{q_i}t_{si}$, with $m_{Li} \le L_{di} \le \gamma_{Li} \le t_i$. The total holding cost HC_{Bi} comprises the normal holding cost NHC_{Bi} and the emergency borrowing holding cost eBC_i [24]. The parameters in Fig. 4 can thus be transformed into those in Fig. 5 as follows:

$$\text{Delay arrival } \textit{screening time} = \frac{q_i - D_{ri}(L_{di} - m_{Li})}{q_i}t_{si}$$
$$= [1 - (L_{di} - m_{Li})]t_{si}$$

Where $q_i = \frac{Q_{pi}}{n_i}$, $Q_{pi} = \frac{D_{ri}T_{pi}}{N_{pi}}$, $T_{pi} = n_i N_{pi}$.

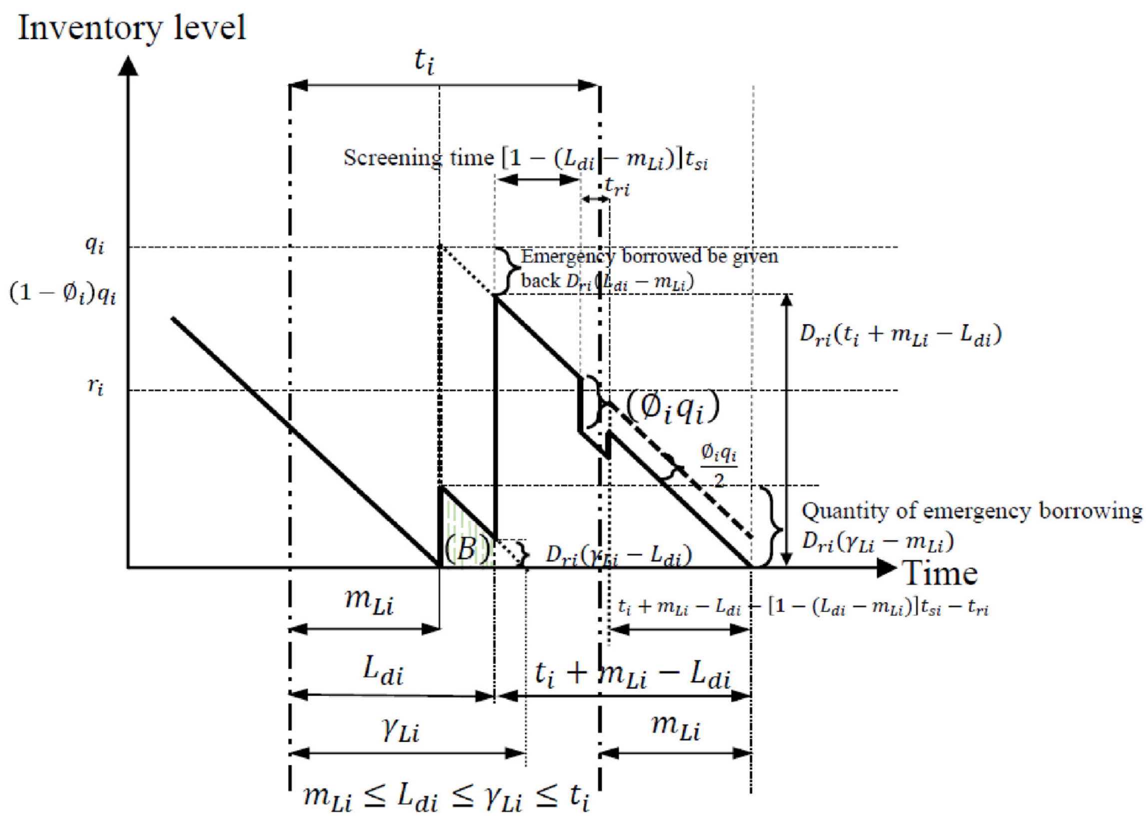

Fig. 5. The unexpected inventory level that is caused by delivery delay.

Shadow area (B) of emergency borrowing and holding cost

$$eBC_i = H_{pi}\left\{\frac{(L_{di}-m_{Li})[D_{ri}(\gamma_{Li}-L_{di})+D_{ri}(\gamma_{Li}-m_{Li})]}{2}\right\}$$
$$= D_{ri}H_{pi}\left\{\frac{(L_{di}-m_{Li})[(\gamma_{Li}-L_{di})+(\gamma_{Li}-m_{Li})]}{2}\right\}$$

Where $\gamma_{Li}-L_{di} : \gamma_{Li}-m_{Li} = x{:}D_{ri}(\gamma_{Li}-m_{Li})$.
$\Rightarrow x = D_{ri}(\gamma_{Li}-L_{di})$.

Normal holding cost

$$(NHC_{Bi}) = H_{pi}\left[\left[\frac{\left(\frac{q_i\emptyset_i}{2}+D_{ri}(t_i+m_{Li}-L_{di})\right)(t_i+m_{Li}-L_{di})}{2}\right] - q_i\emptyset_i t_{ri} - \frac{q_i\emptyset_i}{2}(t_i+m_{Li}-L_{di}-[1-(L_{di}-m_{Li})]t_{si}-t_{ri})\right]$$
$$= D_{ri}H_{pi}\left[\left[\frac{\frac{t_i\emptyset_i}{2}(t_i+m_{Li}-L_{di})+(t_i+m_{Li}-L_{di})^2}{2}\right] - \frac{D_{ri}(\emptyset_i t_i)^2}{\theta_i} - \frac{\emptyset_i t_i}{2}\left(t_i+m_{Li}-L_{di}-[1-(L_{di}-m_{Li})]t_{si} - \frac{\emptyset_i D_{ri} t_i}{\theta_i}\right)\right]$$

$$HC_{Bi}=[(eBC_i)+(NHC_{Bi})]$$

$$HC_{Bi}=D_{ri}H_{pi}\left\{\frac{(L_{di}-m_{Li})[(\gamma_{Li}-L_{di})+(\gamma_{Li}-m_{Li})]}{2}+\left[\frac{\frac{t_i\varnothing_i}{2}(t_i+m_{Li}-L_{di})+(t_i+m_{Li}-L_{di})^2}{2}\right]-\frac{D_{ri}(\varnothing_i t_i)^2}{\theta_i}-\frac{\varnothing_i t_i}{2}\left(t_i+m_{Li}-L_{di}-[1-(L_{di}-m_{Li})]t_{si}-\frac{\varnothing_i D_{ri}t_i}{\theta_i}\right)\right\} \tag{3.2}$$

Therefore, the equations (3.1) and (3.2) must combine to form the expected holding cost (EHC_i) of member i shows as follows:

$$\begin{aligned}EHC_i=&\int_{\lambda_{Li}}^{m_{Li}}HC_{Ai}f(L_{di})dL_{di}+\int_{m_{Li}}^{\gamma_{Li}}HC_{Bi}f(L_{di})dL_{di}=D_{ri}H_{pi}\Bigg[\left(t_i m_{Li}+\left(\frac{t_i^2\varnothing_i}{4}+\frac{t_i^2}{2}\right)-\frac{D_{ri}(\varnothing_i t_i)^2}{\theta_i}\right)\\&\int_{\lambda_{Li}}^{m_{Li}}f(L_{di})dL_{di}+t_i\int_{\lambda_{Li}}^{m_{Li}}L_{di}f(L_{di})dL_{di}-\frac{\varnothing_i t_i}{2}\int_{\lambda_{Li}}^{m_{Li}}\left(t_i-L_{di}-t_{si}-\frac{\varnothing_i D_{ri}t_i}{\theta_i}+m_{Li}\right)f(L_{di})dL_{di}\Bigg]\\&+D_{ri}H_{pi}\Bigg[\int_{m_{Li}}^{\gamma_{Li}}\frac{(L_{di}-m_{Li})[(\gamma_{Li}-L_{di})+(\gamma_{Li}-m_{Li})]}{2}f(L_{di})dL_{di}+\int_{m_{Li}}^{\gamma_{Li}}\left[\frac{\frac{t_i\varnothing_i}{2}(t_i+m_{Li}-L_{di})+(t_i+m_{Li}-L_{di})^2}{2}\right]\\&f(L_{di})dL_{di}-\frac{D_{ri}(\varnothing_i t_i)^2}{\theta_i}\int_{m_{Li}}^{\gamma_{Li}}f(L_{di})dL_{di}-\frac{\varnothing_i t_i}{2}\int_{m_{Li}}^{\gamma_{Li}}\left(t_i+m_{Li}-L_{di}-[1-(L_{di}-m_{Li})]t_{si}-\frac{\varnothing_i D_{ri}t_i}{\theta_i}\right)f(L_{di})dL_{di}\Bigg]\end{aligned} \tag{3.3}$$

