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GLOBAL EXPONENTIAL STABILITY CRITERIA FOR SWITCHED NEUTRAL SYSTEMS WITH INTERVAL TIME-VARYING DELAY

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Key words: switched systems, neutral systems, global exponential stability, interval time-varying state delay, Razumikhin-like approach.

ABSTRACT

The delay-dependent and delay-independent conditions are proposed to guarantee the global exponential stability for uncertain switched neutral system with interval time-varying state delay. New additional nonnegative inequalities are introduced to improve the conservativeness of system. Razumikhin-like approach is used to prove the exponential stability for system. Structured and unstructured uncertainties are investigated in this paper. The solving schemes based on Linear Matrix Inequality (LMI) approach along with the selective examples are presented to demonstrate the improvements achieved.

I. INTRODUCTION

A switched system is a class of hybrid systems which consists of several subsystems and exhibits the switching feature between multi-models, which is usually used to approximate many practical nonlinear systems [10]. It is well known that the existence of time delay in a system may cause instability or bad system performance in feedback control systems. Since time-delay phenomenon may encounter in many practical systems, such as aircraft stabilization, chemical engineering systems, inferred grinding model, neural network, nuclear reactor, population dynamic model, rolling mill, ship stabilization, and systems with lossless transmission lines [2, 3, 5]. Hence stability analysis for switched time-delay systems has been investigated in recent years [4-9, 11, 13].

It is interesting to note that for each stable subsystem cannot imply that the overall system is also stable [7]. Hence we will consider the global exponential stability problem for switched neutral systems with interval time-varying state delay and two classes of perturbations under arbitrary switched signal. Based on Razumikhin-like [12] and LMI approaches [1], delay-dependent and delay-independent results are provided. The LMI approach [1] is an efficient and powerful tool in solving some control problems; such as H_∞ control, stability analysis, guaranteed cost control, state feedback control, static output feedback control, and observer-based control. Hence LMI approach will be used to guarantee the stability problem of systems. Some numerical examples are provided to demonstrate the main proposed results.

The notation used throughout this paper is as follows. For a matrix A, we denote the transpose by A^T, spectral norm by $\|A\|$, minimal (maximal) eigenvalue by $\lambda_{\min}(A)$ $(\lambda_{\max}(A))$, symmetric positive (negative) definite by $A > 0$ $(A > 0)$. $A \le B$ means that matrix $B - A$ is symmetric positive semi-definite. For a vector x, we denote the Euclidean norm by $\|x\|$. For the state x_t of system, we define $x_t(\theta) := x(t+\theta), \forall \theta \in [-H, 0]$ and denote its norm by $\|x_t\|_s = \sup_{-H\le\theta\le 0} \sqrt{\|x(t+\theta)\|^2 + \|\dot{x}(t+\theta)\|^2}$. I denotes the identity matrix.

II. PROBLEM FORMULATION AND MAIN RESULTS

Consider the following uncertain switched neutral system with interval time-varying state delay:

$$\dot{x}(t) - D\dot{x}(t-\tau) = [A_{0\sigma} + \Delta A_{0\sigma}(t)]x(t)$$

$$+ [A_{1\sigma} + \Delta A_{1\sigma}(t)]x(t-h(t)),\ t \ge 0, \tag{1a}$$

$$x(t) = \phi(t),\ t \in [-H, 0], \tag{1b}$$

where $x \in \Re^n$. Switching signal σ may depend on t or x and takes its values in the finite set $\{i = 1, 2, \cdots, N\}$. Interval time-varying delay $h(t)$ satisfies $0 \le h_m < h(t) \le h_M$, $\dot{h}(t) \le h_D$. Constant delay $\tau > 0$ and $H = \max\{h_M, \tau\}$. Matrices D, A_{0i}, and $A_{1i} \in \Re^{n\times n}$, $i = 1, 2, \cdots, N$, are constant, and the initial vector $\phi \in C_1$, where C_1 is the set of differentiable functions from $[-H, 0]$ to $\Re^n$.

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In this paper, we will consider the following two types of perturbations on system:

(A1) Structured perturbations: $\Delta A_{0i}(t)$ and $\Delta A_{1i}(t)$ satisfy the following conditions

$$\left[\Delta A_{0i}(t) \quad \Delta A_{1i}(t)\right] = M_i F_i(t)\left[N_{0i} \quad N_{1i}\right],$$
$$\forall\, i \in \{1, 2, \cdots, N\},\, t \ge 0, \tag{2a}$$

where M_i, N_{0i}, and N_{1i}, $i = 1, 2, \cdots, N$, are some given constant matrices with appropriate dimensions, and $F_i(t)$, $i = 1, 2, \cdots, N$, are unknown matrices representing the parameter perturbation which satisfy

$$F_i^T(t) F_i(t) \le I,\ \forall\, i \in \{1, 2, \cdots, N\},\, t \ge 0\,. \tag{2b}$$

(A2) Unstructured perturbations: $\Delta A_{0i}(t)$ and $\Delta A_{1i}(t)$ satisfy the following conditions

$$\|\Delta A_{0i}(t)\| \le \sigma_{0i} \text{ and } \|\Delta A_{1i}(t)\| \le \sigma_{1i}, \tag{3}$$

where σ_{0i} and σ_{1i}, $i = 1, 2, \cdots, N$, are some given nonnegative constants.

Define the functions $\lambda_i(t)$, $i = 1, 2, \cdots, N$, as follows:

$$\lambda_i(t) = \begin{cases} 1, & \sigma = i, \\ 0, & \text{otherwise}, \end{cases} \quad i = 1, 2, \cdots, N. \tag{4}$$

We can rewrite the switched system (1) to the following form:

$$\dot{x}(t) - D\dot{x}(t-\tau) = \sum_{i=1}^{N} \lambda_i(t)\{[A_{0i} + \Delta A_{0i}(t)]x(t)$$
$$+ [A_{1i} + \Delta A_{1i}(t)]x(t - h(t))\},\ t \ge 0, \tag{5a}$$

$$x(t) = \phi(t), \qquad t \in [-H, 0], \tag{5b}$$

where $\lambda_i(t)$ is defined in (4) and $\sum_{i=1}^{N} \lambda_i(t) = 1$, $\lambda_i^2(t) = \lambda_i(t)$, and $\lambda_i(t) \cdot \lambda_j(t) = 0,\ i \ne j,\ \ \forall\, t \ge 0$.

The following lemma will be used in the proof of our main results.

Lemma 1: [12] Let U, V, W and M be real matrices of appropriate dimensions with M satisfying $M = M^T$, then

$$M + UVW + W^T V^T U^T < 0, \text{ for all } V^T V \le I,$$

if and only if there exists a scalar $\varepsilon > 0$ such that

$$M + \varepsilon^{-1} \cdot UU^T + \varepsilon \cdot W^T W$$
$$= M + \varepsilon^{-1} \cdot UU^T + \varepsilon^{-1} \cdot (\varepsilon W)^T (\varepsilon W) < 0.$$

Definition 1. The system (1) with (A1) or (A2) is said to be the globally exponentially stable with convergence rate $\alpha > 0$, if there are two positive constants α and Ψ such that

$$\|x(t)\| \le \Psi \cdot e^{-\alpha t}, \quad t \ge 0\,.$$

Now we present a delay-dependent condition for stability of system (1) with (A1).

