Breaking Wave Spectrum and Set-Down Set-Up in Shallow Water

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ABSTRACT

To study the mean water surface in the shallow water, it is necessary to compute the wave set-up and set-down of random waves on the surf zone. Therefore, the breaking wave spectra and radiation stress must be calculated from a wide-band breaking wave model.

In this study, we extend the breaking wave spectrum model (Yang, 1991) by using the Wallops spectrum in the deep water. The Wallops spectrum is based on wave dynamics theoretical analysis and no empirical coefficients to be determined by the users. It also contents correct energy (Huang *et al.*, 1981). The breaking wave spectra, radiation stress, and set-down set-up are presented graphically with distance from shoreline. It is shown that the results agree with the model (Yang, 1991) which used Pierson spectrum in the deep water.

INTRODUCTION

In order to study the set-down and set-up of irregular waves on sloping beaches, it must compute the radiation stress and breaking spectrum of waves in the shallow water. The shallow water breaking wave spectrum was studied by Yang (1991) which used Pierson spectrum in the deep water. In this study, we used Wallops spectrum which is involved wave theory analysis and correct energy content (Huang *et al.*, 1981). However, the Wallops spectrum does not consider wave breaking. The modification of breaking Wallops spectrum in deep water was studied by Tung *et al.* (1987). Although, the importance of the breaking waves and radiation stress must be consider in shallow water. There are numbers of publications on the subject of breaking waves in shallow water such as Battjes and Janssen (1978). However these methods do not obtain the breaking wave spectrum directly.

By improving the breaking wave criterion used in Battjes (1974), a wide-band model of breaking waves approach to the shoreline is established. Based on the model and assuming the original ideal (fictitious) non-breaking waves are zero-mean, stationary, and Gaussian, the spectrum of breaking waves is derived and the corresponding radiation stress is computed. The variances of the spectrum and the standard deviations of wave elevation and radiation stress with distance from shoreline are computed and presented graphically. It is shown that the results are in good agreement with those data by Yang (1991) and Yang and Tung (1993).

PROBLEM STATEMENT

Breaking criterion for sinusoidal and finite bandwidth waves is (Yang, 1991)

$$\zeta_b = 0.44 \, d \, \frac{tanh(k_0 d)}{k_0 d} \tag{1}$$

where d is water depth, k_0 is the component wave-number in deep water for waves of finite-bandwidth. For simplicity, it is assumed that k_0 in Eq. (1) is the mean wave-number in deep water denoted $\bar{k}_0$ and is

related to the mean wave-frequency $\overline{\omega}_0$ in deep water by the dispersive relation $\overline{k}_0 = \dfrac{\overline{\omega}_0^2}{g}$, g being gravitational acceleration. Therefore, we assume the ideal (fictitious) wave amplitude breaks and reaches ζ_b if water surface elevation $\zeta(t)$ exceeds ζ_b. The mean water surface elevation $\overline{\zeta}(t)$ can be considered as

$$\overline{\zeta} = \zeta \times H(\zeta_b - \zeta) \times H(\zeta) + \zeta_b \times H(\zeta - \zeta_b) + \zeta$$

$$\times H(d + \zeta) \times H(-\zeta) - d \times H(-d - \zeta) \qquad (2)$$

where $H(\cdot)$ is the Heaviside step function and $\zeta(t)$ is elevation of the ideal waves as in Fig. 1. The wave elevation in Eq. (2) is a nonlinear function of the random process $\zeta(t)$. Assuming that $\zeta(t)$ is zero-mean, stationary, and Gaussian (Phillips 1977). The expected mean water surface considering breaking $E[\overline{\zeta}]$ is

$$E[\overline{\zeta}] = E[\zeta \times H(\zeta) \times H(\zeta_b - \zeta)] + \zeta_b$$

$$\times E[H(\zeta - \zeta_b)] + E[\zeta \times H(-\zeta) \times H(d + \zeta)]$$

$$- d \times E[H(-d - \zeta)] \qquad (3)$$

Here, $E[\cdot]$ is the expected value of the quantity in the bracket. Eq. (3) can be transformed (Yang, 1991) into

$$E[\overline{\zeta}] = \sigma[Z(\overline{\eta}) - Z(\overline{\overline{\eta}}) - \overline{\eta}Q(\overline{\eta}) + \overline{\overline{\eta}}Q(\overline{\overline{\eta}})] \qquad (4)$$

where σ is the standard deviation of $\zeta(t)$, $\overline{\eta} \equiv \dfrac{\zeta_b}{\sigma}$ and $\overline{\overline{\eta}} \equiv \dfrac{d}{\sigma}$ respectively. The function $Z(\lambda)$ and $Q(\lambda)$ are defined as

