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Manipulation of Transfer Function Matrix via State-Space Approach

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MANIPULATION OF TRANSFER FUNCTION MATRIX VIA STATE-SPACE APPROACH

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Keywords: Linear systems, Multivariable systems, Dynamic controllers.

ABSTRACT

Algebraic manipulations of transfer function matrix via state-space approach will be given without referring to the symbolic manipulations involving the indeterminate s, for a linear continuous-time system (or, z, for a linear discrete-time system). The resultant transfer function is closely related to a state-space representation, and therefore is advantageous in realization, implementation, computer simulation and stability analysis. Furthermore, the design, implementation, and analysis of linear dynamic controllers, in the forms of 2-input-1-output and 2D (two degree of freedoms), are illustrated with numerical examples via the method proposed.

INTRODUCTION

For a linear dynamic system {A, B, C}, if a linear state variable feedback (*LSVF*) law u = Fv - Kx, here F and K are constant matrices, is imposed on it, the static-gain feedback system becomes {A - BK, BF, C}. The original system is said to be *stabilizable* if there exists a static gain K such that (A - BK) can be stable. And this is always possible if there is no unstable hidden modes [6] in the system {A, B, C}. For this purpose, the design is addressed as stabilization using LSVF. If a design is required that all the eigenvalues of (A - BK) be assigned to some predetermined locations in the s-plane, the problem is *pole-placement* using LSVF (*eigenvalue assignment problem*). As is well-known to the control society, pole-placement design is possible if, and only if, the pair {A, B} is completely controllable[2,6]. Other purposes of design using LSVF can be found in decoupling[2,5] , LQR [7], etc.

In linear system design, LSVF is a basic discipline and is very important in theoretic discussions provided that all the state variables can be accessible (measurable) for feedback usage. It is not, however, always the case in practically. As is also known[2,6], if the pair {C, A} is completely observable, one can build an asymptotic state estimator (*observer*) in conjunction with the LSVF law to form an *observer-based- controller* (*OBC*) using only output feedback such that the I/O map for the feedback system is the same as that of static feedback system. This is what *separation property* [2] applies.

In this paper we consider dynamic controllers that can achieve the design purposes as provided as by static gain feedback. Basically, theoretical foundation of a dynamic controller is based on OBC, however we extend this concept to a more generalized case of 2D (two-degree of freedom) dynamic controller (*2DDC*). All the algebraic manipulations of TFM will be presented via state-space approach without referring to the symbolic processing involving the indeterminate s, and the resultant TFM is closely related to a state-space representation. This is advantageous in realization, implementation, computer simulation and stability analysis. A 2DDC takes only output as feedback, whereas LSVF static-gain controller takes full state variables. Let x, u, and y be the state, input, and output variables respectively for a linear controlled plant, then a static LSVF law takes the form u = Fv- Kx, (here v the new control variable); whereas a 2DDC is described by u = $G_c(s)$v - Q(s)y, (here, $G_c(s)$ and Q(s) are proper rational functions with real coefficients). Closed-loop pole-assignment is achieved by Q(s), while the choice of $G_c(s)$ assigns the added zeros. In the proposed 2DDC, it is shown that the added stable zeros are to be canceled out by the closed-loop poles, then overall I/O map will be that of using state feedback. The design steps for $G_c(s)$ and Q(s) are independently, that is why 2DDC being addressed.

PRELIMINARIES

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Consider a linear time-invariant (LTI) multivariable system $\{A, B, C, D\}$:

$$\dot{x} = Ax + Bu\ , \tag{1a}$$
$$y = Cx+Du. \tag{1b}$$

The transfer function matrix (TFM) from u to y is denoted by,

$$\mathbf{H}_{yu}(s) = C(sI-A)^{-1}B + D := \begin{bmatrix} A & B \\ C & D \end{bmatrix} . \tag{2}$$

here, the state vector $x \in \mathbb{R}^n$, control vector $u \in \mathbb{R}^m$, and output vector $y \in \mathbb{R}^q$. Constant matrices A, B, C, D are of compatible dimensions. The TFM $\mathbf{H}_{yu}(s) \in \mathbb{R}^{q\times m}(s)$, where the quadruple $\{A, B, C, D\}$, satisfying (2), is a realization of $\mathbf{H}_{yu}(s)$. Note that, without loss of generality, we have $D = 0$ for a dynamic system, and call it the triple $\{A, B, C\}$. The followings are some important mathematical preliminaries needed for the discussions of this paper.

