OPTIMUM DESIGN OF MUFFLERS HYBRIDIZED WITH ONE-CONNECTED CURVED TUBE USING THE BOUNDARY ELEMENT METHOD, NEURAL NETWORKS, AND THE GENETIC ALGORITHM

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ABSTRACT

Recently, research on new techniques for an economical muffler, which is hybridized with a single-connected curved tube, has been applied in the industrial field. Most researchers have explored noise reduction effects based on the transfer matrix method and the stiffness matrix method. However, the maximum noise reduction of a silencer within a constrained space, which frequently occurs in engineering problems, has been neglected. Therefore, the optimum design of mufflers becomes an essential issue. In this paper the shape optimization of a one-connected tube muffler with a fixed length is examined.

In order to speed up the optimization assessment, a simplified objective function (*OBJ*) is established by linking the boundary element model (*BEM*— developed by the commercialized software, *SYSNOISE*) with neural network model (*NNM*). Instead of a complicated mathematical model (*BEM*), a polynomial neural network, which is constructed using a neural network fitted with a series of real data — input design data (muffle dimensions) and output data approximated by *BEM* data in advance. To assess the optimal mufflers, a genetic algorithm (*GA*) is applied. Moreover, the numerical cases of sound elimination with respect to various parameter sets and pure tones (350, 500, and 650) have been exemplified and discussed. Before the *GA* operation can be carried out, the approximation between *BEM* and experimental data is checked. In addition, both the *BEM* and *NNM* are compared. It is found that for a one-connected curved tube muffler the *BEM* and the experimental data are in agreement. Moreover,

the *BEM* and *NNM* conform.

Optimal results reveal that the maximum value of the sound transmission loss (*STL*) can be improved at the desired frequencies. Consequently, the optimum algorithm proposed in this study can provide an efficient way to develop an optimal muffler hybridized with a one-connected curved tube for industry.

I. INTRODUCTION

Muffler research used in engine noise was started by Davis *et al.* [5]. Cummings [4] established the theory and experimental measurements regarding sound wave propagating along a curved tube with a rectangular and circular section. Later, Rostafinski [15] developed the wave propagation formulation for a sound wave transported along a curved tube. By varying the sectional ratio Fuller and Bies [7] investigated the acoustical difference on a straight tube and a curved tube. To [18] analyzed the sound propagation in a piping system by developing a computer program based on the transfer matrix method. Craggs and Stredulinsky [3] analyzed the attenuation of sound wave propagating in a pipe network using a stiffness matrix method in which a two-dimension finite element model (FEM) was used. Later, a general analytical expression for the sound transmission loss for a muffler conjugated with two tubes (the Herschel-Quincke (*HQ*) tube), in which a resonance appears and gives a peak in the transmission loss spectrum, was developed by Selamet *et al.* [16]. However, the attenuation bands were too narrow, as a result, the overall noise reduction of the broadband noise was insufficient. Therefore, Selamet improved the broadband noise reduction by increasing the parallel tubes connected to the muffler. Selamet and Easwaran [17] developed a close form expression for the transmission loss characteristics and resonance locations for an n-duct configuration. Subsequently, Kim and Ih [11] established a sound reduction mathematical model for a sound energy transport along a curved and expanded duct using a four-pole transfer matrix. Dowling and Peat [6] had investigated a general geometry silencer using the transfer matrix approach. Panigrahi and Munjal [13] proposed a transfer
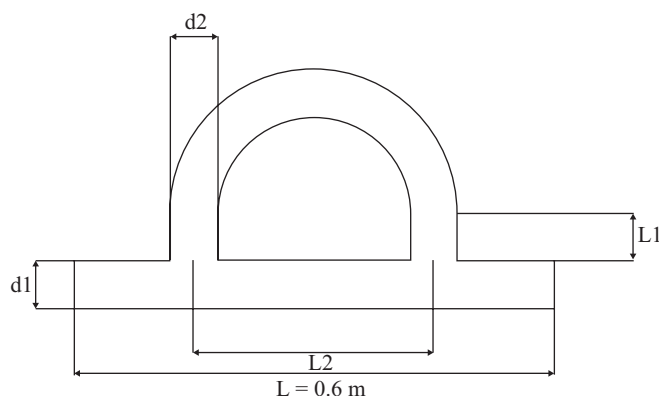
Fig. 1. A one-connected tube muffler.

matrix approach to analyze the generalized m-node, n-branch, and two-port network of ducts.