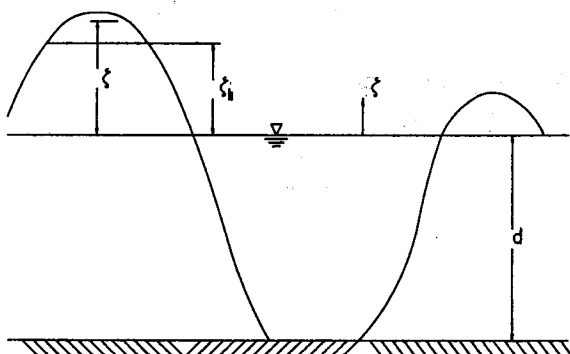$$Z(\lambda) = \frac{1}{(2\pi)^{1/2}} \exp\left[-\frac{\lambda^2}{2}\right] \qquad (5)$$

and

$$Q(\lambda) = \int_\lambda^\infty Z(s)\,ds \qquad (6)$$

The standard deviation of ideal wave elevation at specified water depth is given by

$$\sigma^2 = \int_0^\infty S(\omega)\,d\omega \qquad (7)$$

where $S(\omega)$ is ideal wave spectrum in the specified water depth. In this study, we chose the Wallops spectrum for deep water spectrum. Because it is based on fluid dynamic analysis with no empirical coefficients and correct energy content (Huang et al. 1981). Therefore, the deep water spectrum is

$$S_0(\omega) = \frac{\beta g^2}{\omega^m \omega_0^{5-m}} exp\left[-\frac{m}{4}\left(\frac{\omega_0}{\omega}\right)^4\right] \qquad (8)$$

where β and m are function of the significant slope $\S$ of wave field and ω_0 is the frequency of the waves at spectral peak. The significant slope $\S$ is defined as

$$\S = \frac{M_0^{1/2}}{\lambda_0} \qquad (9)$$

where M_0 is the zero-moment of the Wallops spectrum and $\lambda_0 = \dfrac{2\pi}{k_0}$ is the wave length corresponding to the waves at the spectrum peak. The two parameters, m and β, in the Wallops spectrum then define as

$$m = \frac{\log(\sqrt{2}\,\pi\,\S)^2}{\log 2} \qquad (10)$$

and

$$\beta = \frac{m^{(m-1)/4}}{4^{(m-5)/4}} \frac{(2\pi\S)^2}{\Gamma[\frac{(m-1)}{4}]} \qquad (11)$$

where $\Gamma[\cdot]$ is the Gamma function.

In this study, we consider the beach is infinite long with parallel depth contours monotonically decreasing water depth to the shore ward direction. The incoming waves are long-crested and progressing parallel to the water depth contour. The shoaling spectrum of finite water depth $S(\omega)$ can be obtained from Battjes (1974)

$$S(\omega) = S_0(\omega)\frac{C_{g0}}{C_g} \qquad (12)$$

Fig. 1. Schematic Diagram of Breaking Wave.

where C_g and C_{g0} are group velocities in deep water and finite water depth respectively. Using the approximation of component wave number in deep water k_0 and in the finite water depth k (Tayfun, 1979),

$$k_0 \cong k \tanh^{1/2}(k_0 d) \qquad (13)$$

the shoaling coefficient of wave spectrum can be represented

$$\frac{C_{g0}}{C_g} = \frac{\tanh^{1/2}(k_0 d)}{\tanh(k_0 d) + k_0 d[1 - \tanh(k_0 d)]} \qquad (14)$$

The standard deviation of ideal (fictitious) waves σ can be calculated from Eq. (7), (12) and (14).