The expected borrowing cost (EBC_i) of member i displays as follows:

$$\begin{aligned}EBC_i&=F_{ei}\int_{m_{Li}}^{\gamma_{Li}}f(L_{di})dL_{di}+\beta_i\int_{m_{Li}}^{\gamma_{Li}}(\gamma_{Li}-m_{Li})f(L_{di})dL_{di}\\&=[F_{ei}+\beta_i(\gamma_{Li}-m_{Li})]\int_{m_{Li}}^{\gamma_{Li}}f(L_{di})dL_{di}\end{aligned} \tag{3.4}$$

The EBC_i occurs only at time interval $[m_{Li},\gamma_{Li}]$. $F_{ei}\int_{m_{Li}}^{\gamma_{Li}}f(L_{di})dL_{di}$ means the ordering cost of emergency borrowing, and $\beta_i\int_{m_{Li}}^{\gamma_{Li}}(\gamma_{Li}-m_{Li})f(L_{di})dL_{di}$ represents the exclusion costs of borrowing units.

EC_{pi} has (N_{pi}) purchasing orders, $n_i\times N_{pi}$ delivery receiving times, $n_i\times N_{pi}\times F_{pi}$ delivery cost, $N_{pi}\times F_{oi}$ ordering cost, $n_i\times N_{pi}\times q_i\times S_{ci}$ screening cost and $n_i\times N_{pi}\times q_i\times\varnothing_i\times R_{ci}$ reworking cost respectively during the purchasing time interval (T_{pi}) of each member i. Thus, the expected cost (EC_{pi}) of the member i associated with the purchase is as follows:

$$\begin{aligned}EC_{pi}(n_i,N_{pi},m_{Li})\\&=\text{holding cost}+\text{emergency borrowing cost}\\&\quad+\text{ordering cost}+\text{delivery cost}+\text{screening cost}\\&\quad+\text{reworking cost}\end{aligned}$$

$$
\begin{aligned}
&= n_i\times N_{pi}[EHC_i+EBC_i]+N_{pi}\times F_{oi}+n_i\times N_{pi}\times F_{pi}+n_i\times N_{pi}\times q_i\times S_{ci}+n_i\times N_{pi}\times q_i\times\varnothing_i\times R_{ci}\\
&= n_i\times N_{pi}\times\Bigg[D_{ri}H_{pi}\Bigg[\left(t_i m_{Li}+\left(\frac{t_i^2\varnothing_i}{4}+\frac{t_i^2}{2}\right)-\frac{D_{ri}(\varnothing_i t_i)^2}{\theta_i}\right)\int_{\lambda_{Li}}^{m_{Li}} f(L_{di})dL_{di}+t_i\int_{\lambda_{Li}}^{m_{Li}} L_{di}f(L_{di})dL_{di}\\
&-\frac{\varnothing_i t_i}{2}\int_{\lambda_{Li}}^{m_{Li}}\left(t_i-L_{di}-t_{si}-\frac{\varnothing_i D_{ri}t_i}{\theta_i}+m_{Li}\right)f(L_{di})dL_{di}\Bigg]+D_{ri}H_{pi}\Bigg[\int_{m_{Li}}^{\gamma_{Li}}\frac{(L_{di}-m_{Li})[(\gamma_{Li}-L_{di})+(\gamma_{Li}-m_{Li})]}{2}f(L_{di})dL_{di}\\
&+\int_{m_{Li}}^{\gamma_{Li}}\left[\frac{\frac{t_i\varnothing_i}{2}(t_i+m_{Li}-L_{di})+(t_i+m_{Li}-L_{di})^2}{2}\right]f(L_{di})dL_{di}-\frac{D_{ri}(\varnothing_i t_i)^2}{\theta_i}\int_{m_{Li}}^{\gamma_{Li}} f(L_{di})dL_{di}-\frac{\varnothing_i t_i}{2}\int_{m_{Li}}^{\gamma_{Li}}\Big(t_i+m_{Li}-L_{di}\\
&-[1-(L_{di}-m_{Li})]t_{si}-\frac{\varnothing_i D_{ri}t_i}{\theta_i}\Big)f(L_{di})dL_{di}\Bigg]+[F_{ei}+\beta_i(\gamma_{Li}-m_{Li})]\int_{m_{Li}}^{\gamma_{Li}} f(L_{di})dL_{di}+F_{pi}+q_i^2\times S_{ci}+\varnothing_i\times R_{ci}\Bigg]\\
&+N_{pi}\times F_{oi}
\end{aligned}
\tag{3.5}
$$

3.1.2.2. Manufacturing costs for each member. Each member i implements JIT manufacturing, producing goods and delivering them to member $i-1$ by performing N_{mi} production runs during T_{pi-1}. Each production run begins at $\frac{q_{i-1}}{P_{ri}}$ time units after the previous run. The average amount of goods produced by member i after each run is shown in Fig. 6; this value can be computed by subtracting the cumulative time-weighted delivery quantity (1) (the area of trapezoid T_2) from the cumulative time-weighted production quantity (2) (the area of the staircase shape T_1).

The cumulative time-weighted production quantity of member i is equal to the square of the area of the trapezoid. The cumulative time-weighted production quantity (1) can also be calculated by subtracting the area of the triangle from the area of the rectangle.

The squared measure of the area of a rectangle is $\left[\frac{T_{pi-1}}{N_{mi}}-\left(t_{i-1}-\frac{q_{i-1}}{P_{ri}}\right)\right]\left(\frac{D_{ri-1}T_{pi-1}}{N_{mi}}\right)$ and the square measure of the triangle area is $\frac{1}{2}\left(\frac{D_{ri-1}T_{pi-1}}{P_{ri}N_{mi}}\right)\left(\frac{D_{ri-1}T_{pi-1}}{N_{mi}}\right)$. Therefore, the area of the trapezoid shows as following:

Trapezoid area T_2

$$
\begin{aligned}
&=\left[\frac{T_{pi-1}}{N_{mi}}-\left(t_{i-1}-\frac{q_{i-1}}{P_{ri}}\right)\right]\left(\frac{D_{ri-1}T_{pi-1}}{N_{mi}}\right)-\frac{1}{2}\left(\frac{D_{ri-1}T_{pi-1}}{P_{ri}N_{mi}}\right)\\
&\quad\times\left(\frac{D_{ri-1}T_{pi-1}}{N_{mi}}\right)\\
&=\left(\frac{T_{pi-1}}{N_{mi}}\right)^2 D_{ri-1}\left[1-\left(\frac{1}{D_{mi}}\right.\right.\\
&\quad\left.\left.-\frac{D_{ri-1}}{D_{mi}(1-\varnothing_{i-1}+R_{ri-1})\times P_{ri}}\right)-\frac{1}{2}\left(\frac{D_{ri-1}}{P_{ri}}\right)\right]
\end{aligned}
\tag{3.6}
$$

Where

$$t_{i-1}=\frac{T_{Pi-1}}{N_{pi-1}\times n_{i-1}},n_{i-1}\times N_{pi-1}$$

$$=D_{mi}\times N_{mi}\Rightarrow t_{i-1}=\frac{T_{Pi-1}}{D_{mi}\times N_{mi}}$$

$$T_{pi-1}=\frac{(1-\varnothing_{i-1}+R_{ri-1})q_{i-1}\times n_{i-1}\times N_{pi-1}}{D_{ri-1}}\Rightarrow q_{i-1}$$

$$=\frac{T_{pi-1}D_{ri-1}}{D_{mi}\times N_{mi}(1-\varnothing_{i-1}+R_{ri-1})}$$

During each production run of the member i, there are D_{mi} deliveries and q_{i-1} units of goods are