Since the constrained problem is mostly concerned with the necessity of operation and maintenance in practical engineering work, there is a growing need to optimize the acoustical performance within a limited space. Yet, an investigation of the optimal *HQ* muffler design within space constraints is missing.

In previous work [1, 2, 17, 20], the shape optimizations of mufflers within a constrained space have been discussed. A *GA* based optimization, which can get out of the local optimum, is an excellent choice for seeking a better solution when there is no accurate starting point. In this paper, as indicated in Fig. 1, a muffler equipped with a one-connected tube used for sound elimination at various pure tones (350, 500, and 650 Hz) is exemplified and discussed. In order to speed up the optimal assessment, a simplified objective function (*OBJ*) is established by linking the boundary element model (*BEM*) with the neural network model (*NNM*). Because the design parameters can easily be changed without a total overhaul of the muffler design when using the polynomial neural network instead of the complicated mathematical model (*BEM*), the surrogate model — a trained neural network model (*NNM*) fitted by a series of real data — is established and used as the new *OBJ* function. Because the real data is very close to the theoretical data (*BEM*), and to facilitate the assessment of real data fitted to a *NNM*, the theoretical data is used as the real data.

In this paper, the *BEM* in conjunction with the *NNM* and the *GA* is used to maximize the *STL* by adjusting the dimension of a curved tube connected to the main duct (shown in Fig. 1) within a constrained space.

II. NEURAL NETWORK MODEL (*NNM*)

The neural network used in optimization is widely applied in various fields. It has been found that the neural network provides a great benefit in establishing a *NNM* by imitating a given model. In this paper, a well-known polynomial neural network is adopted and discussed.

1. Concept of the Polynomial Neural Network

Artificial Neural Networks (*ANN*) have been successfully applied in many fields to model complex non–linear relationships. *ANN*s may be viewed as the universal approximators, but the main disadvantage of this approach is that detected dependencies are hidden within the neural network structure. Conversely, a polynomial neural network called Group Method of Data Handling (*GMDH*) was developed by Ivakhnenko [9] while working on an improved model for predicting fish populations in rivers. Ivakhnenko made the neuron a more complex unit featuring a polynomial transfer function. The interconnections between layers of neurons were simplified, and an automatic algorithm for structure design and weight adjustment was developed. The main idea of *GMDH* is the use of feed-forward networks based on short-term polynomial transfer functions whose coefficients are obtained using a regression technique combined with the emulation of the self-organizing activity for the neural network (*NN*) structural learning. The polynomial neural network is one kind of self-organizing adaptive model which establishes the relationship between input and output parameters. The polynomial network is used for recognition in a non-linear system.

The *GMDH* algorithm, a self-organized recognition method in a nonlinear system, establishes an adaptive, monitoring, and learning model. By using monitoring and learning at input and output, the output data is then modeled by the input function. The self-organizing adaptive model *NNM* is organized as follows [12].