Similar to the mean value of breaking wave water surface, the mean-square value of $\bar{\zeta}$ can be given as

$$E[\bar{\zeta}^2] = \sigma^2[1 + (\bar{\eta}^2 - 1) \times Q(\bar{\eta}) + (\bar{\bar{\eta}}^2 - 1)$$

$$\times Q(\bar{\bar{\eta}}) - \bar{\eta} \times Z(\bar{\eta}) - \bar{\bar{\eta}} \times Z(\bar{\bar{\eta}})] \qquad (15)$$

The standard deviation of $\bar{\zeta}$ can be calculated as

$$\sigma_{\bar{\zeta}}^2 = E[\bar{\zeta}^2] - E^2[\bar{\zeta}] \qquad (16)$$

In order to obtain the breaking wave spectrum the auto-correlation function $R_{\bar{\zeta}\bar{\zeta}}$ and covariance function $K_{\bar{\zeta}\bar{\zeta}}$ must be calculated first. Because the $\bar{\zeta}(t)$ is stationary and letting subscripts 1 and 2 refer to time instants $t + \tau$ and t respectively, so

$$R_{\bar{\zeta}\bar{\zeta}}(\tau) = E[\bar{\zeta}_1 \bar{\zeta}_2] = E[(\zeta_1 \times H(\zeta_b - \zeta_1) \times H(\zeta_1)$$

$$+ \zeta_b \times H(\zeta_1 - \zeta_b) + \zeta_1 \times H(d + \zeta_1)$$

$$\times H(-\zeta_1) - d \times H(-d - \zeta_1)) \times (\zeta_2$$

$$\times H(\zeta_b - \zeta_2) \times H(\zeta_2) + \zeta_b \times H(\zeta_2 - \zeta_b)$$

$$+ \zeta_2 \times H(d + \zeta_2) \times H(-\zeta_2) - d$$

$$\times H(-d - \zeta_2))] \qquad (17)$$

Cross-multiplying and taking the expected values on the right hand side of Eq. (17) give rise to sixteen expectations of which, because of symmetry, only ten terms need to be evaluated. While these expected values can all be obtained in a straight forward manner it is expedient to utilize the Hermite polynomial representation of the joint probability density function of the zero-mean Gaussian random variables ζ_1 and ζ_2

$$f_{\zeta_1 \zeta_2}(\zeta_1 \zeta_2) = \frac{1}{2\pi(1 - r^2)^{1/2}\sigma^2}$$

$$EXP[-\frac{1}{2(1-r)^2}[(\frac{\zeta_1}{\sigma})^2 + (\frac{\zeta_2}{\sigma})^2$$

$$-2r(\frac{\zeta_1 \zeta_2}{\sigma^2})]] \qquad (18)$$

where

$$r(\tau) = \frac{E[\zeta_1 \zeta_2]}{\sigma^2} = \frac{R(\tau)}{\sigma^2} \qquad (19)$$

is the correlation coefficient of ζ_1 and ζ_2, $R(t)$ being the auto correlation function of ζ.

From Erdely et al. (1953), the standard joint prob-ability density function $f_{\eta_1 \eta_2}(\eta_1 \eta_2)$ of zero-mean Gaussian random variables η_1 and η_2 may be expressed in term of Hermite polynomial in series form as

$$f_{\eta_1 \eta_2}(\eta_1 \eta_2) = \frac{1}{2\pi(1 - r^2)^{1/2}} EXP[-\frac{1}{2(1-r^2)}$$

$$(\eta_1^2 + \eta_2^2 - 2r\eta_1 \eta_2]] \qquad (20)$$

$$= \sum_{n=1}^{\infty} r^n h_n(\eta_1) h_n(\eta_2) Z(\eta_1) Z(\eta_2) \qquad (21)$$

where

$$h_n(\eta) = \frac{(-1)^2}{(n!)^{1/2}} EXP(\frac{\eta^2}{2}) \frac{d^n}{d\eta^n} EXP(-\frac{\eta^2}{2}) \qquad (22)$$

