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NOTE ON EFFECT OF ELASTIC DEFORMATION AT THE ROOT OF A CANTILEVER BEAM

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Key words: elastic-plastic, plastic hinge.

ABSTRACT

Parkes neglected elastic strains and examined a classical problem of rigid- plastic structural dynamics for finding the deformation of a cantilever beam carrying a mass at its tip which is subjected to a short pulse loading. Symonds and Fleming examined the Parkes problem by making comparison between exact numerical solutions obtained from an ABAQUS program for an elastic-plastic beam and the rigid-plastic solution slightly modified to allow for large deformation. Further observation is acquired by comparision also with a simplified elastic-plastic approach based on treatment of elastic and plastic action in artificially separated stages. Wang and Yu examined the Parkes problem associated with the elastic deformation by introducing root rotational spring of the beam, they indicate that the dynamic response of the spring-rigid plastic system is of three modes : (i) mode I , (ii) mode IIa ;and (iii) mode IIb. This paper introduce and compare the theory between the Parkes , Symonds and Fleming and the Wang and Yu models. Basing on the extension of theoretical derivation one can find the difference among them. This difference can be used three non-dimensional parameters, that is, (1) t/T (The action time divides by the time taken for the plastic hinge to travel from the cantilever tip to the root.), (2) s/L (The distance from the traveling hinge to the tip of the beam divides by the length of the beam.), (3) V/Vo (The absolute velocity of the striker divides by the initial velocity of the striker.) are used to present it. For very small β and small ζ until $t_p/T \rightarrow 0$ of the mode I developed by Wang and Yu is similar to Parkes' models. No matter what value of β , the cantilever behavior of mode IIa is the same as that of mode IIb so long as the value of ζ is equal. The initial value of v* for the Wang-Yu model will approach Parkes only under the conditions of $t^* << 1$ and $\zeta \rightarrow 0$. $\beta \rightarrow \infty$ is approximated elastic analysis while $\beta \rightarrow 0$ tends to plastic analysis . The plastic behavior is very significant in the range of $0.1 < \zeta < 0.5$ for the same value of β in mode II.

INTRODUCTION

Parkes [1] made an analysis for the deformation of a cantilever beam with rigid-plastic material property witch is struck transversely at its tip by moving mass. In the Parkes problem, the effect of elastic deformation has been neglected. Only two non-dimensional parameters affect the response of the cantilever beam, that is, the ratio of the mass of the striker to that of the cantilever beam, and the ratio between the impact energy and the fully plastic bending moment of the beam cross-section. Symonds and Fleming [2] studied the Parkes problem by making comparison between exact and numerical solutions provided by an ABAQUS program (A finite element program designed for general use in nonlinear as well as linear structural programs for an elastic-plastic beam and the rigid-plastic solution slightly modified to allow for large deflections). Moreover insight is obtained by comparison also with a simplified elastic-plastic approach based on treatment of elastic and plastic action in artificially separated stages. In order to investigate the effect of elastic deformation which was ignored by Parkes, Wang and Yu [3] have introduced a rotation spring at the root of the beam, thereby introducing a third non-dimensional parameter, i.e., the ratio of the maximum elastic energy that can be stored in the model to the impact energy. According to the Wang-Yu [3] model, the response of the system can be classified by three different modes (see Figure 9 in Wang and Yu [3]): (i) mode I, a plastic hinge (The presence of the plastic hinge means that the beam will rotate at the hinge cross section while the bending moment remains constant and equal to M_p.) travels from the tip to the root, but before it reaches the root , the root spring enters into its plastic state and creates a new hinge there. After the traveling hinge reaches the root, it becomes the only plastic hinge and absorbs the remaining kinetic energy; (ii) mode IIa, the traveling hinge disappears at an internal point of the beam, and only elastic deformation takes place thereafter; and (iii) mode IIb, the traveling hinge disappears at an internal point at the beam, but the remaining kinetic energy still results in a plastic hinge at the root. In the case of mode I, Wang and Yu pointed out that when the ratio of the maximum elastic energy that can be stored in the model to the impact energy is very small, i.e. β=[Mp/(2K)]/[(GVo)/2]<< 1 the response of the beam is similar to Parkes' model. In this note, a comparison will be made among the Parkes , Symonds-Fleming [2] , and the Wang-Yu [3] model. One will show that the mode I solution of the Wang-Yu [3] model does not agree with Parkes' model for v* and s* when t* <<1. $\beta \rightarrow \infty$ is approximated elastic analysis while $\beta \rightarrow 0$ tends to plastic analysis.

THEORETICAL REVIEW

Wang-Yu [3] considered that a cantilever beam with a rotational spring at the root was subjected to a transverse impact at its tip by a rigid (point) mass G moving at some initial speed Vo, as shown in Fig. 1 (a). After impact, the mass is supposed to adhere to the cantilever tip. The cantilever is of length L, mass m per unit length, and has a plastic bending moment capacity Mp. They also assumed that:

1. the material of the cantilever beam is rigid-perfectly plastic and time-independent;
2. the effect of shear on yielding can be ignored;
3. the deflection of the beam can be considered small;
4. the rotational spring is elastic-perfectly plastic, which has a plastic bending moment capacity Mp as that of the beam.