Theorem 1. System (1) with (A1) and $h_D < 1$ (resp., $h_D \ge 1$ or unknown) is globally exponentially stable with convergence rate $0 < \alpha < -(\ln\|D\|)/\tau$, if $\|D\| < 1$ and there exist some $n \times n$ matrices P, Q_1, Q_2, R_1, R_2, R_3, R_4, S, R_{22}, S_{22}, $T_{22} > 0$, $R_{11}, S_{11}, T_{11} > 0 \in \Re^{7n\times 7n}$ (resp., Q_2 = 0), some matrices U_{1i}, U_{2i}, U_{3i}, U_{4i}, U_{5i}, U_{6i}, $U_{7i} \in \Re^{n\times n}$, $R_{12}, S_{12}, T_{12} \in \Re^{7n\times n}$, and some positive constants ε_i, $i = 1, 2, \cdots, N$, such that the following LMI conditions hold for all $i = 1, 2, \cdots, N$

$$e^{-2\alpha h_m} \cdot R_1 - R_{22} > 0,\ e^{-2\alpha h_M} \cdot R_2 - S_{22} > 0,$$
$$e^{-2\alpha h_M} \cdot R_2 - T_{22} > 0, \tag{6a}$$

$$\begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} > 0, \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} > 0, \begin{bmatrix} T_{11} & T_{12} \\ * & T_{22} \end{bmatrix} > 0, \tag{6b}$$

$$\hat{\Sigma}_i = \begin{bmatrix} \Sigma_{11i} & \Sigma_{12i} & \Sigma_{13i} & \Sigma_{14i} & \Sigma_{15i} & \Sigma_{16i} & \Sigma_{17i} & \Sigma_{18i} & \Sigma_{19i} \\ * & \Sigma_{22i} & \Sigma_{23i} & 0 & 0 & 0 & 0 & \Sigma_{28i} & 0 \\ * & * & \Sigma_{33i} & \Sigma_{34i} & \Sigma_{35i} & \Sigma_{36i} & \Sigma_{37i} & \Sigma_{38i} & \Sigma_{39i} \\ * & * & * & \Sigma_{44i} & \Sigma_{45i} & \Sigma_{46i} & 0 & \Sigma_{48i} & 0 \\ * & * & * & * & \Sigma_{55i} & \Sigma_{56i} & \Sigma_{57i} & \Sigma_{58i} & 0 \\ * & * & * & * & * & \Sigma_{66i} & \Sigma_{67i} & \Sigma_{68i} & 0 \\ * & * & * & * & * & * & \Sigma_{77i} & \Sigma_{78i} & 0 \\ * & * & * & * & * & * & * & \Sigma_{88i} & 0 \\ * & * & * & * & * & * & * & * & \Sigma_{99i} \end{bmatrix}$$
$$+ \begin{bmatrix} \Phi_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} < 0, \tag{6c}$$

where * represents the symmetric form in the matrix and

$$\Sigma_{11i} = 2\alpha \cdot P + PA_{0i} + A_{0i}^T P + Q_1 + Q_2 + U_{2i}^T A_{0i} + A_{0i}^T U_{2i} + S + R_4,$$

$$\Sigma_{12i} = -2\alpha \cdot PD - A_{0i}^T PD,$$

$$\Sigma_{13i} = PA_{1i} + U_{2i}^T A_{1i} + A_{0i}^T U_{3i}, \Sigma_{14i} = A_{0i}^T U_{6i},$$

$$\Sigma_{15i} = A_{0i}^T U_{1i}^T - U_{2i}^T + A_{0i}^T U_{4i},$$

$$\Sigma_{16i} = -A_{0i}^T U_{1i}^T D + U_{2i}^T D + A_{0i}^T U_{5i}, \Sigma_{17i} = A_{0i}^T U_{7i},$$

$$\Sigma_{18i} = PM_i + U_{2i}^T M_i, \Sigma_{19i} = \varepsilon_i \cdot N_{0i}^T,$$

$$\Sigma_{22i} = 2\alpha \cdot D^T PD - e^{-2\alpha\tau} \cdot S, \Sigma_{23i} = -D^T PA_{1i},$$

$$\Sigma_{28i} = -D^T PM_i,$$

$$\Sigma_{33i} = -e^{-2\alpha h_M} \cdot (1-h_D) \cdot Q_2 + U_{3i}^T A_{1i} + A_{1i}^T U_{3i}, \Sigma_{34i} = A_{1i}^T U_{6i},$$

$$\Sigma_{35i} = A_{1i}^T U_{1i}^T - U_{3i}^T + A_{1i}^T U_{4i},$$

$$\Sigma_{36i} = -A_{1i}^T U_{1i}^T D + U_{3i}^T D + A_{1i}^T U_{5i}, \Sigma_{37i} = A_{1i}^T U_{7i}, \Sigma_{38i} = U_{3i}^T M_i,$$

$$\Sigma_{39i} = \varepsilon_i \cdot N_{1i}^T, \Sigma_{44i} = -e^{-2\alpha h_m} \cdot Q_1 + e^{-2\alpha h_m} R_3,$$

$$\Sigma_{45i} = -U_{6i}^T, \Sigma_{46i} = U_{6i}^T D, \Sigma_{48i} = U_{6i}^T M_i,$$

$$\Sigma_{55i} = h_m \cdot R_1 + (h_M - h_m) \cdot R_2 - U_{1i} - U_{1i}^T - U_{4i} - U_{4i}^T,$$

$$\Sigma_{56i} = (U_{1i} + U_{1i}^T)D + U_{4i}^T D - U_{5i}, \Sigma_{57i} = -U_{7i},$$

$$\Sigma_{58i} = U_{1i} M_i + U_{4i}^T M_i,$$

$$\Sigma_{66i} = -D^T (U_{1i} + U_{1i}^T)D + U_{5i}^T D + D^T U_{5i}, \Sigma_{67i} = D^T U_{7i},$$

$$\Sigma_{68i} = -D^T U_{1i} M_i + U_{5i}^T M_i, \Sigma_{77i} = -e^{-2\alpha h_M}(R_3 + R_4),$$

$$\Sigma_{78i} = U_{7i}^T M_i, \Sigma_{88i} = \Sigma_{99i} = -\varepsilon_i \cdot I,$$

$$\Phi_{11} = h_m \cdot R_{11} + R_{12}\Lambda_1 + \Lambda_1^T R_{12}^T + (h_M - h_m) \cdot (T_{11} + S_{11})$$

$$+ S_{12}\Lambda_2 + \Lambda_2^T S_{12}^T + T_{12}\Lambda_3 + \Lambda_3^T T_{12}^T,$$

$$\Lambda_1 = [I \quad 0 \quad 0 \quad -I \quad 0 \quad 0 \quad 0]_{n \times 7n},$$

$$\Lambda_2 = [0 \quad 0 \quad I \quad 0 \quad 0 \quad 0 \quad -I]_{n \times 7n},$$

$$\Lambda_3 = []0 \quad 0 \quad -I \quad I \quad 0 \quad 0 \quad 0_{n \times 7n}.$$