is the Hermite polynomial. Thus, for example,

$$h_0(\eta) = 1, \quad h_1(\eta) = \eta, \quad h_2(\eta) = (-1 + \eta^2)/\sqrt{2}$$

$$h_3(\eta) = -(3\eta - \eta^3)/\sqrt{6} \quad \text{etc.}$$

Upon substituting $\eta_1 = \frac{\zeta_1}{\sigma}$ and $\eta_2 = \frac{\zeta_2}{\sigma}$ and using the expansion in Eq. (21), all the expected values on the right-hand-side of Eq. (17) may be expressed in terms of the following integrals:

$$I_n(\lambda) = \int_\lambda^\infty h_n(\eta) Z(\eta) d\eta \qquad (23)$$

and

$$J_n(\lambda) = \int_0^\lambda \eta h_n(\eta) Z(\eta) d\eta \qquad (24)$$

These integrals may all carried out and expressed in terms of $Z(\cdot)$ and $Q(\cdot)$ and recursive formula may be derived but these are not shown here. The expected values on the right-hand-side of Eq. (17) also involve integrals of the form

$$\int_{-\infty}^{-\lambda} h_n(\eta) Z(\eta) d\eta = (-1)^n I_n(\lambda) \qquad (25)$$

and

$$\int_{-\lambda}^0 \eta h_n(\eta) Z(\eta) d\eta = (-1)^{n+1} J_n(\lambda) \qquad (26)$$

which are therefore expressible in terms of $Z(\cdot)$ and $Q(\cdot)$.

Thus, for example, the expected value

$$E_1 = E[\zeta_1 \zeta_2 \times H(\zeta_b - \zeta_1) \times H(\zeta_b - \zeta_2) \times H(\zeta_1)$$

$$\times H(\zeta_2)]$$

$$= \int_{\zeta_1=0}^{\zeta_b} \int_{\zeta_2=0}^{\zeta_b} \zeta_1 \zeta_2 f_{\zeta_1 \zeta_2}(\zeta_1 \zeta_2) d\zeta_1 d\zeta_2$$

$$= \sigma^2 \int_{\eta_1=0}^{\bar\eta} \int_{\eta_2=0}^{\bar\eta} \eta_1 \eta_2 f_{\eta_1 \eta_2}(\eta_1 \eta_2) d\eta_1 d\eta_2$$

$$= \sigma^2 \sum_{n=0}^\infty r^n [\int_{\eta=0}^{\bar\eta} \eta h_n(\eta) Z(\eta) d\eta]^2$$

$$= \sigma^2 \sum_{n=0}^\infty r^n J_n^2(\bar\eta) \qquad (27)$$

similarly,

$$E_2 = E[\zeta_1 \times H(\zeta_b - \zeta_1) \times H(\zeta_1) \times H(\zeta_2 - \zeta_b)]$$

$$= \int_{\zeta_1=0}^{\zeta_b} \int_{\zeta_2=\zeta_b}^\infty \zeta_1 f_{\zeta_1 \zeta_2}(\zeta_1 \zeta_2) d\zeta_1 d\zeta_1$$

$$= \sigma \int_{\eta_1=0}^{\bar\eta} \int_{\eta_2=\bar\eta}^\infty \eta_1 f_{\eta_1 \eta_2}(\eta_1 \eta_2) d\eta_1 d\eta_2$$

$$= \sigma \sum_{n=0}^\infty r^n J_n(\bar\eta) I_n(\eta) \qquad (28)$$

The remaining expected values on the right-hand-side of Eq. (17) may be all be written down in a routine manner without effort and will not be shown. The final result is

$$R_{\bar\zeta \bar\zeta}(\tau) = \sigma^2 \sum_{n=0}^\infty r^n (\hat I_n + \hat J_n)^2 \qquad (29)$$

where

$$\hat I_n = \bar\eta I_n(\bar\eta) - (-1)^n \bar\eta I_n(\bar\eta) \qquad (30)$$

and

$$\hat J_n = J_n(\bar\eta) - (-1)^n J_n(\bar\eta) \qquad (31)$$

The covariance function $K_{\bar\zeta \bar\zeta}(\tau)$ of $\bar\zeta$ is

$$K_{\bar\zeta \bar\zeta}(\tau) = R_{\bar\zeta \bar\zeta}(\tau) - E^2[\bar\zeta] \qquad (32)$$

