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ROBUST STABILITY BOUNDS FOR LUR'e SYSTEMS WITH PARAMETRIC UNCERTAINTY

Chyun-Chau Fuh* and Pi-Cheng Tung**

Keywords: Robust stability, stability bounds, Lur'e system, parametric uncertainty, Lyapunov approach.

ABSTRACT

The robust stability of Lur'e system with parametric uncertainty is studied through the Popov-Lyapunov approach. Robust stability bounds of such a system are presented. Since we avoid using norm forms and absolute values, the obtained stability bounds are less conservative. An example demonstrating the feasibility of our method is illustrated.

INTRODUCTION

Many significant results on robust control of linear systems with uncertain parameters are recently obtained. This work can be roughly divided into two directions: the frequency domain approach [1-3] and the state-space approach [4-6]. In the former, many researchers proposed various sufficient (and necessary) conditions to ensure stability or to estimate the root distributions of interval polynomials. However, most of these results fail to provide stability bounds (robustness measures) on the uncertain parameters. For the state-space approach, although many results give the range of parameters for stability, they are generally restricted to bounds on the norm forms of an additive uncertainty in the system matrices. Since the norm expression of the uncertain matrices gives typically more conservative results for the estimation of stability, some achievements to decrease conservatism in stability robustness bounds are successively presented. The method proposed by Gao and Antsaklis [6] is one of these less conservative results.

In practice, most physical systems are generally nonlinear and include parametric uncertainty. These nonlinearities and uncertainty may be caused by saturation of actuators, friction forces, backlashes, aging of components, changes in environmental conditions, or calibration errors. Especially, many nonlinear systems can be transformed into the well-known Lur'e systems; that is, they can be decomposed into linear parts and nonlinear parts.

The robust absolute stability of Lur'e systems is of considerable interest to researches. An analytical method for robust absolute stability analysis of Lur'e systems subject to parameter variations was proposed [7], in which the linear plant coefficients are functions of few physical parameters. Two robust stability criteria for Lur'e systems were proposed [8]; in these cases a bounded scalar perturbation function must be given in advance. The classical Popov criterion was generalized to include problems with both parametric uncertainty and bounded perturbations [9]; the main result is that a robust Popov criterion for an interval Lur'e system is given, and the maximum range of the nonlinear sector is also obtained. However this approach cannot estimate stability bounds of uncertain parameters to guarantee the stability of the system; i.e. the stability can only be judged if the bounds of the uncertain parameters are given. Robust Popov theorems were presented in [10-12], but in many cases the uncertain matrices cannot be transformed to feedback forms (i.e. $\Delta A = -BFC$, $0 \leq F \leq K$, see [11]); thus the feasibility of this approach is diminished. Conditions for robust stability of a Lur'e system with multiple nonlinearities was proposed [13]; stability bounds were also given. The defects of this manner is that the calculation is tedious.

In this paper, we consider a control system of Lur'e type in which not only the linear part include parametric uncertainty but also the nonlinear sector bound is unknown. We use Lyapunov's direct method to guarantee the global stability of a Lur'e system with parametric uncertainty, and then a robustness measure that gives a bound on allowable uncertainty for the system is derived, evading expressions for the norm or the absolute value. A feature of the measure is that it is possible to decrease further the conservatism of the

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stability bounds in a class of problems for which knowledge of signs and ranges of uncertain parameters is available. As the plants are specified in the forms of state variables, all system parameters have explicit physical meaning. Therefore, the derived stability bounds can directly help us to select appropriate performance (precision or reliability) for system elements. An example demonstrates the feasibility of our method.