Proof. Define the Lyapunov functional

$$V(x_t) = e^{2\alpha t} \cdot (x(t) - Dx(t-\tau))^T P(x(t) - Dx(t-\tau))$$

$$+ \int_{t-h_m}^{t} e^{2\alpha s} \cdot x^T(s) Q_1 x(s) ds$$

$$+ \int_{t-h(t)}^{t} e^{2\alpha s} \cdot x^T(s) Q_2 x(s) ds$$

$$+ \int_{t-\tau}^{t} e^{2\alpha s} \cdot x^T(s) S x(s) ds$$

$$+ \int_{t-h_m}^{t} e^{2\alpha s} \cdot (s - (t-h_m)) \dot{x}^T(s) R_1 \dot{x}(s) ds$$

$$+ \int_{t-h_M}^{t-h_m} e^{2\alpha s} \cdot (s - (t-h_M)) \dot{x}^T(s) R_2 \dot{x}(s) ds$$

$$+ (h_M - h_m) \cdot \int_{t-h_m}^{t} e^{2\alpha s} \cdot \dot{x}^T(s) R_2 \dot{x}(s) ds$$

$$+ \int_{t-h_M}^{t-h_m} e^{2\alpha s} \cdot x^T(s) R_3 x(s) ds$$

$$+ \int_{t-h_M}^{t} e^{2\alpha s} \cdot x^T(s) R_4 x(s) ds, \tag{7}$$

where $P, Q_1, Q_2, R_1, R_2, R_3, R_4, S > 0$. The time derivatives of $V(x_t)$, along the trajectories of system (5) satisfy

$$\dot{V}(x_t) = e^{2\alpha t} \cdot [2\alpha \cdot (x(t) - Dx(t-\tau))^T P(x(t) - Dx(t-\tau))]$$

$$+ 2e^{2\alpha t} \cdot \sum_{i=1}^{N} \lambda_i(t)[\{[A_{0i} + \Delta A_{0i}(t)]x(t)\}^T P(x(t) - Dx(t-\tau))]$$

$$+ 2e^{2\alpha t} \cdot \sum_{i=1}^{N} \lambda_i(t)[\{[A_{1i} + \Delta A_{1i}(t)]x(t-h(t))\}^T P(x(t) - Dx(t-\tau))]$$

$$+ e^{2\alpha t} \cdot [x^T(t) Q_1 x(t) - e^{-2\alpha h_m} \cdot x^T(t-h_m) Q_1 x(t-h_m)]$$

$$+ e^{2\alpha t} \cdot [x^T(t) Q_2 x(t) - (1 - \dot{h}(t)) \cdot e^{-2\alpha h(t)} \cdot x^T(t-h(t)) Q_2 x(t-h(t))]$$

$$+ e^{2\alpha t} \cdot [x^T(t) S x(t) - e^{-2\alpha\tau} \cdot x^T(t-\tau) S x(t-\tau)]$$

$$+ e^{2\alpha t} \cdot [h_m \cdot \dot{x}^T(t) R_1 \dot{x}(t) - \int_{t-h_m}^{t} e^{2\alpha(s-t)} \cdot \dot{x}^T(s) R_1 \dot{x}(s) ds]$$

$$+ e^{2\alpha t} \cdot [(h_M - h_m) \cdot \dot{x}^T(t) R_2 \dot{x}(t) - \int_{t-h_M}^{t-h_m} e^{2\alpha(s-t)} \cdot \dot{x}^T(s) R_2 \dot{x}(s) ds]$$

$$+ e^{2\alpha t} \cdot [e^{-2\alpha h_m} x^T(t-h_m) R_3 x(t-h_m)$$

$$- e^{-2\alpha h_M} x^T(t-h_M) R_3 x(t-h_M)]$$

$$+ e^{2\alpha t} \cdot [x^T(t) R_4 x(t) - e^{-2\alpha h_M} x^T(t-h_M) R_4 x(t-h_M)]. \tag{8a}$$

By some simple derivatives, we have

$$-\int_{t-h_m}^{t} e^{2\alpha(s-t)}\cdot\dot{x}^T(s)R_1\dot{x}(s)ds \le -e^{-2\alpha h_m}\cdot\int_{t-h_m}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds, \tag{8b}$$

and

$$-\int_{t-h_M}^{t-h_m} e^{2\alpha(s-t)}\cdot\dot{x}^T(s)R_2\dot{x}(s)ds$$

$$=-e^{-2\alpha h_M}\cdot\left(\int_{t-h_M}^{t-h(t)}\dot{x}^T(s)R_2\dot{x}(s)ds+\int_{t-h(t)}^{t-h_m}\dot{x}^T(s)R_2\dot{x}(s)ds\right). \tag{8c}$$

Define

$$\hat{X}^T=[x^T(t)\quad x^T(t-\tau)\quad x^T(t-h(t))\quad x^T(t-h_m)\quad \dot{x}^T(t)\quad \dot{x}^T(t-\tau)\quad x^T(t-h_M)].$$

By Leibniz-Newton formula and LMIs (6b), we have

$$\int_{t-h_m}^{t}\begin{bmatrix}\hat{X}\\ \dot{x}(s)\end{bmatrix}^T\begin{bmatrix}R_{11} & R_{12}\\ * & R_{22}\end{bmatrix}\begin{bmatrix}\hat{X}\\ \dot{x}(s)\end{bmatrix}ds$$

$$=h_m\hat{X}^TR_{11}\hat{X}+2\hat{X}^TR_{12}\left[x(t)-x(t-h_m)\right]$$

$$+\int_{t-h_m}^{t}\dot{x}^T(s)R_{22}\dot{x}(s)ds\ge 0, \tag{8d}$$

$$\int_{t-h_M}^{t-h(t)}\begin{bmatrix}\hat{X}\\ \dot{x}(s)\end{bmatrix}^T\begin{bmatrix}S_{11} & S_{12}\\ * & S_{22}\end{bmatrix}\begin{bmatrix}\hat{X}\\ \dot{x}(s)\end{bmatrix}ds$$

$$=(h_M-h(t))\cdot\hat{X}^TS_{11}\hat{X}+2\hat{X}^TS_{12}[x(t-h(t))-x(t-h_M)]$$

$$+\int_{t-h_M}^{t-h(t)}\dot{x}^T(s)S_{22}\dot{x}(s)ds\ge 0, \tag{8e}$$

$$\int_{t-h(t)}^{t-h_m}\begin{bmatrix}\hat{X}\\ \dot{x}(s)\end{bmatrix}^T\begin{bmatrix}T_{11} & T_{12}\\ * & T_{22}\end{bmatrix}\begin{bmatrix}\hat{X}\\ \dot{x}(s)\end{bmatrix}ds$$

$$=(h(t)-h_m)\cdot\hat{X}^TT_{11}\hat{X}+2\hat{X}^TT_{12}\left[x(t-h_m)-x(t-h(t))\right]$$

$$+\int_{t-h(t)}^{t-h_m}\dot{x}^T(s)T_{22}\dot{x}(s)ds\ge 0, \tag{8f}$$

By system (5), and $\lambda_i^2(t)=\lambda_i(t)$, and $\lambda_i(t)\lambda_j(t)=0, i\ne j$, we have

$$-\sum_{i=1}^{N}\lambda_i(t)(\dot{x}(t)-D\dot{x}(t-\tau))^T(U_{1i}+U_{1i}^T)(\dot{x}(t)-D\dot{x}(t-\tau))$$

$$+\sum_{i=1}^{N}\lambda_i(t)[(\dot{x}(t)-D\dot{x}(t-\tau))^TU_{1i}\{[A_{0i}+\Delta A_{0i}(t)]x(t)+[A_{1i}+\Delta A_{1i}(t)]x(t-h(t))\}]$$

$$+\sum_{i=1}^{N}\lambda_i(t)[\{[A_{0i}+\Delta A_{0i}(t)]x(t)\}^TU_{1i}^T(\dot{x}(t)-D\dot{x}(t-\tau))]$$

$$+\sum_{i=1}^{N}\lambda_i(t)[\{[A_{1i}+\Delta A_{1i}(t)]x(t-h(t))\}^TU_{1i}^T(\dot{x}(t)-D\dot{x}(t-\tau))]$$

$$=0, \tag{8g}$$

$$\sum_{i=1}^{N}\lambda_i(t)\cdot\{x^T(t)U_{2i}^T+x^T(t-h(t))U_{3i}^T+\dot{x}^T(t)U_{4i}^T+\dot{x}^T(t-\tau)U_{5i}^T+x^T(t-h_m)U_{6i}^T+x^T(t-h_M)U_{7i}^T\}$$