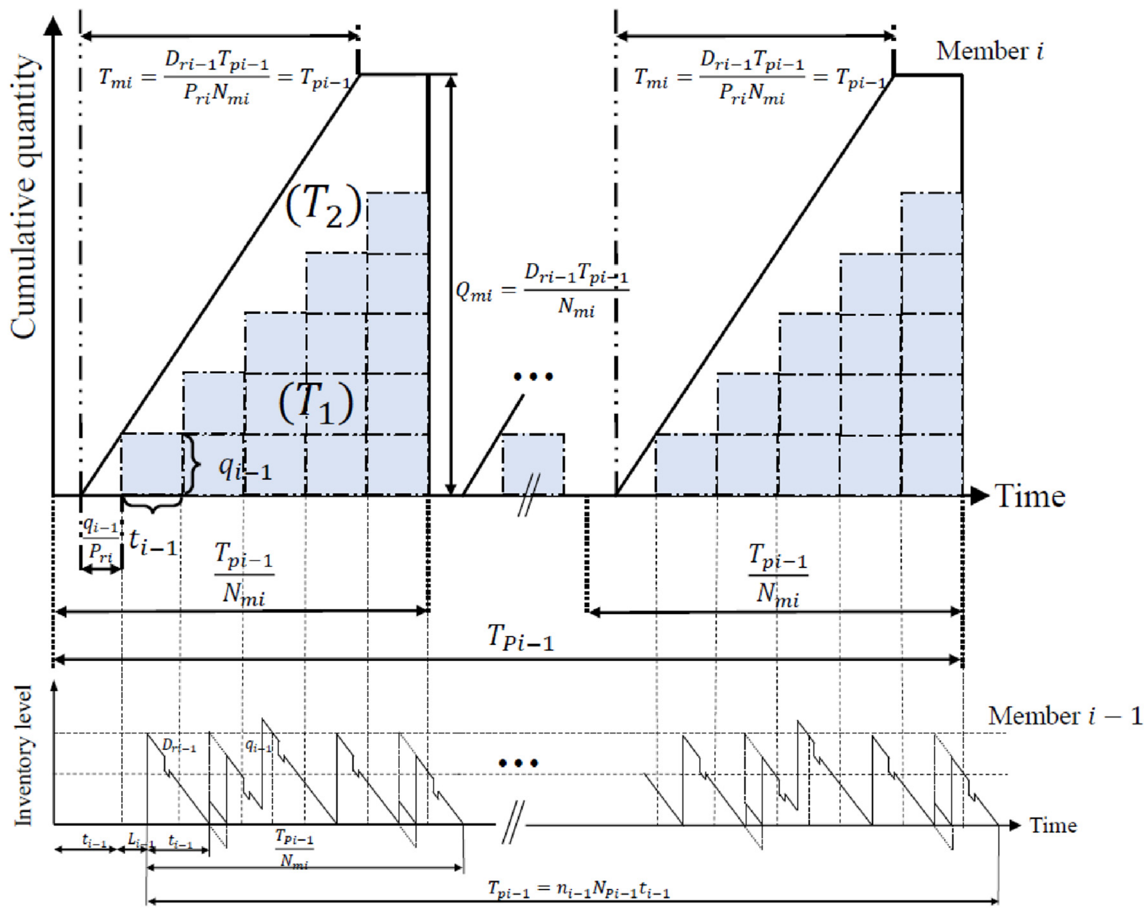

Fig. 6. Average inventory of goods produced by member i.

delivered for each shipment. The time interval between two adjacent deliveries of member $i(t_{i-1})$ must be greater than or equal to γ_{Li-1}. Therefore, the cumulative and time-weighted delivery quantity (2) produced per run for member i is as follows:

Ladder area T_1

$$= \left(\sum_{i=1}^{D_{mi}} I\right) t_{i-1}q_{i-1}$$

$$= (1+2+\ldots+D_{mi})t_{i-1}q_{i-1}$$

$$= \left(\frac{T_{Pi-1}}{N_{mi}}\right)^2 D_{ri-1}\left[\frac{1}{2}\frac{(1+D_{mi})D_{ri-1}}{D_{mi}{}^2(1-\varnothing_{i-1}+R_{ri-1}\varnothing_{i-1})}\right] \tag{3.7}$$

Consequently, the average inventory (AI_i) of the member i during T_{pi-1} can be calculated by subtracting area T_1 from area T_2.

$$AI_i = N_{mi}\{T_2 - T_1\}$$

$$= N_{mi}\left\{\left(\frac{T_{pi-1}}{N_{mi}}\right)^2 D_{ri-1}\left[1-\left(\frac{1}{N_{mi}} - \frac{D_{ri-1}}{D_{mi}(1-\varnothing_{i-1}+R_{ri-1})\times P_{ri}}\right) - \frac{1}{2}\left(\frac{D_{ri-1}}{P_{ri}}\right) - \left[\frac{1}{2}\frac{(1+D_{mi})D_{ri-1}}{D_{mi}(1-\varnothing_{i-1}+R_{ri-1})}\right]\right]\right\} \tag{3.8}$$

So, the production cost of member i (EC_{mi}) during T_{pi-1} is

$$EC_{mi} = (N_{mi}, D_{mi})$$

$$= N_{mi}F_{mi} + H_{mi}AI_i$$

$$= N_{mi}F_{mi} + H_{mi} \times N_{mi}\left\{\left(\frac{T_{pi-1}}{N_{mi}}\right)^2 D_{ri-1}\left[1 - \left(\frac{1}{N_{mi}} - \frac{D_{ri-1}}{D_{mi}(1-\varnothing_{i-1}+R_{ri-1})\times P_{ri}}\right) - \frac{1}{2}\left(\frac{D_{ri-1}}{P_{ri}}\right) - \left[\frac{1}{2}\frac{(1+D_{mi})D_{ri-1}}{D_{mi}(1-\varnothing_{i-1}+R_{ri-1})}\right]\right]\right\} \quad (3.9)$$

3.1.2.3. Joint cost of purchasing for member i and manufacturing for member i + 1. To optimize the costs for buyers and manufacturers, we can calculate the expected purchasing and manufacturing cost for member i. Hence, the joint cost of purchasing for member i and manufacturing for member $i+1$ can be determined [24] as a function of the variables $C_{i,i+1}$ by substituting $n_i \times N_{pi} = D_{mi+1} \times N_{mi+1}$ into (3.5) and (3.9) and substituting $t_i = \frac{T_{pi}}{D_{mi+1}N_{mi+1}}$ into the equation. Thus, the joint total cost of members i and $i+1$ can be calculated as follows:

$$C_{i,i+1} = (m_{Li}, N_{mi+1}, D_{mi+1}) = EC_{pi} + EC_{mi}$$

$$= D_{mi+1}\times N_{mi+1}\left[EHC_i + EBC_i + F_{pi} + q_i^2 \times S_{ci} + \varnothing_i \times R_{ci}\right] + N_{pi}\times F_{oi} + N_{mi+1}F_{mi+1} + H_{mi+1}AI_{i+1}$$