SYSTEM DESCRIPTION

In practical applications, many control systems can be described by a Lur'e system with parametric uncertainty as ,

$$\dot{\boldsymbol{x}} = (\boldsymbol{A} + \Delta\boldsymbol{A})x - (\boldsymbol{b} + \Delta\boldsymbol{b})\phi_0(\sigma), \tag{1a}$$

$$\sigma = \boldsymbol{c}^T\boldsymbol{x} \tag{1b}$$

where $\boldsymbol{x}(t) \in \boldsymbol{R}^n$ is the state; $\sigma(t) \in \boldsymbol{R}$ is the output; $\boldsymbol{A} \in \boldsymbol{R}^{n\times n}$, $\boldsymbol{b} = [b_j] \in \boldsymbol{R}^{n\times 1}$ and $\boldsymbol{c} \in \boldsymbol{R}^{n\times 1}$ are the nominal system matrices; $\Delta\boldsymbol{A} \in \boldsymbol{R}^{n\times n}$ and $\Delta\boldsymbol{b} = [\beta_j] \in \boldsymbol{R}^{n\times 1}$ denote uncertain matrices; and $\phi_0 \in \boldsymbol{R}$ is a sector nonlinearity and belongs to the sector $[0, K_0]$, $K_0 > 0$. Assume that $\Delta\boldsymbol{A}$ and $\Delta\boldsymbol{b}$ take forms

$$\Delta\boldsymbol{A} = \sum_{i=1}^{m} k_i\boldsymbol{E}_i, \tag{2}$$

$$\Delta\boldsymbol{b} = \{[\beta_j]: \beta_j^- \le \beta_j \le \beta_j^+, \text{ if } b_j = 0, \\ \text{then } \beta_j = 0,\ j = 1...n\} \tag{3}$$

in which $\boldsymbol{E}_i$ are given real constant matrices, and k_i are real uncertain parameters. We assume $(b_j + \beta_j^-)(b_j + \beta_j^+) > 0$; this assumption signifies that even if b_j are uncertain, their signs do not be changed. Without loss of generality, we represent system (1) as the following form

$$\dot{\boldsymbol{x}} = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x} - \boldsymbol{b}\phi(\sigma), \tag{4a}$$

$$\sigma = \boldsymbol{c}^T\boldsymbol{x} \tag{4b}$$

where the nonlinearity $\phi(\sigma)$ belongs to the sector $[0, K]$, and K satisfies the following condition

$$K \ge K_0 \max\left\{\max_{\substack{j \\ b_j \ne 0}}\left(\frac{b_j + \beta_j^-}{b_j}\right), \max_{\substack{j \\ b_j \ne 0}}\left(\frac{b_j + \beta_j^+}{b_j}\right)\right\}. \tag{5}$$

In the next section, we consider the stability of Lur'e system with parametric uncertainty and find some less conservative allowable bounds such that the system remains stable.

ANALYSIS OF STABILITY

The following lemmas are useful for stability analysis of Lur'e systems with parametric uncertainty.

Lemma 1 If $\boldsymbol{U} \in \boldsymbol{R}^{n\times n}$ and $\boldsymbol{x} \in \boldsymbol{R}^n$, then

$$\boldsymbol{x}^T\boldsymbol{U}\boldsymbol{x} = \boldsymbol{x}^T\left(\frac{\boldsymbol{U}^T + \boldsymbol{U}}{2}\right)\boldsymbol{x} = \boldsymbol{x}^T\boldsymbol{H}\boldsymbol{x} \tag{6}$$

where $\boldsymbol{H}$ is a symmetric matrix.

Proof: The proof is straightforward and is omitted for brevity.

Lemma 2 For any Hermitian matrices $\boldsymbol{H}_i$, and scalars k_i, $i = 1...m$.

$$\lambda\left(\sum_{i=1}^{m} k_i\boldsymbol{H}_i\right) \le \sum_{i=1}^{m}\lambda_{\max}(k_i\boldsymbol{H}_i). \tag{7}$$

Proof: See [6].

