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ELASTIC MODULUS OF CONCRETE AFFECTED BY ELASTIC MODULI OF MORTAR AND ARTIFICIAL AGGREGATE

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Keywords: concrete, aggregate, mortar, elastic moduli, micromechanics.

ABSTRACT

The effect of two artificial aggregate types and three kinds of mortar on the elastic modulus of concrete are reported. Eighteen batches of concrete with various volume fractions (10%, 20%, and 30%) were selected. Theoretical moduli were calculated based on micromechanics. Following recent studies of concrete as a two-phase composite material, this study examines whether there is a relation between the elastic modulus of the constituents and concrete. From the examination of available experimental results, it is apparent that elastic modulus of concrete is affected by elastic modulus of constituents and volume fraction of aggregate. Test results were compared with the theoretical moduli and the theoretical predictions fairly agree with the experimental data.

INTRODUCTION

Concrete is the world's versatile and most widely produced material because the cost is low compared with other construction materials, the components are available to be produced worldwide for the local market, and it can be cast to produce complex shapes with adequate strength. Despite these remarkable advantages, concrete has been largely ignored by the materials community. The major reason for this is that concrete is usually considered a "low-technology material". Actually, concrete designation as low technology can only be because of its low price [1].

The elastic modulus of concrete is very difficult to predict because the properties of concrete is influenced by the properties and quantities of the components. The extremely complex mechanical behavior of concrete has forced the design engineers to assume that it behaves as a homogeneous, isotropic, elastic material taking an average of elastic moduli and Poisson's ratio.

By considering concrete as a two-phase material, Aitcin and Mehta [2], Baalbaki et al. [3] demonstrated that the elastic modulus of concrete is influenced by the elastic properties and volume fraction of aggregates. Hirsch [4] derived an equation to express the elastic modulus of concrete in terms of an empirical constant, and also provided some experimental results for the elastic moduli of concretes with different aggregates.

The overall mechanical behavior of composite materials has been extensively studied. Voigt's [5] approximation yielded the upper bound and the Reuss's [6] approximation yielded the lower bound of the average elastic moduli. Hashin and Shtrikman [7] proposed the variational principle to find bounds on the average elastic moduli of composite materials which were better than the Voigt and Reuss bounds. Hansen [8] developed mathematical models to predict the elastic moduli of composite materials based on the individual elastic modulus and volume fraction of the components.

Mori and Tanaka [9] applied the concept of average field to analyze macroscopic properties of composite materials. The average field in a body contains inclusions with eigenstrain. In addition, the shape effect of dispersoids was introduced in Eshelby's [10] method to assess the properties of composite materials. The recent development of evaluating overall elastic modulus and overall elastic-plastic behavior was reviewed by Mura [11], Nemat-Nasser and Hori [12]. In addition, Yang et al. [13] proposed a model for approximating elastic modulus of concrete by employing Mori-Tanaka Theory and Eshelby's Method [10].

EXPERIMENTAL PROGRAM

Eighteen batches of concrete were made with

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two different artificial coarse aggregates (W/C=0.4 and 0.6) and three different mortars (W/C=0.3, 0.4 and 0.6). The amount of coarse aggregates used in the concrete varied from 10% to 30% by volume.

In this study, two stages casting procedure were followed. In the first stage, the coarse aggregates were cast in a sphere mold and cured in water until the time of the second stage. And in the second stage, concrete cylinders were cast and cured in water until the time of testing.

Coarse aggregate

Artificial coarse aggregates were cast using cement paste and having a uniform size (diameter = 15 mm). Two different aggregates A and B were made with water-cement ratios of 0.4, and 0.6, respectively. Their moduli of elasticity ranged from 18.76 GPa to 10.19 GPa . Cement paste cylinders (150 x 300 mm) were cast and cured in water. At 28 days, the static modulus of elasticity and compressive strength of the coarse aggregate were determined according to ASTM C469 and C39, respectively. The mechanical characteristics of the coarse aggregate are presented in Table 1.

Concrete

Concrete specimens were made from river sand, Type I cement, and two different aggregates. Concrete mix design is given in Table 2. Three different mortars, M, N, and O were made with water-cement ratios of 0.3, 0.4 and 0.6, respectively (See Table 3). Each mortar was mixed with two different coarse aggregates and the volume fractions of each coarse aggregates are 0%, 10%, 20%, and 30% (See Table 4).