A. Some Operational Definitions [4]

(A1) Let a matrix X be partitioned for some compatible dimensions as follows:

$$\begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}$$

then,

$$\begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} X_{11}A + X_{12}C & X_{11}B + X_{12}D \\ X_{21}A + X_{22}C & X_{21}B + X_{22}D \end{bmatrix} . \tag{3}$$

(A2) Let an LTI system $\{A_1, B_1, C_1, D_1\}$ be cascaded by another LTI system $\{A_2, B_2, C_2, D_2\}$, and

$$G_1 = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}, \quad G_2 = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} ;$$

then the overall TFM for the composite system is,

$$G_1G_2 = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

$$= \begin{bmatrix} A_1 & B_1C_2 & B_1D_2 \\ 0 & A_2 & B_2 \\ C_1 & D_1C_2 & D_1D_2 \end{bmatrix} \tag{4a}$$

$$= \begin{bmatrix} A_2 & 0 & B_2 \\ B_1C_2 & A_1 & B_1D_2 \\ D_1C_2 & C_1 & D_1D_2 \end{bmatrix} . \tag{4b}$$

Note that the resultant TFM (4a) or (4b) may or may not be irreducible, even if both G_1 and G_2 are minimal. This is because there may be pole-zero cancellations between these two systems. We also note that the quadruple $\{A, B, C, D\}$ is a minimal realization for the TFM H_{yu} if, and only if, $\{A, B\}$ is completely controllable and $\{C, A\}$ is completely observable[2], and A has the minimum dimension.

(A3) Suppose $\mathbf{D}^+$ is the right (left) inverse of $\mathbf{D}$, then the right (left) inverse of $\mathbf{H}_{yu}$ is,

$$H^+ = \begin{bmatrix} A - BD^+C & BD^+ \\ -D^+C & D^+ \end{bmatrix} . \tag{5a}$$

Therefore, for a sqare TFM $G = \begin{bmatrix} A & B \\ C & I \end{bmatrix}$, then

$$G^{-1} = \begin{bmatrix} A - BC & B \\ -C & I \end{bmatrix} . \tag{5b}$$

(A4) For the LTI system (1), if the following change of variable is made:

$$x \to \overline{x} = Tx$$
$$y \to \overline{y} = Ry$$
$$u \to \overline{u} = Pu$$

Then the resultant TFM, from $\overline{u}$ to $\overline{y}$, becomes

$$H_{\overline{y}\,\overline{u}} = \begin{bmatrix} \overline{A} & \overline{B} \\ \overline{C} & \overline{D} \end{bmatrix}$$

$$= \begin{bmatrix} T & 0 \\ 0 & R \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} T^{-1} & 0 \\ 0 & P \end{bmatrix}$$

$$= \begin{bmatrix} TAT^{-1} & TBP \\ RCT^{-1} & RDP \end{bmatrix} . \tag{6}$$

(A5) If a linear state variable feedback (LSVF) law

$$u = F v + K x \tag{7}$$

is imposed on the system (1), then the resultant feedback system becomes,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} I & 0 \\ K & F \end{bmatrix} = \begin{bmatrix} A + BK & BF \\ C + DK & DF \end{bmatrix} . \tag{8}$$