Lemma 3 If ε is a strictly positive number, $\boldsymbol{W}_0$ and $\boldsymbol{W}$ are positive-definite symmetric matrices and

$$\varepsilon\boldsymbol{W}_0 + \sum_{i=1}^{m} k_i\boldsymbol{H}_i = \varepsilon\boldsymbol{W} \tag{8}$$

where k_i are real uncertain parameters, and $\boldsymbol{H}_i$ are given real constant Hermitian matrices. Then

$$\sum_{i=1}^{m} k_i\lambda_i < \lambda_{\max}(\varepsilon\boldsymbol{W}) \tag{9}$$

with λ_i defined by

$$\lambda_i = \begin{cases}\lambda_{\max}(\boldsymbol{H}_i) & \text{for } k_i \ge 0 \\ \lambda_{\min}(\boldsymbol{H}_i) & \text{for } k_i < 0\end{cases} \quad i = 1...m. \tag{10}$$

Proof: This lemma is easy to be proved with the aid of Lemma 2.

With the aid of the above lemmas and Lyapunov's direct method, we have the following theorem.

Theorem: If the system described by equation (4) satisfies the assumptions as follows:

(A1) the nonlinearity $\phi(\sigma)$ belongs to the sector $[0, K]$ where K is a positive number;

(A2) let $\boldsymbol{v} = \frac{1}{2}(r\boldsymbol{A}^T\boldsymbol{c} + \boldsymbol{c})$ and $\gamma = r\boldsymbol{c}^T\boldsymbol{b} + \frac{1}{K}$ where r, $K >$ 0are chosen such that $\gamma \ge 0$. Given a symmetric positive-definite matrix $\boldsymbol{W}$, there exists a scalar $\varepsilon > 0$, a vector $\boldsymbol{q}$, and symmetric positive-definite matrices $\boldsymbol{P}$ and $\boldsymbol{W}_0$ satisfying

$$\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} = -\boldsymbol{q}\boldsymbol{q}^T - \varepsilon\boldsymbol{W}, \tag{11}$$

$$\boldsymbol{P}\boldsymbol{b} - \boldsymbol{v} = \sqrt{\gamma}\boldsymbol{q}, \tag{12}$$

$$\varepsilon\boldsymbol{W}_0 + \sum_{i=1}^{m} k_i\boldsymbol{H}_i = \varepsilon\boldsymbol{W} \tag{13}$$

where

$$\boldsymbol{H}_i = \frac{1}{2}(\boldsymbol{U}_i^T + \boldsymbol{U}_i) \tag{14}$$

$$\boldsymbol{U}_i = (2\boldsymbol{P} + rK\boldsymbol{c}\boldsymbol{c}^T)\boldsymbol{E}_i; \tag{15}$$

Then, the point $\boldsymbol{x} = 0$ is globally asymptotically stable.

Proof: Let us consider a Lur'e-Pastnikov Lyapunov function candidate

$$V(\boldsymbol{x}) = \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{x} + r\int_0^\sigma \phi(y)dy, \quad r > 0 \tag{16}$$

Since $\boldsymbol{P}$ is a symmetric positive-definite matrices and the nonlinearity $\phi(\sigma)$ belongs to the sector $[0, K]$, i.e., $0 \le \sigma \cdot \phi(\sigma) \le K\sigma^2$, it implies $V(\boldsymbol{x})$ is positive-definite. From system (4), we derive

$$\begin{aligned}\dot{V}(\boldsymbol{x}) &= \boldsymbol{x}^T(\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A})\boldsymbol{x} - 2\boldsymbol{x}^T(\boldsymbol{P}\boldsymbol{b} - \frac{1}{2}r\boldsymbol{A}^T\boldsymbol{c} - \frac{1}{2}\boldsymbol{c})\phi(\sigma) \\ &\quad - (r\boldsymbol{c}^T\boldsymbol{b} + \frac{1}{K})\phi^2(\sigma) - \frac{1}{K}(K\sigma - \phi(\sigma))\phi(\sigma) \\ &\quad + 2\boldsymbol{x}^T\boldsymbol{P}\Delta\boldsymbol{A}\boldsymbol{x} + r\phi(\sigma)\boldsymbol{c}^T\Delta\boldsymbol{A}\boldsymbol{x}. \end{aligned} \tag{17}$$