$$\cdot\sum_{i=1}^{N}\lambda_i(t)\cdot\{-\dot{x}(t)+D\dot{x}(t-\tau)+[A_{0i}+\Delta A_{0i}(t)]x(t)+[A_{1i}+\Delta A_{1i}(t)]x(t-h(t))\}$$

$$+\sum_{i=1}^{N}\lambda_i(t)\cdot\{-\dot{x}(t)+D\dot{x}(t-\tau)+[A_{0i}+\Delta A_{0i}(t)]x(t)+[A_{1i}+\Delta A_{1i}(t)]x(t-h(t))\}^T$$

$$\cdot\sum_{i=1}^{N}\lambda_i(t)\cdot\{x^T(t)U_{2i}^T+x^T(t-h(t))U_{3i}^T+\dot{x}^T(t)U_{4i}^T+\dot{x}^T(t-\tau)U_{5i}^T+x^T(t-h_m)U_{6i}^T+x^T(t-h_M)U_{7i}^T\}^T$$

$$=\sum_{i=1}^{N}\lambda_i(t)\cdot[\{x^T(t)U_{2i}^T+x^T(t-h(t))U_{3i}^T+\dot{x}^T(t)U_{4i}^T+\dot{x}^T(t-\tau)U_{5i}^T+x^T(t-h_m)U_{6i}^T+x^T(t-h_M)U_{7i}^T\}$$

$$\cdot\{-\dot{x}(t)+D\dot{x}(t-\tau)+[A_{0i}+\Delta A_{0i}(t)]x(t)+[A_{1i}+\Delta A_{1i}(t)]x(t-h(t))\}]$$

$$+\sum_{i=1}^{N}\lambda_i(t)\cdot[\{-\dot{x}(t)+D\dot{x}(t-\tau)+[A_{0i}+\Delta A_{0i}(t)]x(t)+[A_{1i}+\Delta A_{1i}(t)]x(t-h(t))\}^T$$

$$\cdot \{x^T(t)U_{2i}^T + x^T(t-h(t))U_{3i}^T + \dot{x}^T(t)U_{4i}^T + \dot{x}^T(t-\tau)U_{5i}^T$$

$$+ x^T(t-h_m)U_{6i}^T + x^T(t-h_M)U_{7i}^T\}^T] = 0. \quad (8h)$$

By (8a)-(8h), we obtain the following result

$$\dot{V}(x_t) \le e^{2\alpha t} \cdot \left\{ \sum_{i=1}^{N} \lambda_i(t) \hat{X}^T \cdot \Sigma_i \cdot \hat{X} \right.$$

$$- \int_{t-h_m}^{t} \dot{x}^T(s)[e^{-2\alpha h_m} \cdot R_1 - R_{22}]\dot{x}(s)ds$$

$$- \int_{t-h_M}^{t-h(t)} \dot{x}^T(s)[e^{-2\alpha h_M} \cdot R_2 - S_{22}]\dot{x}(s)ds$$

$$\left. - \int_{t-h(t)}^{t-h_m} \dot{x}^T(s)[e^{-2\alpha h_M} \cdot R_2 - T_{22}]\dot{x}(s)ds \right\}, \quad (9)$$

where

$$\Sigma_i = \begin{bmatrix} \Sigma_{11i} & \Sigma_{12i} & \Sigma_{13i} & \Sigma_{14i} & \Sigma_{15i} & \Sigma_{16i} & \Sigma_{17i} \\ * & \Sigma_{22i} & \Sigma_{23i} & 0 & 0 & 0 & 0 \\ * & * & \Sigma_{33i} & \Sigma_{34i} & \Sigma_{35i} & \Sigma_{36i} & \Sigma_{37i} \\ * & * & * & \Sigma_{44i} & \Sigma_{45i} & \Sigma_{46i} & 0 \\ * & * & * & * & \Sigma_{55i} & \Sigma_{56i} & \Sigma_{57i} \\ * & * & * & * & * & \Sigma_{66i} & \Sigma_{67i} \\ * & * & * & * & * & * & \Sigma_{77i} \end{bmatrix}$$

$$+ \Phi_{11} + \begin{bmatrix} PM_i + U_{2i}^T M_i \\ -D^T PM_i \\ U_{3i}^T M_i \\ U_{6i}^T M_i \\ U_{1i} M_i + U_{4i}^T M_i \\ -D^T U_{1i} M_i + U_{5i}^T M_i \\ U_{7i}^T M_i \end{bmatrix} F_i(t) \begin{bmatrix} N_{0i}^T \\ 0 \\ N_{1i}^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T$$

$$+ \begin{bmatrix} N_{0i}^T \\ 0 \\ N_{1i}^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i^T(t) \begin{bmatrix} PM_i + U_{2i}^T M_i \\ -D^T PM_i \\ U_{3i}^T M_i \\ U_{6i}^T M_i \\ U_{1i} M_i + U_{4i}^T M_i \\ -D^T U_{1i} M_i + U_{5i}^T M_i \\ U_{7i}^T M_i \end{bmatrix}^T .$$

By the Lemma 1 and schur complement of [1], conditions $\hat{\Sigma}_i < 0$, $i = 1, 2, \cdots, N$, in (6) will imply $\Sigma_i < 0$ in (9). From $\Sigma_i < 0$ and LMIs (6a), we have

$$V(x_t) \le V(x_0), \cdots t \ge 0 \quad (10)$$

where

$$V(x_0) = (x(0) - Dx(-\tau))^T P(x(0) - Dx(-\tau))$$

$$+ \int_{-h_m}^{0} e^{2\alpha s} \cdot x^T(s)Q_1 x(s)ds$$

$$+ \int_{-h(0)}^{0} e^{2\alpha s} \cdot x^T(s)Q_2 x(s)ds$$

$$+ \int_{-\tau}^{0} e^{2\alpha s} \cdot x^T(s)Sx(s)ds$$

$$+ \int_{-h_m}^{0} e^{2\alpha s} \cdot (s + h_m)\dot{x}^T(s)R_1\dot{x}(s)ds$$

$$+ \int_{-h_M}^{-h_m} e^{2\alpha s} \cdot (s + h_M)\dot{x}^T(s)R_2\dot{x}(s)ds$$

$$+ (h_M - h_m) \cdot \int_{-h_m}^{0} e^{2\alpha s} \cdot \dot{x}^T(s)R_2\dot{x}(s)ds$$

$$+ \int_{-h_M}^{-h_m} e^{2\alpha s} x^T(s)R_3 x(s)ds$$

$$+ \int_{-h_M}^{0} e^{2\alpha s} x^T(s)R_4 x(s)ds$$

$$\le \delta_1 \cdot \|x_0\|_s^2 ,$$

and

$$\delta_1 = \lambda_{\max}(P) \cdot (1 + \|D\|)^2 + h_m \cdot \lambda_{\max}(Q_1) + h_M \cdot \lambda_{\max}(Q_2)$$

$$+ \tau \cdot \lambda_{\max}(S) + h_m^2 \cdot \lambda_{\max}(R_1)$$

$$+ (h_M - h_m)^2 \cdot \lambda_{\max}(R_2) + h_m \cdot (h_M - h_m) \cdot \lambda_{\max}(R_2)$$

$$+ (h_M - h_m) \cdot \lambda_{\max}(R_3) + h_M \cdot \lambda_{\max}(R_4).$$

On the other hand, we have

$$\lambda_{\min}(P) \cdot e^{2\alpha t} \cdot \|\wp(t)\|^2 \le e^{2\alpha t} \cdot \wp^T(t)P\wp(t)$$

$$\le V(x_t) \le V(x_0) \le \delta_1 \cdot \|x_0\|_s^2 , \quad (11)$$

where $\wp(t) = x(t) - Dx(t-\tau)$. From (11), we can obtain the following result

$$\|x(t)\| = \|\wp(t) + Dx(t-\tau)\| \le \|D\| \cdot \|x(t-\tau)\| + \|\wp(t)\|$$

$$\le \|D\| \cdot \|x(t-\tau)\| + \delta_2 \cdot e^{-\alpha t}, \quad t \ge 0,$$

where $\delta_2 = \sqrt{\delta_1 / \lambda_{\min}(P)} \cdot \|x_0\|_s$. Since $\|D\| < 1$ and $\tau > 0$, we can choose a sufficiently small positive constant $\xi = \alpha < -(\ln\|D\|)/\tau$ satisfying $\|D\| \cdot e^{\xi\tau} < 1$. By the Razumikhin-like approach of [12], we obtain the following result

$$\|x(t)\| \le \left[\sup_{-h\le\theta\le 0} \|x(\theta)\| + \frac{\delta_2}{1-\|D\|e^{\xi h}} \right] \cdot e^{-\xi t}$$

$$\le \left[\|x_0\|_s + \frac{\delta_2}{1-\|D\|e^{\xi h}} \right] \cdot e^{-\xi t}, \; t \ge 0.$$

