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## STUDIES OF THE EFFECT OF DIFFUSION-INDUCED CHLORIDE BINDING ON CHLORINATION LIFE PREDICTIONS FOR EXISTING REINFORCED CONCRETE BRIDGES

Ming-Te Liang

Department of Civil Engineering, China University of Science and Technology, Taipei, Taiwan, R.O.C., mtliang@cc.cust.edu.tw

Jiang-Jhy Chang Department of Harbor and River Engineering, National Taiwan Ocean University, Keelung, Taiwan, R.O.C.

Tung-Wei Yu Graduate Student, Department of Harbor and River Engineering, National Taiwan Ocean University, Keelung, Taiwan, R.O.C.

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# STUDIES OF THE EFFECT OF DIFFUSION-INDUCED CHLORIDE BINDING ON CHLORINATION LIFE PREDICTIONS FOR EXISTING REINFORCED CONCRETE BRIDGES

Ming-Te Liang<sup>1</sup>, Jiang-Jhy Chang<sup>2</sup>, and Tung-Wei Yu<sup>3</sup>

Key words: bridge, chloride binding, chlorinational life, Langmuir isotherm, Freundlich isotherm.

## **ABSTRACT**

The chloride ions diffusion is one of important factors of corrosion of reinforced concrete (RC) structures. The total chloride concentration penetrated into concrete can be expressed as the sum of bound and free chloride concentrations. The principal purpose of this paper is to study the chlorination life prediction of existing RC bridges due to chloride binding effect. The investigation method is that the no binding, linear binding, Langmuir and Freundlich adsorption isotherms are substituted into the analytical solutions of linear (diffusivity D = constant) and non-linear ( $D \neq$  constant) diffusion equations associated with initial and boundary conditions, respectively. The chlorination life can be calculated from the analytical solutions through the Mathematica software. These theoretical models are used to apply the existing Lin-bian and Dah-jin RC bridges in Taiwan. The results of the present study show that the Langmuir adsorption isotherms is suitable for bridge deck with small concrete cover while the linear binding is suitable for pier and abutment with large concrete cover. The studied results can be provided as a crucial reference for engineering decision-making for repair, strengthening or demolition of existing RC bridges.

#### **I. INTRODUCTION**

Chloride-induced corrosion of reinforcing steel bars in concrete exposed to marine environments and de-icing salts used in winter has become one of the principal causes of deterioration in many chief facilities made of reinforced concrete (RC) (e.g. bridge decks, piers, parking garages, off-shore platforms, etc.). The corrosion of the steel bars results in concrete fracture by way of cracking, delamination and spalling of the concrete cover, reduction of concrete and steel bars cross sections, loss of bond along the steel-concrete interface as well as reduction in strength and durability [2]. Accordingly, the safety and serviceability of RC structures are reduced.

Concrete is porous. The chloride ions in atmosphere from concrete surface through pore void into the interior concrete can be either physically and chemically bound to the cement hydrates and their surfaces (bound chlorides,  $C<sub>b</sub>$ ), or dissolved in the pore solution (free chlorides,  $C_f$ ). Only the free chlorides dissolved in the pore solution are expected to account for initiating the process of corrosion [10], because it is in this condition that they continue to ingress through the concrete cover.

Sergi *et al*. [13] experimentally verified that the relationship between total chlorides  $(C_t)$  and  $C_f$  and depth can be expressed by Fick's second law. They also verified that the Langmuir adsorption isotherm can be represented the relationship between  $C_b$  and  $C_f$ . Yu *et al.* [18] experimently examined that the relationship between  $C_b$  and  $C_f$  expressed by the Langmuir adsorption isotherm may explain the effect of chloride binding on diffusion. They also verified that Fick's second law derived from Fick's first law and the principle of mass conservation may describe the diffusion characteristics of  $C_f$ . The magnitude of diffusivity of  $C_f$  in concrete not only can reflect the permeability of concrete but also can be used to predict the service life of concrete as the model is based on the corrosion of steel bar. Arya and Newman [1] studied the  $C_f$ content in concrete using extraction and analysis of pore fluid, leaching techniques, quantitative X-ray diffraction analysis, and empirical relationships. Tang and Nilsson [15] experimentally explored the chloride binding capacity by using cement paste and mortar specimen. They pointed out that the Freundlich adsorption isotherm expressed the relation of  $C<sub>b</sub>$ and  $C_f$  at high  $C_f$  concentration (i.e.,  $> 0.01$  mol/l) while the Langmuir adsorption isotherm expressed the relation of  $C<sub>b</sub>$  and