where $\frac{1}{K}(K\sigma - \phi(\sigma))\phi(\sigma) \ge 0$. Let

$$\boldsymbol{v} = \frac{1}{2}(r\boldsymbol{A}^T\boldsymbol{c} + \boldsymbol{c}), \tag{18}$$

$$\gamma = r\boldsymbol{c}^T\boldsymbol{b} + \frac{1}{K}. \tag{19}$$

It follows that

$$\begin{aligned}\dot{V}(\boldsymbol{x}) &\le \boldsymbol{x}^T(\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A})\boldsymbol{x} - 2\boldsymbol{x}^T(\boldsymbol{P}\boldsymbol{b} - \boldsymbol{v})\phi(\sigma) - \gamma\phi^2(\sigma) \\ &\quad + \boldsymbol{x}^T(2\boldsymbol{P}\Delta\boldsymbol{A} + rK\boldsymbol{c}\boldsymbol{c}^T\Delta\boldsymbol{A})\boldsymbol{x} \\ &= \boldsymbol{x}^T(\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A})\boldsymbol{x} - 2\boldsymbol{x}^T(\boldsymbol{P}\boldsymbol{b} - \boldsymbol{v})\phi(\sigma) - \gamma\phi^2(\sigma) \\ &\quad + \boldsymbol{x}^T(2\boldsymbol{P} + rK\boldsymbol{c}\boldsymbol{c}^T)\left(\sum_{i=1}^m k_i\boldsymbol{E}_i\right)\boldsymbol{x}. \end{aligned} \tag{20}$$

Let us define

$$\boldsymbol{U}_i = (2\boldsymbol{P} + rK\boldsymbol{c}\boldsymbol{c}^T)\boldsymbol{E}_i \tag{21}$$

$$\boldsymbol{H}_i = \frac{1}{2}(\boldsymbol{U}_i^T + \boldsymbol{U}_i). \tag{22}$$

According to Lemma 1, we have

$$\begin{aligned}\dot{V}(\boldsymbol{x}) &\le \boldsymbol{x}^T(\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A})\boldsymbol{x} - 2\boldsymbol{x}^T(\boldsymbol{P}\boldsymbol{b} - \boldsymbol{v})\phi(\sigma) - \gamma\phi^2(\sigma) \\ &\quad + \boldsymbol{x}^T\left(\sum_{i=1}^m k_i\boldsymbol{U}_i\right)\boldsymbol{x} \\ &= \boldsymbol{x}^T\left(\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} + \sum_{i=1}^m k_i\boldsymbol{H}_i\right)\boldsymbol{x} - 2\boldsymbol{x}^T(\boldsymbol{P}\boldsymbol{b} - \boldsymbol{v})\phi(\sigma) \\ &\quad - \gamma\phi^2(\sigma). \end{aligned} \tag{23}$$

If equations (11) to (13) are satisfied, then

$$\begin{aligned}\dot{V}(\boldsymbol{x}) &\le -\boldsymbol{x}^T\left(\varepsilon\boldsymbol{W} - \sum_{i=1}^m k_i\boldsymbol{H}_i\right)\boldsymbol{x} - (\boldsymbol{x}^T\boldsymbol{q} + \sqrt{\gamma}\phi(\sigma))^2 \\ &\le -\varepsilon\boldsymbol{x}^T\boldsymbol{W}_0\boldsymbol{x} - (\boldsymbol{x}^T\boldsymbol{q} + \sqrt{\gamma}\phi(\sigma))^2 < 0 \end{aligned} \tag{24}$$

So, $\boldsymbol{x} = 0$ is globally asymptotically stable.

For system (4), if the parameters $\boldsymbol{A}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{E}_i$ are known, the procedure of deriving the robust stability bounds of k_i is as follows:

Step 1. Draw the Popov plot of $\boldsymbol{c}^T(j\omega\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}$. If the Lur'e system is absolute stable, then select appropriate positive numbers r and K.