$$= \left(D_{ri}H_{pi}T_{pi}m_{Li} + D_{ri}H_{pi}\left(\frac{T_{pi}^2}{D_{mi+1}N_{mi+1}}\frac{\varnothing_i}{4} + \frac{T_{pi}^2}{D_{mi+1}N_{mi+1}}\frac{1}{2}\right) - D_{ri}H_{pi}\frac{T_{pi}^2}{D_{mi+1}N_{mi+1}}\frac{D_{ri}\varnothing_i^2}{\theta_i}\right)\int_{\lambda_{Li}}^{m_{Li}} f(L_{di})dL_{di}$$
$$+ D_{ri}H_{pi}T_{pi}\int_{\lambda_{Li}}^{m_{Li}} L_{di}f(L_{di})dL_{di} - D_{ri}H_{pi}T_{pi}\frac{\varnothing_i}{2}\int_{\lambda_{Li}}^{m_{Li}}\left(\frac{T_{pi}}{D_{mi+1}N_{mi+1}} - L_{di} - t_{si} - \frac{T_{pi}}{D_{mi+1}N_{mi+1}}\frac{\varnothing_i D_{ri}}{\theta_i} + m_{Li}\right)f(L_{di})dL_{di}$$
$$+ D_{ri}H_{pi}D_{mi+1}N_{mi+1}\int_{m_{Li}}^{\gamma_{Li}}\frac{(L_{di}-m_{Li})[(\gamma_{Li}-L_{di})+(\gamma_{Li}-m_{Li})]}{2}f(L_{di})dL_{di} + D_{ri}H_{pi}D_{mi+1}N_{mi+1}$$
$$\int_{m_{Li}}^{\gamma_{Li}}\left[\frac{\frac{T_{pi}\varnothing_i}{2(D_{mi+1}N_{mi+1})}\left(\frac{T_{pi}}{D_{mi+1}N_{mi+1}} + m_{Li} - L_{di}\right) + \left(\frac{T_{pi}}{D_{mi+1}N_{mi+1}} + m_{Li} - L_{di}\right)^2}{2}\right]f(L_{di})dL_{di} - D_{ri}H_{pi}\frac{T_{pi}^2}{D_{mi+1}N_{mi+1}}\frac{D_{ri}\varnothing_i^2}{\theta_i}$$
$$\int_{m_{Li}}^{\gamma_{Li}} f(L_{di})dL_{di} - D_{ri}H_{pi}T_{pi}\frac{\varnothing_i}{2}\int_{m_{Li}}^{\gamma_{Li}}\left(\frac{T_{pi}}{D_{mi+1}N_{mi+1}} + m_{Li} - L_{di} - [1-(L_{di}-m_{Li})]t_{si} - \frac{T_{pi}}{D_{mi+1}N_{mi+1}}\frac{\varnothing_i D_{ri}}{\theta_i}\right)f(L_{di})dL_{di}$$
$$+ D_{mi+1}N_{mi+1}[F_{ei} + \beta_i(\gamma_{Li}-m_{Li})]\int_{m_{Li}}^{\gamma_{Li}} f(L_{di})dL_{di} + D_{mi+1}N_{mi+1}F_{pi} + D_{mi+1}N_{mi+1}q_i^2\times S_{ci} + D_{mi+1}N_{mi+1}\varnothing_i\times R_{ci}$$
$$+ N_{pi}\times F_{oi} + \left[N_{mi+1}F_{mi+1} + H_{mi+1}\times N_{mi+1}\left\{\left(\frac{T_{pi}}{N_{mi+1}}\right)^2 D_{ri}\left[1 - \left(\frac{1}{D_{mi+1}} - \frac{D_{ri}}{D_{mi+1}(1-\varnothing_i+R_{ri})\times P_{ri+1}}\right) - \frac{1}{2}\left(\frac{D_{ri}}{P_{ri+1}}\right)\right.\right.\right.$$
$$\left.\left.\left.- \left[\frac{1}{2}\frac{(1+D_{mi+1})D_{ri}}{{D_{mi+1}}^2(1-\varnothing_i+R_{ri}\varnothing_{i-1})}\right]\right]\right\}\right] \quad (3.10)$$

$$
\begin{aligned}
C_{i,i+1} &= (m_{Li}, N_{mi+1}, D_{mi+1}) \\
&= \left(\tau_i T_{pi} m_{Li} + \left(\tau_i \frac{T_{pi}^2}{D_{mi+1}N_{mi+1}}\right)\left(\frac{\varnothing_i}{4}+\frac{1}{2}-\frac{D_{ri}\varnothing_i^2}{\theta_i}\right)\right)\int_{\lambda_{Li}}^{m_{Li}} f(L_{di})dL_{di} + \tau_i T_{pi}\int_{\lambda_{Li}}^{m_{Li}} L_{di}f(L_{di})dL_{di} - \tau_i T_{pi}\frac{\varnothing_i}{2}\int_{\lambda_{Li}}^{m_{Li}}\left(\frac{T_{pi}}{D_{mi+1}N_{mi+1}}\right. \\
&\quad \left. - t_{si} - \frac{T_{pi}}{D_{mi+1}N_{mi+1}}\frac{\varnothing_i D_{ri}}{\theta_i} + \varphi_i\right) f(L_{di})dL_{di} + \tau_i D_{mi+1}N_{mi+1}\int_{m_{Li}}^{\gamma_{Li}} \rho_i f(L_{di})dL_{di} + \tau_i D_{mi+1}N_{mi+1} \\
&\quad \int_{m_{Li}}^{\gamma_{Li}}\left[\frac{\frac{T_{pi}\varnothing_i}{2(D_{mi+1}N_{mi+1})}\left(\frac{T_{pi}}{D_{mi+1}N_{mi+1}}+\varphi_i\right)+\left(\frac{T_{pi}}{D_{mi+1}N_{mi+1}}+\varphi_i\right)^2}{2}\right] f(L_{di})dL_{di} - \tau_i\frac{T_{pi}^2}{D_{mi+1}N_{mi+1}}\frac{D_{ri}\varnothing_i^2}{\theta_i}\int_{m_{Li}}^{\gamma_{Li}} f(L_{di})dL_{di} \\
&\quad - \tau_i T_{pi}\frac{\varnothing_i}{2}\int_{m_{Li}}^{\gamma_{Li}}\left(\frac{T_{pi}}{D_{mi+1}N_{mi+1}}+\varphi_i-[1-(L_{di}-m_{Li})]t_{si}-\frac{T_{pi}}{D_{mi+1}N_{mi+1}}\frac{\varnothing_i D_{ri}}{\theta_i}\right) f(L_{di})dL_{di} + D_{mi+1}N_{mi+1}[F_{ei} \\
&\quad + \beta_i(\gamma_{Li}-m_{Li})]\int_{m_{Li}}^{\gamma_{Li}} f(L_{di})dL_{di} + D_{mi+1}N_{mi+1}\vartheta_i + N_{pi}\times F_{oi} + \left[N_{mi+1}F_{mi+1} + H_{mi+1}\times N_{mi+1}\left\{\left(\frac{T_{pi}}{N_{mi+1}}\right)^2 D_{ri}\omega_i\right\}\right]
\end{aligned} \tag{3.11}
$$

Where $\tau_i = D_{ri}H_{pi}$

$$\rho_i = \frac{(L_{di}-m_{Li})[(\gamma_{Li}-L_{di})+(\gamma_{Li}-m_{Li})]}{2}$$

$$\vartheta_i = F_{pi} + q_i^2 \times S_{ci} + \varnothing_i \times R_{ci}$$

$$\varphi_i = m_{Li} - L_{di}$$

$$\omega_i = 1 - \left(\frac{1}{D_{mi+1}} - \frac{D_{ri}}{D_{mi+1}(1-\varnothing_i+R_{ri})\times P_{ri+1}}\right) - \frac{1}{2}\left(\frac{D_{ri}}{P_{ri+1}}\right) - \left[\frac{1}{2}\frac{(1+D_{mi+1})D_{ri}}{{D_{mi+1}}^2(1-\varnothing_i+R_{ri}\varnothing_i)}\right]$$

Since $T_{pi} = \frac{Q_{mi}}{P_{ri}} = \left(\frac{D_{ri-1}}{P_{ri}N_{mi}}\right)T_{pi-1}$, T_{pi} can be expressed further as

$T_{pi} = \frac{\delta_i}{\prod_{j=2}^{i} N_{mj}} T_{p1}$, $i = 1, 2 \ldots, r-1$, where $\delta = \begin{cases} 1, if\ i = 1, \\ \prod_{j=2}^{i}\frac{D_{rj-1}}{P_{rj}}, if\ i = 2, 3, \ldots K-1 \end{cases}$;. All the T_{pi} in function (3.11) would be replaced as following.

