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ANALYSIS OF ROLLED PIECE DEFORMATION IN STRAIGHTENING PROCESS USING FM-BEM

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Key words: straightening process, plate bending problem, inverse problem, fast multipole boundary element method (FM-BEM), neutral layer offset.

ABSTRACT

Rolled piece deformation in a straightening process is regarded as an inverse problem of the plate bending problem. The straightening force was obtained accurately by applying the fast multipole boundary element method (FM-BEM) to the plate bending problem. Furthermore, the straightening process of a nine-roller straightening roller system with a variable roller distance was simulated using numerical analysis, and a function describing the neutral layer offset was obtained from an analysis of FM-BEM results. In this paper, the principle stress values and neutral layer offset of the microunits in the tension and compression regions are discussed. It is concluded that the neutral layer offset has considerable influence on the straightening force. A neutral layer offset theory that provides a theoretical basis for accurate straightening is presented.

I. INTRODUCTION

A straightening machine plays a crucial role in the plate production process, and it is mainly used to eliminate plate defects such as edge waves introduced in the rolling and heating process. The straightening process ensures that the flatness of plates meets the national standard. For improving the plate quality precision, it is necessary to investigate rolled piece deformation caused by the straightening machine and improve the straightening efficiency. In engineering, the finite element method (FEM) is used to analyze rolled piece deformation and simulate the straightening process [17, 26, 27]. However, the personal error is large because the straightening force is specified on the basis of experience. Rolled piece deformation is crucial in the straightening process and the direct object of straightening effects. Therefore, the cause of the error produced by rolled piece deformation must be determined.

The fast multipole boundary element method (FM-BEM) [20] is a new numerical calculation method that has shorter calculation times and higher accuracy than the FEM. This method is widely applied in engineering. For example, the contact stress distribution during the hydrobulging assembly process was described quantitatively by using the FM-BEM in Ref. [16], and the distribution shows that the sleeve structure assembled using even interference and an elastic joint is not reasonable. In Ref. [5], the three-dimensional elastic contact FM-BEM was used for determining the traction distributions in the press-down screw pairs of a 3500 heavy plate mill. The pressure and friction distributions between the contact surface in the radial direction were given, and the advantages in computer memory size provided by the algorithm under the same conditions were illustrated. In Ref. [25], the fast multipole method based on the Taylor expansion was combined with the BEM to determine thin plate and shell structures in three-dimensional elastostatic problems. The study showed that, for analyzing thin structures, the FM-BEM was considerably more efficient than the conventional BEM. The accuracy achieved was sufficient for engineering applications. However, no research reports on the straightening machine that involve the FM-BEM can be found.

According to the classical theory of the straightening process, the neutral layer of a material during bending coincides with the middle layer of the material in the thickness direction. However, there exists an offset deviation between the neutral layer and the middle layer in the practical straightening process. In this study, the rolled piece deformation in the straightening process was regarded as the inverse problem of the plate bending problem. In Section 2, the boundary integral equation (BIE) for the plate bending problem is established, and the straightening force is obtained for the inverse problem of

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FM-BEM. Section 3 presents an FM-BEM-based analysis of the straightening process, a fundamental solution satisfying the requirement of the fast multipole method, and the discretization of the BIE. Section 4 provides an analysis of the forces acting on the microunits in the tension and compression regions as well as the principal stress values and neutral layer offset formula. Section 5 presents an FM-BEM-based analysis of a nine-roller straightening roller system for a variable roller distance; in addition, a neutral layer offset theory based on the results of the FM-BEM analysis is presented along with a function for calculating the neutral layer offset. Through numerical analysis, it is shown that the neutral layer offset has a considerable influence on the straightening force.

II. BOUNDARY INTEGRAL EQUATION FOR STRAIGHTENING PROCESS

The rolled piece deformation in the straightening process is regarded as an inverse problem of the plate bending problem, and this problem is solved to obtain the bending deflection, which applied a vertical force b on plate surface. The straightening process is opposite to the plate bending process, and therefore, it is regarded as the inverse problem of the plate bending problem. The BIE for the straightening process can be obtained by using the established BIE for the plate bending problem.