This completes the proof. □

If $D = 0$, Theorem 1 can be rewritten in the following result with $S = U_{5i} = 0$.

Corollary 1. System (1) with $D = 0$ and (A1), $h_D < 1$ (resp., $h_D \ge 1$ or unknown) is globally exponentially stable with convergence rate $\alpha > 0$, if there exist some $n \times n$ matrices P, Q_1, Q_2, R_1, R_2, R_3, R_4, R_{22}, S_{22}, $T_{22} > 0$, R_{11}, S_{11}, $T_{11} > 0 \in \Re^{5n \times 5n}$ (resp., $Q_2 = 0$), some matrices U_{1i}, U_{2i}, U_{3i}, U_{4i}, U_{6i}, $U_{7i} \in \Re^{n \times n}$, R_{12}, S_{12}, $T_{12} \in \Re^{5n \times n}$, and some positive constants ε_i, $i = 1, 2, \cdots, N$, such that the following LMI conditions hold for all $i = 1, 2, \cdots, N$

$$e^{-2\alpha h_m} \cdot R_1 - R_{22} > 0, \; e^{-2\alpha h_M} \cdot R_2 - S_{22} > 0, \; e^{-2\alpha h_M} \cdot R_2 - T_{22} > 0,$$

$$\begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} > 0, \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} > 0, \begin{bmatrix} T_{11} & T_{12} \\ * & T_{22} \end{bmatrix} > 0,$$

$$\Xi_i = \begin{bmatrix} \Xi_{11i} & \Xi_{12i} & \Xi_{13i} & \Xi_{14i} & \Xi_{15i} & \Xi_{16i} & \Xi_{17i} \\ * & \Xi_{22i} & \Xi_{23i} & \Xi_{24i} & \Xi_{25i} & \Xi_{26i} & \Xi_{27i} \\ * & * & \Xi_{33i} & \Xi_{34i} & 0 & \Xi_{36i} & 0 \\ * & * & * & \Xi_{44i} & \Xi_{45i} & \Xi_{46i} & 0 \\ * & * & * & * & \Xi_{55i} & \Xi_{56i} & 0 \\ * & * & * & * & * & \Xi_{66i} & 0 \\ * & * & * & * & * & * & \Xi_{77i} \end{bmatrix}$$

$$+ \begin{bmatrix} \tilde{\Phi}_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} < 0, \tag{12}$$

where

$$\Xi_{11i} = 2\alpha \cdot P + PA_{0i} + A_{0i}^T P + Q_1 + Q_2 + U_{2i}^T A_{0i} + A_{0i}^T U_{2i} + R_4,$$

$$\Xi_{12i} = PA_{1i} + U_{2i}^T A_{1i} + A_{0i}^T U_{3i}, \; \Xi_{13i} = A_{0i}^T U_{6i},$$

$$\Xi_{14i} = A_{0i}^T U_{1i}^T - U_{2i}^T + A_{0i}^T U_{4i}, \; \Xi_{15i} = A_{0i}^T U_{7i},$$

$$\Xi_{16i} = PM_i + U_{2i}^T M_i, \; \Xi_{17i} = \varepsilon_i \cdot N_{0i}^T,$$

$$\Xi_{22i} = -e^{-2\alpha h_M} \cdot (1-h_D) \cdot Q_2 + U_{3i}^T A_{1i} + A_{1i}^T U_{3i}, \; \Xi_{23i} = A_{1i}^T U_{6i},$$

$$\Xi_{24i} = A_{1i}^T U_{1i}^T - U_{3i}^T + A_{1i}^T U_{4i}, \; \Xi_{25i} = A_{1i}^T U_{7i},$$

$$\Xi_{26i} = U_{3i}^T M_i, \; \Xi_{27i} = \varepsilon_i \cdot N_{1i}^T, \; \Xi_{33i} = -e^{-2\alpha h_m} \cdot Q_1 + e^{-2\alpha h_m} R_3,$$

$$\Xi_{34i} = -U_{6i}^T, \; \Xi_{36i} = U_{6i}^T M_i,$$

$$\Xi_{44i} = h_m \cdot R_1 + (h_M - h_m) \cdot R_2 - U_{1i} - U_{1i}^T - U_{4i} - U_{4i}^T,$$

$$\Xi_{45i} = -U_{7i}, \; \Xi_{46i} = U_{1i} M_i + U_{4i}^T M_i, \; \Xi_{55i} = -e^{-2\alpha h_M} (R_3 + R_4),$$

$$\Xi_{56i} = U_{7i}^T M_i, \; \Xi_{66i} = \Xi_{77i} = -\varepsilon_i \cdot I,$$

$$\tilde{\Phi}_{11} = h_m \cdot R_{11} + R_{12}\tilde{\Lambda}_1 + \tilde{\Lambda}_1^T R_{12}^T + (h_M - h_m) \cdot (T_{11} + S_{11})$$

$$+ S_{12}\tilde{\Lambda}_2 + \tilde{\Lambda}_2^T S_{12}^T + T_{12}\tilde{\Lambda}_3 + \tilde{\Lambda}_3^T T_{12}^T,$$

$$\tilde{\Lambda}_1 = [I \quad 0 \quad -I \quad 0 \quad 0]_{n\times 5n}, \; \tilde{\Lambda}_2 = [0 \quad I \quad 0 \quad 0 \quad -I]_{n\times 5n},$$

$$\tilde{\Lambda}_3 = [0 \quad -I \quad I \quad 0 \quad 0]_{n\times 5n}.$$

In the next, we will consider the delay-dependent condition for stability of system (1) with (A2).