The bending moment at the root of the beam is

$$M_r = \begin{cases} K\,\phi_e, & \text{elastic or unloading} \\ M_p \text{ or } -M_p & \text{otherwise} \end{cases} \qquad (1)$$

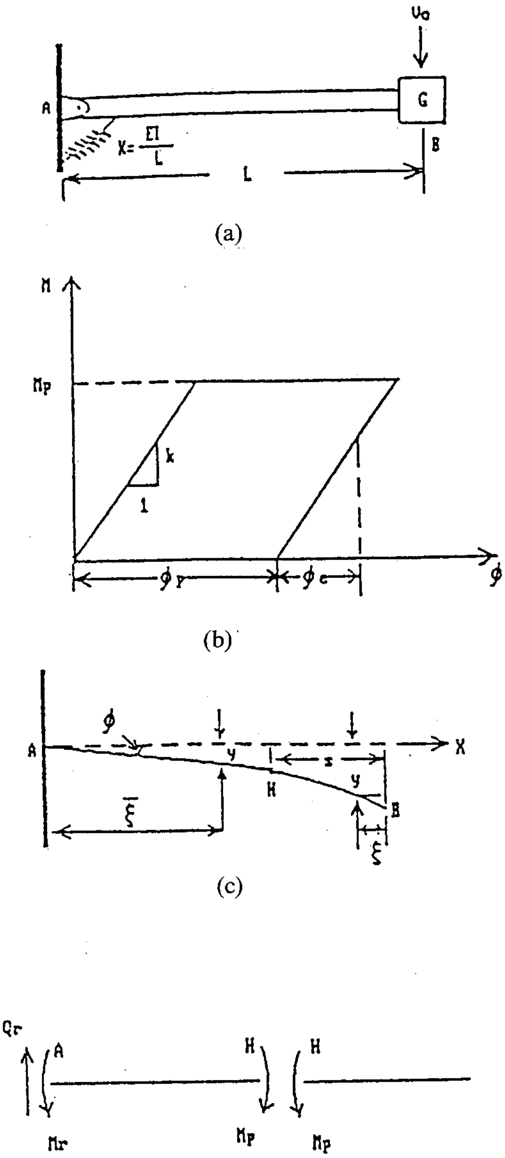


Fig. 1. Wang-Yu cantilever problem:
(a) A beam struck by a rigid mass at its tip with root spring.
(b) The constitutive relation of the root spring.
(c) Deformation mechanism of the beam.
(d) Forces acting on free body AH and BH.

in which K = EI/L is the elastic constant of the root spring, $\phi_e=\phi-\phi_p$ is the elastic rotation angle of the spring and ϕ_p is the plastic rotation angle, as shown in Fig. 1(b). Based on the above assumptions, the impact by the striker at the tip causes plastic bending of the beam and the elastic-plastic deformation of rotational spring at the root. The former will induce a traveling plastic hinge as shown by Parkes [1]. Assume that at time t a plastic hinge builds up at H located at x = L - s(t) as shown in Fig. 1(c). Segment AH has a rigid-body rotation about point A with angular velocity $\dot{\phi}$, where ϕ is the angle between AH and the X-axis as shown in Fig. 1(c). The velocity of a point x = $\overline{\xi}$ in segment AH is

$$\dot{y}=\dot{\phi}\,\overline{\xi} \tag{2}$$

Assume that the mass G has a relative velocity v to the reference frame fixed on AH. Then segment HB has a rigid-body rotation with relative angular velocity v/s. Hence the velocity of a point x = L - $\overline{\xi}$ in segment BH is

$$\dot{y}=\dot{\phi}(L-\xi)+v(1-\xi/s) \tag{3}$$

Accordingly, the transverse acceleration of the beam can be obtained as

$$\ddot{y}=\ddot{\phi}\,\overline{\xi} \tag{4}$$

for segment AH and

$$\ddot{y}=\ddot{\phi}(L-\xi)+\dot{v}(1-\overline{\xi}/s)+\xi v\dot{s}/s^2 \tag{5}$$

for segment HB.

The shear force Q_H=0 at hinge H since the bending moment takes a local maximum there. By using the equilibrium conditions of two segments AH and HB, respectively, as shown in Fig. 1(d), the equilibrium equations result in

$$M_r-M_p+\int_0^{L-s} m\ddot{\phi}\,\overline{\xi}\,\overline{\xi}\,d\overline{\xi}=0 \tag{6}$$

$$G\ddot{y}|_{\overline{\xi}=0}+\int_0^{s} m\ddot{y}\,d\xi=0 \tag{7}$$

$$-M_p+\int_0^{s} m\ddot{y}\,d\xi=0 \tag{8}$$

$$M_r-M_p+m(L-s)^3\ddot{\phi}/3=0 \tag{9}$$

$$(G+sm/2)\dot{v}+[LG+ms(L-S/2)]\ddot{\phi}+v\dot{s}m/2=0 \tag{10}$$

$$s^2\dot{v}m/6+(L/2-s/3)ms^2\ddot{\phi}+s\dot{s}vm/3-M_p=0 \tag{11}$$

It is convenient to use the non-dimensional parameter as follows:

$$s^*=s/L,\ V^*=v/V_o,\ \phi^*=\phi,$$

$$t^*=tV_o/L,\ M_r^*=M_r/mLV_0^2$$

$$\alpha=\frac{1}{2}GV_0^2/M_p,\ \beta=\frac{M_p^2}{2k}/\frac{1}{2}GV_0^2,\ \varsigma=G/mL$$