*Paper submitted 08/27/10; revised 01/12/11; accepted 03/18/11. Author for correspondence: Ming-Te Liang (e-mail: mtliang@cc.cust.edu.tw).* 

*<sup>1</sup> Department of Civil Engineering, China University of Science and Technology, Taipei, Taiwan, R.O.C.* 

*<sup>2</sup> Department of Harbor and River Engineering, National Taiwan Ocean University, Keelung, Taiwan, R.O.C.* 

*<sup>3</sup> Graduate Student, Department of Harbor and River Engineering, National Taiwan Ocean University, Keelung, Taiwan, R.O.C.* 

C<sub>f</sub> at low C<sub>b</sub> concentration (i.e., < 0.05 mol/l). Mart'in-Pérez *et al*. [10] substituted four mathematical models, i.e., no binding, linear binding, Langmuir and Freundlich adsorption isotherms, into 1-D nonlinear Fick's second law and used the finite difference method to predict the service life of concrete specimen. Jheng [4] employed the Kirchhoff transformation associated with the Laplace transformation to obtain the analytical solution of the 1-D nonlinear Fick's second law with initial and boundary conditions. Using the same input data as Marťin-Pérez *et al*., the predicted results obtained from the analytical solution have difference from those of Marťin-Pérez *et al*. calculated the approximate solution by the finite difference method. Lu *et al.* [10] obtained the relation  $C_t \approx 3C_f$  and  $D_t \approx$  $3D_f$ , in which  $D_t$  and  $D_f$  are the coefficients of diffusion of  $C_t$ and  $C_f$ , on the basis of the experiment of concrete specimen. Jeng [4] theoretically examined the exactly experimental results obtained by Lu *et al*. [9]. The literature mentioned above has not studied the effect of chloride binding on the service life prediction of existing RC bridges. This is a shortcoming. This article will conduct the effect of diffusion-induced chloride binding on chlorination life prediction for existing RC bridges.

The main aim of this paper is to use four mathematiccal models, i.e., no binding, linear binding, Langmuir and Freundlich adsorption isotherms substituting into the analytical solutions of linear (coefficient of diffusion  $D = constant$ ) and nonlinear ( $D \neq$  constant) Fick's second law with initial and boundary conditions. In order to verify those feasible and reliable theoretical models, the existing Dah-jin and Lin-bian RC bridges in Taiwan are used as illustrative examples. For calculating the chlorination lives of bridge deck, pier and abutment of existing RC bridges, the commercial software "Mathematica" [17] is used as an algorithm tool. The results of present study may offer as an engineering reference of decision making for repair, strengthening or demolition for existing RC bridges.

#### **II. THEORETICAL MODEL**

#### **1. Linear Diffusion Equation**

Provided that concrete is in a saturated state, chloride ions penetrate the concrete by ionic diffusion resulting from the existing concentration gradient between the exposed surface and the pore solution of the cement matrix. This process due to diffusion driving force is usually expressed by Fick's first law of diffusion [10]

$$
J_c = -D_c w_e \frac{\partial C_f}{\partial x} = -\overline{D}_c \frac{\partial C_f}{\partial x}
$$
 (1)

where  $J_c$  is the flux of chloride ions due to diffusion,  $D_c$  is the effective diffusion coefficient when the concentration is represented in kilograms per cubic meter of concrete,  $\overline{D}_c$  is the effective diffusion coefficient when the concentration is expressed in kilograms per cubic meter of pore solution,  $w_e$  is the

evaporable water content ( $w_e = 0.8\%$  [10]), and C<sub>f</sub> is the free chloride concentration at depth *x*.

Based on the concrete in a saturated situation, the mass conservation of chloride ions in concrete leads to

$$
\frac{\partial C_t}{\partial t} = -\frac{\partial J_c}{\partial x} \tag{2}
$$

where  $C_t$  is the total chloride concentration and t is the time.

The relationship among the total, bound, and free chloride concentrations in concrete can be expressed as

$$
C_t = C_b + w_e C_f \tag{3}
$$

where  $C_b$  is the concentration of bound chlorides.