Step 2. According to equations (3) and (5), the stability bounds of β_j and K_0 can be found.

Step 3. Evaluate $\boldsymbol{v}$ and γ from equations (18) and (19), respectively .

Step 4. Select a symmetric positive-definite matrix $\boldsymbol{W}$ and a positive real number ε, then $\boldsymbol{P}$ can be obtained by solving the following algebraic Riccati equation

$$\boldsymbol{A}_r^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_r - \boldsymbol{P}\boldsymbol{R}_r\boldsymbol{P} + \boldsymbol{Q}_r = 0 \tag{25}$$

in which

$$\boldsymbol{A}_r = \boldsymbol{A} - \frac{1}{\gamma}\boldsymbol{b}\boldsymbol{v}^T, \tag{26}$$

$$\boldsymbol{Q}_r = \varepsilon\boldsymbol{W} + \frac{\boldsymbol{v}\boldsymbol{v}^T}{\gamma}, \tag{27}$$

$$\boldsymbol{R}_r = \frac{\boldsymbol{b}\boldsymbol{b}^T}{\gamma}. \tag{28}$$

Of course, solving the above Riccati equation is very easy with the aid of many package software such as MATLAB. If $\boldsymbol{P}$ is nonpositive-definite, then select different ε or $\boldsymbol{W}$ until $\boldsymbol{P}$ has a positive-definite solution. For convenience, we can set $\varepsilon\boldsymbol{W} = \varepsilon\boldsymbol{I}$.

Step 5. Since $\lambda_{\max}(\varepsilon\boldsymbol{W}) = \lambda_{\max}(\varepsilon\boldsymbol{I}) = \varepsilon$ and λ_i can be found by equations (10), (12) and (13). Thus, the allowable bounds of k_i can be estimated from inequality (9).

Since a parameter varies in various directions, it affects the system stability variously [6]. One of the significance of the proposed method is that the conservatism is apparently decreased as a result of avoiding using norms and absolute values. Some remarks concerning the proposed method are as follows.

Remark 1 The stability bound on $\Delta\boldsymbol{A}$ is obviously dependent also on the size of K (i.e. the bound of $\Delta\boldsymbol{b}$ and the size of K_0). In general, if K is large, then large uncertainty in the entries of $\Delta\boldsymbol{A}$ is not allowable.

Remark 2 The proposed method only provides a sufficient condition to guarantee the stability of a Lur'e system with parametric uncertainty. If various values

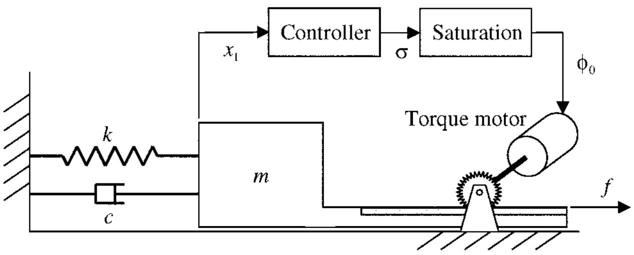

Fig. 1. A schematic description of the mass-damper-spring system.

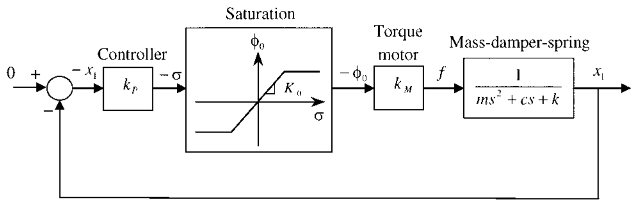

Fig. 2. Equivalent block diagram of the mass-damper-spring system.

of ε are selected in Step 3, then the corresponding stability bounds can be found, respectively. The system which parameters are varied in the union region of all bounds is still stable. Thus the conservatism is decreased further.