1. Control Equation for Plate Bending

Suppose that Γ is the boundary of the plate, Ω is the domain of the plate, and w denotes the deflection of the plate surface. The control equation for the plate bending problem can then be written as

$$-D\nabla^4 w + b = 0 \tag{1}$$

where $D = \dfrac{Eh}{12(1-\nu^2)}$ denotes the bending rigidity of the plate, E and ν denote the elastic modulus and Poisson ratio respectively, and b denotes the vertical force.

The boundary conditions for the plate can be written as

$$\text{In } \Gamma_1: \begin{cases} w-\bar{w}=0 \\ \beta_n-\bar{\beta}_n=0 \\ \beta_s-\bar{\beta}_s=0 \end{cases} \qquad \text{In } \Gamma_2: \begin{cases} q-\bar{q}=0 \\ m_n-\bar{m}_n=0 \\ m_s-\bar{m}_s=0 \end{cases} \tag{2}$$

where β_n and β_s denote the normal angle and tangential angle at an arbitrary point on the boundary, respectively; $\bar{\beta}_n$, and $\bar{\beta}_s$ are the known quantities of β_n and β_s; m_n and m_s denote the bending moment and torsional moment in a cross section of the plate, respectively; $\bar{m}_n$ and $\bar{m}_s$ are the known quantities of m_n and m_s; q is the shear force; and $\bar{q}$ is the known quantity of q.

2. Fundamental Solution for Plate Bending

The fundamental solution of the equation for plate bending can be written as

$$-D\nabla^4 w^* + \Delta(\xi, x) = 0 \tag{3}$$

where w^* denotes the deflection at an arbitrary point on the plate surface when a unit concentrated force acts on an infinite thin plate in the direction of x^3 at point ξ.

The fundamental solution can be written as

$$w^* = \frac{r^2}{8\pi D}\ln r \tag{4}$$

where r is the distance between x and ξ.

3. Boundary Integral Equation for Plate Bending

The weighted residual method is used to solve the boundary value problems in (1) and (2), which can be written in integral form as follows:

$$\begin{aligned} &\int_\Omega \left(\frac{\partial q_1}{\partial x_1}+\frac{\partial q_2}{\partial x_2}+b\right)w^* d\Omega \\ &= \int_{\Gamma_2}[(m_n-\bar{m}_n)\beta_n^* + (m_s-\bar{m}_s)\beta_s^* + (q-\bar{q})w^*]\,d\Gamma \\ &\quad + \int_{\Gamma_1}[(\beta_n-\bar{\beta}_n)m_n^* + (\beta_s-\bar{\beta}_s)m_s^* + (w-\bar{w})q^*]\,d\Gamma \end{aligned} \tag{5}$$

Eq. (5) can be written as follows by using integration by parts:

$$\begin{aligned} -w_i + \int_\Omega bw^* d\Omega &= \int_\Gamma (m_n\beta_n^* + m_s\beta_s^* + qw^*)\,d\Gamma \\ &\quad + \int_\Gamma (\beta_n m_n^* + \beta_s m_s^* + wq^*)\,d\Gamma \end{aligned} \tag{6}$$

where $q_1^* = \dfrac{\partial m_{11}^*}{\partial x_1} + \dfrac{\partial m_{12}^*}{\partial x_2}$, $q_2^* = \dfrac{\partial m_{12}^*}{\partial x_1} + \dfrac{\partial m_{22}^*}{\partial x_2}$

Therefore, the BIE can be written as

$$\begin{aligned} \int_\Omega bw^* d\Omega &= Cw_i + \int_\Gamma (m_n^*\beta_n + t^* w)\,d\Gamma + \int_\Gamma \left(\frac{\partial m_s^*}{\partial s}w + m_s^*\frac{\partial w}{\partial s}\right)d\Gamma \\ &\quad - \int_\Gamma (m_n\beta_n^* + tw^*)\,d\Gamma - \int_\Gamma \left(\frac{\partial m_s}{\partial s}w^* + m_s\frac{\partial w^*}{\partial s}\right)d\Gamma \end{aligned} \tag{7}$$

where $C = \begin{cases} 1 & \text{interior} \\ \dfrac{1}{2} & \text{smooth boundary} \end{cases}$, $t = q + \dfrac{\partial m_s}{\partial s}$

