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Key words: MHD squeeze films, electrically conducting fluids, non-Newtonian couple stresses, curved circular plates.

ABSTRACT

The squeeze film performances between curved circular plates lubricated with an electrically conducting non-Newtonian fluid in the presence of external magnetic fields are investigated in this paper. Based upon the magneto-hydrodynamic flow theory together with the Stokes microcontinuum theory, the magneto-hydrodynamic non-Newtonian Reynolds equation is derived and applied to predict the curved circular squeeze film behaviors. Comparing with the hydrodynamic Newtonian case, the squeeze film characteristics for curved circular plates are improved by the use of an electrically conducting non-Newtonian fluid in the presence of external magnetic fields. Numerical values of the load capacity and the approaching time are provided in Tables for engineering applications.

I. INTRODUCTION

Squeeze film technology plays an important role in many areas of engineering practice and applied science. In order to prevent the unexpected variation of lubricant viscosity due to the change of temperature, the increasing use of an electrically conducting fluid as the lubricant has become of interest. In the presence of an external magnetic field, the load-carrying capacity of magneto-hydrodynamic (MHD) squeeze films is increased. MHD squeeze film performances are then improved as compared to the conventional non-conducting case. Several studies have investigated the MHD effects on the lubrication performances of the journal bearings by Malik and Singh [9], the slider bearings by Lin [7], and the squeeze film bearings by Bujurke and Kudenatti [3], Usha and Vimala [14] and Lin *et al*. [8]. On the other hand, the presence of additives is found to ha beneficial effects on the load capacity and bearing characteristics by the experimental works by Oliver [12]. Some non-Newtonian microcontinuum theories have been developed to describe the intrinsic motion of particle constituents, such as the simple microfluid model by Eringen [5] and the couple stress fluid model by Stokes [13]. In these theories, the microcontinuum theory of couple stress fluids developed by Stokes [13] presents the elegant generation from the traditional Newtonian theory. Several articles have applied this non-Newtonian (NN) model of couple stress fluids to investigate various squeeze film systems, such as the squeeze film mechanism with reference to human joints by Ahmad and Singh [1] and Bujurke and Jayaraman [2], the sphere-plate film by El-sharkawy and AL-Fadhalah [4], and the circular stepped plates by Naduvinamani and Siddangouda [11]. According to their results, the NN influences of couple stresses increase the load capacity and lengthen the response time for squeeze films. From the above studies, when an electrically conducting fluid is mixed with small amount of long-chained additives, the NN effects of couple stresses would appear in squeeze films. According to the study on squeeze film plates with a non-conducting lubricant by Murti [10], the curved circular mechanism is important for engineering applications. Therefore, a further investigation is motivated in the present study.

In the present study, the squeeze film performances between curved circular plates lubricated with an electrically conducting non-Newtonian fluid in the presence of external magnetic fields are mainly concerned. Based upon the magneto-hydrodynamic flow theory incorporating the Stokes microcontinuum theory, the modified Reynolds equation is derived and applied to evaluate the squeeze film behaviors. Comparing with the hydrodynamic Newtonian (HN) case, the combined magneto-hydrodynamic non-Newtonian (MHNN) effects on the squeeze film characteristics are described for different values of the magnetic Hartmann parameter, the non-Newtonian couple-stress parameter and the shape parameter of curved circular disks.

II. ANALYSIS

Fig. 1 describes the squeeze film geometry between curved

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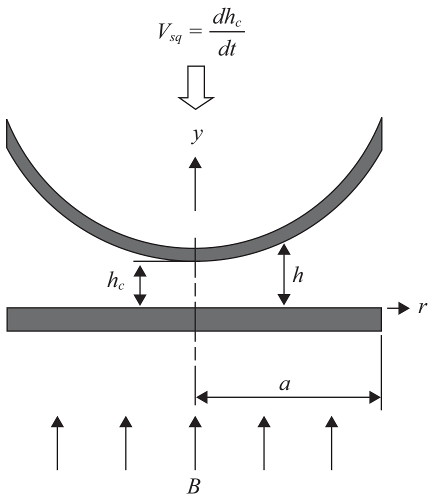

Fig. 1. Magneto-hydromagnetic non-Newtonian squeeze film geometry between exponential curved plates in the presence of a transversely uniform magnetic field.

circular plates of radius a lubricated with an electrically conducting non-Newtonian couple-stress fluid. An external magnetic field B is applied in the y – direction. The film shape h is taken to be an exponential type as Murti [10].