It may be verified that the term corresponding to $n=o$ in Eq. (29) of $R_{\bar\zeta \bar\zeta}(\tau)$ is in fact $E^2[\bar\zeta]$ so that

$$K_{\bar\zeta \bar\zeta}(\tau) = \sigma^2 \sum_{n=1}^\infty r^n (\hat I_n + \hat J_n)^2 \qquad (33)$$

The terms in Eq. (33) converge rapidly so that only the first few terms should suffice. Retaining the first two terms in Eq. (33)

$$K_{\bar\zeta \bar\zeta}(\tau) = \sigma^2 [r[1 - Q(\bar\eta) - Q(\bar\eta)]]^2 + \frac{r^2}{2}$$

$$[Z(\bar\eta) - Z(\bar\eta)]^2] \qquad (34)$$

The approximate spectrum of $\bar\zeta$ is simply the Fourier transform of the approximate $K_{\bar\zeta \bar\zeta}(\tau)$ and is

$$S_{\bar\zeta \bar\zeta}(\omega) \triangleq S(\omega)[1 - Q(\bar\eta) - Q(\eta)]^2 + \frac{S(\omega) * S(\omega)}{\sigma^2}$$

$$\frac{[Z(\bar\eta) - Z(\bar\eta)]^2}{2} \qquad (35)$$

Where ω is frequency, $S(\omega)$ is frequency spectrum of ζ and is the fourier transform of $R(\tau)$ in Eq. (17). The symbol * denotes convolution.

Having found the spectrum $S_{\bar\zeta \bar\zeta}(\omega)$, we may compute

the radiation stress component S_{11} given by Battjes (1974)

$$S_{11} = \rho g \int_0^\infty [\frac{1}{2} + \frac{2kd}{sinh2kd}] S_{\bar{\zeta}\bar{\zeta}}(\omega) \, d\omega \qquad (36)$$

where ρ is density of water. From Eq. (13), the wave number of component wave in the finite water depth k rederived as function of k_o, the component wave number in deep water. Then, applying the dispersion relationship $k_0 = \frac{\omega^2}{g}$ the Eq. (36) becomes function of water depth d and wave frequency ω. The integration of Eq. (36) can be carried out numerically at given water depth.

NUMERICAL RESULTS

The expected values $E[\bar{\zeta}]$ of $\bar{\zeta}$ are plotted as a function of water depth d for $\omega_0 = 0.981, 0.654, 0.4905$ Hz and § = 0.008 in Fig. 2. Similar to the result of Yang (1991), the expected values $E[\bar{\zeta}]$ is non-positive in the whole range of water depth. This result can be explained by referring Eq. (3). It can be seen that in deep water breaking waves affected by the water depth is negligible, there the variation of $\bar{\zeta}$ from ζ is due to breaking wave in the range of $\zeta > \zeta_b$ so that the expected value of $\bar{\zeta}$ is small negative value called wave set-down. As wave propagate shoreward, magnitude of the expected value of $E[\bar{\zeta}]$ increase due to vigorous breaking. After breaking inception, $E[\bar{\zeta}]$ vanish as water depth approaches to zero. The expected water surface elevation in the surf zone is called wave set-up.

As mention above, we choose the Wallops spectrum as the deep water spectrum. Therefore, the variance of ideal wave