Assume the effective diffusion coefficient,  $D<sub>c</sub>$ , to be constant. Inserting Eqs. (1) and (3) into Eq. (2) and using chain rule, we obtain

$$
(\frac{\partial C_b}{\partial C_f} + w_e) \frac{\partial C_f}{\partial t} = D_c w_e \frac{\partial^2 C_f}{\partial x^2}
$$
 (4)

where  $\partial C_b/\partial C_f$  is the binding capacity of the concrete binder [10]. Eq. (4) can be rewritten as

$$
\frac{\partial C_f}{\partial t} = D_c^* \frac{\partial^2 C_f}{\partial x^2}, D_c^* = \frac{D_c w_e}{\frac{\partial C_b}{\partial C_f} + w_e} = \frac{D_c}{1 + \frac{1}{w_e} \frac{\partial C_b}{\partial C_f}}
$$
(5a)

where  $D_c^*$  is the apparent diffusion coefficient.

The associated with initial and boundary conditions of Eq. (5a) are

$$
C_f(x, 0) = 0 \tag{5b}
$$

$$
C_f(0, t) = C_0 \tag{5c}
$$

$$
C_f(x \to \infty, t) = 0 \tag{5d}
$$

where  $C_0$  is the chloride concentration on the concrete surface.

By applying the method of Laplace transform [12] to Eq. (5), the analytical solution of Eq. (5) [3] yields

$$
C_{\rm f}(x,t) = C_0 \,\text{erfc}(\frac{x}{\sqrt{4D_{\rm c}^*t}})
$$
\n<sup>(6)</sup>

where erfc is the complementary error function.

#### **2. Nonlinear Diffusion Equation**

If the coefficient of diffusion,  $D_c$ , is not constant, then the substitution of Eq. (1) into Eq. (2) yields

$$
\frac{\partial C_t}{\partial t} = \frac{\partial}{\partial x} (D_c w_e \frac{\partial C_f}{\partial x})
$$
(7)

Inserting Eq.  $(3)$  into Eq.  $(7)$ , we have

$$
\frac{\partial C_f}{\partial t} = \frac{\partial}{\partial x} (D_c^* \frac{\partial C_f}{\partial x})
$$
 (8a)

where

$$
D_c^* = \frac{D_c}{1 + \frac{1}{w_e} \frac{\partial C_b}{\partial C_f}}
$$
 (8b)

associated with initial and boundary conditions

$$
C_f(x, 0) = 0 \tag{8c}
$$

$$
C_f(0, t) = C_0 \tag{8d}
$$

$$
C_f(x \to \infty, t) = 0
$$
 (8e)

Using the Kirchhoff transformation in conjunction with the Laplace transformation [14], the analytical solution of Eq. (8) can be obtained

$$
\int_{C_R}^{C_f} D_c^*(C_f) dC_f = \int_{C_R}^{0} D_c^*(C_f) dC_f
$$
\n
$$
+ \left[ \int_{C_R}^{C_0} D_c^*(C_f) dC_f - \int_{C_R}^{0} D_c^*(C_f) dC_f \right] erfc(\frac{x}{\sqrt{4D_c^*t}})
$$
\n(9)

where  $C_R$  is the any reference value of chloride concentration. For convenience, we may take  $C_R = 0$ . Thus, Eq. (9) can be rewritten as

$$
\frac{\int_0^{C_f} D_c^*(C_f) dC_f}{\int_0^{C_0} D_c^*(C_f) dC_f} = \text{erfc}(\frac{x}{\sqrt{4D_c^*t}})
$$
\n(10)

Eq. (10) provides the relationship among apparent diffusion coefficient  $D_c^*$ , free chloride concentration  $C_f$ , penetration depth *x*, and time *t*. The left side of Eq. (10) purely represents the area ratio between areas under the  $D_c^* - C_f$  curve from  $C_R = 0$  to  $C_f$  and  $C_R = 0$  to  $C_0$ , respectively.

#### **3. Chloride Binding Isotherm**

The chloride binding isotherm is defined as the described relationship between the bound and free chloride concentrations in concrete under a given temperature condition. Now a brief description is introduced in the following [10]:

• No binding

$$
C_b = 0, \frac{\partial C_b}{\partial C_f} = 0, D_c^* = D_c \tag{11}
$$

• Linear binding

$$
C_{b} = \alpha C_{f}, \frac{\partial C_{b}}{\partial C_{f}} = \alpha, D_{c}^{*} = \frac{D_{c}}{1 + \frac{\alpha}{w_{e}}}
$$
(12)

where  $\alpha$  is the binding capacity. • Langmuir isotherm

$$
\mathcal{L}^{\mathcal{L}}(\mathcal{L}
$$

$$
C_b = \frac{\alpha C_f}{1 + \beta C_f}, \frac{\partial C_b}{\partial C_f} = \frac{\alpha}{\left(1 + \beta C_f\right)^2}, D_c^* = \frac{D_c}{1 + \frac{\alpha}{w_e \left(1 + \beta C_f\right)^2}}
$$
\n(13)

where  $\alpha$  and  $\beta$  are the binding constants. • Freundlich isotherm

$$
C_b = \alpha C_f^{\beta}, \frac{\partial C_b}{\partial C_f} = \alpha \beta C_f^{\beta - 1}, D_c^* = \frac{D_c}{1 + \frac{\alpha \beta}{w_e} C_f^{\beta - 1}} \qquad (14)
$$

where  $\alpha$  and  $\beta$  are the binding constants.

Substituting the values of  $C_0$  and  $D_c^*$  represented by Eqs. (11)-(14) into Eqs. (6) and (10), we can obtain the free chloride concentrations at depth x and time t of the analytic solutions of linear and nonlinear diffusion equations, respectively.

#### **4. Fick's Second Diffusion Law**

First, Fick's second law of linear diffusion equation in a semi-infinite quasi homogeneous and isotropic concrete is used to state the free chloride concentration profile. This equation with initial boundary conditions can be expressed as [12]

$$
\frac{\partial C_f}{\partial t} = D_f \frac{\partial^2 C_f}{\partial x^2}, D_f = \text{constant}
$$
 (15a)

$$
C_f(x, 0) = 0 \tag{15b}
$$

$$
C_f(0, t) = C_{0f} \tag{15c}
$$

$$
C_f(x \to \infty, t) = 0 \tag{15d}
$$

where  $C_{0f}$  is the free chloride concentration on the concrete surface.

Using the method of Laplace transform [12], the analytic solution [3] of Eq. (15) can be obtained

$$
C_{f}(x, t) = C_{0f} \left[ 1 - erf \left( \frac{x}{\sqrt{4D_{f}t}} \right) \right] = C_{0f} erfc \left( \frac{x}{\sqrt{4D_{f}t}} \right) \qquad (16)
$$

where erf is the error function and erfc is the complementary error function.



**Fig. 1. Schematic diagram of component number system of the Lin-bian bridge.** 

Finally, Fick's second law of linear diffusion equation in a semi-infinite quasi-homogeneous and isotropic concrete is employed to describe the total chloride concentration profile. This equation with initial and boundary conditions can be put forward as

$$
\frac{\partial C_t}{\partial t} = D_t \frac{\partial^2 C_t}{\partial x^2}, D_t = \text{constant}
$$
 (17a)

$$
Ct(x, 0) = 0
$$
 (17b)

$$
\mathbf{C}_{t}(0, t) = \mathbf{C}_{0t} \tag{17c}
$$

$$
C_t(x \to \infty, t) = 0 \tag{17d}
$$

In a similar way, the analytic solution of Eq. (17) can be stated as

$$
C_{t}(x, t) = C_{0t} \left[ 1 - erf\left(\frac{x}{\sqrt{4D_{t}t}}\right) \right] = C_{0t} erfc\left(\frac{x}{\sqrt{4D_{t}t}}\right)
$$
 (18)

#### **III. ILLUSTRATIVE EXAMPLES**

In order to verify the feasibility and reliability of the theories mentioned above, both the Lin-bian [5] and Dah-jin [6] bridges in Taiwan are used to do illustrative examples. Figs. 1 and 2 demonstrate the schematic diagram of component number system of both bridges. The results predicted from the analytic solution of Eqs. (6), (10), (16), and (18) can be calculated by the Mathematica software [17]. Tables 1 and 2 show the basic data of the Lin-bian and Dah-jin bridges, respectively. Inserting the relative parameters obtained from



**Fig. 2. Schematic diagram of Component number system of the Dah-jin bridge.** 

field measure of the two bridges into the analytical solutions of Eqs.  $(6)$ ,  $(10)$ ,  $(16)$  and  $(18)$ , the service lives of the two bridges can be predicted by the Mathematica software.