AN ILLUSTRATIVE EXAMPLE

In order to show the feasibility of the proposed method, we consider a mass-damper-spring system, in which the mass is driven by a torque motor. The schematic description of the system and the equivalent block diagram are shown in Figures 1 and 2, respectively. Assume that the time constant of the motor is much smaller than the time constant of the mass-damper-spring system such that the relationship between the motor command $-\phi_0$ and the driving force f can be regarded as a constant gain, i.e., $f = -k_M\phi_0$. Additionally, the motor is controlled by a proportional controller with saturation. The dynamic equations of the nonlinear system are as follows:

$$\dot{x} = x_2, \tag{29a}$$

$$\dot{x} = -\frac{k}{m}x_1 - \frac{c}{m}x_2 + \frac{1}{m}f$$
$$= -\frac{k}{m}x_1 - \frac{c}{m}x_2 - \frac{k_M}{m}\phi_0(\sigma), \tag{29b}$$

$$\sigma = k_P x_1. \tag{29c}$$

where ϕ_0 is a nonlinear element that belongs to the sector $[0, K_0]$. For practice applications, the parameters m, c, k, k_M and K_0 are difficult to be estimated exactly or may be variable. In this example, we suppose

$$\frac{k}{m} = 1 - k_1, \tag{30}$$

$$\frac{c}{m} = 3 - k_2, \tag{31}$$

$$\frac{k_M}{m} = 1 + \beta_2, \tag{32}$$

$$k_P = 1. \tag{33}$$

Rewriting equation (29) as the form of equation (1), we have

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ -1 + k_1 & -3 + k_2 \end{bmatrix}\boldsymbol{x} - \begin{bmatrix} 0 \\ 1 + \beta_2 \end{bmatrix}\phi_0(\sigma), \tag{34a}$$

$$\sigma = [1 \;\; 0]\boldsymbol{x}, \tag{34b}$$

According to the previous definition, the matrices $\boldsymbol{A}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\Delta\boldsymbol{A}$, $\Delta\boldsymbol{b}$, and $\boldsymbol{E}_i$ are as follows:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix}, \tag{35}$$

$$\boldsymbol{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{36}$$

$$\boldsymbol{c} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{37}$$

$$\Delta\boldsymbol{A} = k_1\boldsymbol{E}_1 + k_2\boldsymbol{E}_2 = k_1\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + k_2\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \tag{38}$$

$$\Delta\boldsymbol{b} = \begin{bmatrix} 0 \\ \beta_2 \end{bmatrix}. \tag{39}$$

Our purpose is to find the stability bounds of K_0, k_1, k_2, and β_2. According to the Popov plot of the nominal system $\boldsymbol{c}^T(j\omega\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}$ shown in Figure 3, let us consider the case of $r = 1$ and $K = 1$. Since the bounds of $\Delta\boldsymbol{A}$ and K are notably dependent as described in Remark 1. The larger the value of K is, the smaller the allowable range of $\Delta\boldsymbol{A}$. Along the previous mention and equations (3) and (5), K_0 and β_2 have to satisfy the following conditions

$$K_0 > 0, \tag{40}$$

$$1 + \beta_2 > 0, \tag{41}$$

$$K_0(1 + \beta_2) \le 1. \tag{42}$$

The stability bounds of K_0 and β_2 are shown in Figure 4. Next, we let ε be equal to 0.5, 1 and 2, respectively. The corresponding stability bounds are shown in Figure 5, and the stability region is the union of these bounds. The stability region according to our method is appar-