In terms of these parameter, Eqns (9)-(11) can be rewritten:

$$\frac{1}{3}(1-s^*)^3\ddot{\phi}^*+M_r^*-\varsigma/2\alpha=0 \tag{12}$$

$$(\varsigma+s^*/2)\dot{V}^*V^*+[\varsigma+s^*(1-s^*/2)]\ddot{\phi}^*+\frac{1}{2}V^*\dot{s}^*=0 \tag{13}$$

$$\dot{V}^*s^{*2}/6+(1/-s^*/3)s^{*2}\ddot{\phi}+\dot{s}^*V^*s^*/3-\varsigma/2\alpha=0 \tag{14}$$

Eqns.(1) is non-dimensionalized as

$$M_r^*=\begin{cases}\varsigma\phi_e^*/4\alpha^2\beta & \textit{elastic or unloading}\\ \varsigma/2\alpha \text{ or } -\varsigma/2\alpha & \textit{otherwise}\end{cases} \tag{15}$$

in which $\phi_e^*=\phi_e$. The problem is now changed to solve Equations (12)-(14) associated with the initial conditions V*(0)=1, s*(0)=0, ϕ*(0)=0 and $\dot{\phi}^*$(0)=0. Rearranging the above equations, one gets

$$\ddot{\phi}^*=\frac{3}{(1-s^*)^3}(\varsigma/2\alpha-M_r^*) \tag{16}$$

$$s^*=W_2/W_1 \tag{17}$$

$$\dot{V}^*=W_3 \tag{18}$$

in which

$$W_1=s^*V^*/3-s^{*2}V^*/(\varsigma+s^*/2)/12$$

$$W_2=\varsigma/2\alpha-(3/2-s^*)s^{*2}(\varsigma/2\alpha-M_r^*)/(1-s^*)^3$$
$$+s^{*2}[\varsigma+s^*(1-s^*)](\varsigma/2\alpha-M_r^*)$$
$$/(\varsigma+s^*/2)/(1-s^*)^3/2$$

$$W_3=-\{3[\varsigma+s^*(1-s^*/2)](\varsigma/2\alpha-M_r^*)$$
$$/(1-s^*)^3+V^*W_2/W_1/2\}(\varsigma+s^*/2)$$

For t* << 1, Eqns (16)-(18) become

$$\ddot{\phi}^*=3\varsigma/2\alpha \tag{19}$$

$$\dot{s}^*=3\varsigma/2\alpha s^* \tag{20}$$

$$\dot{V}^*=-3/4\alpha s^* \tag{21}$$

By integration,

$$\dot{\phi}^*=3\varsigma t^*/2\alpha \tag{22}$$

$$\phi^*=3\varsigma t^{*2}/4\alpha \tag{23}$$

$$V^* = 1 - \frac{1}{2}\sqrt{\frac{3t^*}{\alpha\varsigma}} \tag{24}$$

$$s^* = \sqrt{\frac{3t^*}{\alpha}} \tag{25}$$

Wang and Yu [3] pointed out that the results obtained from Eqns (22)-(25) agree with Parkes' model for V* and s* for t* << 1. Based on Eqns (22)-(25), they used Eqns (16)-(18) to carry out the numerical calculation for various parameters (α, β, ζ) in order to illustrate the dynamic response of the spring-beam system. On the numerical results, they classified the response by three different modes. For a specified ζ (see Figure 9 in Wang and Yu [3]), the response become Mode I when β tends to zero. When $\beta > 1$, the response is always in Mode IIa. And for a specified $\beta < 1$, the response tends to take Mode IIb.

EXTENSIVE COMPARISON

Wang and Yu pointed out that the system of ordinary differential equations (16)-(18) can be integrated numerically step by step using the well-known Rugge-Kutta method by taking the values of ϕ^*, $\dot{\phi}^*$, V^* and s* given by Eqns. (22)-(25), for t* << 1, as the initial conditions. Substituting Eqns. (25) into Eqns. (24), one gets

$$V^* = 1 - (s^*/\varsigma)/2 \tag{26}$$

Eqns. (25), in dimensional form , is

$$s^2 = \frac{6M_p t}{mV_o} \tag{27}$$

When the plastic hinge reaches the root of the beam, that is s = L , one obtains

$$T = \frac{mV_o L^2}{6M_p} \tag{28}$$

Combining Equations. (27) and (28) in the non-dimensional form yields

$$\frac{s}{L} = S_w(\varsigma, \frac{t}{T}) = \sqrt{\frac{t}{T}} \tag{29}$$

Eqns. (26) can be written as

$$\frac{v}{V} = 1 - (\frac{2}{\varsigma})(\frac{s}{L}) = 1 - \frac{2}{\varsigma}\sqrt{\frac{t}{T}} \tag{30}$$

However, the absolute velocity at the free end is

$$\frac{V}{V_o} = \frac{v}{V_o} + (1 - s^*)\dot{\phi}^* \tag{31}$$

Hence

$$\frac{V}{V} = V_w(\varsigma, \frac{t}{T}) = 1 - (\frac{2}{\varsigma})\sqrt{\frac{t}{T}} + \frac{1}{2}(\frac{t}{T})(1 - \sqrt{\frac{t}{T}}) \tag{32}$$

According to Johnson [4], one gets the equation from the Parkes model

$$\frac{V}{V_o} = \frac{1}{1 + (\frac{2}{\varsigma})(\frac{s}{L})} \tag{33}$$

and

$$t = \frac{ms^2}{6M_p} \frac{V_o}{1 + (\frac{2}{\varsigma})(\frac{s}{L})} \tag{34}$$