Concrete cylinders (150 x 300 mm) were cast and cured in water. At 28 days, the static modulus of elasticity and compressive strength were measured according to ASTM C469 and C39, respectively. The moduli of elasticity of concrete were obtained directly from the stress/strain curves. The Poisson's ratio were also measured and recorded by use of lateral strain gages. The modulus of elasticity of concrete, E_c, and Poisson's ratio of concrete, v_c, are shown in Table 4. All the test results are the average of three specimens. The typical stress-strain curves for matrix, aggregate, and concrete are shown in Fig. 1. It shows that the modulus of elasticity of concrete is between matrix and aggregate.

Notation for the specimens is as follows: the first letter indicates the three different mortars M, N, and O; the second letter is the two different artificial coarse aggregates A and B; and the last number is the volume fractions of coarse aggregates.

Table 1. W/C and mechanical properties of coarse aggregate

	A	B
Water-cement ratio (W/C)	0.4	0.6
Compressive strength, (MPa)	55.46	28.26
Modulus of elasticity, E_a, (GPa)	18.76	10.19
Poisson's ratio, v_a	0.21	0.20

Table 2. Mix Proportions (kg/m^3)

Materials	Mortar		
Water	M	N	O
Cement	233.1	5288.5	400.7
Sand	1247	1150	996
Silica fume	66.5	61.4	53.1
Superplasticizer	17.4	20.2	0

MODEL OF APPROXIMATE ELASTIC MODULI

A sufficiently large body D is considered to contain the inclusions. The inclusions are with the

Table 3. W/C and mechanical properties of matrix

	M	N	O
Water-cement ratio (W/C)	0.3	0.4	0.6
Compressive strength, MPa	68.21	50.19	68.21
Elastic modulus, E_m, (GPa)	24.28	21.29	16.19
Poisson's ratio, v_m	0.22	0.21	0.19

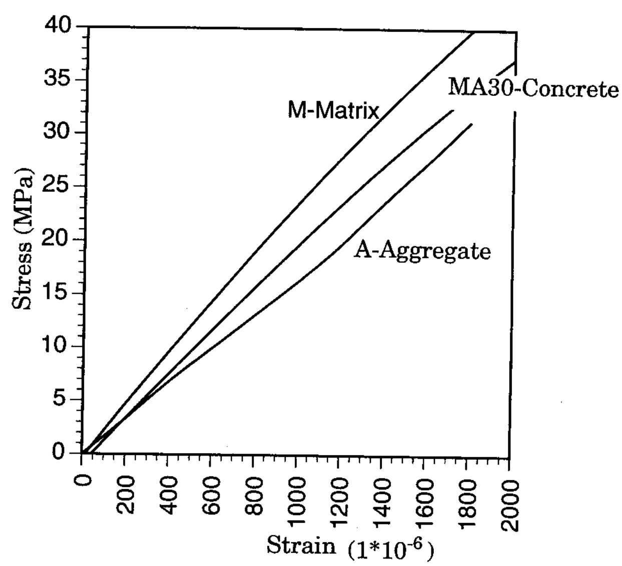

Fig. 1. A typical stress-strain curve for mortar, aggregate, and concrete.

sphere shape and distributed randomly. Consider coarse aggregate as the inclusions, $\Omega = \sum_{i=1}^{N} \Omega_i$, with elastic moduli $\underset{\sim}{C}^*$ and volume fraction f, and mortar as the matrix, with elastic moduli $\underset{\sim}{C}$. As a result, the average elastic moduli of concrete $\overline{\underset{\sim}{C}}$, can be evaluated.

Consider concrete subjected to an applied uniform stress $\underset{\sim}{\sigma}^0$ at infinity. In the so-called equivalent inclusions method [11], concrete is simulated by a homogeneous material with uniform stiffness $\underset{\sim}{C}$ and distributing eigenstrains $\underset{\sim}{\varepsilon}^*$ in Ω's. The disturbances of the stress caused by the inhomogeneities in the concrete i.e., deviations from $\underset{\sim}{\sigma}^0$ are denoted by $\underset{\sim}{\sigma}^\Omega$, and $\underset{\sim}{\sigma}^M$ for inclusions and matrix, respectively. Mura [11] proposed that the strain deviation ($\Delta\underset{\sim}{\gamma}$) caused by the eigenstrain in the inclusion and applied Eshelby's Method [10] can be obtained as follows:

$$\Delta\gamma_{ij}(x) = -\int_{\Omega_1} C_{klmn}\,\varepsilon^*_{nm}(x')\,\frac{1}{2}\,\{G_{ik,lj}(x-x') + G_{jk,li}(x-x')\}\,dx' = \underset{\sim}{S}\,\underset{\sim}{\varepsilon}^*, \tag{1}$$

where $\underset{\sim}{S}$ is the Eshelby tensor for sphere inclusion when it isolately exists in an infinite homogeneous medium and $\underset{\sim}{I}$ is the unit tensor. The Green's function $G_{ij}(x-x')$ is the displacement component in the x_i-direction at point x when a unit body force in the x_j-direction is applied at point x' in the infinitely extended material.