Theorem 2. System (1) with (A2) and $h_D < 1$ (resp., $h_D \ge 1$ or unknown) is globally exponentially stable with convergence rate $0 < \alpha < -(\ln\|D\|)/\tau$, if $\|D\| < 1$ and there exist some $n \times n$ matrices P, Q_1, Q_2, R_1, R_2, R_3, R_4, S R_{22}, S_{22}, $T_{22} > 0$, R_{11}, S_{11}, $T_{11} > 0 \in \Re^{7n \times 7n}$ (resp., $Q_2 = 0$), some matrices U_{1i}, U_{2i}, U_{3i}, U_{4i}, U_{5i}, U_{6i}, $U_{7i} \in \Re^{n \times n}$, R_{12}, S_{12}, $T_{12} \in \Re^{7n \times n}$, and some positive constants μ_{0i}, μ_{1i}, $i = 1, 2, \cdots, N$, such that the following LMI conditions hold for all $i = 1, 2, \cdots, N$

$$e^{-2\alpha h_m} \cdot R_1 - R_{22} > 0, \; e^{-2\alpha h_M} \cdot R_2 - S_{22} > 0, \; e^{-2\alpha h_M} \cdot R_2 - T_{22} > 0,$$

$$\begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} > 0, \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} > 0, \begin{bmatrix} T_{11} & T_{12} \\ * & T_{22} \end{bmatrix} > 0,$$

$$\Psi_i = \begin{bmatrix} \Psi_{11i} & \Psi_{12i} & \Psi_{13i} & \Psi_{14i} & \Psi_{15i} & \Psi_{16i} & \Psi_{17i} & \Psi_{18i} & \Psi_{19i} \\ * & \Psi_{22i} & \Psi_{23i} & 0 & 0 & 0 & 0 & \Psi_{28i} & \Psi_{29i} \\ * & * & \Psi_{33i} & \Psi_{34i} & \Psi_{35i} & \Psi_{36i} & \Psi_{37i} & \Psi_{38i} & \Psi_{39i} \\ * & * & * & \Psi_{44i} & \Psi_{45i} & \Psi_{46i} & 0 & \Psi_{48i} & \Psi_{49i} \\ * & * & * & * & \Psi_{55i} & \Psi_{56i} & \Psi_{57i} & \Psi_{58i} & \Psi_{59i} \\ * & * & * & * & * & \Psi_{66i} & \Psi_{67i} & \Psi_{68i} & \Psi_{69i} \\ * & * & * & * & * & * & \Psi_{77i} & \Psi_{78i} & \Psi_{79i} \\ * & * & * & * & * & * & * & \Psi_{88i} & 0 \\ * & * & * & * & * & * & * & * & \Psi_{99i} \end{bmatrix}$$

$$+ \begin{bmatrix} \Phi_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} < 0, \qquad (13)$$

where

$$\Psi_{11i} = 2\alpha \cdot P + PA_{0i} + A_{0i}^T P + Q_1 + Q_2$$

$$+ U_{2i}^T A_{0i} + A_{0i}^T U_{2i} + S + \mu_{0i} \cdot \sigma_{0i}^2 \cdot I + R_4,$$

$$\Psi_{12i} = -2\alpha \cdot PD - A_{0i}^T PD,$$

$$\Psi_{13i} = PA_{1i} + U_{2i}^T A_{1i} + A_{0i}^T U_{3i}, \Psi_{14i} = A_{0i}^T U_{6i},$$

$$\Psi_{15i} = A_{0i}^T U_{1i}^T - U_{2i}^T + A_{0i}^T U_{4i},$$

$$\Psi_{16i} = -A_{0i}^T U_{1i}^T D + U_{2i}^T D + A_{0i}^T U_{5i}, \Psi_{17i} = A_{0i}^T U_{7i},$$

$$\Psi_{18i} = \Psi_{19i} = P + U_{2i}^T, \Psi_{22i} = 2\alpha \cdot D^T PD - e^{-2\alpha\tau} \cdot S,$$

$$\Psi_{23i} = -D^T PA_{1i}, \Psi_{28i} = \Psi_{29i} = -D^T P,$$

$$\Psi_{33i} = -e^{-2\alpha h_M} \cdot (1 - h_D) \cdot Q_2 + U_{3i}^T A_{1i} + A_{1i}^T U_{3i} + \mu_{1i} \cdot \sigma_{1i}^2 \cdot I,$$

$$\Psi_{34i} = A_{1i}^T U_{6i}, \Psi_{35i} = A_{1i}^T U_{1i}^T - U_{3i}^T + A_{1i}^T U_{4i},$$

$$\Psi_{36i} = -A_{1i}^T U_{1i}^T D + U_{3i}^T D + A_{1i}^T U_{5i}, \Psi_{37i} = A_{1i}^T U_{7i},$$

$$\Psi_{38i} = \Psi_{39i} = U_{3i}^T, \Psi_{44i} = -e^{-2\alpha h_m} \cdot Q_1 + e^{-2\alpha h_m} R_3, \Psi_{45i} = -U_{6i}^T,$$

$$\Psi_{46i} = U_{6i}^T D, \Psi_{48i} = \Psi_{49i} = U_{6i}^T,$$

$$\Psi_{55i} = h_m \cdot R_1 + (h_M - h_m) \cdot R_2 - U_{1i} - U_{1i}^T - U_{4i} - U_{4i}^T,$$

$$\Psi_{56i} = (U_{1i} + U_{1i}^T) D + U_{4i}^T D - U_{5i}, \Psi_{57i} = -U_{7i},$$

$$\Psi_{58i} = \Psi_{59i} = U_{1i} + U_{4i}^T,$$

$$\Psi_{66i} = -D^T (U_{1i} + U_{1i}^T) D + U_{5i}^T D + D^T U_{5i}, \Psi_{67i} = D^T U_{7i},$$

$$\Psi_{68i} = \Psi_{69i} = -D^T U_{1i} + U_{5i}^T, \Psi_{77i} = -e^{-2\alpha h_M} (R_3 + R_4),$$

$$\Psi_{78i} = \Psi_{79i} = U_{7i}^T, \Psi_{88i} = -\mu_{0i} \cdot I, \Psi_{99i} = -\mu_{1i} \cdot I,$$

$$\Phi_{11} = h_m \cdot R_{11} + R_{12}\Lambda_1 + \Lambda_1^T R_{12}^T + (h_M - h_m) \cdot (T_{11} + S_{11})$$

$$+ S_{12}\Lambda_2 + \Lambda_2^T S_{12}^T + T_{12}\Lambda_3 + \Lambda_3^T T_{12}^T,$$

$$\Lambda_1 = [I \quad 0 \quad 0 \quad -I \quad 0 \quad 0 \quad 0]_{n \times 7n},$$

$$\Lambda_2 = [0 \quad 0 \quad I \quad 0 \quad 0 \quad 0 \quad -I]_{n \times 7n},$$

$$\Lambda_3 = [0 \quad 0 \quad -I \quad I \quad 0 \quad 0 \quad 0]_{n \times 7n}.$$

Proof. From the assumption in (A2), we have

$$\sigma_{0i}^2 x^T(t) x(t) - x^T(t) \Delta A_{0i}^T(t) \Delta A_{0i}(t) x(t) \geq 0, \qquad (14a)$$

$$\sigma_{1i}^2 x^T(t - h(t)) x(t - h(t)),$$

$$-x^T(t - h(t)) \Delta A_{1i}^T(t) \Delta A_{1i}(t) x(t - h(t)) \geq 0, \ i = 1, 2, \cdots, N. \qquad (14b)$$

By (8a)-(8h), we can obtain the following result

$$\dot{V}(x_t) + e^{2\alpha t} \cdot \sum_{i=1}^{N} \lambda_i(t) \cdot \mu_{0i} \cdot [\sigma_{0i}^2 x^T(t) x(t)$$

$$- x^T(t) \Delta A_{0i}^T(t) \Delta A_{0i}(t) x(t)]$$

$$+ e^{2\alpha t} \cdot \sum_{i=1}^{N} \lambda_i(t) \cdot \mu_{1i} \cdot [\sigma_{1i}^2 x^T(t - h(t)) x(t - h(t))$$

$$- x^T(t - h(t)) \Delta A_{1i}^T(t) \Delta A_{1i}(t) x(t - h(t))]$$

$$\leq e^{2\alpha t} \cdot \left\{ \sum_{i=1}^{n} Z_i^T \cdot \Psi_i \cdot Z_i \right.$$

$$- \int_{t-h_m}^{t} \dot{x}^T(s) [e^{-2\alpha h_m} \cdot R_1 - R_{22}] \dot{x}(s) ds$$

$$- \int_{t-h_M}^{t-h(t)} \dot{x}^T(s) [e^{-2\alpha h_M} \cdot R_2 - S_{22}] \dot{x}(s) ds$$

$$\left. - \int_{t-h(t)}^{t-h_m} \dot{x}^T(s) [e^{-2\alpha h_M} \cdot R_2 - T_{22}] \dot{x}(s) ds \right\}, \qquad (15)$$

where matrices Ψ_i, $i = 1, 2, \cdots, N$, are defined in (13), and

$$Z_i^T = [x^T(t) \quad x^T(t-\tau) \quad x^T(t-h(t)) \quad x^T(t-h_m) \quad \dot{x}^T(t)$$

$$\dot{x}^T(t-\tau) \quad x^T(t-h_M) \quad x^T(t)\Delta A_{0i}^T(t) \quad x^T(t-h(t))\Delta A_{1i}^T(t)]$$

From conditions in (13)-(15), we can achieve that the condition (10) is satisfied. By the same derivation of Theorem 1, we can complete this proof. □

If $D = 0$, Theorem 2 can be rewritten in the following result with $S = U_{5i} = 0$.