Cored cylindrical specimens from abutment, pier, and bridge deck components of the existing Lin-bian and Dah-jin bridges in Taiwan and used acid-soluble method for determining chloride content, the concentration measured from the method is total chloride concentration. Lu *et al*. [9] pointed out that total chloride concentration is about 2.2~3.2 times free chloride concentration. In present study, the total chloride concentration is taken as 2.5 times the free chloride concentration. As to the effective coefficient of diffusion, Lu *et al*. [9] also pointed out that the total diffusivity of chloride concentration is 2.8 times the diffusivity of free chloride concentration. Arya and Newman [1] pointed out that both the surface chloride concentration and coefficient of diffusion can be predicted from the relation of chloride concentration versus chlorination depth under given service life. Since the relationship between chloride concentration and chlorination depth has not carried out by the Join Engineering Consultant Company  $[5, 6]$ ,  $C_f$ concentration, surface concentration  $(C_0)$ , effective coefficient of diffusion (D<sub>c</sub>), α and β values used for four mathematical models are cited from Mart'in-Pérez *et al.* [10]. When  $D_c \neq$ constant, numerical simulation was adopted. That is, substituting the service life t obtained from Eq. (6) into Eq. (10), the relation of inconstant coefficient of diffusion, time and depth can be drawn after putting relative parameters into Eq. (10). As a result, the surface chloride concentrations and true diffusivity of the components such as bridge deck, pier and abutment of existing RC bridges can be obtained.

Tables 3 and 4 show the total and surface Cl concentrations and cover thickness of the existing Lin-bian and Dah-jin bridges, respectively. It is worthwhile to point out that the total Cl<sup>-</sup> concentration are directly measured from concrete specimen cored from field. The  $C_f$  concentrations are

Item	Description			
Bride site	This bridge is located between $274K+387.54$ and $274K+846.54$ of the $17th$ line of provincial road in Taiwan, strides			
	over the Lin-bian river, and connects the Lin-bian and Jia-dong countries.			
Date of completed construction	This bridge was completed in April, 1976. The expanded engineering of single side was performed in October 1982.			
Material strength	1. Plain concrete: 1:3:6 concrete			
	2. Reinforced concrete (RC): $fc' = 210 \text{ kgf/cm}^2$			
	3. Prestressed concrete beam: $fc' = 350 \text{ kgf/cm}^2$			
	4. Deformed steel : $f_y = 2800 \text{ kgf/cm}^2$			
	5. Pre-stressed tendon: 12-7 mm $\Phi$ HTW (Girder used), 17-5 mm $\Phi$ HTW (Transverse diaphragm used), $f_v = 14500$ kgf/cm <sup>2</sup> , fs' = 16500 kgf/cm <sup>2</sup> (7 mm $\Phi$ ) f <sub>y</sub> = 15500 kgf/cm <sup>2</sup> , fs' = 17500 kgf/cm <sup>2</sup> (5 mm $\Phi$ )			
Concrete cover	1. Bridge deck: 2.9 cm			
	2. Balustrade: 2.0 cm			
	3. Beam and diaphragm: 3.3 cm			
	4. Pier: 7.5 cm			
	5. Abutment: 7.0 cm			
Simply structural	1. The superstructure is constructed by the inverse T-type beams of RC and is divided into 15 spans with total			
introduction	459 m. The allocation of stridden length is 15@30.6 m. The width is 20 m after the expanded engineering. The thickness of bridge deck is 17 cm and is of 5 cm design friction layer.			
	2. The substructure is constructed by the single column RC pier and abutment with pile foundations.			
Geological condition	Silty sand and clay.			

**Table 1. Overall structural condition of the Lin-bian bridge.** 





Bridge member	<b>Tested Points</b>	Total Cl concentration	*Free Cl <sup>-</sup> Concentration	Cover thickness c (mm)	
		(% mass of concrete)	(% mass of concrete)		
	$A1-1$	0.0084	0.0034	70	
Abutment	$A2-1$	0.0082	0.0033	70	
	$A2-2$	0.0086	0.0034	70	
	1S4-8	0.0195	0.0078	29	
	$2S1-8$	0.0168	0.0067	29	
	S <sub>3</sub>	0.0070	0.0028	29	
	S <sub>4</sub>	0.0096	0.0038	29	
Slab	S <sub>5</sub>	0.0082	0.0033	29	
	S <sub>6</sub>	0.0071	0.0028	29	
	S7	0.0052	0.0021	29	
	S8	0.0073	0.0029	29	
	S <sub>9</sub>	0.0074	29 0.0030 29 0.0019 $\overline{75}$ 0.0026 75 0.0026		
	<b>S10</b>	0.0047			
	$P1-1$	0.0066			
	$P1-2$	0.0066			
	$P1-3$	0.0062	0.0025	75	
	$P2-1$	0.0068	0.0027	75	
	$P2-2$	0.0068	0.0027	75	
	$P2-3$	0.0075	0.0030	75	
	P12-1	0.0093	0.0037	75	
	P12-2	0.0147	0.0059	75	
Pier	$P12-3$	0.0065	0.0026	75	
	P12-3 Cap beam	0.0074	0.0030	75	
	$P13-1$	0.0104	0.0042	75	
	P13-2	0.0070	0.0028	75	
	P13-2 Cap beam	0.0063	0.0025	75	
	P <sub>13</sub> -3	0.0097	0.0039	75	
	P14-1 Column	0.0067	0.0027	75	
	P14-2 Column	0.0060	0.0024	75	
	P14-3 Column	0.0062	0.0025	75	
	P14-3 Cap beam	0.0070	0.0028	75	