$$\sigma^2 = \int S(\omega) \, d\omega = \int S_o(\omega) \frac{C_{go}(\omega)}{C_g(\omega)} d\omega \qquad (37)$$
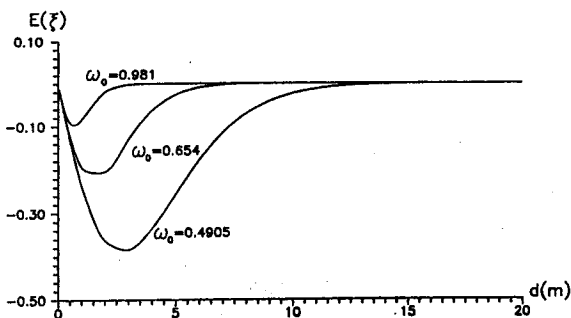
The variances $\sigma^2_{\bar{\zeta}}$ and σ^2 are plotted as function of water 10832 and frequency of the waves at spectral peak $\omega_0 = 0.981, 0.654$, and 0.4905 Hz in Fig. 3. The variances are increasing as water depth decreasing due to shoaling. The variance of ideal (fictitious) wave goes to infinite and $\sigma^2_{\bar{\zeta}}$ diminishes because of breaking. The breaking points are corresponding to the curves of slope equal to zero in Fig. 2 and 3.

The breaking spectra $S_{\bar{\zeta}\bar{\zeta}}(\omega)$ from Eq. (35) are plotted for $\xi = 0.008$ and $\omega_0 = 0.981, 0.654$, and 0.4505 Hz in Fig. 4, 5, and 6 respectively. These spectra are given at specified water depth $d = 20, 5, 1$, and 0.5m. Since the wave energy dissipated at the surf zone, it is seen that the spectra vanishes at the water depth less than the depth of breaking points. Also, the main energy of breaking spectra move to low frequency as Hatori et al., (1981).

In Fig.7 the radiation stress S_{11} calculated from Eq.(36) are presented as functions of water depth for $\omega_o = 0.981, 0.654$, and 0.4509 respectively. The radiation stress obtained by Battjes (1974)

$$S_{11} = [1 - (2\pi)^{1/2} Z(\sqrt{2}\,\bar{\eta})] S_{11f} \qquad (37)$$

and

$$S_{11f} = \rho g \sigma^2 (2\bar{\bar{n}} - \frac{1}{2}) \qquad (38)$$

$$\bar{\bar{n}} = [\int_0^\infty [\frac{1}{2} + \frac{kd}{sinh(2kd)}] S(\omega) d\omega]/\sigma^2 \qquad (39)$$

also plotted in Fig. 7. The variations of radiation stress for Battjes (1974) and this study are due to the wide-band breaking model used in this study. By the mean time, the maghitude of these differences are increasing as energy spectra increasing. By the definition of the radiation stress

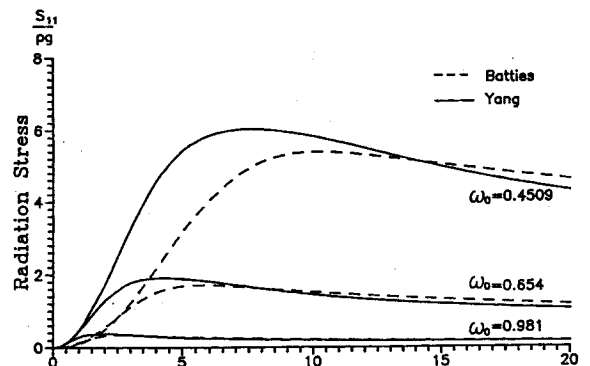$$\frac{d\bar{\zeta}}{dx} = -\frac{1}{\rho g(h + \bar{\zeta})} \frac{dS_{11}}{dx} \qquad (40)$$

Fig. 2. Expected Values $E(\bar{\zeta})$ of $\bar{\zeta}$ as Function of Water Depth.

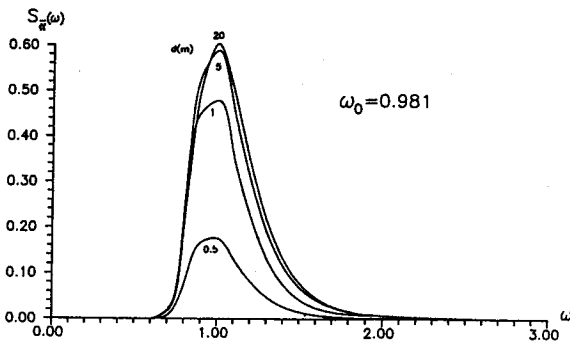Fig. 3. Variances $\sigma^2_{\bar{\zeta}}$ and σ^2 of $\bar{\zeta}$ and ζ as Function of Water Depth.