When the plastic hinge happens at time t and reaches the cantilever root, s = L, one has

$$T = \frac{mL^2}{6M_p} \frac{V_o}{1 + (\frac{2}{\varsigma})} \tag{35}$$

Eqns. (34) divided by Eqn.(35) one gets

$$\frac{t}{T} = (\frac{s}{L})^2 \frac{1 + (\frac{2}{\varsigma})}{1 + (\frac{2}{\varsigma})(\frac{s}{L})} \tag{36}$$

The above Eqns. can be rewritten as

$$1 + (\frac{2}{\varsigma})](\frac{s}{L})^2 - (\frac{2}{\varsigma})(\frac{t}{T})(\frac{s}{L}) - (\frac{t}{T}) = 0 \tag{37}$$

This is a quadratic equation in terms of s/L. The solution is

$$(\frac{s}{L}) = S_p(\varsigma, \frac{t}{T})$$

$$= \frac{(\frac{2}{\varsigma})(\frac{t}{T}) + \sqrt{(\frac{2}{\varsigma})^2(\frac{t}{T})^2 + 4(\frac{t}{T})[1 + (\frac{2}{\varsigma})]}}{2[1 + (\frac{2}{\varsigma})]} \tag{38}$$

Substituting Eqn. (38) into Eqn. (33), one gets

$$\frac{V}{V_o} = V_p(\varsigma, \frac{t}{T}) = \frac{2[1 + (\frac{2}{\varsigma})]}{2[1 + (\frac{2}{\varsigma})] + (\frac{2}{\varsigma})^2(\frac{t}{T}) + (\frac{2}{\varsigma})\sqrt{(\frac{2}{\varsigma})^2(\frac{t}{T})^2 + 4(\frac{t}{T})[1 + (\frac{2}{\varsigma})]}} \tag{39}$$

Taking $\zeta \rightarrow 0$ for finding the limit of S_w-S_p and V_p-V_w, yields

$$\lim_{\varsigma \to 0}(S_w - S_p) = \sqrt{\frac{t}{T}} - \frac{t}{T} \tag{40}$$

and

$$\lim_{\varsigma \to 0}(V_p - V_w) = \lim_{\varsigma \to 0}\left(\frac{2}{\varsigma}\right)\sqrt{\frac{t}{T}} = \infty \tag{41}$$

Based on the results of Eqns (40) and (41), one knows that the solution of Eqns (22)-(25) associated with the initial condition t* << 1 is corresponding with the value of s* obtained by Parkes. However, if considers the influence of ζ under the conditions of $t/T \rightarrow 0$ and $\zeta \rightarrow 0$, to the parts of V*, one finds that the difference between the Wang and Yu and the Parkes model is the same as study the value of ∞/∞ instead of 0/0 pointed out by Wang and Yu. In order to describe this special feature one takes the difference of either Eqns (31) and (38) or Eqns (29) and (37) with respect to the value of t/T and ζ as shown in Fig. 2 and Fig. 3. Fig. 2(b) shows that the limit value of S_w-S_p approaches the constant value of $\sqrt{t/T} - t/T$ when ζ << 1. Fig. 3(b) indicates that the limit value of V_p-V_w tends to infinite when ζ << 1.

At present one considers again the initial value problem developed by Wang and Yu instead of $\zeta \rightarrow 0$. The condition of t/T << 1 in the Eqns. (22) -(25) is given a very small value for application. One should estimate the influence due to the change of t/T value. Fig. 2(a) and 3(a) can be shown this phenomena. It is found out that if one considers the deformation of a cantilever beam struck transversely at its tip on the small value of ζ then one should choose the small value of t/T in order to avoid the large influence caused by the error of initial value.