$$h = h_c \exp(-kr^2 / a^2) \tag{1}$$

where h_c is the central minimum film thickness and k is the curved shape parameter. Assume that the thin film lubrication theory as Hamrock [6] is applicable and the induced magnetic field is small as compared to the induced magnetic field is small when comparing with the applied magnetic field. Following the MHD flow equations of Lin [7] and the Stokes microcontinuum theory [13], the magneto-hydrodynamic non-Newtonian couple-stress momentum equations and the continuity equation can be expressed in axially cylindrical coordinates as follows.

$$\frac{\partial p}{\partial r} = \mu \frac{\partial^2 u}{\partial y^2} - \eta_c \frac{\partial^4 u}{\partial y^4} - \sigma B^2 u \tag{2}$$

$$\frac{\partial p}{\partial y} = 0 \tag{3}$$

$$\frac{1}{r}\frac{\partial (ru)}{\partial r} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

where p is the hydrodynamic film pressure, u and v are the velocity components in the r – and y – directions respectively, μ is the lubricant viscosity, σ denotes the electrical conductivity of the lubricant, and η_c represents a material constant responsible for the non-Newtonian couple stress fluid. The velocity boundary conditions are

$$u\big|_{y=0} = 0,\ u\big|_{y=h} = 0 \tag{5}$$

$$v\big|_{y=0} = 0,\ v\big|_{y=h} = \frac{dh_c}{dt} \tag{6}$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = 0,\ \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=h} = 0 \tag{7}$$

The conditions in Eqs. (5) and (6) are the conventional non-slip conditions. The conditions in Eq. (7) come from the vanishing of couple stresses at the solid boundary [13]. Applying the boundary conditions (5) and (7), the velocity component in the r – direction is obtained by integrating Eq. (2).

$$u = -\frac{1}{\mu}\frac{\partial p}{\partial r}\frac{{h_{c0}}^2}{M^2({\alpha_1}^2 - {\alpha_2}^2)} \cdot \left\{ \begin{array}{l} {\alpha_1}^2 - {\alpha_2}^2 - {\alpha_1}^2 \dfrac{\sinh(\alpha_2 y) + \sinh[\alpha_2(h-y)]}{\sinh(\alpha_2 h)} \\ + {\alpha_2}^2 \dfrac{\sinh(\alpha_1 y) + \sinh[\alpha_1(h-y)]}{\sinh(\alpha_1 h)} \end{array} \right\} \tag{8}$$

where

$$\alpha_1 = \sqrt{1 + \sqrt{1 - 4l^2 M^2 / {h_{c0}}^2}} \,/ \left(\sqrt{2}l\right) \tag{9}$$

$$\alpha_2 = \sqrt{1 - \sqrt{1 - 4l^2 M^2 / {h_{c0}}^2}} \,/ \left(\sqrt{2}l\right) \tag{10}$$

$$l = \sqrt{\eta_c / \mu} \tag{11}$$

$$M = B h_{c0} \sqrt{\sigma / \mu} \tag{12}$$

In addition, h_{c0} denotes the initial central film height, and M describes the Hartmann parameter measuring the strength of applied magnetic field. Integrating the continuity Eq. (4) across the film thickness gives

$$\frac{1}{r}\int_{y=0}^{h} \frac{\partial (ru)}{\partial r} dy = -\int_{y=0}^{h} \frac{\partial v}{\partial y} dy \tag{13}$$

Substituting the expression of the velocity component (8) and applying the boundary conditions (5) and (6), the MHD non-Newtonian Reynolds equation for the curved circular squeeze film plates can be derived.

$$\frac{1}{r}\frac{d}{dr}\left\{ f(M, l, h) r \frac{dp}{dr} \right\} = 12\mu \frac{dh_c}{dt} \tag{14}$$

where

$$f(M,l,h)=\frac{12h_{c0}{}^{2}h}{M^{2}}$$

$$\cdot\left[1-\frac{2\alpha_1{}^3\tanh(0.5\alpha_2 h)-2\alpha_2{}^3\tanh(0.5\alpha_1 h)}{\alpha_1\alpha_2(\alpha_1{}^2-\alpha_2{}^2)h}\right] \tag{15}$$

The film pressure can be obtained by solving this MHD non-Newtonian Reynolds equation. Therefore, the squeeze film characteristics can be predicted.