A. Divide the original data into two groups — a training data group and a testing data group. The training data group is used for evaluating the weight of the neural network. In addition, a testing data group is used for the function test in the neural network.
B. Create a variable set in each layer.
   An input variable set in each layer is created. The assembled number is $p!/[(p-r)!r!]$ where p is the number of input variables and r is normally set to be two.
C. Best organization of the neural cell.
   To describe the organization of the neural cell with a partial differential characteristic in a nonlinear system, a recursive analysis and data training is utilized. By using AIC (Akaike's information criterion), the domain variable is selected. The output variable of the best neural cell is called the intermediate variable. The best cell is selected from various neural structures in the *GMDH* neural network.
D. Select the intermediate variable.
   The L number of the created intermediate variables with the smallest AIC is selected. To minimize AIC, a larger L is required.
E. Stop the internal calculation between layers.
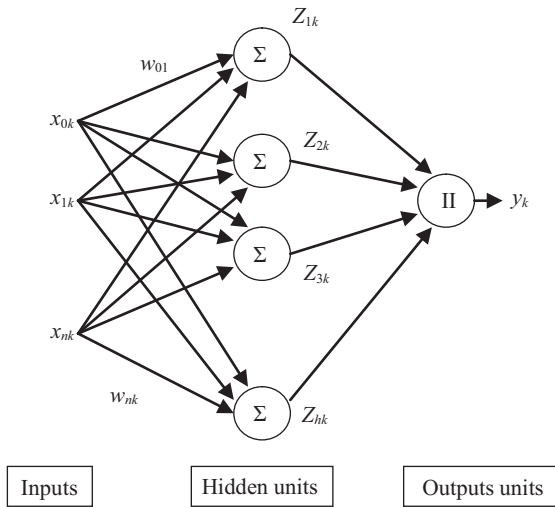   When the decrement of error in each layer stops, the

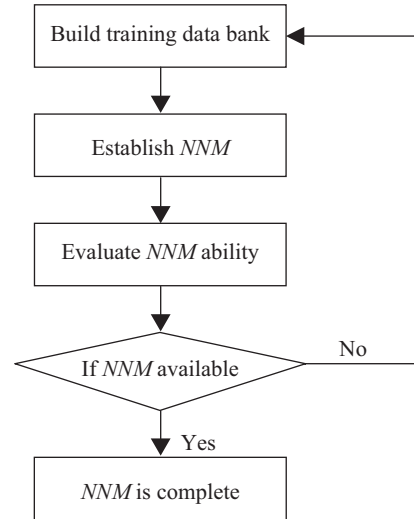**Fig. 2. The organization of polynomial neural network.**

Fig. 3 The steps in *NNM*.

internal calculation will terminate. The complete neural network in a non-linear system can be constructed by the created neural cells in each layer.

2. Polynomial Neural Network Build Up

As indicated in Fig. 2, the polynomial neural network is composed of an input layer, hidden layer Σ (summation), and output layer (product), where the hidden layer is the weight summation, the output layer is the product of the input and weighted value, and w_{nk} is the weighted value [14]. Therefore, the jth output —z_{jk} is

$$z_{jk} = \sum_{i=0}^{n} W_{ij} X_{ij} \tag{1}$$

The total output of the neural network is expressed as

$$y_k = \prod_{j=1}^{h} z_{jk} \tag{2}$$

where h is the unit's number in a hidden layer.

Combining Eqs. (1) and (2) yields

$$y_k = B_0 + \sum_{i=1}^{n} B_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij} x_i x_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} B_{ijk} x_i x_j x_k + \ldots\ldots\ldots \tag{3}$$

where y_k is the output value, x_i, x_j, x_k is the input data, and B_0, B_i, B_{ij}, and B_{ijk} are the coefficient of the node function.

3. System Training on *NNM*

To obtain the *NNM* by using the theoretical data of *BEM* as the input data (silencer dimensions such as d_1, d_2, and L_2) and the output data (*STL*) in the proposed *NNM*, the trained *NNM*

is achieved by the training data bank and polynomial calculation with the *PSE* standard (deviation of mean square).

PSE is expressed as

$$PSE = FSE + k_p \tag{4}$$

$$FSE = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{y}_i - y_i\right)^2 \tag{5}$$

where *FSE* is the deviation of mean square, k_p is the penalty function, N is the number of training data, $\hat{y}_i$ is the required data, and y_i is the predicted data for *NNM*.

The penalty function k_p can be expressed as

$$k_p = CPM \frac{2\sigma p^2 Q}{N} \tag{6}$$

where *CPM* is the product of penalty function, Q the number of network's coefficients, and σp^2 the error variation.

The steps of *NNM* construction shown in Fig. 3 include the following.