B. Irreducibility

A TFM $G(s) = C(s\mathbf{I} - A)^{-1}B$ is irreducible (minimal) if, and only if, the dimension of matrix A in the realization $\{A,B,C\}$ is minimal[2]. Consider a dynamic system:

$$\left\{\begin{bmatrix} A_1 & A_{12} \\ 0 & A_2 \end{bmatrix}, \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \begin{bmatrix} C_1 & C_2 \end{bmatrix}\right\} ,$$

the controllable and observable subsystem is $\{A_1, B_1, C_1\}$, therefore

$$\begin{bmatrix} A_1 & A_{12} & B_1 \\ 0 & A_2 & 0 \\ C_1 & C_2 & 0 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & 0 \end{bmatrix} . \tag{9}$$

The right-hand side of the above equation denotes the minimal representation of the TFM. Next, consider the following dynamic system:

$$\left\{ \begin{bmatrix} A_1 & 0 \\ A_{21} & A_2 \end{bmatrix}, \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, [C_1 \ 0] \right\} ,$$

the controllable and observable subsystem is $\{A_1, B_1, C_1\}$, so

$$\begin{bmatrix} A_1 & 0 & B_1 \\ A_{21} & A_2 & B_2 \\ C_1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & 0 \end{bmatrix} . \tag{10}$$

Finally, based on the facts discussed above:

$$\begin{bmatrix} A_1 & 0 & A_{13} & B_1 \\ A_{21} & A_2 & A_{23} & B_2 \\ 0 & 0 & A_3 & 0 \\ C_1 & 0 & C_3 & 0 \end{bmatrix} \tag{11a}$$

$$= \begin{bmatrix} A_1 & 0 & B_1 \\ A_{21} & A_2 & B_2 \\ C_1 & 0 & 0 \end{bmatrix} \tag{11b}$$

$$= \begin{bmatrix} A_1 & A_{13} & B_1 \\ 0 & A_3 & 0 \\ C_1 & C_3 & 0 \end{bmatrix} \tag{11c}$$

$$= \begin{bmatrix} A_1 & B_1 \\ C_1 & 0 \end{bmatrix} . \tag{11d}$$

Note that the system (11a) is neither completely controllable, nor observable, therefore not minimal. (11b) represents an incompletely observable equivalence, and (11c) incompletely controllable. Finally, (11d) is the minimal representation of (11a), i.e., the system $\{A_1, B_1, C_1\}$ is completely controllable and observable, therefore a minimal realization. The TFM is irreducible only if its realization is minimal.

2D DYNAMIC CONTROLLERS

In this section we consider a minimal LTI dynamic system $\{A, B, C\}$, i.e.,

$$\dot{x} = Ax + Bu \quad , \tag{12a}$$

$$y = Cx, \tag{12b}$$

with the following TFM:

$$P(s) = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = C(sI - A)^{-1} B \ . \tag{13}$$

Assume that all state variables can be used for feedback and let an LSVF law be imposed for some control design purpose, such as pole-placement, stabilization, decoupling, LQR optimal control, etc.,

$$u = v - Kx, \tag{14}$$

then the feedback system becomes $\{A\text{-}BK, B, C\}$, i.e., the TFM from v to y is

$$\begin{bmatrix} A - BK & B \\ C & 0 \end{bmatrix} . \tag{15}$$

A. Observer-Based Controller (OBC)

In practical applications, however, only the measurable output variable y(t) can be used for feedback, then the following full-order observer is made:

$$\dot{\hat{x}} = (A - LC)\hat{x} + Bu + Ly \tag{16}$$

here, L is chosen such that (A-LC) can be stable. This is always possible for {C, A} being detectable and $\hat{x} \in \mathbb{R}^n$ is used to estimate the state vector x(t) asymptotically, ie., $\lim_{t\to\infty} \hat{x}(t) = x(t)$. If it is required that eigenvalues of (A-LC) be completely specified, then the existence of L is guaranteed by the fact of {C,A} being completely observable[6]. By separational principle[2] the feedback control law (14) is reiterated by the following:

$$u = v - K\hat{x} \ . \tag{17}$$

The observer dynamics (16) along with the feedback control law (17) constitute a so called OBC, and it has been shown that [2] the TFM form v to y is also that of (15). In Fig. 1, the dotted block shows this OBC configuration.