III. MHD NON-NEWTONIAN CHARACTERISTICS

To analyze the squeeze film characteristics, the MHD non-Newtonian Reynolds equation is expressed in a non-dimensional form.

$$\frac{1}{r^*}\frac{d}{dr^*}\left\{f^*(M,N,h^*)r^*\frac{dp^*}{dr^*}\right\}=-12 \tag{16}$$

where

$$f^*(M,N,h^*)=\frac{12h^*}{M^2}$$

$$\cdot\left[1-\frac{2\beta_1{}^3\tanh(0.5\beta_2 h^*)-2\beta_2{}^3\tanh(0.5\beta_1 h^*)}{\beta_1\beta_2(\beta_1{}^2-\beta_2{}^2)h^*}\right] \tag{17}$$

$$h^*=\frac{h}{h_{c0}}=h_c{}^*\exp(-kr^{*2}) \tag{18}$$

$$h_c{}^*=\frac{h_c}{h_{c0}},\ r^*=\frac{r}{a},\ p^*=\frac{ph_{c0}{}^3}{\mu a^2(-dh_c/dt)} \tag{19}$$

$$\beta_1=\sqrt{1+\sqrt{1-4M^2N^2}}\,/\left(\sqrt{2}N\right) \tag{20}$$

$$\beta_2=\sqrt{1-\sqrt{1-4M^2N^2}}\,/\left(\sqrt{2}N\right) \tag{21}$$

$$N=l/h_{c0}, \tag{22}$$

Since the dimension of l defined in Eq. (11) is of length, it can be identified as the characteristic material length of the suspended particles. Therefore, the non-Newtonian influences on the squeeze film are characterized by the non-Newtonian parameter of couple stresses, N.

The non-dimensional film pressure can be derived by solving the MHD non-Newtonian Reynolds Eq. (16) subject to the boundary conditions: $dp^*/dr^* = 0$ at $r^* = 0$, and $p^* = 0$ at $r^* = 1$.

$$p^*=6\int_{r^*=r^*}^{r^*=1}\frac{r^*}{f^*(M,N,h^*)}dr^* \tag{23}$$

Integrating the film pressure over the film region, the load-carrying capacity is obtained.

$$W=2\pi\int_{r=0}^{a}prdr \tag{24}$$

Performing the integration, the non-dimensional load-carrying capacity is then expressed as the follows.

$$W^*=\frac{Wh_{c0}{}^3}{\mu a^4(-dh_c/dt)}=6\pi\int_{r^*=0}^{1}\frac{r^{*3}}{f^*(M,N,h^*)}dr^* \tag{25}$$

Introduce a non-dimensional approaching time for the curved circular squeeze film.

$$t^*=\frac{Wh_{c0}{}^2}{\mu a^4}t \tag{26}$$

Then the differential equation governing the film thickness changing with the approaching time can be derived from Eq. (25).

$$\frac{dh_c{}^*}{dt^*}=-\left[6\pi\int_{r^*=0}^{1}\frac{r^{*3}}{f^*(M,N,h^*)}dr^*\right]^{-1} \tag{27}$$

Integrating the differential equation and applying the initial condition $h_c^*(t^* = 0) = 1$, the non-dimensional approaching time for the squeeze film is obtained.

$$t^*=\int_{h_c^*}^{h_c^*=1}\int_{r^*=0}^{1}\frac{r^{*3}}{f^*(M,N,h^*)}dr^*dh_c{}^* \tag{28}$$

Although the values of the squeeze film pressure (23), load-carrying capacity (25) and approaching time (28) cannot be calculated by direct integration, they could be numerically obtained from the Gaussian Quadrature Method.

IV. RESULTS AND DISCUSSION

According to the present study, some special cases in the literature can be obtained from specific values of the Hartmann parameter measuring the strength of applied magnetic field M, the non-Newtonian couple-stress parameter N, and the curved shape parameter k.

Case 1: $M \to 0$, $N \to 0$, $k \to 0$, it is the HN (hydrodynamic Newtonian) case of parallel circular squeeze films. The non-dimensional function derived in the Reynolds Eq. (16) reduces to the identical results by Hamrock [6].