A. Building up the data bank for network training.
   A data bank is used to construct a polynomial neural network. It can be divided into two parts — the training data and the testing data. One is adopted for the training of the *NNM*, and the other is adopted for evaluating the *NNM*.
B. Building up the neural network model.
   By selecting the number and type of layer and using the training data bank in the chosen network, a neural network model is built.
C. Evaluating the ability of *NNM*.
   After the *NNM* is established, a function test with testing data is required for evaluating the ability of the *NNM*.
D. Using the *NNM*.

Table 1. Accuracy comparison of *STL* (a one-connected curved tube muffler at 500 Hz) between *BEM* and *NNM*.

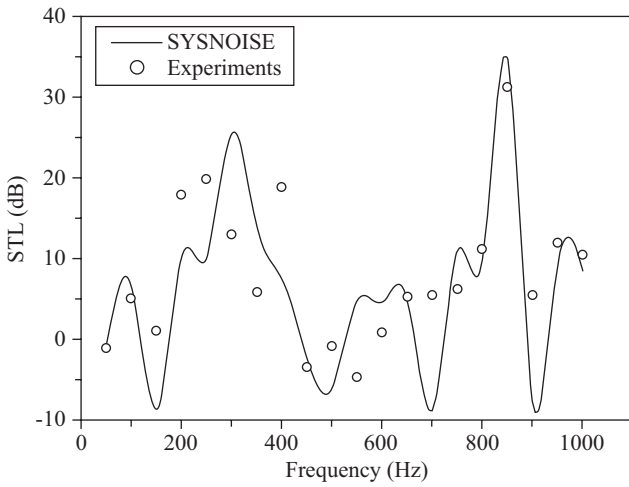|		*STL* (dB)	Deviation (%)
500 Hz (d_1 = 0.06 (m); d_2 = 0.06 (m); L_2 = 0.3 (m))	Theoretical Mathematical Model (*BEM*)	21.37	5.05
	Neural Network Model (*NNM*)	20.29	

Fig. 4. Performance of a muffler equipped with a one-connected curved tube [*M* = 0, d_1 = 0.0493 (m), d_2 = 0.1016 (m), L_1 = 0.1286 (m), L_2 = 0.1 (m), *Lo* = 0.6 (m)].

III. MODEL CHECK

1. *BEM* Accuracy Check

Before performing the *GA* optimal simulation on mufflers, an accuracy check of the mathematical model of the *BEM* on the fundamental element (a muffler equipped with a single-connected curved tube shown in Fig. 1) was performed using experimental data. As revealed in Fig. 4, accuracy between the *BEM* and the experimental data for the muffler equipped with a one-connected curved tube is in agreement. Consequently, the developed *BEM* model of multi-connected tube mufflers linked by the numerical method is applied to the shape optimization in the following section.

2. *NNM* Accuracy Check

Before using the *NNM* as an *OBJ* function in the *GA* optimization, an accuracy check of the *NNM* at 500 Hz is performed and shown in Table 1. The result reveals that *NNM* and *BEM* are accurate within 94.95%. This is acceptable.

IV. GENETIC ALGORITHM

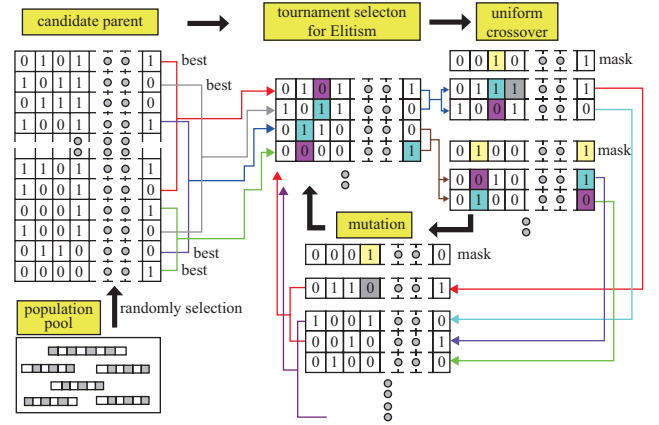The concept of Genetic Algorithms, first formalized by Holland [8] and later extended to functional optimization by

Fig. 5. Operations in the *GA* method.

