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NEAR-FIELD SOURCE LOCALIZATION USING MUSIC WITH POLYNOMIAL ROOTING

Hsien-Sen Hung,* Shun-Hsyung Chang** and Chien-Hsing Wu***

Keywords: Near field, Polynomial rooting, MUSIC, Location estimation

ABSTRACT

In this paper, the *bearing, elevation*, and *range* estimation of near-field narrow-band sources from data observed across an array of sensors is considered. For high-resolution source localization, the multiple signal classification (MUSIC) algorithm is modified and extended to its 3-D version by accounting for spherical curvature and spreading factor in the array manifold. However, the simultaneous estimation of bearing, elevation, and range from 3-D MUSIC spectrum requires exhaustive multidimensional search. As to alleviate computational load, an alternative algorithm is proposed. The proposed algorithm involves search in the range domain combined with polynomial rooting, which replaces the search in the azimuth-elevation domain, for bearing and elevation estimation. Simulation results are provided to show the efficacy of the proposed algorithm.

INTRODUCTION

Localization of radiating sources by passive sensor array is an important problem in a variety of applications, such as radar, sonar, seismology, and radio astronomy. Various algorithms have been proposed for bearing estimation of multiple sources which are assumed to be located in the far field so that the propagating waves emanating from them are essentially planar when they reach the array. The plane wave approximation to the actual spherical wavefront generated by a point source is appropriate when the distance is sufficiently large and the aperture is sufficiently small. In essence, the wavefront curvature is spherical in the near-field region, quadratic in Fresnel region, and planar in far-field (Fraunhofer) region. The near field is usually characterized by a distance, R, also known as the near-field to far-field transition distance, given by $R = 2D^2/\lambda$, where D is the array aperture measured in the unit of wavelength λ [1]. For large aperture arrays as used in sonar systems where D is on the order of several tens or more, the distance R is sufficiently large (on the order of several hundreds or more) so that sources are often located in the near field.

For narrow-band sources in the near-field region, Huang and Barkat [2] proposed a two-dimensional (2-D) version of MUSIC algorithm in which an exhaustive search is required in bearing-range domain. Weiss and Friedlander [3] examined an efficient algorithm which involves search in the range direction combined with polynomial rooting, which replaces the search in the bearing direction. As a consequence, computational load can be significantly reduced. For narrow-band sources in Fresnel region, Starer and Nehorai [4] developed an algorithm based on path-following (or homotoby), which is limited to uniform linear arrays. All the algorithms mentioned above were developed under the assumption that all the sources and sensors are on the same plane so that only bearing and range are estimated.

In this paper, we examine a source localization problem, of which the data model is more general and accurate than the ones considered in [2-4], for simultaneous estimation of bearing, elevation, and range of narrow-band sources located in the near-field region. The MUSIC algorithm [5] is modified to its three-dimensional (3-D) version to estimate the near-field range, bearing and elevation of sources. A computationally efficient algorithm, extending the work of [3], is proposed, which involves search in the range domain combined with two-stage polynomial rooting for bearing and elevation estimation. The proposed algorithm requires a smaller amount of computation than algorithms based on three-dimensional search. The performance of the proposed algorithm is evaluated by Monte-Carlo simulation.

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PROBLEM FORMULATION

In this section, we formulate the problem for the near-field localization of narrow-band sources. By near field, we mean that the wavefront of the propagating wave emitted by a source is spherical as opposed to plane wave for far-field source.

A passive 3-D sensor-array geometry with sources in near field, measured in spherical coordinate system, is shown in figure 1. Suppose that M sources are located at unknown locations (r_m, θ_m, ϕ_m), $m = 1, ..., M$, where r_m, θ_m, and ϕ_m represent the range, bearing (azimuth) and elevation angles for the mth source, respectively. The wavefronts of the propagating waves emitted by the sources are assumed to be *spherical*. The L $(L > M)$ sensors are deployed at sites of $(\tilde{r}_l, \tilde{\theta}_l, \tilde{\phi}_l)$, $l = 1, ..., L$, arbitrary but known to the processor. Assuming the medium for propagating waves is homogeneous and nondispersive, the direction and shape of wavefronts arriving at each sensor are unaltered in the near-field region. In the presence of noise, the observed data at the ith sensor can be expressed as

$$x_i(t) = \sum_{m=1}^{M} \frac{1}{z_{im}} s_m(t - \tau_{im}) + n_i(t),\ i = 1, 2, ..., L. \quad (1)$$