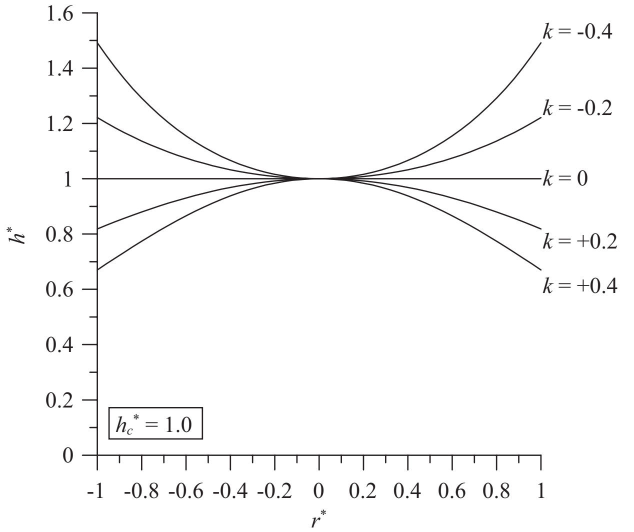

Fig. 2. Squeeze film shapes of exponential curved plates for different values of the shape parameter *k*.

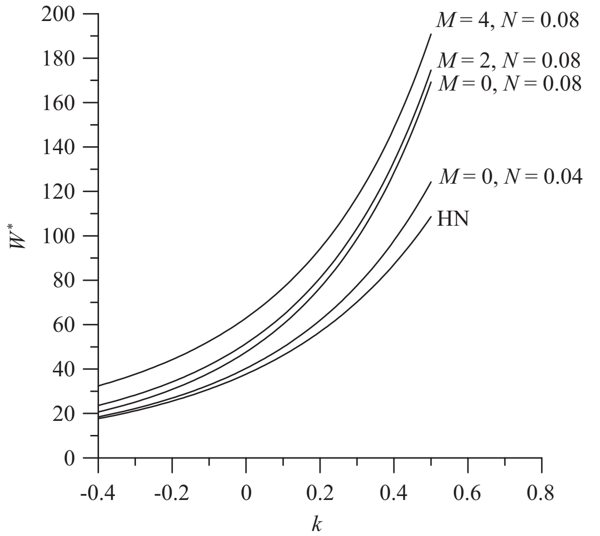

Fig. 3. Non-dimensional load capacity W^* as a function of *k* under h_c^* = 0.5.

$$\lim_{M\to 0, N\to 0, k\to 0} f^* = h^{*3} : h^* \text{ is independent of } r^* \tag{29}$$

Case 2: $M \to 0$, $N \to 0$, $k \neq 0$, it is the HN (hydrodynamic Newtonian) case of curved circular squeeze films. The non-dimensional function f^* reduces to the one described by Murti [10], where the non-dimensional film shape is described in Eq. (18).

$$\lim_{M\to 0, N\to 0, k\neq 0} f^* = h^{*3} : h^* \text{ dependent on } r^* \tag{30}$$

Case 3: $M \neq 0$, $N \to 0$, $k \neq 0$, it is the MHD case of curved circular squeeze films. The reduced non-dimensional function f^* is the same as the one derived by Lin *et al*. [8], in which the curved annular plates using Eq. (18) are investigated.

$$\lim_{M\neq 0, N\to 0, k\neq 0} f^* = \frac{12Mh^* - 24\tanh(0.5Hh^*)}{M^3} \tag{31}$$

In the present study ($M \neq 0$, $N \neq 0$, $k \neq 0$), the MHNN (magneto-hydrodynamic non-Newtonian) effects on the curved circular squeeze film characteristics are considered. Fig. 2 shows the squeeze film shapes of exponential curved circular plates for different values of the shape parameter k. For $k < 0$, convex films are obtained. For $k > 0$, concave films are generated. For $k = 0$, the geometry between parallel circular plates is recovered as Hamrock [6].

Fig. 3 shows the non-dimensional load capacity W^* as a function of k under $h_c^* = 0.5$. It is observed that the load capacity increases with increasing values of the curved shape parameter k. In other words, the values of the load capacity for convex films ($k < 0$) are smaller than those of the concave films ($k > 0$). Comparing with the HN (hydrodynamic Newtonian) case, the non-Newtonian influences ($M = 0$, $N = 0.04$) result in a higher load capacity. Increasing values of the couple-stress parameter ($M = 0$, $N = 0.08$) increases the non-Newtonian effects on the load capacity. By the use of an electrically conducting non-Newtonian couple-stress fluid ($M = 2$, $N = 0.08$; $M = 4$, $N = 0.08$), the increments of the load capacity are enlarged. Totally, the MHNN (magneto-hydrodynamic non-Newtonian) effects on the squeeze film load are further emphasized for a larger value of the Hartmann parameter, the non-Newtonian parameter, and the curved shape parameter. Fig. 4 describes the non-dimensional response time t^* as a function of k under $h_c^* = 0.5$. The curved circular squeeze film lubricated with an electrically conducting non-Newtonian