$$
\begin{aligned}
&C_{i,i+1} = (m_{Li}, N_{mi+1}, D_{mi+1}) \\
&= \left(\tau_i \frac{\delta_i}{\prod_{j=2}^{i} N_{mj}} T_{p1} m_{Li} + \left(\tau_i \frac{(\delta_i T_{p1})^2}{\left(\prod_{j=2}^{i} N_{mj}\right) D_{mi+1}} \right) \left(\frac{\varnothing_i}{4} + \frac{1}{2} - \frac{D_{ri}\varnothing_i^2}{\theta_i} \right) \right) \int_{\lambda_{Li}}^{m_{Li}} f(L_{di}) dL_{di} + \tau_i \frac{\delta_i}{\prod_{j=2}^{i} N_{mj}} T_{p1} \int_{\lambda_{Li}}^{m_{Li}} L_{di} f(L_{di}) dL_{di} \\
&- \tau_i \frac{\delta_i}{\prod_{j=2}^{i} N_{mj}} T_{p1} \frac{\varnothing_i}{2} \int_{\lambda_{Li}}^{m_{Li}} \left(\frac{\delta_i}{\prod_{j=2}^{i} N_{mj} D_{mi+1}} T_{p1} - t_{si} - \frac{\delta_i}{\prod_{j=2}^{i} N_{mj} D_{mi+1}} T_{p1} \frac{\varnothing_i D_{ri}}{\theta_i} + \varphi_i \right) f(L_{di}) dL_{di} + \tau_i D_{mi+1} N_{mi+1} \int_{m_{Li}}^{\gamma_{Li}} \rho_i f(L_{di}) dL_{di} \\
&- \tau_i \frac{(\delta_i T_{p1})^2}{\left(\prod_{j=2}^{i} N_{mj}\right) D_{mi+1}} \frac{D_{ri}\varnothing_i^2}{\theta_i} \int_{m_{Li}}^{\gamma_{Li}} f(L_{di}) dL_{di} - \tau_i T_{pi} \frac{\varnothing_i}{2} \int_{m_{Li}}^{\gamma_{Li}} (\varphi_i - [1 - (L_{di} - m_{Li})] t_{si}) f(L_{di}) dL_{di} + D_{mi+1} N_{mi+1} [F_{ei} + \beta_i (\gamma_{Li} \\
&- m_{Li})] \int_{m_{Li}}^{\gamma_{Li}} f(L_{di}) dL_{di} + D_{mi+1} N_{mi+1} \vartheta_i + N_{pi} \times F_{oi} + \left[N_{mi+1} F_{mi+1} + H_{mi+1} \times N_{mi+1} \left\{ \left(\frac{(\delta_i T_{p1})^2}{\left(\prod_{j=2}^{i} N_{mj}\right) D_{mi+1}} \right) D_{ri} \omega_i \right\} \right]
\end{aligned}
\tag{3.12}
$$

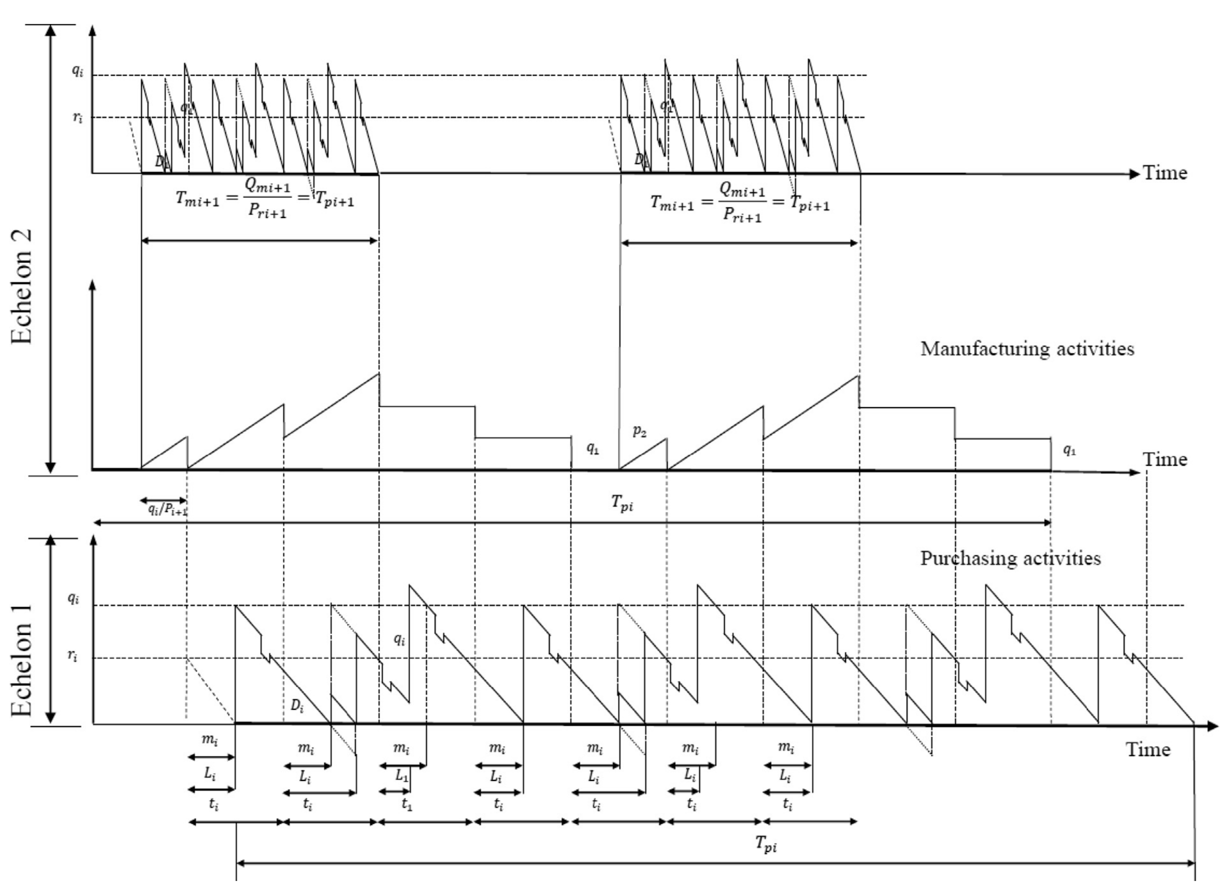

Fig. 7. The relationship between purchasing and manufacturing activities.

The K-echelon inventory model can be constructed from the combined joint cost functions of echelon $K-1$. $JC_{i,K}$ indicates the total joint cost from member i to member K. Because the purchase time of member $i+1$ is N_{mi+1} in T_{pi}, a 4-echelon inventory model can be formulated as follows. The relationship between purchasing and manufacturing activities is shown in Fig. 7

$$
\begin{aligned}
JC_{1,4}(m_{Li},N_{mi+1},D_{mi+1}) \\
&= C_{1,2}(m_{L1},N_{m2}D_{m2})+N_{m2}C_{2,3}(m_{L2},N_{m3}D_{m3}) \\
&\quad +N_{m2}N_{m3}C_{3,4}(m_{L3},N_{m4}D_{m4}) \\
&= \sum_{i=1}^{3} N_{mi}!C_{i,i+1}(m_{Li},N_{mi+1}D_{mi+1}) \qquad (3.13)
\end{aligned}
$$

Where

$$
\begin{cases} i=1, \aleph N_{mi}!\ ex\text{: } 1{*}C_{1,2}(N_{p1},m_{L1},N_{m2}D_{m2}) \\ i=2\ldots k-1, \aleph N_{mi}!\ ex: N_{m3}N_{m2}N_{m1}{*}C_{3,4}(N_{p3},m_{L3},N_{m4}D_{m4}). \end{cases}
$$