Fig. 4. Spectra $S_{\bar{\zeta}\bar{\zeta}}(\omega)$ Water Depth $d = 20, 5, 1, 0.5$ m at $w_0 = 0.981$.
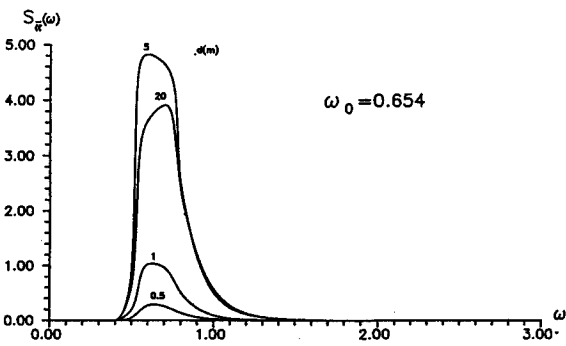

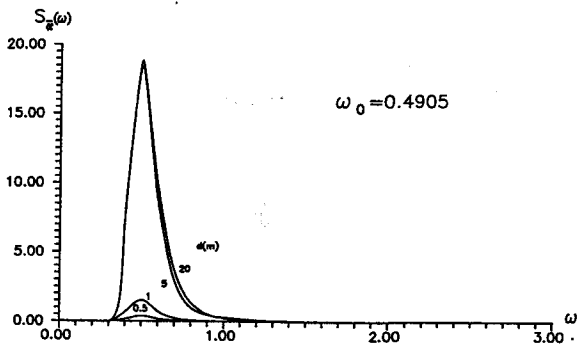Fig. 5. Spectra $S_{\bar{\zeta}\bar{\zeta}}(\omega)$ Water Depth $d = 20, 5, 1, 0.5$ m at $w_0 = 0.654$.

Fig. 6. Spectra $S_{\bar{\zeta}\bar{\zeta}}(\omega)$ Water Depth $d = 20, 5, 1, 0.5$ m at $w_0 = 0.4905$.

it increase when the wave propagate from deep water to shallow water, the expected value of $\bar{\zeta}$ decrease, the water surface is set-down as Fig. 2. After breaking taking place, the wave energy is dissipated, therefore, radiation stress decrease. This is the wave set-up which is the mean water surface elevation in the surf zone.

CONCLUSION AND DISCUSSION

The breaking wave criterion for wave of finite band-width is establish (Eq. 1) by modifying the narrow-band breaking wave criterion used in Battjes
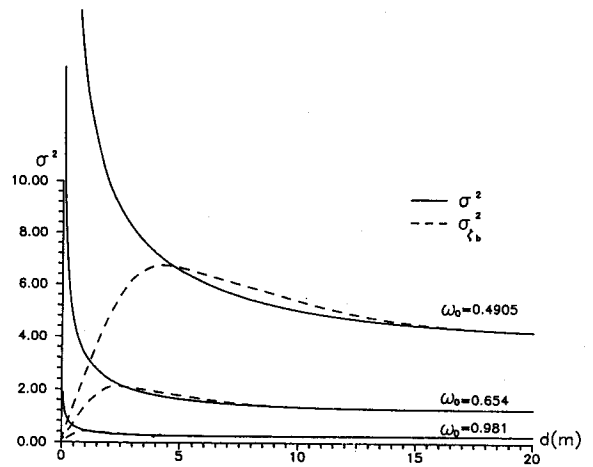

Fig 7. Radiation Stress S_{11} as Function of Water Depth.

(1974) and the expression of component wave-number in deep water in terms of its in water of finite depth (Tayfun, 1979). Base on the breaking wave criterion, a model for breaking waves is proposed (Eq. 2). Assuming the ideal waves are stationary, zero-mean, and Gaussian (Pillips, 1977), the breaking wave spectrum is obtained (Eq. 35). It is noted that the spectrum $S_{\bar{\zeta}\bar{\zeta}}(\omega)$ of broken waves $\bar{\zeta}$ as given in Eq. (35) consists of two parts. Similar to the results of Yang (1991), the second part is negligible small compare with the first part so that $S_{\bar{\zeta}\bar{\zeta}}(\omega)$ is essentially given by

$$S_{\bar{\zeta}\bar{\zeta}}(\omega) = S(\omega)[1 - Q(\bar{\eta}) - Q(\bar{\bar{\eta}})]^2$$

This mean that wave breaking reduces the spectrum $S(\omega)$ of the ideal wave ζ by the factor $[1 - Q(\bar{\eta}) - Q(\bar{\bar{\eta}})]^2$.