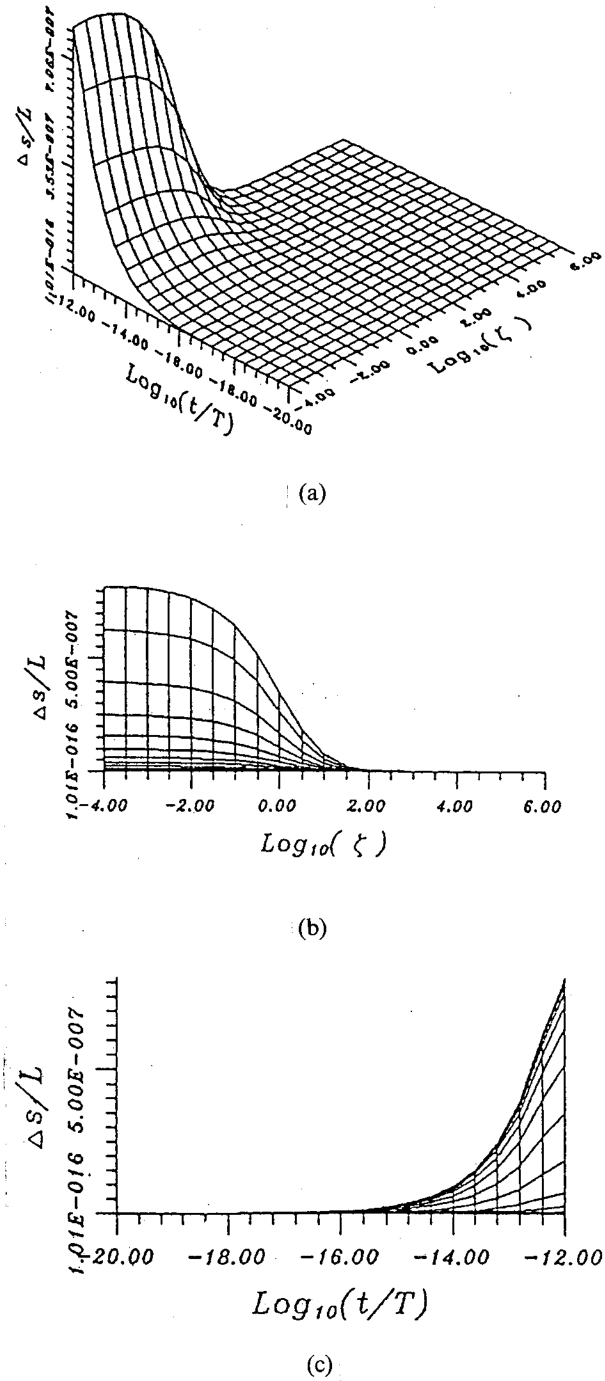
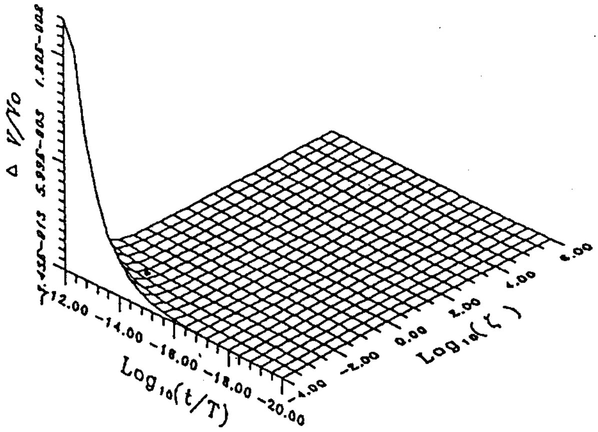


Fig. 2. The relationship among s/L, t/T and ζ.

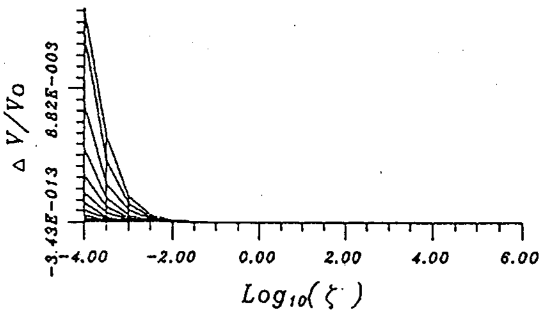
EXTENSIVE INVESTIGATION

The s/L phenomena affected by the value of t/T has been shown in Figs. 2(c) and 3(c). For the sake of studying in advance, one treats again the Eqns. (22)-(25) associated the initial condition t* << 1 with Rugge-Kutta method under the conditions of $0^3 < \varsigma < 10^5$ and $t/T = 0^{-18}$. One can compare the difference between the Parkes and the Wang and Yu model through the numerical operation.

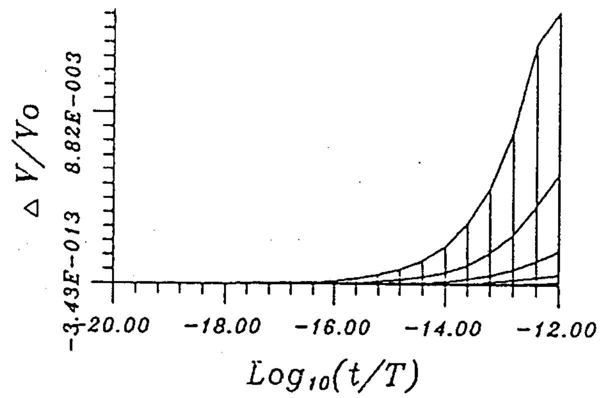
One takes t/T, s* and $V^* + (1 - s^*)\phi^*$ of mode I in the Wang-Yu problem for comparing the Parkes problem. Fig. 4 indicates that the difference of dynamic energy is small and V/Vo is very closed between the Wang and Yu and Parkes problem when the elastic modules K is increased (i.e. the stored elastic strain energy and β are decreased). This implies that the influence of elastic modules K of spring at the root of cantilever is suggested by Wang and Yu. However, this is not agreed that the mode I in the Wang-Yu problem is similar to Parkes' rigid-plastic model if the value of β is very small. Both the Wang-Yu and the Parkes model are similar only at $\beta = 0$. Fig. 5 shows that the position of plastic hinge of mode I in the Wang and Yu model at the small value of β and the same time t/T is quite different. This also indicates that both the velocity dis-



(a)



(b)



(c)

Fig. 3. The relationship among V/Vo, t/T and ζ.

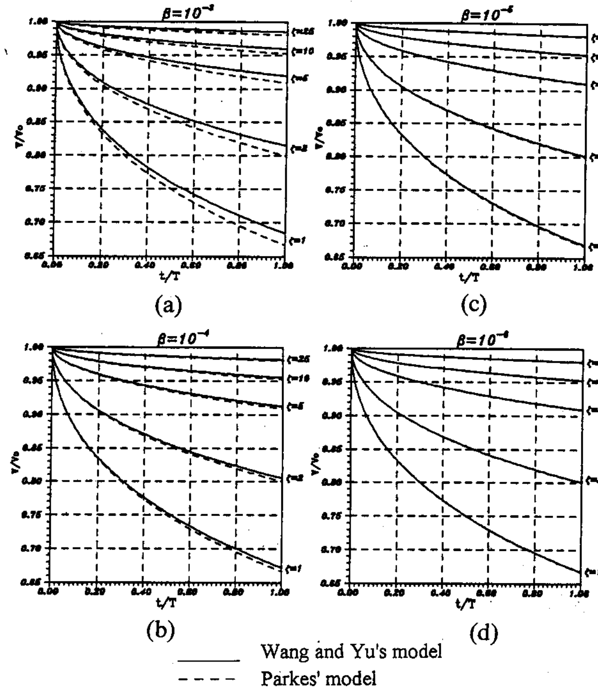


Fig. 4. Comparison between the Parkes and Wang-Yu model.