In the straightening process, the straightening force b is an unknown quantity, while deflection is a known quantity. Suppose that the straightening force is constant; Eq. (7) can then be written as

$$b\int_{\Omega} w^* d\Omega = Cw_i + \int_{\Gamma}(m_n^*\beta_n + t^* w)\, d\Gamma + \int_{\Gamma}\left(\frac{\partial m_s^*}{\partial s} w + m_s^* \frac{\partial w}{\partial s}\right) d\Gamma$$
$$- \int_{\Gamma}(m_n \beta_n^* + t w^*)\, d\Gamma - \int_{\Gamma}\left(\frac{\partial m_s}{\partial s} w^* + m_s \frac{\partial w^*}{\partial s}\right) d\Gamma \quad (8)$$

III. ANALYSIS OF STRAIGHTENING PROCESS WITH FAST MULTIPOLE BOUNDARY ELEMENT METHOD

1. Decomposition of Fundamental Solution

Suppose that the source point is x and the field points are $y_1, y_2, \ldots, y_n$. The logarithmic function of r can then be decomposed as [13, 14]

$$\ln r = \ln|y_i - x| = \ln y_i - \sum_{k=1}^{\infty} f_k(x) g_k(y_i) \quad i = 1, 2, \cdots, n \quad (9)$$

where $f_k(x) = \frac{x^k}{k}$, $g_k(y_i) = \frac{1}{y_i^k}$.

The fundamental solution can be written as

$$w^* = \frac{r^2}{8\pi D}\ln r = \frac{r^2}{8\pi D}(\ln y_i - \sum_{k=1}^{\infty} f_k(x) g_k(y_i)) \quad (10)$$

2. Discretization of Boundary Integral Equation

The boundary is discretized through regular subdivision, and the BIE is written as

$$\sum_{l=1}^{M} b_l \int_{\Omega_l} w^* d\Omega = Cw_i + \sum_{j=1}^{NE}\int_{\Gamma_j}(m_n^*\beta_n + t^* w)\, d\Gamma + \sum_{k=1}^{L} t_k^{c*} w_k$$
$$- \sum_{j=1}^{NE}\int_{\Gamma_j}(m_n \beta_n^* + t w^*)\, d\Gamma - \sum_{k=1}^{L} t_k^c w_k^* \quad (11)$$

where M is the number of elements in the region, NE is the number of elements on the boundary, and L is the number of Gauss integration points.

The BIE is discretized in matrix form by using boundary conditions:

$$\mathbf{BX} = \mathbf{W} \quad (12)$$

where **B** denotes the coefficient matrix of unknown variables and **W** denotes the known variables.

The equation system can be solved using the GMRES(m) method, and the straightening force can be obtained.

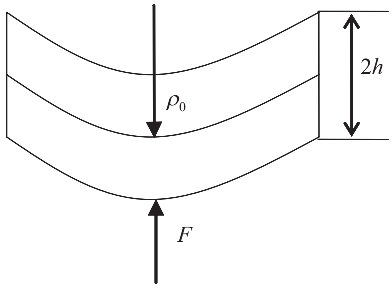

Fig. 1. Initial state of the plate.

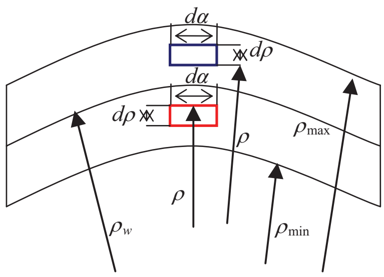

Fig. 2. State of the plate after bending.

IV. ANALYSIS OF NEUTRAL LAYER OFFSET

A rolled piece section is considered, and the force acting on it is analyzed (Fig. 1). Suppose that the original curvature of the rolled piece is $\frac{1}{\rho_0}$, The state of the rolled piece deformation under a force can then be depicted as shown in Fig. 2. The bending curvature is $\frac{1}{\rho_w}$.

The force follows the von Mises yield criterion:

$$(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 = 2\sigma_s^2 \quad (13)$$

where σ_x, σ_y, and σ_z denote the principle stress, and σ_s denotes the yield stress.

For the plane stress problem, Eq. (13) can be simplified:

$$\sigma_x - \sigma_y = \frac{2}{\sqrt{3}}\sigma_s \quad (14)$$

Suppose that the curvature radius is ρ when plastic deformation occurs. The size of the microunits is $d\rho \times d\alpha$. The forces acting on the microunits are analyzed in the tension and compression regions, as shown in Fig. 2. In this section, the principal stress values are analyzed.