Table 4. Physical properties of concretes

Notation	volume fractions of coarse aggregate (%)	W/C	E_c (GPa)	v_c
MA10	10		23.69	0.19
MA20	20	0.3	23.43	0.21
MA30	30		21.86	0.23
NA10	10		21.29	0.23
NA20	20	0.4	20.83	0.21
NA30	30		20.08	0.21
OA10	10		1.10	0.19
OA20	20	0.6	16.47	0.19
OA30	30		16.87	0.21
MB10	10		22.66	0.21
MB20	20	0.3	21.72	0.21
MB30	30		21.04	0.22
NB10	10		20.25	0.19
NB20	20	0.4	19.92	0.20
NB30	30		18.66	0.19
OB10	10		16.19	0.19
OB20	20	0.6	15.31	0.18
OB30	30		12.75	0.17

The solution for the equivalent inclusion problem which simulates the actual elastic state in the concrete is obtained by applying Hooke's law and employing the average stress quantity in Ω:

$$\begin{aligned}\underset{\sim}{\sigma}^0 + \langle\underset{\sim}{\sigma}^\Omega\rangle &= \underset{\sim}{C}\{\underset{\sim}{C}^{-1}(\underset{\sim}{\sigma}^0 + \langle\underset{\sim}{\sigma}^M\rangle) + (\underset{\sim}{S} - \underset{\sim}{I})\langle\underset{\sim}{\varepsilon}^*\rangle\} \\ &= \underset{\sim}{C}^*\{\underset{\sim}{C}^{-1}(\underset{\sim}{\sigma}^0 + \langle\underset{\sim}{\sigma}^M\rangle) + \underset{\sim}{S}\langle\underset{\sim}{\varepsilon}^*\rangle\},\end{aligned} \tag{2}$$

where "⟨ ⟩" is the notation for the average over Ω.

From Eq. (2), it follows that

$$\langle\underset{\sim}{\sigma}^\Omega\rangle = \langle\underset{\sim}{\sigma}^M\rangle + \underset{\sim}{C}(\underset{\sim}{S} - \underset{\sim}{I})\langle\underset{\sim}{\varepsilon}^*\rangle. \tag{3}$$

Since the average of the stress disturbance must be zero, namely,

$$f\langle\underset{\sim}{\sigma}^\Omega\rangle + (1-f)\langle\underset{\sim}{\sigma}^M\rangle = 0. \tag{4}$$

From Eqns. (3) and (4), it follows that

$$\langle\underset{\sim}{\sigma}^M\rangle = -f\,\underset{\sim}{C}(\underset{\sim}{S} - \underset{\sim}{I})\langle\underset{\sim}{\varepsilon}^*\rangle. \tag{5}$$

By substituting Eqns. (3) and (5) into Eq. (2), then solving Eq. (2) for $\langle\underset{\sim}{\varepsilon}^*\rangle$ yields

$$\langle\underset{\sim}{\varepsilon}^*\rangle = \{(1-f)(\underset{\sim}{C}^* - \underset{\sim}{C})\underset{\sim}{S} - f(\underset{\sim}{C} - \underset{\sim}{C}^*) + \underset{\sim}{C}\}^{-1}(\underset{\sim}{C} - \underset{\sim}{C}^*)\underset{\sim}{C}^{-1}\underset{\sim}{\sigma}^0. \tag{6}$$

From Eqns. (3) and (6), the total average strain $\langle\underset{\sim}{\gamma}\rangle$ of concrete is given as

$$\begin{aligned}\langle\underset{\sim}{\gamma}\rangle &= \underset{\sim}{\gamma}^0 + f\langle\underset{\sim}{\gamma}^\Omega\rangle + (1-f)\langle\underset{\sim}{\gamma}^M\rangle \\ &= \underset{\sim}{\gamma}^0 + f(\underset{\sim}{C}^{-1}\langle\underset{\sim}{\gamma}^M\rangle + \underset{\sim}{S}\langle\underset{\sim}{\varepsilon}^*\rangle) + (1-f)\underset{\sim}{C}^{-1}\langle\underset{\sim}{\sigma}^M\rangle \\ &= \underset{\sim}{C}^{-1}\underset{\sim}{\sigma}^0 + f\{(1-f)(\underset{\sim}{C}^* - \underset{\sim}{C})\underset{\sim}{S} - f(\underset{\sim}{C} - \underset{\sim}{C}^*) \\ &\quad + \underset{\sim}{C}\}^{-1}(\underset{\sim}{C} - \underset{\sim}{C}^*)\underset{\sim}{C}^{-1}\underset{\sim}{\sigma}^0 \\ &= \overline{\underset{\sim}{C}}^{-1}\underset{\sim}{\sigma}^0,\end{aligned} \tag{7}$$