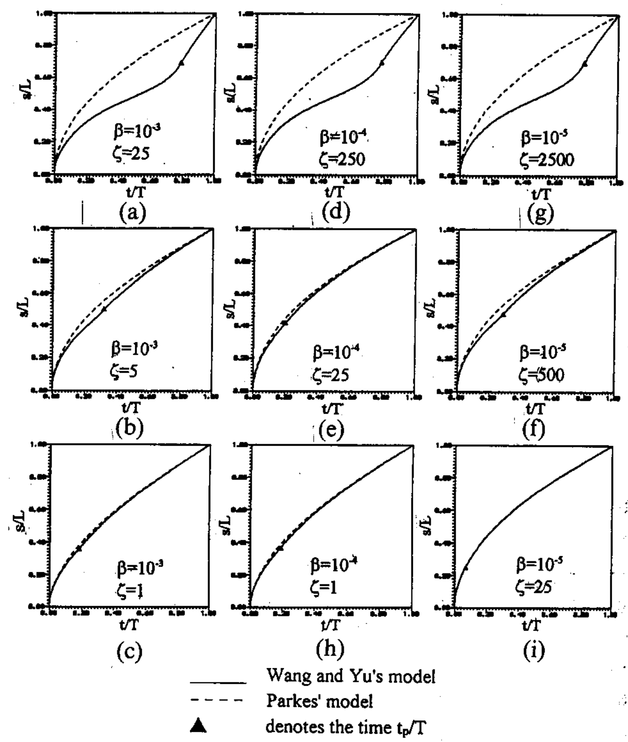


Fig. 5. Comparison between the Parkes and Wang-Yu model.

tribution curve and the deflection curve of cantilever are completely different. Thus, one can find out that both the Wang-Yu and the Parkes model are similar at the cantilever of $\beta \rightarrow 0$ by means of searching for the moving velocity factor of the two plastic hinges. Fig. 5 reveals that the t_p/T at the root rotating spring in the model I of the Wang-Yu problem is really influence the movement of plastic hinge. It is obvious that the value of t/T is influenced by ζ. Fig. 6 obtained from Fig. 7 can be explained that if the ζ increases then the t_p/T decreases under the same condition of β. This means that the cantilever behavior described by Wang

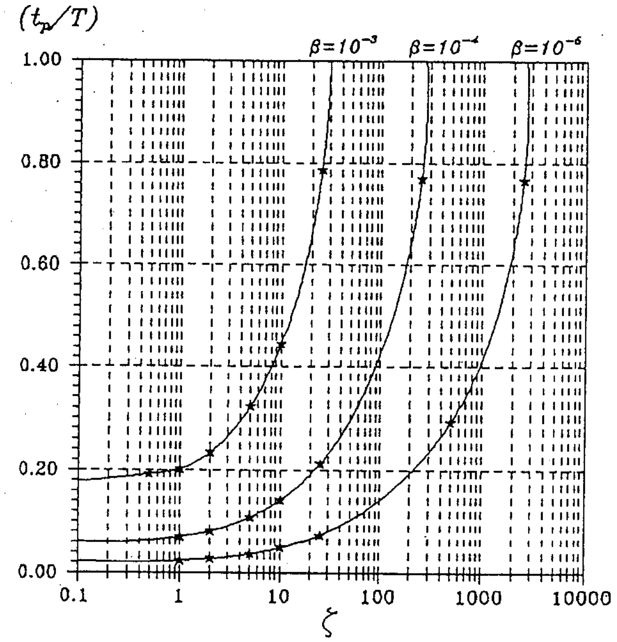


Fig. 6. The variation of time, tp/T, with ζ.

and Yu and Parkes is quite different. The point of view obtained by Wang and Yu should be modified that for very small β and small ζ until $t_p/T \to 0$ of the mode I developed by Wang and Yu is similar to Parkes' model.

Moreover, it is worth to point out that the cantilever behavior of mode IIa is similar to mode IIb only using the same value of ζ no matter what value of β. Fig. 7 indicates that the position of plastic hinge and the tip velocity both mode IIa and mode IIb are very closed under the same value of ζ and t*.

The phenomena of Mode IIa and Mode IIb in the Wang and Yu model will continuously study here. For the impact theory of elastic-plastic cantilever beam, the beam in time occurs free vibration with nature frequency after the dissipation of plastic behavior. Now one investigates the relationship between (α, β, ζ) and t_f/t_w which is the ratio of the time from impact to dissipated plastic behavior to that of period of natural frequency. Table 1 indicates that the value of t_f/t_w is free of relation to α, However, it is very obvious that $t_f/t_w \to \infty$ when $\beta \to 0$ while $t_f/t_w \to 0.25$ when $\beta \to \infty$ (see Fig. 8). It may be said that $\beta \to 0$ tends to plastic analysis , that is $t_f/t_w \to \infty$ approach to plastic analysis. It is approximated elastic analysis $t_f/t_w \to 0.25$, that is $\beta \to \infty$. The relationship between ζ and tf/tw (see Fig. 9) is very interested that the maximum value of t_f/t_w occurs in the range of $0.1 < \zeta < 0.5$ for the same value of β in Mode II. This imply that the plastic behavior is very significant in the range of $0.1 < \zeta < 0.5$.