(1) Tensile region

The forces acting on a microunit are analyzed in the tensile region, as shown in Fig. 3. According to the equilibrium condition of the microunit, they can be written as

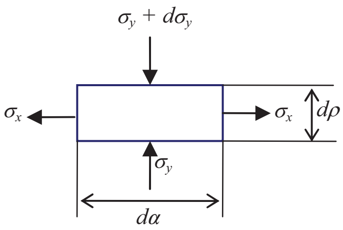

Fig. 3. Force analysis of the tension region.

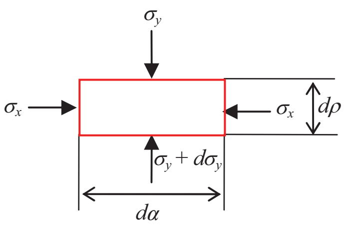

Fig. 4. Force analysis of the compression region.

$$2d\rho\sigma_x \sin\frac{d\alpha}{2} + \sigma_y(\rho_{la} + d\rho)d\alpha - (\sigma_y + d\sigma_y)\rho_{la}d\alpha = 0$$

If it is assumed that $d\alpha$ has a low value and if the boundary condition $(\sigma_y)_{\rho_{la}=\rho_{\max}} = 0$ is considered, then σ_y can be written as

$$\sigma_y = \frac{2}{\sqrt{3}}\sigma_s \ln\frac{\rho_{la}}{\rho_{\max}} \tag{15}$$

From Eq. (14), the following expression can be obtained:

$$\sigma_x = \frac{2}{\sqrt{3}}\sigma_s(1 + \ln\frac{\rho_{la}}{\rho_{\max}})$$

$$\sigma_z = \frac{\sigma_x + \sigma_y}{2} = \frac{1}{\sqrt{3}}\sigma_s(1 + 2\ln\frac{\rho_{la}}{\rho_{\max}}) \tag{16}$$

Here, σ_y is the compressive stress, and σ_x and σ_z are the tensile stresses.

(2) Compression region

The forces acting on a microunit in the compression region are shown in Fig. 4. According to the equilibrium condition of the microunit, they can be expressed as

$$2d\rho\sigma_x \sin\frac{d\alpha}{2} + \sigma_y(\rho_{ya} - d\rho)d\alpha - (\sigma_y + d\sigma_y)\rho_{ya}d\alpha = 0 \tag{17}$$

If it is assumed that $d\alpha$ has a small value and if the boundary condition $(\sigma_y)_{\rho_{ya}=\rho_{\min}} = \bar{P}$, is considered, then the following expression holds:

$$\sigma_y = \frac{2}{\sqrt{3}}\sigma_s \ln\frac{\rho_{\min}}{\rho_{ya}} - \bar{P} \tag{18}$$

From Eq. (14), the following expressions can be obtained:

$$\sigma_x = \frac{2}{\sqrt{3}}\sigma_s(1 + \ln\frac{\rho_{\min}}{\rho_{ya}}) - \bar{P}$$

$$\sigma_z = \frac{1}{\sqrt{3}}\sigma_s(1 + 2\ln\frac{\rho_{\min}}{\rho_{ya}}) - \frac{1}{2}\bar{P} \tag{19}$$

To ensure plastic deformation and achieve the purpose of straightening, the condition $\bar{P} > \frac{2}{\sqrt{3}}\sigma_s$. must be satisfied. Therefore, σ_x, σ_y, and σ_z are compressive stresses.

Suppose that the plastic deformation in the neutral layer is identical to that in tension and compression regions. The following expression can then be obtained:

$$\frac{2}{\sqrt{3}}\sigma_s \ln\frac{\rho}{\rho_{\max}} = \frac{2}{\sqrt{3}}\sigma_s \ln\frac{\rho_{\min}}{\rho} - \bar{P} \tag{20}$$

Then,

$$\rho^2 = \exp(-\frac{\bar{P}}{\frac{2}{\sqrt{3}}\sigma_s})\rho_{\max}\rho_{\min} \tag{21}$$

Suppose that the plate thickness is H. The maximum curvature radius can then be written as $\rho_{\max} = \rho_{\min} + H$, and the central layer of the plate is given by the expression $\rho' = \frac{1}{2}(\rho_{\max} + \rho_{\min}) = \rho_{\min} + \frac{1}{2}H$.