The model can be extended as the presence of current. The wave-current interaction in an opposing direction, the wave amplitude increase and wave-length decrease (Longuet-Higgins *et al.*, 1961). As these circumstances, the sea surface becomes choppier and vigorous breaking takes place which increase the spectrum in high frequency (Huang *et al.* 1972). Wave breaking limits wave growth and reduces the spectral ordinates that evolves wave component into low frequency and has a similar spectral structure (Hatori, *et al.*, 1981).

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NOTATION

a_b	Amplitude of breaking wave
C_{go}, C_g	Group velocity in deep water and water of finite depth respectively
d	Water depth
$E[\cdot]$	Expected value
E_1, E_2	Expected values computed in Eqs.(25) and (26)
$f_\zeta(\cdot)$	Probability density function of zero-mean Gaussian process
$f_{\zeta_1\zeta_2}(\cdot,\cdot)$	Joint probability density function of zero-mean Gaussian processes ζ_1 and ζ_2
$f_{\eta_1\eta_2}(\cdot,\cdot)$	Joint probability density function of zero-mean Gaussian processes η_1 and η_2 with unit standard deviation
g	Gravitational acceleration
$H(\cdot)$	Heaviside step function
$h_\eta(\cdot)$	Hermite polynomial
$K_{\bar\zeta\bar\zeta}(\tau)$	Covariance function of $\bar\zeta$
k	Wave-number
k_o	Wave-number in deep water
$\bar k$	$\dfrac{\bar\omega_0}{g}$
n	Integer index
$\bar n$	Quantity defined in Eq.(43)
$Q(\cdot)$	Function defined in Eq.(12)
$R(\cdot)$	Autocorrelation function of ζ (see Eq.(18))
$R_{\bar\zeta\bar\zeta}(\tau)$	Autocorrelation function of $\bar\zeta$
$r(\tau)$	Correlation coefficient of ζ
$S(\omega)$	Spectrum of ζ
$S_{\bar\zeta\bar\zeta}(\omega)$	Spectrum of $\bar\zeta$
$S_o(\omega)$	Spectrum of ζ in deep water (see Eq.(34))
S_{11}	Radiation stress component (see Eqs.(40) and (41))
S_{1f}	Radiation stress of fictitious wave ζ (see Eq. (42))
t	Time
U	Wind speed
$Z(\cdot)$	Standard zero-mean Gaussian probability density function
α	Nondimensional constant in Eq.(34)
β	Nondimensional constant in Eq.(34)
ζ	Elevation of ideal wave
ζ_b	Breaking limit of wave elevation
ζ_1, ζ_2	Elevation of ideal wave at time instants $t+\tau$ and t
$\bar\zeta$	Elevation of breaking wave
$\bar\zeta_1, \bar\zeta_2$	Elevation of breaking wave at time instants $t+\tau$ and t
η	Dummy variable
η_1, η_2	Dummy variable; zero-mean Gaussian processes with unit standard deviation at time instants $t+\tau$ and t
$\bar\eta$	$\dfrac{\zeta_b}{\sigma}$
$\bar{\bar\eta}$	$\dfrac{d}{\sigma}$
λ	Dummy variable
ρ	Density of water
σ	Standard deviation of ζ
$\sigma_{\bar\zeta}$	Standard deviation of $\bar\zeta$
τ	Time lag
ω	Radian frequency
$\bar\omega$	$\dfrac{g}{U}$
$\bar\omega_0$	Mean wave-frequency in deep water
$I_n(\cdot), J_n(\cdot)$	Quantities defined in Eqs.(21) and (22)
$I_n(\cdot), J_n(\cdot)$	Quantities defined in Eqs.(28) and (29)