Jong [10], involves the use of optimization search strategies patterned after the Darwinian notion of natural selection.

For the optimization of the objective function (*OBJ*), the design parameters of (X_1, X_2, ..., X_k) were determined. When the *chrmlength* (the bit length of the chromosome) and the popsize (population number) were chosen, the interval of the design parameter (X_k) with $[Lb, Ub]_k$ was then mapped to the band of the binary value. The mapping system between the variable interval of $[Lb, Ub]_k$ and the kth binary chromosome of

$$[\underbrace{0\ 0\ 0\ 0\ \bullet\ \bullet\ \bullet\ 0\ 0\ 0}_{chrmlength} \sim \underbrace{1\ 1\ 1\ 1\ \bullet\ \bullet\ \bullet\ 1\ 1\ 1}_{chrmlength}]$$

was then built. The encoding from *x* to *B2D* (binary to decimal) is performed as

$$B2D_k = \text{integer}\left\{\frac{x_k - Lb_k}{Ub_k - Lb_k}(2^{chrmlength} - 1)\right\} \qquad (7)$$

The initial population was built up by randomization. The parameter set was encoded to form a string which represented the chromosome. By evaluating the objective function (*OBJ*), the whole set of chromosomes [$B2D_1$, $B2D_2$, ..., $B2D_k$] that changed from binary form to decimal form was then assigned a fitness by decoding the transformation system.

$$fitness = OBJ(X_1, X_2, ..., X_k); \qquad (8)$$

where $X_k = B2D_k * (Ub_k - Lb_k)/(2^{chrmlength} - 1) + Lb_k$.

As indicated in Fig. 5, during the *GA* optimization, one pair of offspring was generated from the selected parent using uniform crossover with a probability of *pc*.

Genetically, mutation occurred with a probability of *pm* where the new and unexpected point was brought into the *GA* optimizer's search domain. To prevent the best gene from disappearing and to improve the accuracy of optimization during reproduction, the elitism scheme of keeping the best gene (one pair) in the parent generation using a tournament
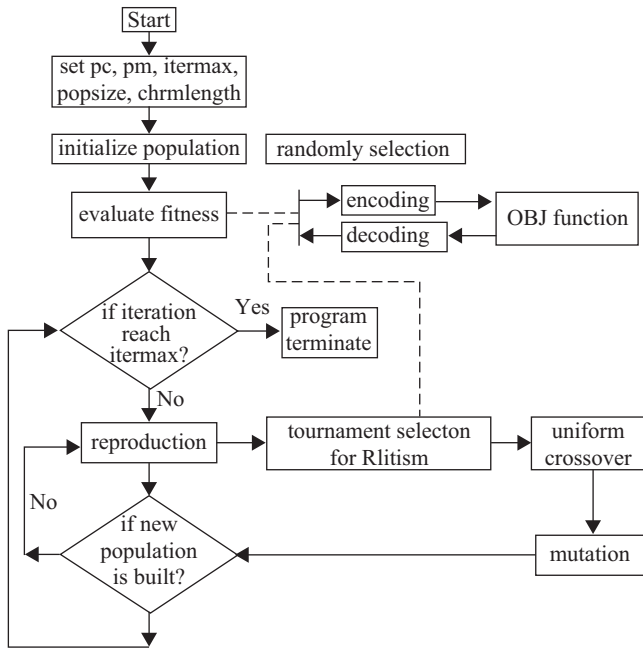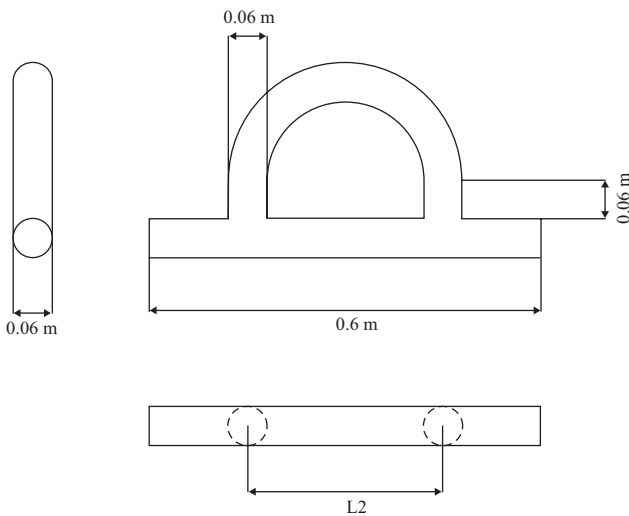
Fig. 6. Flow chart of the *GA*.