Herein, $s_m(t)$, $m = 1, ..., M$ are the radiated signals and $n_i(t)$, $i = 1, ..., L$ are some noise process. z_{im} denotes the distance from the mth source to the ith sensor, defined as

$$z_{im} = \{\tilde{r}_i^2 + r_m^2 - 2\tilde{r}_i r_m[\cos\tilde{\phi}_i \cos\phi_m + \sin\tilde{\phi}_i \sin\phi_m \cos(\tilde{\theta}_i - \theta_m)]\}^{1/2}, \quad (2)$$

and its reciprocal represents the spherical spreading factor. This factor describes the fact that the amplitude of a spherical wave decays as the wave propagates away from the source, and is inversely proportional to the propagating distance. The parameter τ_{im} is the delay associated with the signal propagation time from the mth source travelling to the ith sensor, and equals z_{im}/c, where c is the wave's propagation speed.

We assume that the signals are narrow-band and they all have a common frequency f_o. Therefore, the effect of a time delay on the received waveforms is simply a phase shift. Thus Eq. (1) becomes

$$x_i(t) = \sum_{m=1}^{M} \frac{1}{z_{im}} e^{-j2\pi f_o \tau_{im}} s_m(t) + n_i(t). \quad (3)$$

Define

$$\vec{x}(t) = [x_1(t), ..., x_L(t)]^T,$$

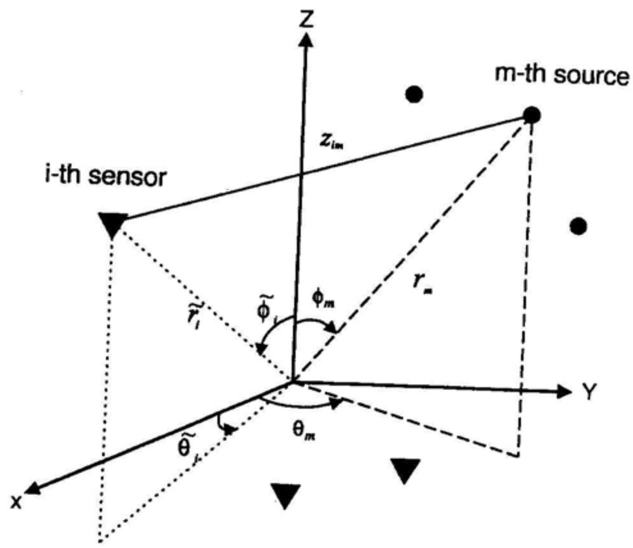


Fig. 1. A passive three-dimensional array geometry with sources in the near field, measured in spherical coordinate system.

$$\vec{s}(t) = [s_1(t), ..., s_M(t)]^T,$$

$$\vec{n}(t) = [n_1(t), ..., n_L(t)]^T,$$

where the superscript T denotes the transpose. Equation (3) can be expressed in vector-matrix form as

$$\vec{x}(t) = \vec{A}\vec{s}(t) + \vec{n}(t), \quad (4)$$

where $\vec{A}$ is the array *location matrix*, whose (i, m) element is given by

$$[\vec{A}]_{im} = \frac{1}{z_{im}} e^{-j2\pi z_{im}/\lambda},\ i = 1, ..., L;\ m = 1, ..., M, \quad (5)$$

where λ is the wave length.

The problem that we are addressing here is how to estimate $\{r_m, \theta_m, \phi_m\}$, $m = 1, ..., M$, given the K snapshot data $\vec{x}(t)$, $t = 1, ..., K$.

3-D MUSIC ALGORITHM

The MUSIC algorithm [5] can be modified in a straightforward way to estimate bearing, elevation and range as follows. Let $\vec{\tilde{R}}$ be the sample correlation matrix given by

$$\vec{\tilde{R}} = \frac{1}{K} \sum_{t=1}^{K} \vec{x}(t)\vec{x}^H(t), \quad (6)$$

where the superscript H denotes the conjugate transposition. To determine the number of sources, we rank the eigenvalues λ_i of the matrix $\vec{\tilde{R}}$ in decending order to obtain