Corollary 2. System (1) with $D = 0$ and (A2), $h_D < 1$ (resp., $h_D \geq 1$ or unknown) is globally exponentially stable with convergence rate $\alpha > 0$, if there exist some $n \times n$ matrices P, Q_1, Q_2, R_1, R_2, R_3, R_4, R_{22}, S_{22}, $T_{22} > 0$, R_{11}, S_{11}, $T_{11} > 0 \in \Re^{5n \times 5n}$, (resp., $Q_2 = 0$), some matrices U_{1i}, U_{2i}, U_{3i}, U_{4i}, U_{6i}, $U_{7i} \in \Re^{n \times n}$, R_{12}, S_{12}, $T_{12} \in \Re^{5n \times n}$, and some positive constants μ_{0i}, μ_{1i},, $i = 1, 2, \cdots, N$, such that the following LMI conditions hold for all $i = 1, 2, \cdots, N$

$$e^{-2\alpha h_m} \cdot R_1 - R_{22} > 0, \; e^{-2\alpha h_M} \cdot R_2 - S_{22} > 0, \; e^{-2\alpha h_M} \cdot R_2 - T_{22} > 0,$$

$$\begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} > 0, \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} > 0, \begin{bmatrix} T_{11} & T_{12} \\ * & T_{22} \end{bmatrix} > 0,$$

$$\hat{\Psi}_i = \begin{bmatrix} \hat{\Psi}_{11i} & \hat{\Psi}_{12i} & \hat{\Psi}_{13i} & \hat{\Psi}_{14i} & \hat{\Psi}_{15i} & \hat{\Psi}_{16i} & \hat{\Psi}_{17i} \\ * & \hat{\Psi}_{22i} & \hat{\Psi}_{23i} & \hat{\Psi}_{24i} & \hat{\Psi}_{25i} & \hat{\Psi}_{26i} & \hat{\Psi}_{27i} \\ * & * & \hat{\Psi}_{33i} & \hat{\Psi}_{34i} & 0 & \hat{\Psi}_{36i} & \hat{\Psi}_{37i} \\ * & * & * & \hat{\Psi}_{44i} & \hat{\Psi}_{45i} & \hat{\Psi}_{46i} & \hat{\Psi}_{47i} \\ * & * & * & * & \hat{\Psi}_{55i} & \hat{\Psi}_{56i} & \hat{\Psi}_{57i} \\ * & * & * & * & * & \hat{\Psi}_{66i} & 0 \\ * & * & * & * & * & * & \hat{\Psi}_{77i} \end{bmatrix}$$

$$+ \begin{bmatrix} \tilde{\Phi}_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} < 0, \tag{16}$$

where

$$\hat{\Psi}_{11i} = 2\alpha \cdot P + PA_{0i} + A_{0i}^T P + Q_1 + Q_2$$

$$+ U_{2i}^T A_{0i} + A_{0i}^T U_{2i} + \mu_{0i} \cdot \sigma_{0i}^2 \cdot I + R_4,$$

$$\hat{\Psi}_{12i} = PA_{1i} + U_{2i}^T A_{1i} + A_{0i}^T U_{3i},$$

$$\hat{\Psi}_{13i} = A_{0i}^T U_{6i}, \; \hat{\Psi}_{14i} = A_{0i}^T U_{1i}^T - U_{2i}^T + A_{0i}^T U_{4i}, \; \hat{\Psi}_{15i} = A_{0i}^T U_{7i},$$

$$\hat{\Psi}_{16i} = \hat{\Psi}_{17i} = P + U_{2i}^T,$$

$$\hat{\Psi}_{22i} = -e^{-2\alpha h_M} \cdot (1 - h_D) \cdot Q_2 + U_{3i}^T A_{1i} + A_{1i}^T U_{3i} + \mu_{1i} \cdot \sigma_{1i}^2 \cdot I,$$

$$\hat{\Psi}_{23i} = A_{1i}^T U_{6i}, \; \hat{\Psi}_{24i} = A_{1i}^T U_{1i}^T - U_{3i}^T + A_{1i}^T U_{4i},$$

$$\hat{\Psi}_{25i} = A_{1i}^T U_{7i}, \; \hat{\Psi}_{26i} = \hat{\Psi}_{27i} = U_{3i}^T,$$

$$\hat{\Psi}_{33i} = -e^{-2\alpha h_m} \cdot Q_1 + e^{-2\alpha h_m} R_3, \; \hat{\Psi}_{34i} = -U_{6i}^T, \; \hat{\Psi}_{36i} = \hat{\Psi}_{37i} = U_{6i}^T,$$

$$\hat{\Psi}_{44i} = h_m \cdot R_1 + (h_M - h_m) \cdot R_2 - U_{1i} - U_{1i}^T - U_{4i} - U_{4i}^T,$$

$$\hat{\Psi}_{45i} = -U_{7i}, \; \hat{\Psi}_{46i} = \hat{\Psi}_{47i} = U_{1i} + U_{4i}^T,$$

$$\hat{\Psi}_{55i} = -e^{-2\alpha h_M}(R_3 + R_4), \; \hat{\Psi}_{56i} = \hat{\Psi}_{57i} = U_{7i}^T,$$

$$\hat{\Psi}_{66i} = -\mu_{0i} \cdot I, \; \hat{\Psi}_{77i} = -\mu_{1i} \cdot I,$$

$$\tilde{\Phi}_{11} = h_m \cdot R_{11} + R_{12}\tilde{\Lambda}_1 + \tilde{\Lambda}_1^T R_{12}^T + (h_M - h_m) \cdot (T_{11} + S_{11})$$

$$+ S_{12}\tilde{\Lambda}_2 + \tilde{\Lambda}_2^T S_{12}^T + T_{12}\tilde{\Lambda}_3 + \tilde{\Lambda}_3^T T_{12}^T,$$

$$\tilde{\Lambda}_1 = [I \quad 0 \quad -I \quad 0 \quad 0]_{n \times 5n}, \; \tilde{\Lambda}_2 = [0 \quad I \quad 0 \quad 0 \quad -I]_{n \times 5n},$$

$$\tilde{\Lambda}_3 = [0 \quad -I \quad I \quad 0 \quad 0]_{n \times 5n}.$$

If $D = 0$, $\Delta A_{0i}(t) = 0$, and $\Delta A_{1i}(t) = 0$, Corollaries 1 and 2 can be rewritten in the following result.