Fig. 7. An original muffler equipped with a one-connected curved tube without shape optimization.

strategy was developed. The process was terminated when the number of generations exceeded a pre-selected value of $iter_{max}$. The block diagram of *GA* optimization on mufflers is depicted in Fig. 6.

V. CASE STUDIES

In this paper, an original muffler without shape optimization shown in Fig. 7 is introduced. To achieve a higher acoustical performance (*STL*), a muffler hybridized with a one-connected curved tube in reducing various target tones (350, 500, and 650 Hz) is exemplified below.

Table 2. Design data sets used for *NNM*'s establishment (a one-connected curved tube muffler).

Design data set	d_1(m)	d_2(m)	L_2(m)
1	0.06	0.03	0.225
2	0.06	0.03	0.3
3	0.06	0.03	0.375
4	0.06	0.045	0.225
5	0.06	0.045	0.3
6	0.06	0.045	0.375
7	0.06	0.06	0.225
8	0.06	0.06	0.3
9	0.06	0.06	0.375
10	0.075	0.03	0.225
11	0.075	0.03	0.3
12	0.075	0.03	0.375
13	0.075	0.045	0.225
14	0.075	0.045	0.3
15	0.075	0.045	0.375
16	0.075	0.06	0.225
17	0.075	0.06	0.3
18	0.075	0.06	0.375
19	0.09	0.03	0.225
20	0.09	0.03	0.3
21	0.09	0.03	0.375
22	0.09	0.045	0.225
23	0.09	0.045	0.3
24	0.09	0.045	0.375
25	0.09	0.06	0.225
26	0.09	0.06	0.3
27	0.09	0.06	0.375

To simplify the optimization, an assumption that the dimension of L_1 is fixed as 0.06 (M) and the one-connected curved tube is symmetrical to the muffler is made in advance. To appreciate the acoustical performance under the limited space (Lo = 0.6 M), three kinds of design parameters — d_1, d_2, and L_2 — are chosen as the tuned variables. Therefore, the *STL* with respect to twenty seven design parameter sets shown in Table 2 is calculated by the *BEM*.

Using d_1, d_2, and L_2 as the input data and the *STL* as the output data in the *NNM*, and taking a series of training data into the *NNM* system, the fitness functions with respect to the targeted frequency (500 Hz) is established and shown in Eq. (9).

$$N1_{500\,HZ} = -6.00925 + 80.1234 \times d_1 \qquad (9a)$$

$$N2_{500\,HZ} = -3.60555 + 80.1234 \times d_2 \qquad (9b)$$

$$N3_{500\,HZ} = -4.8074 + 16.0247 \times L_2 \qquad (9c)$$

$$N6_{500\,HZ} = -0.559315 \times N1_{500\,HZ} + 0.68598 \times N2_{500\,HZ} +$$

$$0.11297 \times N3_{500\,HZ} \qquad (9d)$$

Table 3. Constrained condition for a one-connected curved tube muffler.

	Min. (m)	Max. (m)
d_1	0.06	0.09
d_2	0.03	0.06
L_2	0.225	0.375

Table 4. Selected *GA* parameters during shape optimization.