The neutral layer offset can be written as

$$e = \rho - \rho' = \frac{H^2}{4\left[\rho' + \exp\left(\frac{\bar{P}}{\frac{2}{\sqrt{3}}\sigma_s}\right)\rho\right]} + \left[\exp\left(\frac{\bar{P}}{\frac{2}{\sqrt{3}}\sigma_s}\right) - 1\right]\rho \tag{22}$$

V. NUMERICAL ANALYSIS

In this study, the roller straightening model used was a nine-roller straightening roller system with a variable roller distance; the roll diameter and roll distance are shown (in

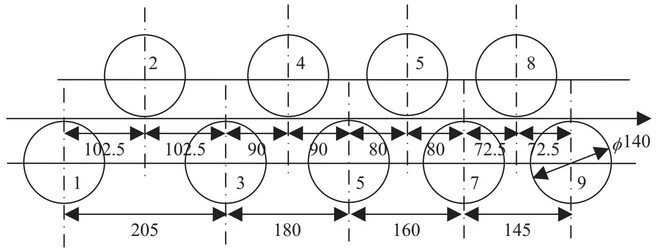

Fig. 5. Nine-roller straightening roller system with a variable roller distance.

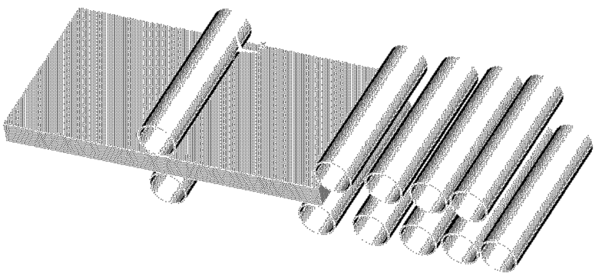

Fig. 6. Calculation model.

millimeters) in Fig. 5. The calculation model is shown in Fig. 6. The elastic modulus, Poisson ratio, and yield strength were 2.1E+11 Pa, 0.3, and 400 MPa, respectively. Furthermore, the static friction coefficient, sliding friction coefficient, and straightening rate of the piece were 0.3, 0.25, and 100 m/min, respectively.

In the calculation model, the lower straightening rollers were stationary, and the upper straightening rollers were subjected to forces of 250 and 500 kN. The plastic deformation on a cross section of the rolled piece is discussed. As shown in Figs. 7(a) and 7(b), when the straightening force was 250 kN, the plastic deformation in a cross section was symmetric to the neutral axis. The maximum plastic deformation occurred on both sides of the neutral axis, and the plastic depth at the neutral axis was lesser than those on both sides. With an increase in the straightening force, the plastic deformation at the neutral axis converged with that on both sides. When the straightening force was 500 kN, the plastic deformation at the neutral axis was identical to that on both sides. In the compression region, the plastic deformation at the top and bottom gradually became flat with an increase in the straightening force. This is because plastic deformation occurs in a region farther from the center of the plate, causing plastic deformation to increase rapidly in the length direction and increase gradually in the thickness direction.

The equivalent strains in the upper surface, lower surface, and central layer were compared by considering the results obtained using the FM-BEM, as shown in Fig. 8. This figure shows a comparison of the strain at a force of 500 kN in the straightening process. The strain in the lower surface was offset relative to the neutral axis. In addition, strain was

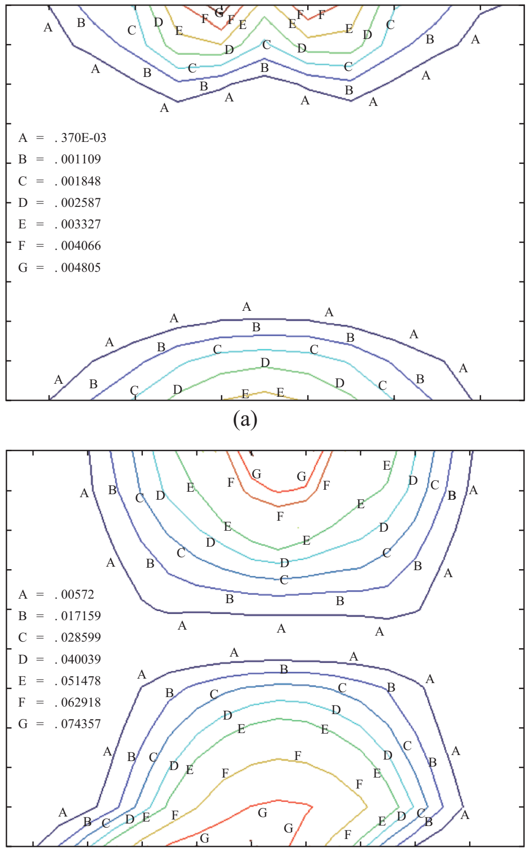

Fig. 7. (a) Plastic deformation under a 250-kN force in a cross section. (b) Plastic deformation under a 500-kN force in a cross section.