Corollary 3. System (1) with $D = 0$, $\Delta A_{0i}(t) = 0$, $\Delta A_{1i}(t) = 0$, and $h_D < 1$ (resp., $h_D \geq 1$ or unknown) is globally exponentially stable with convergence rate $\alpha > 0$, if there exist some $n \times n$ matrices P, Q_1, Q_2, R_1, R_2, R_3, R_4, R_{22}, S_{22}, $T_{22} > 0$, R_{11}, S_{11}, $T_{11} > 0 \in \Re^{5n \times 5n}$, (resp., $Q_2 = 0$), some matrices U_{1i}, U_{2i}, U_{3i}, U_{4i}, U_{6i}, $U_{7i} \in \Re^{n \times n}$, R_{12}, S_{12}, $T_{12} \in \Re^{5n \times n}$, such that the following LMI conditions hold for all $i = 1, 2, \cdots, N$

$$e^{-2\alpha h_m} \cdot R_1 - R_{22} > 0, \; e^{-2\alpha h_M} \cdot R_2 - S_{22} > 0, \; e^{-2\alpha h_M} \cdot R_2 - T_{22} > 0,$$

$$\begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} > 0, \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} > 0, \begin{bmatrix} T_{11} & T_{12} \\ * & T_{22} \end{bmatrix} > 0,$$

$$\hat{\Xi}_i = \begin{bmatrix} \Xi_{11i} & \Xi_{12i} & \Xi_{13i} & \Xi_{14i} & \Xi_{15i} \\ * & \Xi_{22i} & \Xi_{23i} & \Xi_{24i} & \Xi_{25i} \\ * & * & \Xi_{33i} & \Xi_{34i} & 0 \\ * & * & * & \Xi_{44i} & \Xi_{45i} \\ * & * & * & * & \Xi_{55i} \end{bmatrix} + \tilde{\Phi}_{11} < 0, \tag{17}$$

$$\tilde{\Phi}_{11} = h_m \cdot R_{11} + R_{12}\tilde{\Lambda}_1 + \tilde{\Lambda}_1^T R_{12}^T + (h_M - h_m)\cdot(T_{11} + S_{11})$$
$$+ S_{12}\tilde{\Lambda}_2 + \tilde{\Lambda}_2^T S_{12}^T + T_{12}\tilde{\Lambda}_3 + \tilde{\Lambda}_3^T T_{12}^T,$$

$$\tilde{\Lambda}_1 = [I \quad 0 \quad -I \quad 0 \quad 0]_{n\times 5n},\ \tilde{\Lambda}_2 = [0 \quad I \quad 0 \quad 0 \quad -I]_{n\times 5n},$$

$$\tilde{\Lambda}_3 = [0 \quad -I \quad I \quad 0 \quad 0]_{n\times 5n},$$

where Ξ_{kli}, $k, l = 1, 2,\ldots, 5$, $i = 1, 2,\ldots, N$, are defined in (12).

Remark 1. In [7] and [11], the slow variation condition $\dot{h}(t) \le h_D < 1$ is constrained in their considered systems. The hard constraint $h_D < 1$ is not imposed on our results.

Remark 2. By setting $\alpha = 0$ in Theorems 1-2 and Corollaries 1-3, the global asymptotic stability for system (1) can be guaranteed. By setting $\alpha = 0$, $R_1 = 0$, and $R_2 = 0$, $R_{11} = T_{11} = S_{11} = 0$ in Theorems 1-2 and Corollaries 1-3, we can obtain the delay-independent stability result for system (1).

III. NUMERICAL EXAMPLES

Example 1. Consider the system (1) with $D = 0$, $\Delta A_{0i}(t) = 0$, and $\Delta A_{1i}(t) = 0$ and the following parameters: (Example 1 of [9])

$$N = 3,\ A_{01} = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix},\ A_{11} = \begin{bmatrix} -1 & -1 \\ 0 & -0.5 \end{bmatrix},\ A_{02} = \begin{bmatrix} -1.5 & 1 \\ 0 & -1 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 0.6 & -1 \\ 0 & -0.4 \end{bmatrix},\ A_{03} = \begin{bmatrix} -0.5 & 0 \\ 0 & -3 \end{bmatrix},\ A_{13} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}. \quad (18)$$

By Corollary 3, LMI (17) with (18) for $\alpha = 0.1$, $h_D < 0.2$, $h_m < 0.1$, $h_M < 1.1$ has a feasible solution. This implies that the system (1) with $D = 1$, $\Delta A_{0i}(t) = 0$, $\Delta A_{1i}(t) = 0$, $0.1 \le h(t) \le 1.1$, $\dot{h}(t) \le 0.2$, and (18) is globally exponentially stable with convergence rate $\alpha = 0.1$. Some comparisons of the obtained results for switched system (1) with (18) are made in Table 1. The results of this paper provide a larger allowable upper bound for time delay to guarantee the global asymptotic stability of system (1) with (18).

Example 2. Consider the system (1) with $D = 0$, $\Delta A_{0i}(t) = 0$, and $\Delta A_{1i}(t) = 0$ and the following parameters: (Example 1 of [8])

$$N = 2,\ A_{01} = A_{02} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},\ A_{11} = \begin{bmatrix} 0 & 0.5 \\ -1 & 0 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 0 & 1 \\ -0.5 & 0 \end{bmatrix}. \quad (19)$$

Table 1. Comparing some previous results with this paper.

Some results of delay that guarantee global asymptotic stability ($\alpha = 0$) of the system (1) with (18)		
Results	[9]	Our result
$h_D = 0$ (Constant delay)	No results	$0 \le h \le 1.5004$
$h_D = 0.4$		$0 \le h(t) \le 1.277$
$h_D = 0.8$		$0 \le h(t) \le 1.1235$
$h_D = 0.9$		$0 \le h(t) \le 1.1209$
$h_D \ge 1$ or unknown	$0 \le h(t) \le 0.9339$	$0 \le h(t) \le 1.12$
$h_D \ge 1$ or unknown		$0.3 \le h(t) \le 1.17$
$h_D \ge 1$ or unknown		$0.6 \le h(t) \le 1.21$
$h_D \ge 1$ or unknown		$0.9 \le h(t) \le 1.28$

Table 2. Comparing some previous results with this paper.

Some upper bounds of delay that guarantee the global exponential stability of the system (1) with (19)		
Results	convergence rate α and h_D	Bounds of delay h_m and h_M
[8]	$\alpha = 0.3$, $h_D = 0.1$	$h_m = 0$, $h_M = 0.4$
Our results	$\alpha = 0.3$, $h_D = 0.1$	$h_m = 0$, $h_M = 0.497$
	$\alpha = 0.3$, ($h_D \ge 1$ or unknown)	$h_m = 0$, $h_M = 0.475$
		$h_m = 0.3$, $h_M = 0.511$
		$h_m = 0.4$, $h_M = 0.519$
		$h_m = 0.5$, $h_M = 0.525$
Some upper bounds of delay that guarantee the global asymptotic stability of the system (1) with (19)		
[8]	$\alpha = 0$, $h_D = 0.1$	$h_m = 0$, $h_M = 1$
Our results	$\alpha = 0$, $h_D = 0.1$	$h_m = 0$, $h_M = 3.585$
	$\alpha = 0$ ($h_D \ge 1$ or unknown)	$h_m = 0$, $h_M = 0.895$
		$h_m = 0.4$, $h_M = 1.08$
		$h_m = 0.8$, $h_M = 1.35$
		$h_m = 1$, $h_M = 1.501$

Some comparisons of the obtained results for switched system (1) with (19) are made in Table 2. The results of this paper provide a larger allowable upper bound for time delay to guarantee the global asymptotic and exponential stability of system (1) with (19).

IV. CONCLUSIONS

In this paper, global exponential stability for uncertain switched neutral systems with interval time-varying state delay and arbitrary switching signal has been considered. Structured and unstructured perturbations of systems have been investigated. LMI and Razumikhin-like approaches have been used to improve our results. The obtained results are less conservative than previous ones via the numerical simulation.

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