According to (3.13), the total joint cost function of the serial K-echelon inventory model is presented in equation (3.14)

$$
\begin{aligned}
&JC_{1,K}(m_{Li},N_{mi+1},D_{mi+1}) \\
&= \sum_{i=1}^{K-1} \aleph N_{mi}!C_{i,i+1}(m_{Li},N_{mi+1}D_{mi+1}) \\
&= \sum_{i=1}^{K-1} \aleph N_{mi}! \Bigg[\Bigg(\tau_i \frac{\delta_i}{\prod_{j=2}^{i} N_{mj}} T_{p1} m_{Li} + \left(\tau_i \frac{(\delta_i T_{p1})^2}{\left(\prod_{j=2}^{i} N_{mj}\right) D_{mi+1}} \right) \left(\frac{\varnothing_i}{4} + \frac{1}{2} - \frac{D_{ri}\varnothing_i^2}{\theta_i} \right) \Bigg) \int_{\lambda_{Li}}^{m_{Li}} f(L_{di})dL_{di} \\
&\quad + \tau_i \frac{\delta_i}{\prod_{j=2}^{i} N_{mj}} T_{p1} \int_{\lambda_{Li}}^{m_{Li}} L_{di} f(L_{di})dL_{di} - \tau_i \frac{\delta_i}{\prod_{j=2}^{i} N_{mj}} T_{p1} \frac{\varnothing_i}{2} \int_{\lambda_{Li}}^{m_{Li}} \left(\frac{\delta_i}{\prod_{j=2}^{i} N_{mj} D_{mi+1}} T_{p1} - t_{si} - \frac{\delta_i}{\prod_{j=2}^{i} N_{mj} D_{mi+1}} T_{p1} \frac{\varnothing_i D_{ri}}{\theta_i} + \varphi_i \right) f(L_{di})dL_{di} \\
&\quad + \tau_i D_{mi+1} N_{mi+1} \int_{m_{Li}}^{\gamma_{Li}} \rho_i f(L_{di})dL_{di} - \tau_i \frac{(\delta_i T_{p1})^2}{\left(\prod_{j=2}^{i} N_{mj}\right) D_{mi+1}} \frac{D_{ri}\varnothing_i^2}{\theta_i} \int_{m_{Li}}^{\gamma_{Li}} f(L_{di})dL_{di} - \tau_i \frac{\delta_i}{\prod_{j=2}^{i} N_{mj}} T_{p1} \frac{\varnothing_i}{2} \int_{m_{Li}}^{\gamma_{Li}} (\varphi_i - [1-(L_{di} \\
&\quad - m_{Li})] t_{si}) f(L_{di})dL_{di} + D_{mi+1}N_{mi+1}[F_{ei} + \beta_i(\gamma_{Li} - m_{Li})] \int_{m_{Li}}^{\gamma_{Li}} f(L_{di})dL_{di} + D_{mi+1}N_{mi+1}\vartheta_i + N_{pi} \times F_{oi} \\
&\quad + \left[N_{mi+1}F_{mi+1} + H_{mi+1} \times N_{mi+1} \left\{ \left(\frac{(\delta_i T_{p1})^2}{\left(\prod_{j=2}^{i} N_{mj}\right) D_{mi+1}} \right) D_{ri}\omega_i \right\} \right] \Bigg]
\end{aligned} \qquad (3.14)
$$

The constraints of delivery times $\begin{cases} 0 \le \lambda_{Li} \le m_{Li} \le \gamma_{Li} \le t_i \\ i=1,2,\ldots,K-1 \end{cases}$ and variables are considered for the inventory model. In the end, the K-echelon inventory model becomes as following.

Minimize

$$
JC_{1,K}(m_{Li},N_{mi+1},D_{mi+1}) = \sum_{i=1}^{K-1} \aleph N_{mi}!C_{i,i+1}(m_{Li},N_{mi+1}D_{mi+1})
$$

$$=\sum_{i=1}^{K-1}\aleph N_{mi}!\left[\left(\left(\tau_i\frac{\delta_i}{\prod_{j=2}^{i}N_{mj}}T_{p1}m_{Li}+\left(\tau_i\frac{(\delta_iT_{p1})^2}{\left(\prod_{j=2}^{i}N_{mj}\right)D_{mi+1}}\right)\left(\frac{\varnothing_i}{4}+\frac{1}{2}-\frac{D_{ri}\varnothing_i^2}{\theta_i}\right)\right)\int_{\lambda_{Li}}^{m_{Li}}f(L_{di})dL_{di}\right.\right.$$

$$+\tau_i\frac{\delta_i}{\prod_{j=2}^{i}N_{mj}}T_{p1}\int_{\lambda_{Li}}^{m_{Li}}L_{di}f(L_{di})dL_{di}-\tau_i\frac{\delta_i}{\prod_{j=2}^{i}N_{mj}}T_{p1}\frac{\varnothing_i}{2}\int_{\lambda_{Li}}^{m_{Li}}\left(\frac{\delta_i}{\prod_{j=2}^{i}N_{mj}D_{mi+1}}T_{p1}-t_{si}-\frac{\delta_i}{\prod_{j=2}^{i}N_{mj}D_{mi+1}}T_{p1}\frac{\varnothing_iD_{ri}}{\theta_i}+\varphi_i\right)f(L_{di})dL_{di}$$

$$+\tau_iD_{mi+1}N_{mi+1}\int_{m_{Li}}^{\gamma_{Li}}\rho_if(L_{di})dL_{di}-\tau_i\frac{(\delta_iT_{p1})^2}{\left(\prod_{j=2}^{i}N_{mj}\right)D_{mi+1}}\frac{D_{ri}\varnothing_i^2}{\theta_i}\int_{m_{Li}}^{\gamma_{Li}}f(L_{di})dL_{di}-\tau_i\frac{\delta_i}{\prod_{j=2}^{i}N_{mj}}T_{p1}\frac{\varnothing_i}{2}$$

$$\int_{m_{Li}}^{\gamma_{Li}}(\varphi_i-[1-(L_{di}-m_{Li})]t_{si})f(L_{di})dL_{di}+D_{mi+1}N_{mi+1}[F_{ei}+\beta_i(\gamma_{Li}-m_{Li})]\int_{m_{Li}}^{\gamma_{Li}}f(L_{di})dL_{di}+D_{mi+1}N_{mi+1}\vartheta_i+N_{pi}\times F_{oi}$$

$$\left.\left.+\left[N_{mi+1}F_{mi+1}+H_{mi+1}\times N_{mi+1}\left\{\left(\frac{(\delta_iT_{p1})^2}{\left(\prod_{j=2}^{i}N_{mj}\right)D_{mi+1}}\right)D_{ri}\omega_i\right\}\right]\right]\right]$$

Subject to $0\le\lambda_{Li}\le m_{Li}\le\gamma_{Li}\le t_i$ for $i=1,2,\ldots,K-1$;

$D_{mi+1}\le\frac{1}{\varnothing_i}-1\,N_{pi},D_{mi+1}$ and N_{mi+1} are positive integers, for $i=1,2,3,\ldots,K-1$

(3.15)

4. Experimental results and sensitivity analysis

This section describes the implementation of a GA to solve the inventory model. In the first experiment, the performance of the GA in obtaining a solution to the study problem was evaluated; this solution was compared with that obtained using another method. The performance metrics were the total number of iterations, total computer processing unit (CPU) time, and performance of the solution for the K-echelon inventory model. In the second experiment, the GA was used to obtain numerous solutions. The solutions were analyzed to determine whether the cost of the near-optimal solutions would decrease. Both experiments were performed on a computer with an Intel X3470 at 2.93 GHz; the programs were written in Python 3.7.

$$f(L_{di})=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)(L_{di}-\lambda_{Li})}\left(\frac{L_{di}-\lambda_{Li}}{\gamma_{Li}-\lambda_{Li}}\right)^{\alpha-1}\left(1-\frac{L_{di}-\lambda_{Li}}{\gamma_{Li}-\lambda_{Li}}\right)^{\beta-1}$$

$$=\frac{6(L_{di}-\lambda_{Li})}{(\gamma_{Li}-\lambda_{Li})^2}\left(1-\frac{L_{di}-\lambda_{Li}}{\gamma_{Li}-\lambda_{Li}}\right)$$

$$\int_{\lambda_{Li}}^{m_{Li}}\frac{6(L_{di}-\lambda_{Li})}{(\gamma_{Li}-\lambda_{Li})^2}\left(1-\frac{L_{di}-\lambda_{Li}}{\gamma_{Li}-\lambda_{Li}}\right)dL_{di}$$

$$=\frac{6m_{Li}(L_{di}-\gamma_{Li})(L_{di}-\lambda_{Li})-6\lambda_{Li}(L_{di}-\gamma_{Li})(L_{di}-\lambda_{Li})}{(\gamma_{Li}-\lambda_{Li})^3}$$

When $\mu_{Ldi}=0.0090,\mathrm{k}=2,\sigma_{Ldi}=0.0020$ (4.1)

4.1. GA experiment

For the first experiment, the number of echelons K was set to 7. Moreover, member i was assumed to purchase goods from member $i + 1$ for a finite purchase period T_{pi} of 1.2 years. The delivery lead time was defined as a beta distribution with $\alpha = 2$ and $\beta = 2$, for $i = 1\ldots6$, and the equation (4.1) illustrates the *pdf* of L_{di} and $L_{di} = \mu_{Ldi} + k\sigma_{Ldi}$.