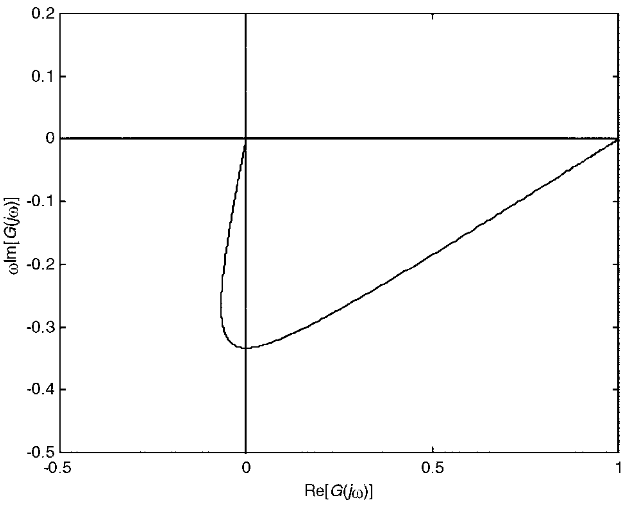

Fig. 3. Popov plot of the nominal system $\boldsymbol{c}^T(j\omega\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}$.

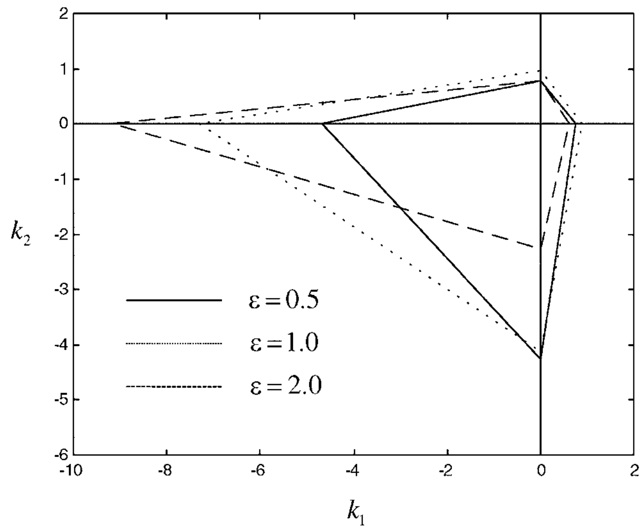

Fig. 5. Stability bounds of k_1 and k_2.

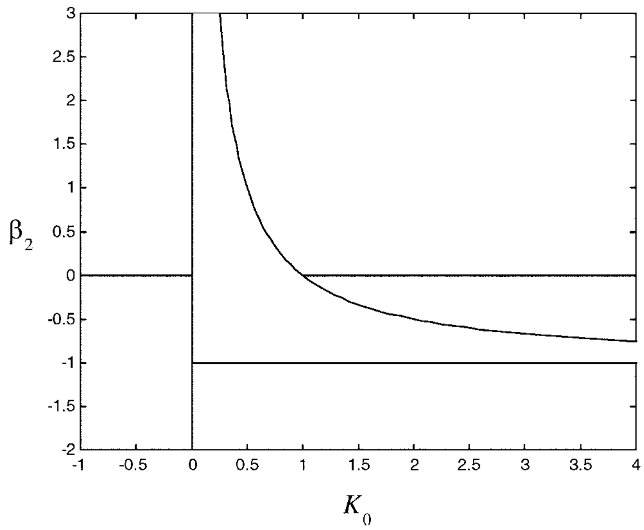

Fig. 4. Stability bounds of K_0 and β_2 when $K = 1$.

ently asymmetric about the origin.

CONCLUSIONS

We assess the stability of Lur'e systems with linear uncertainty and/or unknown nonlinear sectors. Robustness stability bounds for Lur'e systems with parametric uncertainty are presented. These bounds are less conservative and easily obtained due to avoiding norm forms. According to the proposed method, the information how various uncertain parameters affect the stability of system can be described clearly. From the illustrative example, we demonstrate that our method is feasible and useful for practical applications.

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Lur'e系統在參數擾動情況下之強健穩定條件

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摘 要

本文以Popov-Lyapunov法探討具不確定參數之Lur'e系統之強健穩定性，並提出在穩定情況下，參數可允許的變動範圍。因沒有使用norm及絕對值的運算，因此所得結果較不保守。最後，本文以一實際機械系統為實例，除詳細說明演算步驟外，亦同時驗證所提方法的可行性及實用性。