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_L. \tag{7}$$

The estimated number of sources, $\hat{M}$, is then determined as the value for which either the Akaike information criterion or minimum description length function is minimized [6]. Let $\vec{u}_{\hat{M}+1}, \ldots, \vec{u}_L$ be the noise eigenvectors associated with the $L-\hat{M}$ smallest eigenvalues, $\lambda_{\hat{M}+1}, \ldots, \lambda_L$. The column span of the noise-subspace is then constructed as

$$\vec{U}_n = [\vec{u}_{\hat{M}+1}, \ldots, \vec{u}_L]. \tag{8}$$

The array manifold, $\vec{a}(r,\theta,\phi)$, consists of all possible near-field steering vectors, that is

$$\vec{a}(r,\theta,\phi) = \left[\frac{1}{z_1}\exp\left(-j\frac{2\pi z_1}{\lambda}\right), \ldots, \frac{1}{z_L}\exp\left(-j\frac{2\pi z_L}{\lambda}\right)\right]^T, \tag{9}$$

where z_i, $i = 1, \ldots, L$, is

$$z_i(r,\theta,\phi) = \{\tilde{r}_i^2 + r^2 - 2\tilde{r}_i r[\cos\tilde{\phi}_i \cos\phi + \sin\tilde{\phi}_i \sin\phi \cos(\tilde{\theta}_i - \theta)]\}^{1/2}. \tag{10}$$

The estimates of the source locations are defined as the minimizers of the null spectrum

$$P_1(r,\theta,\phi) = \vec{a}^H(r,\theta,\phi)\vec{U}_n\vec{U}_n^H\vec{a}(r,\theta,\phi), \tag{11}$$

or the normalized null spectrum

$$P_2(r,\theta,\phi) = \frac{\vec{a}^H(r,\theta,\phi)\vec{U}_n\vec{U}_n^H\vec{a}(r,\theta,\phi)}{\vec{a}^H(r,\theta,\phi)\vec{a}(r,\theta,\phi)}. \tag{12}$$

From the viewpoint of localization, P_2 is favorably suggested for the reasons stated as follows. As a listening device, the array senses sources that are close to it much more easily than those far away because of spherical spreading. Thus, if we identify a source by a null (inverted peak) in P_1 spectrum there is an intrinsic bias toward positions where the source level is higher. And this intrinsic bias can be corrected by scaling the null spectrum in accordance with the intensity variation caused by spherical spreading as done in the normalized null spectrum P_2.

Alternatively, P_2 in (12) can be expressed in the same form as in P_1 of (11), provided that elements of the array manifold $\vec{a}$ in (9) are multiplied by appropriate normalization factors. It should be noted that both the array manifold and the normalized array manifold have elements which are periodic with period 2π in either elevation or bearing domain. This property facilitates polynomial rooting for both elevation and bearing as to be described in the next section. To estmate source locations, the minima of $P_2(r, \theta, \phi)$ can be found by performing an exhaustive search in range-bearing-elevation domain.

3-D MUSIC WITH POLYNOMIAL ROOTING

In this section, we propose an approach which involves one-dimensional (1-D) search in range domain combined with polynomial rooting in elevation and bearing domains. Because the elements of the normalized array manifold are periodic functions with period 2π in both elevation and bearing angles, we can represent the *ith* element of the normalized array manifold, for a given r, by the Fourier series:

$$a_i(r,\theta,\phi) = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} C_{ikl}(r)e^{j(k\theta+l\phi)},$$
$$i = 1, \ldots, L, \tag{13}$$

where

$$C_{ikl}(r) = 1/(4\pi^2)\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} a_i(r,\theta,\phi)e^{-j(k\theta+l\phi)}d\theta d\phi. \tag{14}$$

Since $a_i(r, \theta, \phi)$ is a smooth function, it can be approximated quite well using a finite number of Fourier coefficients. Suppose that $(2N_1+1)x(2N_2+1)$ coefficients are sufficient to describe all the L elements $a_i(r, \theta, \phi)$. Then construct the matrices $\vec{C}_i(r)$ as

$$[\vec{C}_i(r)]_{kl} = C_{ikl}(r), \quad i = 1, \ldots, L, \tag{15}$$

and the vectors

$$\vec{d}(\phi) = [e^{-jN_2\phi}, \ldots, e^{iN_2\phi}]^T, \tag{16}$$

$$\vec{b}(\theta) = [e^{-jN_1\theta}, \ldots, e^{iN_1\theta}]^T. \tag{17}$$

So, (13) becomes

$$a_i(r,\theta,\phi) \approx \vec{b}^T(\theta)\vec{C}_i(r)\vec{d}(\phi), \tag{18}$$

and the normalized array manifold vector is

$$\vec{a}(r,\theta,\phi) \approx \begin{bmatrix} \vec{b}^T(\theta)\vec{C}_1(r)\vec{d}(\phi) \\ \cdot \\ \cdot \\ \cdot \\ \vec{b}^T(\theta)\vec{C}_L(r)\vec{d}(\phi) \end{bmatrix}. \tag{19}$$