where $\underset{\sim}{\gamma}^0$, $\langle\underset{\sim}{\gamma}^M\rangle$, and $\langle\underset{\sim}{\gamma}^\Omega\rangle$ are the applied strain, the average elastic strain in matrix, and the average elastic strain in Ω's, respectively. The average elastic compliance $\overline{\underset{\sim}{C}}^{-1}$ is evaluated from eqn (7). Therefore, the average elastic moduli tensor of concrete is given by

$$\overline{\underset{\sim}{C}} = \{\underset{\sim}{C}^{-1} + f[\{(1-f)(\underset{\sim}{C}^* - \underset{\sim}{C})\underset{\sim}{S} - f(\underset{\sim}{C} - \underset{\sim}{C}^*)$$

$$+\underset{\sim}{C}\}^{-1}]^{-1}(\underset{\sim}{C}-\underset{\sim}{C}^{*})\underset{\sim}{C}^{-1}\}. \quad (8)$$

COMPARISON OF THEORY WITH EXPERIMENTAL RESULTS

For elasticity calculation, elastic moduli of aggregate $\underset{\sim}{C}^{*}$ are calculated from Table 1, and elastic moduli of mortar $\underset{\sim}{C}$ are calculated from Table 3. The volume fraction f are from 0 to 3.0. The Eshelby's tensor $\underset{\sim}{S}$ for sphere inclusion is listed below [11]:

$$S_{1111}=S_{2222}=S_{3333}=\frac{7-5\,\nu_m}{15\,(1-\nu_m)},$$

$$S_{1122}=S_{2233}=S_{3311}=S_{1133}=S_{2211}=S_{3322}=\frac{5\,\nu_m-1}{15\,(1-\nu_m)},$$

$$S_{1212}=S_{2323}=S_{3131}\frac{4-5\,\nu_m}{15\,(1-\nu_m)}.$$

Substitute $\underset{\sim}{C}^{*}, \underset{\sim}{C}$, f, and $\underset{\sim}{S}$ into Eq. (8), the average elastic moduli of concrete $\overline{\underset{\sim}{C}}$ can be obtained, and the elastic modulus of concrete, E_c^{*}, can be calculated.

The results of the theoretical model and experiment are listed in Table 5. Table 5 shows that for the theoretical elastic modulus of concrete are close to experimental results except for the concrete with high water-cement ratio of constituents and volume ratio (e.g. OB30). The reason for the poor consistency can be accounted for the high porosity of bulk cement paste, mortar, and the transition zone. Fig. 2 and Fig. 3 illustrates the relationships between volume fraction and elastic modulus of concrete with A and B aggregate (W/C= 0.4 and 0.6) and various mortar (M, N, and O). In Fig. 2 and Fig. 3, elastic

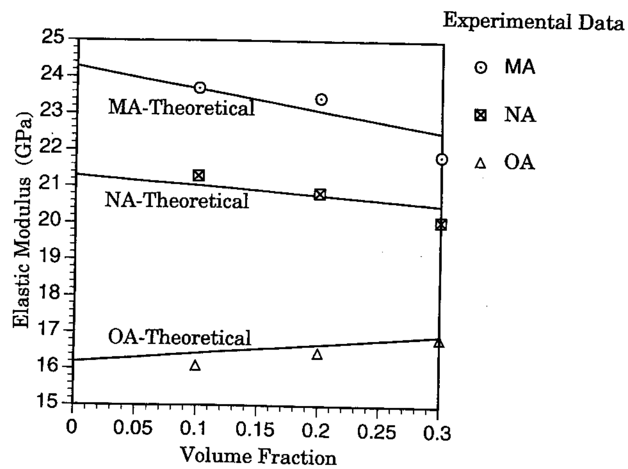

Fig. 2. Volume fraction vs. elastic modulus curves for concrete with A aggregate (W/C=0.4) and various mortar.