By simple TFM manipulations, as presented in the last section of this paper, this OBC is a so called *2-input, 1-output dynamic controller*, as shown as in

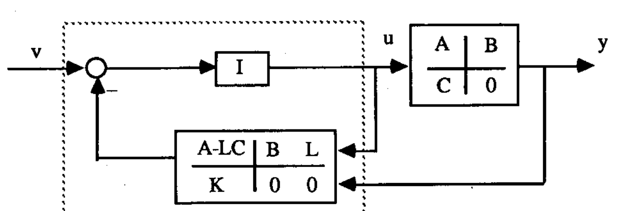

Fig. 1. OBC configuration.

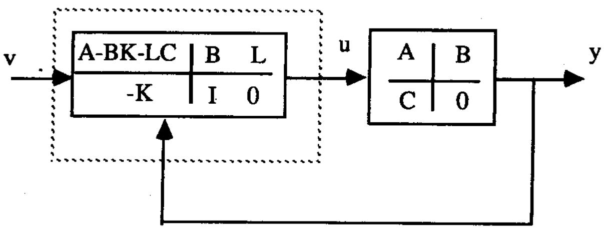

Fig. 2. 2-input, 1-output dynamic controller.

Fig. 2. This is a good way for realization and implementation of the controller, and also for computer simulations.

Proposition 1 The 2-input, 1-output OBC which achieves the task as the feedback control specified by (14) is

$$Q_c(s)=\left[\begin{array}{c|cc} A-BK-LC & B & L \\ \hline -K & I & 0 \end{array}\right]. \quad (18)$$

proof: From (16) and (17), we have

$$u = v - K(sI-A+LC)^{-1}Bu - K(sI-A+LC)^{-1}Ly;$$

$$\therefore\ u=\left[\begin{array}{c|c} A-LC & B \\ \hline K & I \end{array}\right]^{-1}\left(v-\left[\begin{array}{c|c} A-LC & L \\ \hline K & 0 \end{array}\right]y\right) \quad (19)$$

$$=\left[\begin{array}{c|c} A-BK-LC & B \\ \hline -K & I \end{array}\right]v$$

$$+\left[\begin{array}{c|c} A-BK-LC & L \\ \hline -K & 0 \end{array}\right]y, \text{ by (5)} \quad (20)$$

$$=\left[\begin{array}{c|cc} A-BK-LC & B & L \\ \hline -K & I & 0 \end{array}\right]\begin{bmatrix} v \\ y \end{bmatrix};$$

$$=Q_c(s)\begin{bmatrix} v \\ y \end{bmatrix}, \text{ qed.}$$

Example 1 Consider an LTI system,

$$P(s)=\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 0 & -2 \\ \hline 1 & 0 & 0 \end{array}\right]=\frac{2(s-1)}{s^2-s-2}.$$

One finds $K = [4\ \ 2]$ such that $\lambda_i\{A-BK\} = \{-1, -2\}$; and gets $L = [8\ \ 14]^T$ such that $\lambda_i\{A-LC\} = \{-3,-4\}$. Therefore, the 2-input, 1-output OBC is,

$$Q_c(s)=\left[\begin{array}{cc|cc} -15 & -3 & 2 & 8 \\ -4 & 4 & -2 & 14 \\ \hline -4 & -2 & 1 & 0 \end{array}\right].$$

Corollary 1 The TFM for the closed-loop system is $\left[\begin{array}{c|c} A-BK & B \\ \hline C & 0 \end{array}\right]$.

proof: Let the state variable vector of OBC be z(t), from Fig. 2, one gets the following state-space description for the composite systems:

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix}=\begin{bmatrix} A & -BK \\ LC & A-BK-LC \end{bmatrix}\begin{bmatrix} x \\ z \end{bmatrix}+\begin{bmatrix} B \\ B \end{bmatrix}v,$$

$$y=[C\ \ 0]\begin{bmatrix} x \\ z \end{bmatrix}$$

therefore,

$$H_{yv}=\left[\begin{array}{cc|c} A & -BK & B \\ LC & A-BK-LC & B \\ \hline C & 0 & 0 \end{array}\right]$$

$$=\left[\begin{array}{cc|c} A & -BK & B \\ 0 & A-LC & 0 \\ \hline C & 0 & 0 \end{array}\right]$$

$$=\left[\begin{array}{c|c} A-BK & B \\ \hline C & 0 \end{array}\right], \text{ qed.}$$

Remarks: From the above discussion one knows that the subsystem $\dot{e}=(A-LC)e$ is neither controllable nor observable, and the canceled poles are those of the eigenvalues of (A-LC), which are stable.