Thus the normalized null spectrum can be expressed as

$$P_2(r,\theta,\phi) = \vec{d}^H(\phi)\vec{G}(r,\theta)\vec{d}(\phi)\,, \tag{20}$$

where the matrix $\vec{G}$ is given by

$$\vec{G}(r,\theta) \approx [\vec{C}_1^H(r)\vec{b}^*(\theta), ..., \vec{C}_L^H(r)\vec{b}^*(\theta)]\vec{U}_n\vec{U}_n^H \begin{bmatrix} \vec{b}^T(\theta)\vec{C}_1(r) \\ \cdot \\ \cdot \\ \vec{b}^T(\theta)\vec{C}_L(r) \end{bmatrix}, \tag{21}$$

where the superscript * denotes the complex conjugation. By defining $z_1 = e^{j\phi}$, the normalized null spectrum $P_2(r, \theta, z_1) = \vec{d}^H(z_1)\vec{G}(r,\theta)\vec{d}(z_1)$ is a polynomial in z_1 for fixed values of θ and r. If the Fourier series has zero truncation error and infinite data is available, then the roots of $P_2(r, \theta, z_1) = 0$ for the true r and θ are on the unit circle, and they correspond to $z_1 = e^{j\phi}$, where ϕ is the true elevation angle. However, the roots may move off the unit circle due to finite sampling error and the truncation of the Fourier series. Therefore, by defining a set of discrete points over a region of range-bearing of interest, we find the roots of the normalized null spectrum for every precalculated $G(r, \theta)$'s, and choose the roots that are closest to the unit circle as the candidates. From these candidates, $\hat{M}$ roots that are closest to the unit circle are selected and the angles of these selected roots are the estimates of elevation angles, $\hat{\phi}_i$, $i = 1, ..., \hat{M}$.

On the other hand, the normalized null spectrum can also be expressed as

$$P_2(r,\theta,\phi) = \vec{b}^H(\theta)\vec{Q}(r,\phi)\vec{b}(\theta)\,, \tag{22}$$

where $\vec{Q}(r,\phi)$ is given by

$$\vec{Q}(r,\phi) \approx [\vec{C}_1^H(r)\vec{d}^*(\phi)\,, ..., \vec{C}_L^H(r)\vec{d}^*(\phi)]\vec{U}_n\vec{U}_n^H \begin{bmatrix} d^T(\phi)\vec{C}_1(r) \\ \cdot \\ \cdot \\ \cdot \\ \vec{d}^T(\phi)\vec{C}_L(r) \end{bmatrix}. \tag{23}$$

To refine the estimate of θ, we substitute each estimated elevation angle, $\hat{\phi}_i$, into the normalized null spectrum to obtain $P_2(r,\theta,\hat{\phi}_i)$ for every discrete location r. By defining $z_2 = e^{j\theta}$, $P_2(r, z_2,\hat{\phi}_i)$ becomes a polynomial in z_2. Thus, the estimate $\hat{\theta}_i$ of the bearing can be obtained as the angle of the root of $P_2(r, z_2,\hat{\phi}_i) = 0$ that is closest to the unit circle.

To refine the range estimates, we substitute each estimated bearing-elevation pair, $(\hat{\theta}_i, \hat{\phi}_i)$, into the normalized null spectrum to obtain the 1-D function $P_2(r,\hat{\theta}_i,\hat{\phi}_i)$. A line search for the minimum of this function is then performed over the range using Newton gradient procedure. The minimizer $\hat{r}_i$ is the range estimate.

The algorithm presented here extends the work of Weiss and Friedlander [3] which is limited to estimation of range and bearing only and to the case where spherical spreading is of no concern in the data model.