COMPARISON AND DISCUSSION

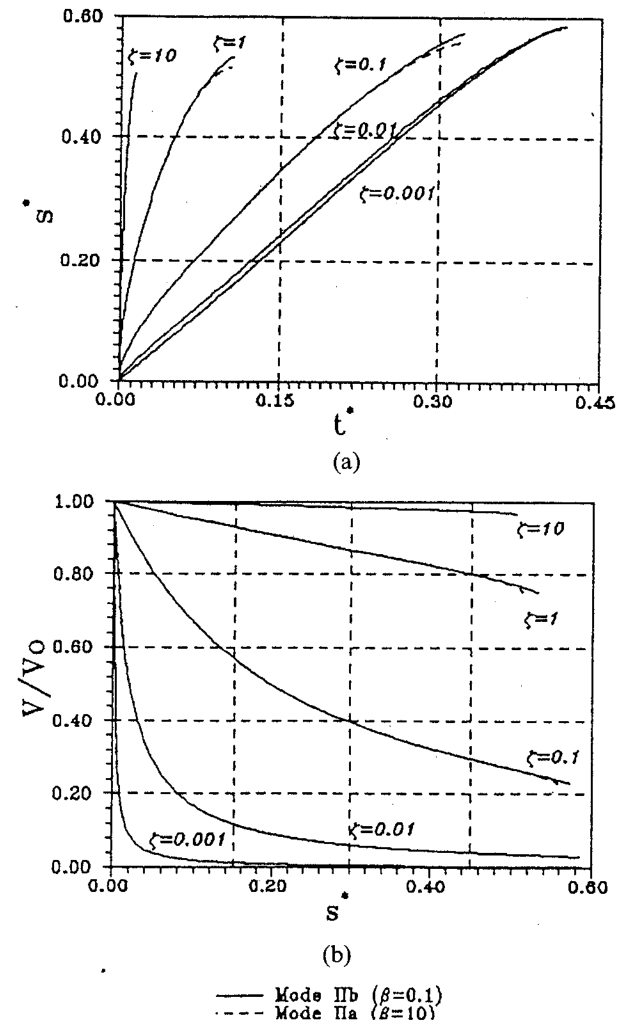


Fig. 7. (a) The variation of travelling hinge location with time.
(b) The variation of tip velocity with travelling hinge location.

Table 1. The relationship among α、β、ζ and t_f/t_w

α	β	ζ	t_f/t_w
.10E+01	.10E+00	.10E+00	.2774E+00
.30E+01	.10E+00	.10E+00	.2774E+00
.50E+01	.10E+00	.10E+00	.2774E+00
.70E+01	.10E+00	.10E+00	.2774E+00
.10E+01	.10E+00	.10E+00	.2774E+00
.30E+01	.10E+00	.10E+00	.2774E+00
.50E+01	.10E+00	.10E+00	.2774E+00
.70E+01	.10E+00	.10E+00	.2774E+00
.10E+01	.20E+00	.10E+00	.2774E+00
.30E+01	.20E+00	.10E+00	.2774E+00
.50E+01	.20E+00	.10E+00	.2774E+00
.70E+01	.20E+00	.10E+00	.2774E+00
.10E+01	.20E+00	.10E+00	.2774E+00
.30E+01	.20E+00	.10E+00	.2774E+00
.50E+01	.20E+00	.10E+00	.2774E+00
.70E+01	.20E+00	.10E+00	.2774E+00

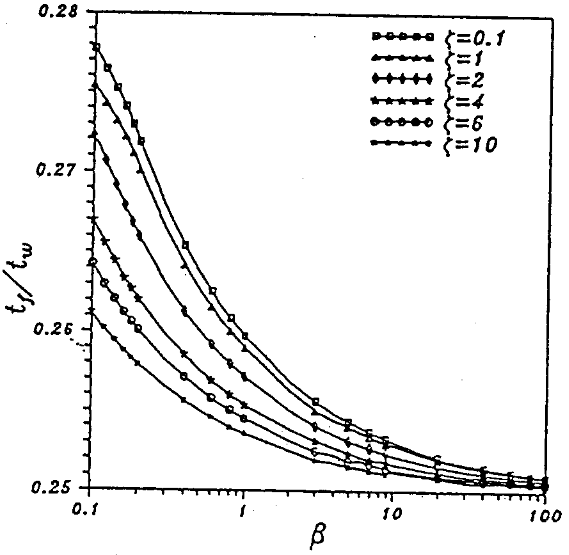


Fig. 8. The relationship between β and t_f/t_w.