B. 2DDC(2D Dynamic Controller)

We consider the dynamic controller as shown in Fig. 3, here $G_c(s)$ and H(s) are two dynamic subsystems, addressed as the series and feedback compensator respectively, and there is no unstable pole-zero cancellations between these two sybsystems. This overcomes the design difficulties as in [3]. Note that only the output y(t) is used for feedback.

The OBC configuration in Fig. 1 can easily be converted into the form of 2DDC as shown in Fig. 3. This is presented as follows,

Proposition 2 Let

$$G_c(s)=\left[\begin{array}{c|c} A-LC-BK & B \\ \hline -K & I \end{array}\right] \quad (21)$$

and,

$$H(s)=\left[\begin{array}{c|c} A-LC & L \\ \hline K & 0 \end{array}\right] \quad (22)$$

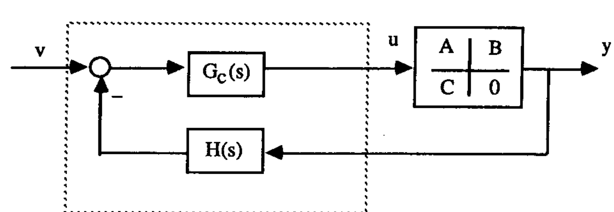

Fig. 3. 2DDC configuration.

in Fig. 3, then

1). $Q(s) := G_c(s)H(s) = \begin{bmatrix} A-LC-BK & L \\ K & 0 \end{bmatrix}$. (23)

2). The control law is,

$$u = G_c(s)[\ v - H(s)\ y\ ]. \tag{24}$$

3). The TFM from v to y is that of (15).

proof:

1). $Q(s) = G_c(s)H(s)$

$$= \begin{bmatrix} A-BK-LC & -BK & 0 \\ 0 & A-LC & L \\ K & K & 0 \end{bmatrix}, \text{ by (4a)}$$

$$= \begin{bmatrix} A-BK-LC & -BK & 0 \\ 0 & A-BK-LC & L \\ 0 & K & 0 \end{bmatrix}, \text{ by (5)}$$

$$= \begin{bmatrix} A-LC-BK & L \\ K & 0 \end{bmatrix}, \text{ by (10), } \textbf{qed}.$$

2). This follows directly from (19).

3). Firstly,

$$PG_c = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} A-BK-LC & B \\ -K & I \end{bmatrix}$$

$$= \begin{bmatrix} A-BK & BK & B \\ LC & A-LC & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Secondly,

$$(I+PQ)^{-1} = \begin{bmatrix} A & BK & 0 \\ 0 & A-LC-BK & L \\ C & 0 & I \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} A & BK & 0 \\ -LC & A-LC-BK & -L \\ C & 0 & I \end{bmatrix}$$

$$= \begin{bmatrix} A-BK & BK & 0 \\ 0 & A-LC & -L \\ C & 0 & I \end{bmatrix}.$$

Therefore,

$H_{yv}(s) = [I + PQ]^{-1}PG_c$

$$= \begin{bmatrix} A-BK & BK & 0 & 0 & 0 \\ 0 & A-LC & -LC & 0 & 0 \\ 0 & 0 & A-BK & BK & B \\ 0 & 0 & LC & A-LC & 0 \\ C & 0 & C & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} A-BK & 0 & BK & 0 & 0 \\ 0 & A-BK & 0 & BK & B \\ 0 & -LC & A-LC & 0 & 0 \\ 0 & LC & 0 & A-LC & 0 \\ C & C & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} A-BK & 0 & 0 \\ 0 & A-BK & B \\ C & C & 0 \end{bmatrix}$$

$$= \begin{bmatrix} A-BK & B \\ C & 0 \end{bmatrix}, \textbf{ qed.}$$

Corollary 2 The closed-loop poles are specified by the following set of eigenvalues:

$$\{\lambda_i(A\text{-}BK)\} \cup \{\lambda_i(A\text{-}LC)\}.$$

proof: This follows directly from the proof of $(I+PQ)^{-1}$, as in (25).