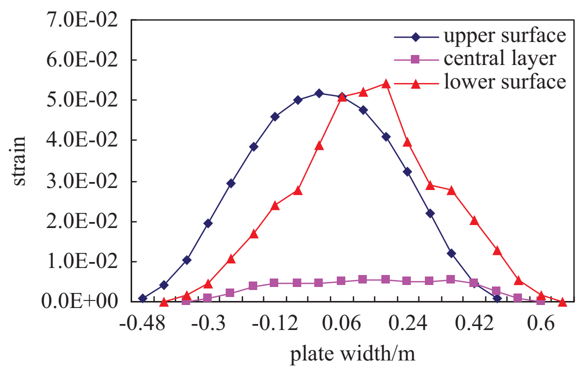

Fig. 8. Comparison of strains at a force of 500 kN.

present in the central layer, and therefore, the neutral layer was not coincident with the central layer.

Fig. 9 shows the normal stress distribution at the neutral axis in the thickness direction. Point A denotes the point of intersection between the central layer and the neutral axis. The normal stress in the central layer was not zero; this result is in agreement with those shown in Fig. 8. The zero-stress point was behind point A, namely the neutral layer is under the central layer. The normal stress distribution in the X direction

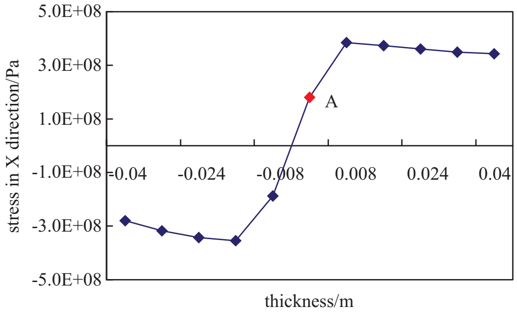

Fig. 9. Stress distribution in the thickness direction.

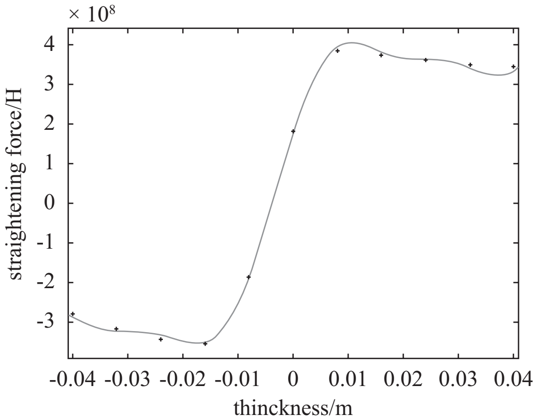

Fig. 10. Fitting curve.

of the neutral axis nodes is a trigonometric function. A curve function is fitted to determine the neutral layer offset. The fitting curve is shown in Fig. 10, and the fitting accuracy is 93.11%.

$$f(x) = 5.093e^8 \sin(0.05615x - 3.131) + 1.56e^9 \sin(0.00272x + 0.04763) + 1.08e^8 \sin(0.1391x + 2.17) \tag{23}$$

VI. CONCLUSIONS

(1) The rolled piece deformation in the straightening process was regarded as an inverse problem of the plate bending problem and analyzed using the FM-BEM. The straightening force was obtained in the analysis.

(2) Through a force analysis of the microunits of a rolled piece, the principle stress values and neutral layer offset were obtained. The obtained values were considered along with the results obtained using the FM-BEM, and it was concluded that the neutral layer is not coincident with the central layer in the straightening process; an offset exists in the compression region.

(3) The nine-roller straightening roller system with a variable roller distance was simulated, and the calculation function of the neutral layer offset was obtained through curve fitting. This function has some advantages when used for obtaining an accurate straightening force, thereby facilitating accurate straightening and improving the productivity of enterprises.

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