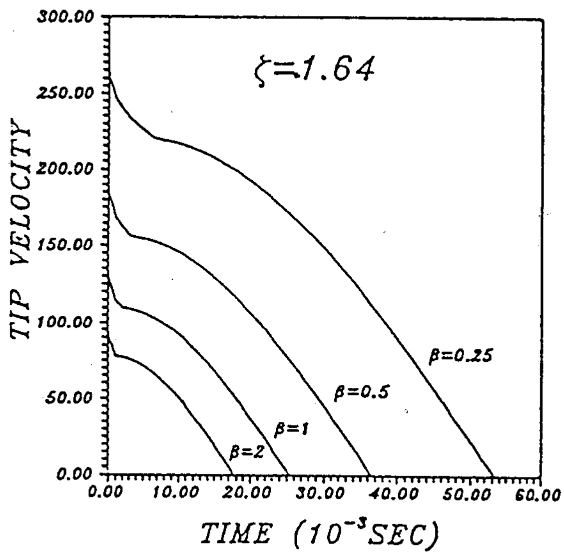


Fig. 10. The curve of tip velocity versus time.

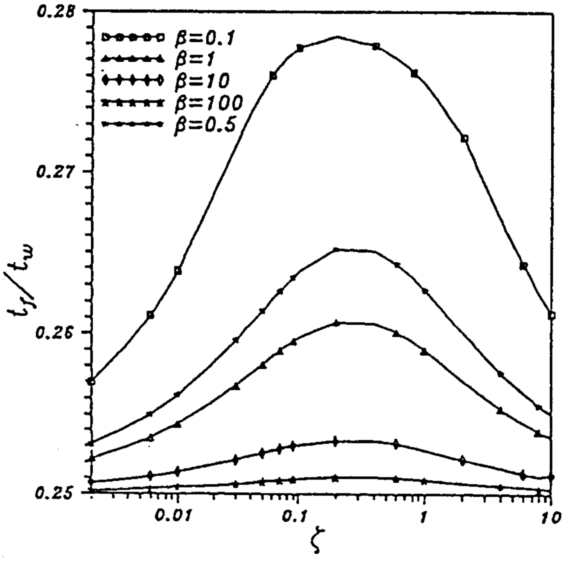


Fig. 9. The relationship between β and t_f/t_w.

Symonds and Fleming [2] used five methods: 1. rigid perfectly-plastic analysis, 2. Simple mode approximation technique (A means of avoiding the travelling hinge phase in the complete rigid plastic analysis is provided by the so-called mode approximation technique.), 3. multistage mode approximation technique (The multistage mode approximation technique has as its basis a bilinear elastic plastic constitutive relationship for moment versus curvature.), 4. linear acceleration lumped mass formulation (The mass matrix in this formulation is based on an acceleration field assumed to vary linearly along the length of the cantilever beam.), 5. ABAQUS program, to study a cantilever beam with tip mass. In order to compare the results of elastic-plastic state between the Wang-Yu model and the Symonds-Fleming model , one substituted the beam dimensions and properties (see Table 1 in Symonds and Fleming [2]) into the theory of Wang-Yu model with $R=1/\beta$. Thus, the curve of tip velocity versus time is shown as in Fig. 10. If compare Fig. 10 with Fig. 5(a) in Symonds and Fleming(2), it is found out that the tip velocity versus time in Wang-Yu model is very similar to multistage mode approximation technique and is approached to linear acceleration lumped mass formulation. Table 2 indicates the time value at the tip velocity with zero. The present solution based on Wang-Yu model approaches the each solution obtained from Symonds and Fleming [2].

CONCLUSIONS

Numerical calculation for various parameters α, β, ζ have been carried out to illustrate the dynamic response of the spring-beam system. This note has shown that the dynamic behavior of the Wang-Yu problem is in general very different from that of the Parkes problem as is evident from the theoretical formula derived for the two problems. For very small β and small ζ until $tp/T \rightarrow 0$ of the mode I developed by Wang and Yu is similar to Parkes' model. No matter what value of β, the cantilever behavior of mode IIa is the same as that of mode IIb so long as the value of ζ is equal. In addition, it is evident from the results both theoretical and numerical analysis that the solution of the initial value problem of V* considered by Wang and Yu is not agreed with the Parkes model at the condition of $t^* \ll 1$. The initial value problem of V* for the Wang-

Table 2. The time value at tip velocity with zero

β	ABAQUS	multistage Mode approximation technique	linear acceleration lumped mass formulation	Wang and Yu model
2	37	24.9	24.9	17.735
1	31	30.0	29.8	25.389
0.5	44	38.0	38.6	36.523
0.25	55	53.0	52.0	53.364
0.125	75	73.5	71.9	92.764

Yu model will approach Parkes only under the conditions of $t^* << 1$ and $\zeta \to 0$. $\beta \to \infty$ is approximated elastic analysis while $\beta \to 0$ tents to plastic analysis. The plastic behavior is very significant in the range of $0.1 < \zeta < 0.5$ for the same value of β in Mode II.

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NOTATION

- G mass of the striker (Wang-Yu's model)
- K root rotating spring constant
- L length of the beam
- m mass per unit length of the beam
- M mass of the striker (Parkes' model)
- Mp fully plastic bending moment of the beam
- s distance from the traveling hinge to the tip of the beam
- s* s/L
- t time
- t* t Vo/L
- tp time of root rotating spring entering to plastic state
- T time for the plastic hinge to travel from the tip to the root
- v relative velocity of the striker to the rotating reference frame
- V absolute velocity of the striker
- Vo initial velocity of the striker
- V* v/Vo
- y deflection
- α ratio of the impact energy to Mp
- β [Mp/(2K)]/[(GVo)/2]
- β^* mL/2M Parkes' model)
  mL/2G (Wang-Yu's model)
- ζ ratio of the mass of the striker to the beam
- ϕ rotation angle of the beam about the root

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