From the equation (4.1), we obtain the probability of delivery lead time which is substituted for $f(L_{di})$ in equation (3.15).

Subject to

$0 \le \lambda_{Li} \le m_{Li} \le \gamma_{Li} \le t_i$ for $i = 1, 2, \ldots, K-1$; $D_{mi+1} \le \frac{1}{\varnothing_i} - 1$.

N_{pi}, D_{mi+1} and N_{mi+1} as positive integers, for $i = 1, 2, \ldots, K-1$.

In addition to the objective function, we need the values of others notations that cannot be obtained by the equation. Therefore, all of these numerical values of notations only are established by us. When these numerical values are set, the procedure will load the numerical values. The numerical values of purchasing and manufacture behavior are shown in Table 1 and Table 2.

Minimize

$$
\begin{aligned}
&JC_{1,S}(m_{Li},N_{mi+1},D_{mi+1})=\sum_{i=1}^{S-1}\aleph N_{mi}!\,C_{i,i+1}(m_{Li},N_{mi+1}D_{mi+1})\\
&=\sum_{i=1}^{S-1}\aleph N_{mi}!\Bigg[\Bigg[\Bigg(\tau_i\frac{\delta_i}{\prod_{j=2}^{i}N_{mj}}T_{p1}m_{Li}+\Bigg(\tau_i\frac{(\delta_iT_{p1})^2}{\left(\prod_{j=2}^{i}N_{mj}\right)D_{mi+1}}\Bigg)\left(\frac{\varnothing_i}{4}+\frac{1}{2}-\frac{D_{ri}\varnothing_i^2}{\theta_i}\right)\Bigg)+\tau_iD_{mi+1}N_{mi+1}\rho_i\\
&-\tau_i\frac{(\delta_iT_{p1})^2}{\left(\prod_{j=2}^{i}N_{mj}\right)D_{mi+1}}\frac{D_{ri}\varnothing_i^2}{\theta_i}+D_{mi+1}N_{mi+1}[F_{ei}+\beta_i(\gamma_{Li}-m_{Li})]\\
&-\tau_i\frac{\delta_i}{\prod_{j=2}^{i}N_{mj}}T_{p1}\frac{\varnothing_i}{2}\Bigg(\frac{\delta_i}{\prod_{j=2}^{i}N_{mj}D_{mi+1}}T_{p1}-t_{si}-\frac{\delta_i}{\prod_{j=2}^{i}N_{mj}D_{mi+1}}T_{p1}\frac{\varnothing_iD_{ri}}{\theta_i}+\varphi_i\Bigg)\\
&-\tau_i\frac{\delta_i}{\prod_{j=2}^{i}N_{mj}}T_{p1}\frac{\varnothing_i}{2}(\varphi_i-[1-(L_{di}-m_{Li})]t_{si})\Bigg]\frac{6m_{Li}(L_{di}-\gamma_{Li})(L_{di}-\lambda_{Li})-6\lambda_{Li}(L_{di}-\gamma_{Li})(L_{di}-\lambda_{Li})}{(\gamma_{Li}-\lambda_{Li})^3}\\
&+\tau_i\frac{\delta_i}{\prod_{j=2}^{i}N_{mj}}T_{p1}\frac{3{m_{Li}}^2(L_{di}-\gamma_{Li})(L_{di}-\lambda_{Li})-3{\lambda_{Li}}^2(L_{di}-\gamma_{Li})(L_{di}-\lambda_{Li})}{(\gamma_{Li}-\lambda_{Li})^3}\\
&+D_{mi+1}N_{mi+1}\vartheta_i+N_{pi}\times F_{oi}\\
&+\Bigg[N_{mi+1}F_{mi+1}+H_{mi+1}\times N_{mi+1}\Bigg\{\Bigg(\frac{(\delta_iT_{p1})^2}{\left(\prod_{j=2}^{i}N_{mj}\right)D_{mi+1}}\Bigg)D_{ri}\omega_i\Bigg\}\Bigg]\Bigg]
\end{aligned}
\tag{4.2}
$$

Table 1. Setting up data of purchase activities.

Echelon	D_{ri}	F_{oi}	H_{pi}	F_{ei}	β_i	F_{pi}	λ_{Li}	γ_{Li}
1	28	2.5	2.8	2.9	3.2	7	0.001	0.026
2	31	2.8	2.2	3.2	2.6	5	0.004	0.019
3	35	3.2	1.7	3.8	2.1	4.5	0.002	0.029
4	42	3.6	1.2	4.2	1.6	5.5	0.004	0.023
5	53	3.5	0.8	3.7	1.2	8	0.003	0.024
6	60	3.8	0.6	4.0	2	6.5	0.004	0.034
7	–	–	–	–	–	–	–	–
Echelon	μ_{Ldi}	σ_{Ldi}	t_{si}	$\varnothing_i$	S_{ci}	R_{ci}	q_i	N_{pi}
1	0.0090	0.0020	0.013	0.2	10	12	1	6
2	0.0064	0.0012	0.009	0.1	12	11	1	3
3	0.0054	0.0012	0.005	0.2	15	13	3	9
4	0.0072	0.0016	0.011	0.25	11	9	2	5
5	0.0085	0.0020	0.012	0.13	13	8	4	7
6	0.0080	0.0020	0.020	0.15	30	10	5	2
7	–		–	–	–	–	–	–

Table 2. Setting up data of manufacture activities.

Echelon	P_{ri}	F_{mi}	H_{mi}
1	–	–	–
2	31	2.8	2.5
3	35	2	1.9
4	42	3.2	1.4
5	53	3.6	1
6	60	3.8	0.9
7	69	4.2	0.5

Table 3. The values which are used in experiment are suitable.

	Number of loops	Solution numbers	k (L_{di})
GA	1000 times	400 (Numbers)	2
	The upper and lower bound of N_{mi} and D_{mi}	Special factor in each algorism	
GA	1–40	1 % (Mutation rate) 60 % (Crossover rate)	

To evaluate the effectiveness of GA. We built a model on 7-echelon inventory which used by the GA algorism to conduct it, but some parameters that didn't cover it in this experiment are presented in Table 3. Table 3 lists some information, not only the conditions are included in all algorithms but also some special factors in each algorithm.

Table 4. Each method performs on 3-echelon inventory.

echelon	GA			Lingo		
	m_{Li}	N_{mi}	D_{mi}	m_{Li}	N_{mi}	D_{mi}
1	0.0045	–	–	0.0045	–	–
2	0.0084	4	1	0.0084	4	1
3	–	1	2	–	1	2
Optimal Cost	113			113		
Total CPU time (second)	1.46			–		
Average Iterations	1000			–		

Table 5. The efficacy of genetic algorithms search capability.

echelon	GA		
	m_{Li}	N_{mi}	D_{mi}
1	0.0202	–	–
2	0.0043	1	22
3	0.0114	4	19
4	0.0133	5	4
5	0.0230	2	2
6	0.0131	3	14
7	–	2	8
Optimal Cost	109,683		
Total CPU time (second)	83.929		
Average Iterations	1000		

The GA was used to find solutions for 3- and 7-echelon inventory models with the same parameters. Small ranges for N_{mi} and D_{mi} were selected; excessive upper-bound values can result in the algorithm producing redundant solutions, wasting CPU time. Table 4 and Table 5 present the results of experiments for the 3- and 7-echelon inventory models, respectively.