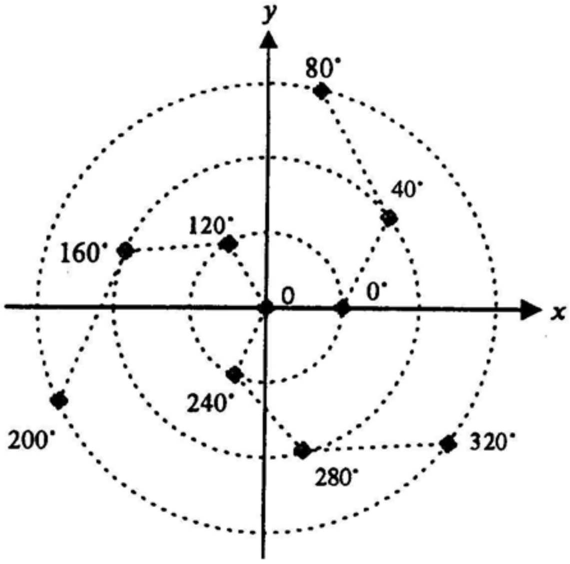
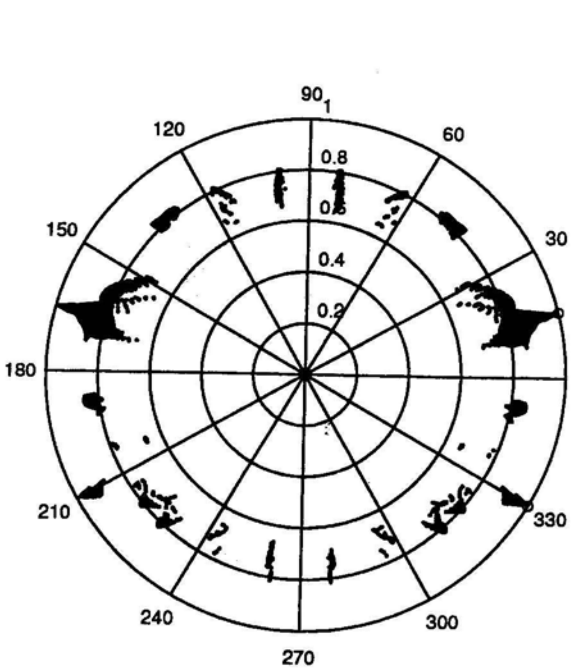


Fig. 2. Sensor array configuration.

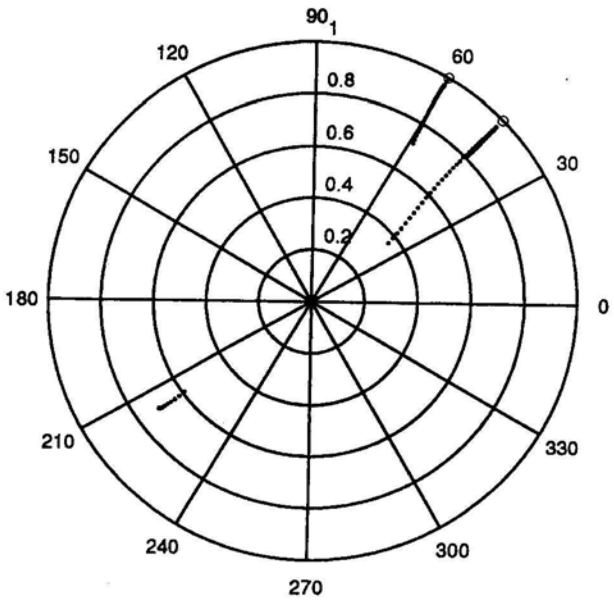
SIMULATION RESULTS

As shown in Fig. 2, we consider an array of 10 sensors deployed on three concentric circles whose radii are $\frac{1}{2}\lambda$, λ, and $\frac{3}{2}\lambda$, respectively, on the x-y plane. Two uncorrelated narrow-band zero-mean Gaussian sources are present: the first source located at the site of bearing angle 45°, elevation angle 15° , and range 4.5λ; and the second source located at the site of bearing angle 60°, elevation angle −30° and range 6.5λ. The noise is a zero-mean Gaussian white random process, uncorrelated with the signals. The signal-to-noise ratio (SNR) of both signals is 20 dB. The array is assumed to collect 500 snapshots.