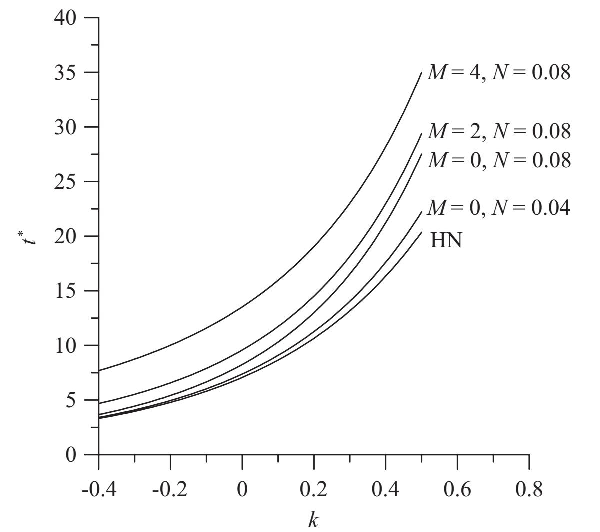

Fig. 4. Non-dimensional response time t^* as a function of *k* under h_c^* = 0.5.

Table 1. Comparison of the MHNN response time t^* with the HN results under h_c^* = 0.6.

	HN results	Present study: MHNN response time t^*			
	Murti [10]	M = 0, N = 0	M = 1, N = 0	M = 1, N = 0.02	M = 1, N = 0.04
k = -0.2 (convex)	2.8368	2.8368	3.0475	3.0670	3.1228
k = 0 (circular)	4.1888	4.1888	4.4293	4.4660	4.5703
k = +0.2 (concave)	6.3100	6.3100	33.7627	34.3120	35.8663

Table 2. Comparison of the MHNN load capacity W^* with the HN results under h_c^* = 0.6.

	HN results	Present study: MHNN load capacity W^*			
	Murti [10]	M = 0, N = 0	M = 1, N = 0	M = 1, N = 0.02	M = 1, N = 0.04
k = -0.2 (convex)	14.7749	14.7749	15.4626	15.6104	16.0305
k = 0 (circular)	21.8166	21.8166	22.6017	22.8790	23.6655
k = +0.2 (concave)	32.8646	32.8646	33.7627	34.3120	35.8663

fluid effects are observed to provide an increase in the response time. Since MHNN effects (M = 2, N = 0.08; M = 4, N = 0.08) result in a higher load-carrying capacity, a higher film thickness would be attained for the same time to be taken as compared to the HN case. Therefore, a longer response time is predicted for the MHD non-Newtonian squeeze film. On the whole, the magneto-hydrodynamic non-Newtonian effects on the squeeze film behavior between curved circular plates are apparent. The response times of curved circular squeeze films are significantly lengthened by the use of an electrically conducting non-Newtonian fluid in the presence of external magnetic fields.

Tables 1 and 2 illustrate the MHNN load capacity and the MHNN approaching time of curved circular squeeze films under h_c^* = 0.5. The HN results of Murti [10] are also included for comparison. For the parameters under M = 0 and N = 0, the calculated values of the load capacity and the approaching time of the present study agree well with the HN case. It is also observed that the MHNN effects provide increased values of W^* and t^*. The squeeze film characteristics for curved circular plates are improved by the use of an electrically conducting non-Newtonian fluid in the presence of external magnetic fields as compared to the case of a conventional hydrodynamic Newtonian squeeze film.

V. CONCLUSIONS

On the ground of the magneto-hydrodynamic flow theory incorporating the Stokes microcontinuum theory, the curved circular squeeze-film plates lubricated with an electrically conducting non-Newtonian fluid in the presence of external magnetic fields have been presented. Analytical expressions of the film pressure have been derived from the magneto-hydrodynamic non-Newtonian Reynolds equation. Comparing with the conventional hydrodynamic Newtonian case, the curved circular squeeze film characteristics are improved by the use of an electrically conducting non-Newtonian fluid in the presence of external magnetic fields. Calculated values of the load capacity and the approaching time are included in Tables for engineering application.

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