Table 5. The calculated results and experimental data for elastic modulus of concrete

	f (%)	E_m (GPa)	E_a (GPa)	(Meeasured) E_c (GPa)	(Calculated) E_c^{*} (GPa)	$\frac{E_c^{*}-E_c}{E_c}$ (%)
MA10	10			23.69	23.67	-0.08
MA20	20	24.28	18.76	23.43	23.07	-1.54
MA30	30			21.86	22.48	2.61
NA10	10			21.29	21.02	-1.27
NA20	20	21.29	18.76	20.83	20.76	-0.34
NA30	30			20.08	20.50	2.09
OA10	10			16.10	16.43	2.05
OA20	20	16.19	18.76	16.47	16.67	1.21
OA30	30			16.87	16.92	0.30
MB10	10			22.66	22.37	-1.28
MB20	20	24.28	10.19	21.72	20.61	-5.11
MB30	30			21.04	18.98	-9.79
NB10	10			20.25	19.84	-2.02
NB20	20	21.29	10.19	19.92	18.49	-7.18
NB30	30			18.66	17.22	-7.72
OB10	10			16.19	15.47	-4.45
OB20	20	16.19	10.19	15.31	14.78	-3.46
OB30	30			12.75	14.12	10.75

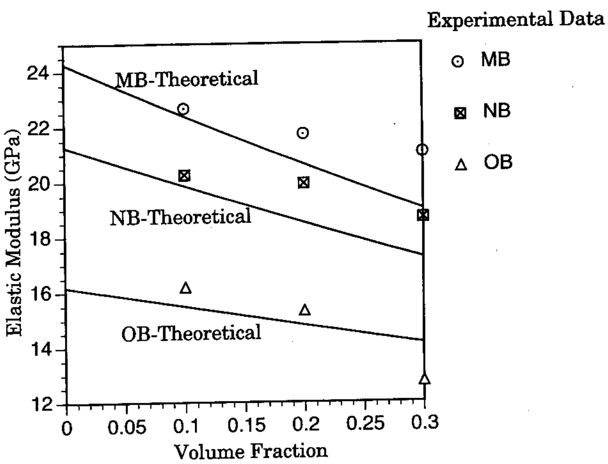

Fig. 3. Volume fraction vs. elastic modulus curves for concrete with B aggregate (W/C=0.6) and various mortar.

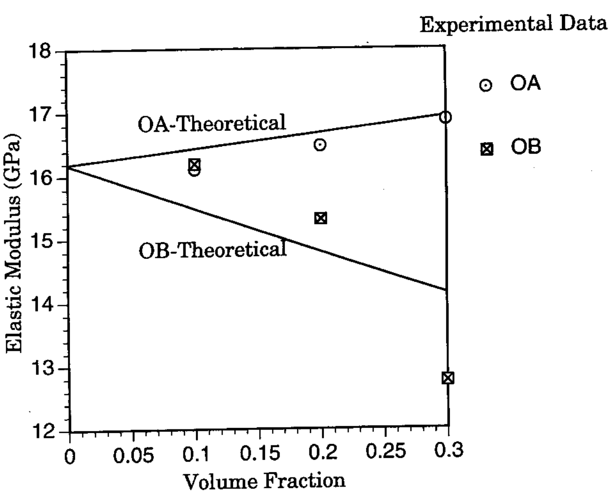

Fig. 4. The elastic modulus vs. volume fraction curves for the concrete with vairous aggregate.

modulus of concrete decreases as volume fraction of aggregate increased except for OA-concrete. Because the elastic modulus of aggregate is higher than mortar, the elastic modulus of OA-concrete increases as volume fraction increases. In Figures 2 and 3 also show that the elastic modulus of concrete increases as long as the elastic modulus of moratr increases. Fig. 4 are the elastic modulus vs. volume fraction curves for the concrete with various aggregate. It shows that the aggregate properties have significant effect on the elastic modulus of concrete.

CONCLUSIONS

Based on the theoretical prediction and experimental results carried out using different mortar and aggregate, the following conclusions can be drawn:

1. The theoretical model for approximating elastic modulus of concrete can be modeled for the concrete with low water-cement ratio.
2. The elastic modulus of concrete decreases with increasing volume fraction, when the elastic modulus of mortar is higher than aggregate.
3. The elastic modulus of aggregate and mortar significantly affected the elastic modulus of concrete.
4. For normal-strength concrete (high water-cement ratio), the theoretical model need to consider the phase of porosity.

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水泥砂漿及人造骨材性質對混凝土彈性模數之影響

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摘 要

本研究中製作兩種人造骨材及使用三種不同的水泥砂漿配合不同的骨材體積比(10%,20%,30%)拌合18組的混凝土材料，探討水泥砂漿及骨材對混凝土彈性模數之影響。理論解的探討是將混凝土視為兩相之複合材料，利用微觀力學理論，推導混凝土的彈性模數。實驗及理論解中均顯示出，混凝土的 彈性模數受水泥砂漿及骨材之影響甚巨。本研究中並將實驗及理論加以印証比較，兩者之結果甚佳。