GA parameters	Value (or condition)
design variables	3
chrmlength	20
pop	50
elitism	(tournament)
crossover	(uniform crossover)
pc	0.8
pm	0.05
$iter_{max}$	5000

Table 5. Comparison of acoustical performance with and without shape optimization at various frequencies.

Targeted frequency	Optimized Muffler				Original muffler
	d_1 (m)	d_2 (m)	L_2 (m)	*STL* (dB)	*STL* (dB)
350 Hz	0.0877	0.06	0.2733	17.51	11.39
500 Hz	0.0601	0.0597	0.3416	45.87	21.37
650 Hz	0.0605	0.0598	0.2256	23.28	9.42
Note: for original muffler: $d_1 = 0.06$; $d_2 = 0.06$; $L_2 = 0.3$					

$$\text{N5}_{500\,\text{HZ}} = -0.320339 + 0.855784 \times \text{N6}_{500\,\text{HZ}} +$$

$$0.417828 \times \text{N6}_{500\,\text{HZ}}{}^2 + 0.0838251 \times \text{N6}_{500\,\text{HZ}}{}^3 \quad (9e)$$

$$STL_{500\,\text{HZ}} = 6.93825 + 6.1319 \times \text{N5}_{500\,\text{HZ}} \quad (9f)$$

Similarly, the fitness functions with respect to the targeted frequencies of 350 Hz and 650 Hz are also built.

In addition, the searching range of d_1, d_2, and L_2 is illustrated in Table 3.

VI. RESULTS AND DISCUSSION

1. Results

By using the trained *NNM* in conjunction with the *GA* optimizer, a series of optimized results are obtained. The selected *GA* parameters are shown in Table 4. The resultant optimizations with respect to the targeted tones (350, 500, and 650 Hz) are shown in Table 5. Moreover, their *STL* curves, with and without optimization at various designed frequencies are plotted in Figs. 8~10. As indicated in Table 5, it is obvious
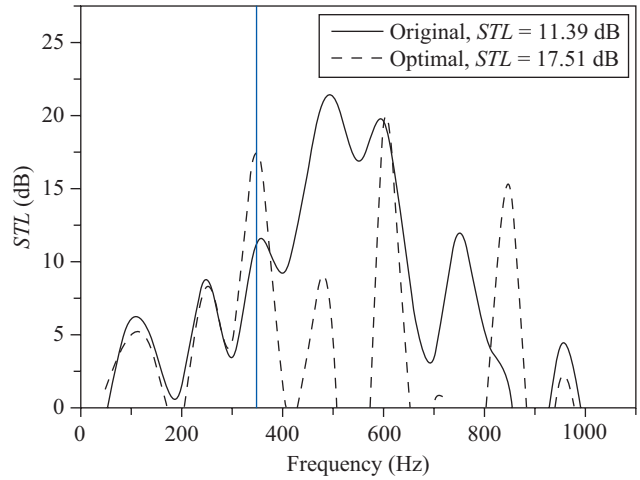

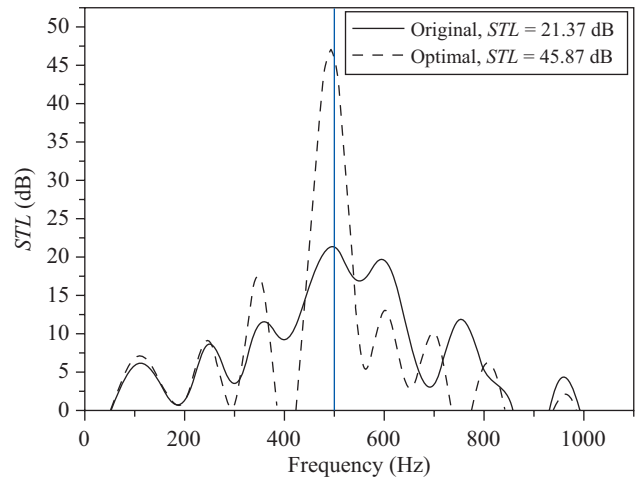**Fig. 8.** *STL* **with and without optimization (at targeted frequency of 350 Hz).**