To have better understandings of elevation estimation in the proposed algorithm, a rectangular grid was placed on the region spanned by $1\lambda \le r \le 8\lambda$ in range and $-90° \le \theta \le 90°$ in bearing on the range-bearing parameter plane. The grid intervals are 0.1λ in range and 1° in bearing. Fig. 3(a) shows the locations of the polynomial roots for each grid point,



(a)



(b)

Fig. 3. Distribution of polynomial roots closest to the unit circle (a):for each grid point in the bearing-range plane for elevation estimation, (b):for each grid point in the range line for bearing estimation.

which are closest to the unit circle. Among these roots, only the two roots that are closest to the unit circle were selected and the angles of these two roots closely matched the true elevation angles as indicated by small circles on the unit circle. For bearing estimation, a linear grid with 0.1λ of grid interval was placed on the region spanned by $1\lambda \le r \le 8\lambda$ in range. Fig. 3(b) depicts the locations of the polynomial roots for

Table 1. Estimator statistics in terms of bias, standard deviation (STD), and root mean square error (RMSE). (a): the first source, and (b): the second source.

(a)

1st SOURCE	REARING	ELEVATION	RANGE
TRUE	45°	15°	4.5λ
MEAN	44.9839°	14.9888°	4.5163λ
STD	0.0666°	0.0307°	0.0318λ
BIAS	0.0161°	0.0112°	-0.0163λ
RMSE	0.0864°	0.0327°	0.0357λ

(b)

2nd SOURCE	REARING	ELEVATION	RANGE
TRUE	60°	–30°	6.5λ
MEAN	60.0009°	–29.9484°	6.5367λ
STD	0.0445°	0.1093°	0.0792λ
BIAS	0.0009°	–0.0516°	-0.0376λ
RMSE	0.0444°	0.1207°	0.0872λ

each grid point, which are closest to the unit circle. Among these roots, only the two roots that are closest to the unit circle were chosen. The bearing angles, as indicated by two small circles on the unit circle, were accurately estimated as the angles of these two roots.

Based on 300 Monte Carlo trials, the estimator statistics in terms of bias, standard deviation (STD), and root mean square error (RMSE) for the two sources are given in Table 1.

Next, estimation performance was evaluated based on 100 Monte Carlo trials for each different SNR and snapshot number. The RMSE of elevation, bearing, and range estimates for the first source were plotted in Fig. 4. Similar behaviors were also observed for the second source, thus not shown in the paper. We note that the accuracy of elevation, bearing, and range estimates improves as SNR increases and/or snapshot numbers increases.

Finally, in order to compare estimation performance using normalized and unnormalized MUSIC spectra, a single source, located at bearing 45° and elevation 15°, has its range varing from 1λ to 18λ. The signal-to-noise ratio is 20 dB. The number of snapshots is 500 and Monte Carlo trials is 100.

Fig. 5 plots the RMSE of elevation, bearing and range estimates as a function of range. The bearing and elevation accuracy degrades as the source is approaching the array. On the contrary, the range accuracy improves as the source is closer to the array. In addition, the proposed algorithm using normalized spectrum has smaller RMSE than unnormalized spectrum.