Definition 1 [1]

1. $\{\lambda_i(A\text{-}BK)\}$ are the set of *regular poles,*
2. $\{\lambda_i(A\text{-}LC)\}$ are the set of *observer poles.*

Definition 2 Q(s) is the *pole-placement dynamic controller* that places closed-loop poles to the regular poles and to the observer poles.

Note that the subsystem $\{(A\text{-}LC), \text{-}BK, 0\}$ is not observable, and the canceled poles are in $\lambda_i(A\text{-}LC)$, therefore the stable pole-zero cancellation is permissible because L has been chosen such that (A-LC) is stable. We also see that in Fig. 3, H(s) is stable and can be chosen strictly proper in order to reject disturbance and feedback noise from the output.

Example 2 As the system shown in example 1, we have

$$G_c(S) = \begin{bmatrix} A-LC-BK & B \\ -K & I \end{bmatrix} = \begin{bmatrix} -15 & -3 & 2 \\ -4 & 4 & -2 \\ -4 & -2 & 1 \end{bmatrix}$$

$$= \frac{s^2+7s+12}{s^2+11s-72},$$

$$H(s) = \begin{bmatrix} A-LC & L \\ K & 0 \end{bmatrix} = \begin{bmatrix} -7 & 1 & 8 \\ -12 & 0 & 14 \\ 4 & 2 & 0 \end{bmatrix}$$

$$= \frac{60s+60}{s^2+7s+12},$$

$$Q(S) = \begin{bmatrix} A-LC-BK & L \\ K & 0 \end{bmatrix} = \begin{bmatrix} -15 & -3 & 8 \\ -4 & 4 & 14 \\ 4 & 2 & 0 \end{bmatrix}$$

$$= \frac{60s+60}{s^2+11s-72},$$

therefore,

$$H_{yv}(s) = \frac{2(s-1)}{s^2+3s+2}.$$

CONCLUDING REMARKS

1. Algebraic manipulations of transfer function ma-

trix via state-space approach has been given without referring to the symbolic manipulations involving the indeterminate s, for a linear continuous-time system (or, z, for a linear discrete-time system). The resultant transfer function is closely related to a state-space representation, and therefore is advantageous in realization, implementation, computer simulation and stability analysis.
2. The proposed method is good for the TFM manipulation to do with the design and implementation of linear dynamic controller.
3. OBC has been extensively studied, and mathematical equivalence of 2-input, 1-output controller and 2DDC have been discussed. Stable pole-zero cancellation is an inherent property in the 2DDC design.
4. By the dynamic controller $u = G_c(s)v - Q(s)y$, $Q(s)$ is chosen to do pole-placement, and $G_c(s)$ can be chosen to meet other control purpose. This unique feature is what 2DDC renders.
5. In 2DDC design H(s) can be chosen strictly proper and stable (low-pass) so as to reject disturbance and noise from output. This will be our research topic in the near future.

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轉移函數矩陣的狀態空間式運算

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摘 要

本文介紹多變數線性系統的轉移函數矩陣的狀態空間式運算，在此法中不須涉及符號s（連續時間系統）或z（離散時間系統）的處理。於代數的運算過程中所有的轉移函數矩陣皆伴隨有狀態空間數值矩陣實現式，使得控制系統的實現、合成、計算機模擬、及做穩定度分析時皆甚方便且簡易。其次，利用本文方法，我們亦介紹2輸入1-輸出式及2D（雙自由度）線性動態控制器的設計及合成。