Table 3. CI concentration and cover thickness of the Lin-bian bridge.

\* Free Cl- concentrations calculated from the relationship between the total and free Cl- concentrations suggested by Lu *et al*. [9].





\* Free Cl- concentrations calculated from the relationship between the total and free Cl- concentrations suggested by Lu *et al*. [9].



## **Table 5. The requirement substitution parameters of calculation chlorination life for the Lin-bian bridge under total chloride with constant diffusivity.**

\* The value of  $C_t$  (% mass of concrete) of each member is the average of  $C_t$  (% mass of concrete) listed in Table 3.

### **Table 6. The requirement substitution parameters of calculation chlorination life for the Lin-bian bridge under free chloride with constant diffusivity.**



\* The value of  $C_f$  (% mass of concrete) of each member is the average of  $C_f$  (% mass of concrete) listed in Table 3.

calculated by using of the multiple relation formula of  $C_f$  and  $C_t$  suggested by Lu *et al.* [9].

In the case of Eq. (6), the value of  $D_{cf} = 1.0 \times 12^{-12}$  m<sup>2</sup>/s is cited from Marťin- Pérez *et al*. [10]. Marťin-Pérez *et al*. [10] obtained the value of  $D_{cf}$  in concrete samples by chloride binding mechanism. While the value of  $D_{ct} = 2.8 \times 10^{-12}$  m<sup>2</sup>/s is obtained from the relation formula of  $D_{ct} = 2.8$   $D_{cf}$  which was established by Lu *et al*. [9] through the test results of

concrete samples using an electrochemical method. Thus, the requirement substitution parameters of calculation chlorination life for the Lin-bian bridge under total and free chloride with constant diffusivity were obtained and listed in Tables 5 and 6, respectively. In the same way, we gained the requirement substitution parameters of calculation chlorination life for the Dah-jin bridge under total and free chloride with constant diffusivity and listed in Tables 7 and 8, respectively.



## **Table 7. The requirement substitution parameters of calculation chlorination life for the Dah-jin bridge under total chloride with constant diffusivity.**

 $*$  The value of C<sub>t</sub> (% mass of concrete) of each member is the average of C<sub>t</sub> (% mass of concrete) listed in Table 4.

## **Table 8. The requirement substitution parameters of calculation chlorination life for the Dah-jin bridge under free chloride with constant diffusivity.**



\* The value of  $C_f$  (% mass of concrete) of each member is the average of  $C_f$  (% mass of concrete) listed in Table 4.

## **Table 9. The requirement substitution parameters of calculation chlorination life for the Lin-bian bridge under total chloride with inconstant diffusivity.**





## **Table 10. The requirement substitution parameters of calculation chlorination life for the Lin-bian bridge under free chloride with inconstant diffusivity.**

## **Table 11. The requirement substitution parameters of calculation chlorination life for the Dah-jin bridge under total chloride with inconstant diffusivity.**



Inserting the values of  $D_c$ ,  $C_f$ ,  $w_e$ , α and β as listed in Tables 5-8 into Eqs. (11)-(14), we can calculate  $D_c^*$ . Putting the values of  $C_0$ ,  $C_f$ ,  $D_c^*$  and  $x = c$  (c = cover thickness), the chlorination lives of components of existing RC bridges under  $D_c$  = constant can be found.