CONCLUSIONS

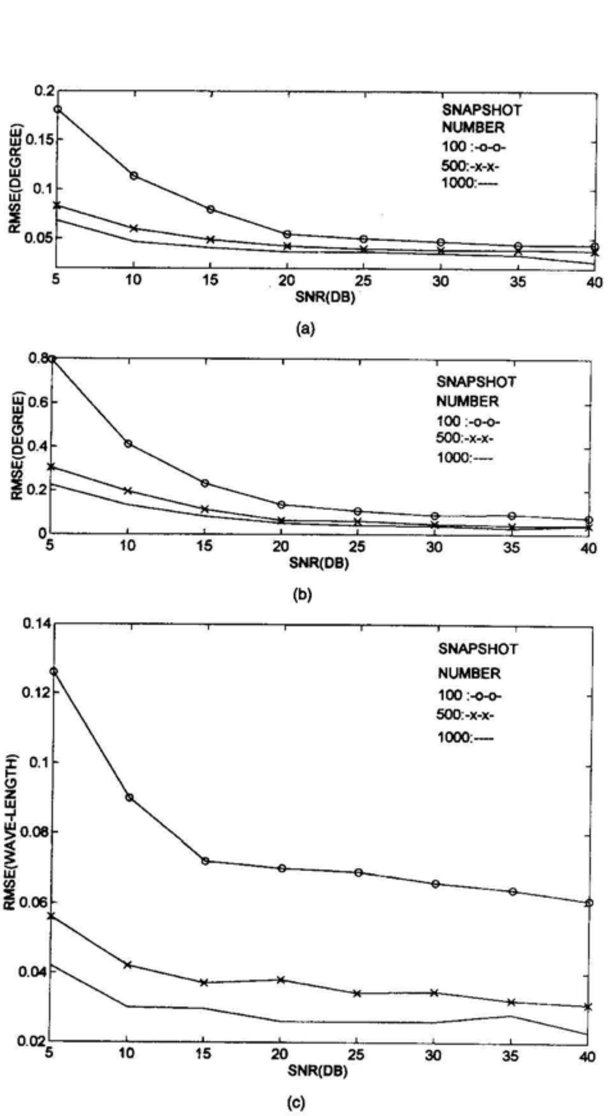


Fig. 4. Root mean square error (RMSE) versus signal-to-noise ratio (SNR) with varing snapshot numbers for the first source. (a): elevation estimate, (b): bearing estimate, (c): range estimate.

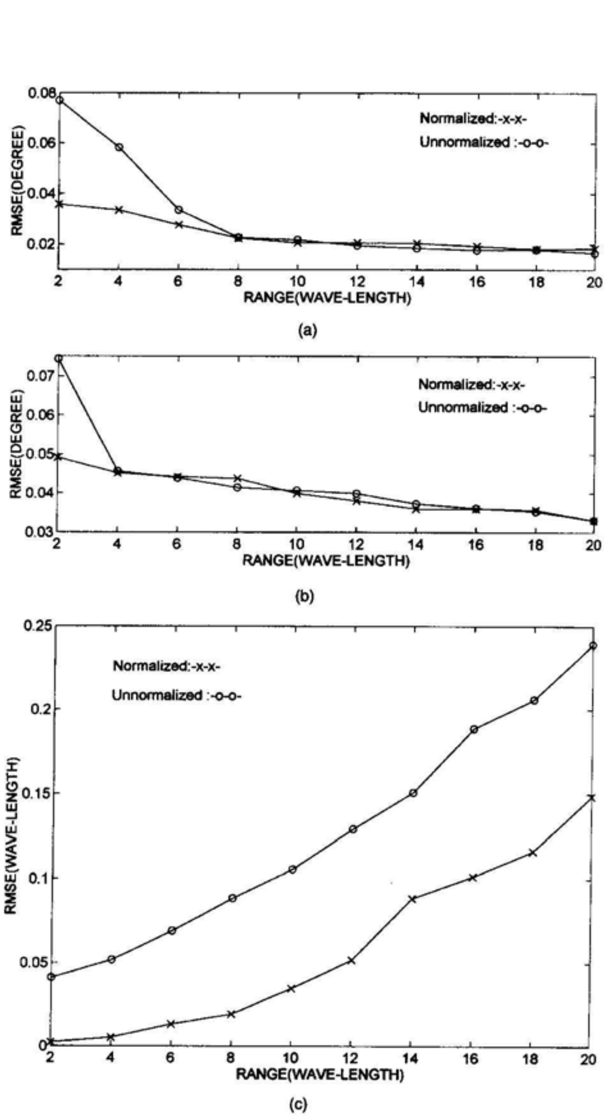


Fig. 5. Root mean square error (RMSE) comparison for normalized and nonnormalized MUSIC spectra for the case of a single source with varing range. (a): elevation estimate, (b): bearing estimate, (c): range estimate.

We presented an algorithm for simultaneous estimation of elevation, bearing and range for near-field narrowband sources. The algorithm is based on MUSIC algorithm. In general, estimation of elevation, bearing and range requires a three-dimensional search. We have shown here that the search in bearing and elevation domains can be replaced by polynomial rooting in two stages. The search in the range direction is performed by a Newton gradient method at the final stage. The proposed algorithm was tested by Monte Carlo simulations.

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多項式根值之MUSIC演算法應用於近場訊號源之位置估計

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摘 要

本論文研究從感測陣列所觀測的資料來估計近場窄頻訊號源之方位角，仰角和距離。為了高解析度訊號源之定位，多重訊號分類(MUSIC)演算法，在考量球面曲度和擴散因子於陣列多維度空間的情況下，可被變更和擴展至三度空間。然而從三維MUSIC頻譜同時估計方位角，仰角和源距需要非常耗時的多維度搜尋。為了減輕計算量之負擔，本文提出一個替代的演算法。所提出的方法先利用多項式根值法來估計方位角和仰角，以取代在這兩個角度範圍內的搜尋，然後才在距離範圍中搜尋頻譜峰值來估計源距。最後以電腦模擬的結果來驗證所提演算法的效能。