Fig. 9. *STL* **with and without optimization (at targeted frequency of 500 Hz).**
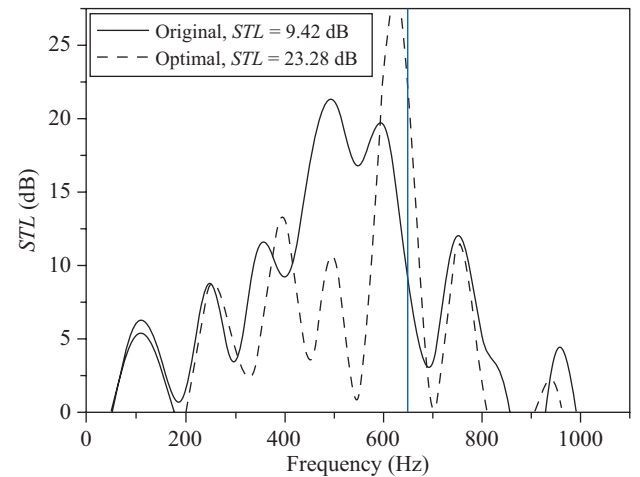

Fig. 10. *STL* **with and without optimization (at targeted frequency of 650 Hz).**

that the acoustical performance (*STL*) at the targeted 350, 500, 650 Hz are improved from 11.39 to 17.51 dB, 21.37 to 45.87 dB, and 9.42 to 23.28 dB.

2. Discussion

As indicated in Table 5, d_1 will be decreased and d_2 will be increased simultaneously during the *GA* optimization. However, the variety of L_2 will depend on the selected targeted frequency. Also, Table 5 indicates that for the one-connected curved tube muffler the best acoustical performances are 17.51~45.87 dB with respect to the targeted tones (350~650 Hz). As can be noted, comparisons of the acoustical performance of mufflers before and after optimization at various targeted tones (350, 500, and 650 Hz) are assessed and plotted in Figs. 8~10. Figs. 8~10 indicate that the *STL*s are precisely maximized at the desired frequency. Therefore, using the *GA* optimization for finding a better design solution of a one-connected curved tube muffler is reliable and efficient.

VII. CONCLUSION

The present paper indicates that a one-connected curved tube muffler can be precisely optimized at a targeted frequency with the *NNM* and the *GA* method by adjusting the muffler shape within a constrained space.

As the design parameters could easily be changed without a total overhaul of the muffler design when we use the polynomial neural network instead of the full model (*BEM*), the surrogate model — a trained neural network model (*NNM*) fitted by a series of real data — is established and used as the new *OBJ* function. Before optimization is performed, the accuracy of the boundary element method (*BEM*) is checked by the experimental data and found to be accurate. As the real data is very close to the *BEM* data, to facilitate the assessment of the real data fitted to a *NNM*, the *BEM* data is thus used as the real data. Moreover, the similarity of the *STL* related to the *BEM* and *NNM* is sufficient. Furthermore, the optimal values of the *STL* achieved at the target frequencies reveal that the NNM along with the *GA* optimizer in the one-connected curved tube mufflers were applicable. Results reveal that the *STL* will increase if d_1 is decreased and d_2 is increased. Moreover, the variety of L_2 will depend on the selected targeted frequency. Consequently, the use of the *GA* optimization in conjunction with *NNM* and *BEM* in the one-connected curved tube mufflers' shape design is more efficient than the complicated models (transfer matrix method and stiffness matrix method) or the redundant tests conducted in the laboratory.

ACKNOWLEDGMENTS

NOMENCLATURE

This paper is constructed on the basis of the following notations:

bit: bit length of chromosome
d_1: diameter of the main duct (m)
d_2: diameter of the connected curved tube (m)
$iter_{max}$: maximum iteration during *GA* optimization
L_1: lengths of straight part for a connected tube (m)
L_2: span between the outlet and inlet of a connected tube along the main duct (m)
pc: crossover ratio
pm: mutation ratio
pop: number of population
STL: sound transmission loss (dB)

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