When the values of  $D<sub>c</sub>$  is inconstant, the numerical simulation technique should be adopted and has been mentioned above. Tables 9 and 10 indicate the requirement substitution parameters of calculation chlorination life for the Lin-bian bridge under total and free chloride with inconstant diffusivity, respectively. Tables 11and 12 depict the requirement substitution parameters of calculation chlorination life for the Dah-jin bridge under total and free chloride with inconstant diffusivity, respectively. Substituting the values of  $D_c$ ,  $C_f$ ,  $W_e$ , α and β as listed in Tables 9-12 into Eqs. (11)-(14), we may

find  $D_c^*$ . Inserting the values of  $C_0$ ,  $C_f$ ,  $D_c^*$  and  $x = c$  into Eq. (10), the chlorination lives of components of existing RC bridges under  $D_c \neq$  constant can be found.

Using the four mathematical models of chloride binding isotherm and employing Eqs. (6), (10), (16) and (18), the chlorination lives of the components of the existing Lin-bian and Dah-jin bridges were predicted as shown in Figs. 3 and 4, respectively. In such measure as free chloride, the abutment and pier of the Lin-bian bridge have chlorination lives 80 and 88 years predicted by the linear binding while the bridge deck of the Lin-bian bridge has chlorination life 55 years predicted by the Langmuir isotherm as shown in Fig. 3. The pier of the Dah-jin bridge has 71 years calculated by the linear binding whereas the bridge deck of the Dah-jin bridge has 50 years calculated by the Langmuir isotherm as sketched in Fig. 4.







**Fig. 3. Chlorination life predictions for the different binding isotherms**  and different Cl<sup>-</sup> concentrations and considering two different diffusivities  $D_c^* =$  Constant and  $D_c^* \neq$  constant for the Lin-bian **bridge exposed to the environmental condition with 0.05763 (% mass of concrete) of Cl- .** 



**Fig. 4. Chlorination life predictions for the different binding isotherms and different Cl- concentrations and considering two different**  diffusivities  $D_c^*$  = constant and  $D_c^* \neq$  constant for the Dah-jin **bridge exposed to the environmental condition with 0.01130 (% mass of concrete) of Cl- .** 

## **IV. DISCUSSION**

## **1. Definition of Total Service Life and Degree of Deterioration**

The deterioration process of RC structures subjected to corrosion media attack is depicted in Fig. 5 [7, 8, 16]. The corrosion process in Fig. 5 is considered as three stages,



**Fig. 5. Deterioration process of reinforced concrete structures due to corrosion.** 

initiation time ( $t_i = t_c$ ), depassivation time ( $t_p$ ), and corrosion (or propagation) time  $(t_{corr})$ . The initiation time is defined as the time for chloride ions to penetrate from the concrete surface to the surface of the passive film. The depassivation time is defined as depassivation normally provided to the steel bars by the alkaline hydrated cement matrix is destroyed locally leading to pitting corrosion. The corrosion time extends to from the time when corrosion products form to the stage where they generate sufficient stress to disrupt the concrete cover by cracking or spalling, or when local attack on the reinforcement becomes sufficiently severe to impair its load-carrying capacity. Thus, the total service life of RC structure can be represented as [7]

$$
t = t_c + t_p + t_{corr}
$$
 (19)

It is worthy to point out that the corrosion time  $(t_{\text{corr}})$  is defined up to degree of deterioration  $D_d = 0.8$  (see Fig. 5). This value of  $D_d$  is chosen just for the conservation to structural safety. The  $D_d$  can be defined as

$$
D_d = 1 - \frac{I}{100}
$$
 (20)

where I stands for the integrity of RC structures. For instance, if RC structure is very integrated, i.e.,  $I = 100$ , then  $D_d = 0$ . If RC structure is collapsed, i.e.,  $I = 0$ , then  $D_d = 1$ . As  $t_{corr} = \frac{1}{4} t_c$  and  $t_p = 10$  years [8], Eq. (15) may be predicted the total chlorination lives for the existing RC bridges.

#### **2. Linear Diffusion Equation**

In the case of linear binding, the Lin-bian bridge has  $t_{max}$  = 121 years and  $t_{min} = 29$  years predicted from Eq. (6). In sofar as linear binding, the Dah-jin bridge has  $t_{max} = 100$  years and  $t_{min} = 26$  years calculated from Eq. (6).

Because the coefficient of diffusion in concrete actually varies with depth and time due to hydration, service life of bridge predicted by  $D_c \neq$  constant is better than that of  $D_c$  = constant. than that of  $D_c$  = constant. Accordingly, assume that

the nonlinear life is correct. As to the larger chlorination life, the reason is caused that the  $t<sub>c</sub>$  time may be increased due to abutment and pier with more thickness of concrete cover. Table 13 summarizes percentage errors of chlorination lives predicted by linear and nonlinear diffusion equations for the Lin-bian and Dah-jin bridges. It is worthwhile to notice that the percentage error ranking from small to large describes as follows: Langmuir isotherm, linear binding, Freundlich isotherm and no binding under total and free chlorides conditions. It is also noted from Table 13 that the percentage errors predicted by total chlorides are larger than those of free chlorides. Only the free chlorides are dissolved in the pore solution of concrete and continued to penetrate through the concrete cover. Consequently, it is reasonable that the chlorination life prediction may be used by the free chlorides.