As indicated in Table 5, the genetic algorithm not only required less CPU time, but also obtained the minimum-cost solution. The minimal cost is \$109,683, $m_{Li} = \{0.0202, 0.0043, 0.0114, 0.0133, 0.0230, 0.0131, -\}, N_{mi} = \{-, 1, 4, 5, 2, 3, 2\}$ and $D_{mi} = \{-, 22, 19, 4, 2, 14, 8\}$.

4.2. Experiment of solution numbers

We adjusted the number of solutions to investigate its effect on the result and CPU time. All other parameters were unchanged; the values are listed in Table 5. In the GA, the number of solutions may not be an integer and can be calculated as follows: $\begin{cases} n + \left(\frac{(n+n^2)}{2} - n\right) \times 2 = n^2 \\ \text{where } n = 2 \ldots k \end{cases}$. Hence, if $n = 2$, the number of solutions should be $2^2 = 4$. The results are presented in Fig. 8; optimal costs exceeding 1 million are shown as 1 million in this figure. The actual optimal values are listed in Appendix A.

Some observations regarding the experimental results shown in Fig. 8 are as follows.

- Total CPU time and performance for different numbers of solutions

The required CPU time for the GA was low; only 40 s was required for 256 solutions (Fig. 8). However,

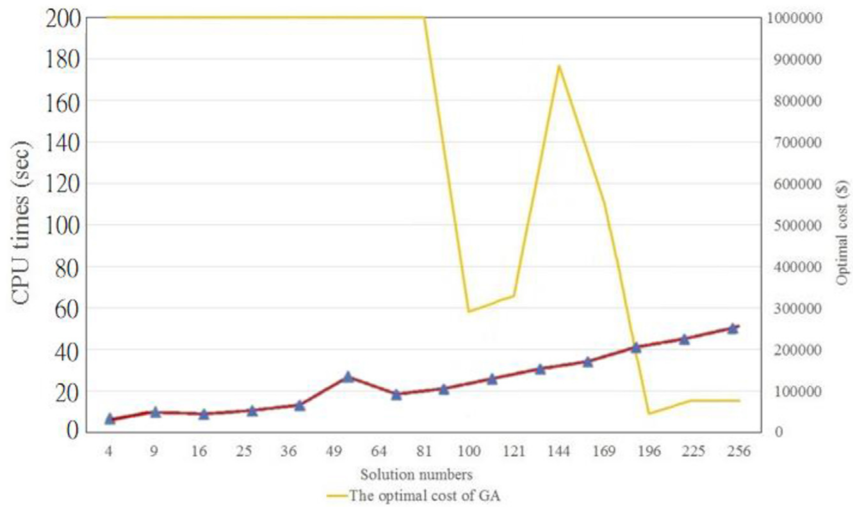

Fig. 8. The result of different solution numbers.

a high number of solutions was required to identify the optimal cost.

- ◆ The total CPU time and optimal cost may suddenly increase or decrease.

Although the total CPU time and number of solutions were expected to be positively correlated, the CPU time increased sharply and then decreased as the number of solutions was increased from 36 to 64 (Fig. 8); otherwise, the total CPU time increased monotonically. This unexpected increase may be attributable to system processes, such as installing updates or running antivirus software.

The optimal cost was expected to decrease as the number of solutions increased because the number of solutions is related to the probability of finding a solution. However, the cost increased anomalously when the number of solutions ranged between 100 and 225, suggesting that this relationship did not hold in this range. This was attributed to luck; the obtained solution can be inconsistent, such as for 100–144 solutions.

5. Conclusion

We propose a JIT inventory model that considers uncertain lead times, unreliable quality, and imperfect rework, with the goal of minimizing the joint total cost of the supplier and the buyer. The model was solved using the GA. We first solved the 3-echelon inventory model and compared the results with those of the optimized solution; the solutions were consistent, and the GA optimization process was faster. Hence, the GA was determined to be effective. We then used the GA to solve the 7-echelon inventory model and performed a sensitivity analysis for different initial solutions provided by the genetic algorithm.

Conflict of interest

The authors declare no conflict of interest.

Appendix

Appendix A. Genetic algorithm adjust solution numbers.

Solution numbers	4			9			16			25		
	m_{Li}	N_{mi}	D_{mi}	m_{Li}	N_{mi}	D_{mi}	m_{Li}	N_{mi}	D_{mi}	m_{Li}	N_{mi}	D_{mi}
1	0.0167	–	–	0.0234	–	–	0.0110	–	–	0.0153	–	–
Echelon												
2	0.0175	12	34	0.0132	3	11	0.0157	5	15	0.0189	4	28
3	0.0281	1	18	0.0156	2	16	0.0095	8	1	0.0242	1	2
4	0.0048	4	33	0.0051	8	27	0.0106	1	13	0.0227	4	32
5	0.0036	17	39	0.0205	24	25	0.0041	5	17	0.0063	2	24
6	0.0317	21	6	0.0136	2	3	0.0147	9	21	0.0156	17	23
7	–	2	29	–	3	29	–	1	29	–	6	19
Optimal Cost	5,279,400,338			178,511,207			6,291,316			59,847,993		
Average CPU time (second)	4.486			5.896			5.126			6.554		

Solution numbers	36			49			64			81		
	m_{Li}	N_{mi}	D_{mi}	m_{Li}	N_{mi}	D_{mi}	m_{Li}	N_{mi}	D_{mi}	m_{Li}	N_{mi}	D_{mi}
1	0.0080	–	–	0.0079	–	–	0.0143	–	–	0.0229	–	–
Echelon												
2	0.0073	8	2	0.0050	1	20	0.0045	1	3	0.0139	3	8
3	0.0172	7	5	0.0148	13	3	0.0117	34	5	0.0234	4	26
4	0.0151	1	8	0.0107	3	23	0.0183	3	11	0.0133	4	7
5	0.0136	2	26	0.0151	4	15	0.0071	1	14	0.0176	4	13
6	0.0254	9	28	0.0177	3	2	0.0137	1	30	0.0172	2	22
7	–	4	7	–	5	13	–	8	8	–	8	3
Optimal Cost	29,112,885			2,243,778			2,494,852			7,087,878		
Average CPU time (second)	8.369			17.947			12.005			14.131		
Solution numbers	100			121			144			169		
	m_{Li}	N_{mi}	D_{mi}	m_{Li}	N_{mi}	D_{mi}	m_{Li}	N_{mi}	D_{mi}	m_{Li}	N_{mi}	D_{mi}
1	0.0151	–	–	0.0126	–	–	0.0141	–	–	0.0097	–	–
Echelon												
2	0.0176	8	2	0.0184	4	13	0.0156	1	11	0.0081	5	7
3	0.0060	2	5	0.0041	2	26	0.0256	1	10	0.0021	1	7
4	0.0117	4	1	0.0207	1	12	0.0067	1	8	0.0194	13	11
5	0.0170	2	18	0.0061	1	3	0.0113	3	19	0.0123	2	17
6	0.0301	1	23	0.0090	6	22	0.0062	8	18	0.0335	1	19
7	–	1	5	–	4	8	–	16	3	–	3	2
Optimal Cost	290,037			328,458			883,105			551,933		
Average CPU time (second)	17.46			20.781			23.473			28.257		
Solution numbers	196			225			256					
	m_{Li}	N_{mi}	D_{mi}	m_{Li}	N_{mi}	D_{mi}	m_{Li}	N_{mi}	D_{mi}			
1	0.0147			0.0082	–	–	0.0168	–	–			
Echelon												
2	0.0149	3	7	0.0044	2	15	0.0111	2	9			
3	0.0280	5	3	0.0261	5	8	0.0055	1	16			
4	0.0154	1	15	0.0160	3	19	0.0183	12	1			
5	0.0228	3	22	0.0101	2	14	0.0101	3	7			
6	0.0185	1	3	0.0086	2	23	0.0057	2	17			
7	–	3	12	–	1	17	–	1	6			
Optimal Cost	44,384			78,529			76,754					
Average CPU time (second)	31.128			35.054			39.727			GA		

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