#### **3. Nonlinear Diffusion Equation**

In the case of Langmuir isotherm, the Lin-bian bridge has  $t_{\text{max}} = 438$  years and  $t_{\text{min}} = 79$  years predicted from Eq. (10). In sofar as Langmuir isotherm, the Dah-jin bridge has  $t_{max} = 361$ years and  $t_{min} = 73$  years calculated from Eq. (10). Table 13 and Figs. 3 and 4 show that the Langmuir isotherm is very suitable for predicting the bridge deck with small concrete cover by the nonlinear diffusion model (see Eq. (10)) due to free chlorides diffusion. As to abutment and pier with larger concrete cover, the mathematical model of linear binding is a suitable prediction method.

Moreover, Eq. (12) is very proper for calculating the coefficient of diffusion in the more thicker concrete cover. However, the Freundlich isotherm is not fitting for calculating the coefficient of diffusion in the concrete cover with larger thickness. This reason may be that the effective coefficient of diffusion predicted by Eq. (14) may be reduced and the chlorination life prediction may be increased. From Table 13 and Figs. 3 and 4, we know that the mathematical model of no binding is the worst one due to only posing total chloride diffusion.

#### **4. Other Factors**

It was worthwhile to notice that some piers among the 12th and 20th of the Dah-jin bridge subjected to debris flow occurred from the typhoon Morakot on August 8, 2009 in Taiwan had been collapsed. However, they will be soon rehabilitated for keeping transportation flow.

#### **V. CONCLUSIONS**

In this paper, four mathematical models of chloride binding isotherm substituting into the analytical solutions of linear and nonlinear diffusion equations associated with initial and boundary conditions have been described. These theories also have been applied to predicting the chlorination lives of the components of the existing Lin-bian and Dah-jin bridges in Taiwan. After this investigation, some important conclusions may be drawn as follows:

Mathematical model	Lin-bian bridge		Dah-jin bridge	
	*Percentage error $(\%)$		Percentage error (%)	
	Total chloride	Free chloride	Total chloride	Free choride
No binding	1.214	-----	-----	-----
Linear binding	0.997	0.394		
Langmuir isotherm	0.830	0.386	-----	-----
Freundlich isotherm	1.181	0.401	-----	-----
No binding	0.387	-----	0.407	-----
Linear binding	0.285	0.245	0.396	0.232
Langmuir isotherm	0.277	0.229	0.351	0.212
Freundlich isotherm	0.306	0.253	0.404	0.312
No binding	1.605	-----	2.100	-----
Linear binding	1.070	0.403	1.101	0.508
Langmuir isotherm	0.905	0.397	1.071	0.498
Freundlich isotherm	1.277	0.415	1.123	0.510

**Table 13. Percentage errors of chlorination life predicted by linear to nonlinear diffusion equations for the Lin-bian and Dah-jin bridges.** 

\* Percentage error  $(\%) = \frac{\text{linear life - nonlinear life}}{\text{nonlinear life}} \times 100$ .

- 1. The difference of chlorination lives predicted by the linear and nonlinear diffusion models for the existing RC bridges is very small. The percentage errors of chlorination lives predicted by the linear and nonlinear diffusion models used free chlorides for the existing RC bridges are less than those of total chlorides. The chlorination life predicted by free chlorides is more suitable than that of total chlorides.
- 2. The chlorination life forecasted by the nonlinear diffusion model is better than that of linear diffusion model. Generally specking, the constant value of coefficient of diffusion used is just for convenient. In fact, the coefficient of diffusion is dependent on time and space.
- 3. The ranking from small to large of percentage error of linear to nonlinear diffusion model is Langmuir isotherm, linear binding, Freundlich isotherm and no binding.
- 4. The chlorination lives of abutment and pier with larger concrete cover of existing RC bridges are suitably predicted by the mathematical model of linear binding while the chlorination life of bridge deck with small concrete cover of exciting RC bridge is properly predicted by the mathematical model of Langmuir isotherm.

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