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Experimental Nonlinear Modeling of a Rotating Machine with an Oil Film Bearing

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EXPERIMENTAL NONLINEAR MODELING OF A ROTATING MACHINE WITH AN OIL FILM BEARING

Chiou-Fong Chung, Pi-Cheng Tung, and Chiang-Nan Chang

Key words: identification, oil whirl, unbalanced mass.

ABSTRACT

In this study the system identification method is applied to obtain a nonlinear equation for an oil film rotating system. The centrifugal force induced by an unbalanced mass is used as the input signal for identification, and the phase between the input signal and the measured output vibration amplitude is calculated to perform the identification. Stability analysis and system performance are evaluated by using the root locus and the Floquet-Liapunov theorem. Based on the model, the oil whirl of the journal bearing can be predicted. A comparison between the simulation results and some experimented data shows the feasibility of the proposed approach.

INTRODUCTION

The fluid model of an oil-film bearing system has been studied by several workers [1-3]. Muszynska *et al.* [4,5] established the fluid force system model in 1986. In practical applications, these mathematical models can not be used to predict the occurrence of the oil whirl because they are all derived theoretically, based on certain simple assumptions, and do not identify the system with the experiment results. Thus mathematical models are only used for theoretical analysis and simulation, not for on line diagnosis. Several different time-domain and frequency-domain techniques [6-11] have been developed for identifying the oil-film coefficients of the bearing, but these studies only identified the linear parts of the system or a system that was linearized. They did not identify the nonlinear parts of the system. Some scholars have identified a nonlinear oil-film bearing system [12-15], using the traditional excitation methods, such as impacts or sweep sines to perturb the experimental oil film bearing system, for the purposes of identification. However, these methods are not suitable for use in practical applications, because a rotating machine does not permit an external force to disturb or change the system once it is in motion.

Zhao *et al.*[16] pointed out that the fluid force induced by the oil in the bearing is related to the operating speed. Thus, we deal with the model in the frequency domain for such reasons as less susceptibility to noise, more convenience of identification, and easier correction of rotating machinery malfunctions. For exhaustive observation of the signals system is input and output, we transform the time domain data to frequency domain data, by using the Discrete Fourier Transform (DFT). The system identification method is applied based on the principle of harmonic balance [17-19]. In order to perform system identification, the input signal and the output signal should be obtained simultaneously. An unbalanced mass force is used as the system excitation force. We developed a method to calculate the phase between the input unbalanced force and the measured vibration amplitude. After obtaining the phase, we can establish the identification method, obtaining a mathematical model by the principle of harmonic balance. By performing stability analysis [20], we can predict the speed at which an oil whirl occurs.

ALGORITHM OF IDENTIFICATION

The equation for a single-degree-of-freedom can be written as

$$m\ddot{x} + c\dot{x} + kx + \tilde{N} = f\,, \tag{1}$$

where m, c, and k denote the mass, the damping and the stiffness in the linear part of the system. The parameter $\tilde{N}$, represents the nonlinear part of the system, and it is a function of x. f is the external force in equation (1). The algorithm of identification is described as follows [21]:

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Step 1:

Assume that the nonlinear term $\tilde{N}$ is approximated by a polynomial of x:

$$\tilde{N} = \alpha x^2 + \beta x^2 + \ldots, \tag{2}$$

where α, β...are unknown coefficients. When this assumption is made, the problem of identification is reduced to the determination of the parameters m, c, and k in equation (1), and the unknown coefficient α, β...in equation (2).

Step 2:

When a periodic external force f, with a period $T = \frac{2\pi}{\omega}$, is applied to the system, the steady-state responses x, with the same period as the external force, are induced. Then, both the external force f and the steady-state response x are recorded within the time interval of one period.

Step 3:

Transfer the external force f and the steady-state response x into the Fourier series [8].

Step 4:

Express the input force term and each term in the equation of motion including the nonlinear terms in Fourier series. Then, equation (1) and equation (2) can be expressed as a matrix in the form of :

$$[A]\{X\} = \{F\}, \tag{3}$$

where:

$$[A] = \begin{bmatrix} 0 & 0 & x_{OR} & (x^2)_{OR} & (x^3)_{OR} & \cdots \\ -\omega^2 x_{1R} & \omega x_{1I} & x_{1R} & (x^2)_{1R} & (x^3)_{1R} & \cdots \\ -\omega^2 x_{1I} & -\omega x_{1R} & x_{1I} & (x^2)_{1I} & (x^3)_{1I} & \cdots \\ -4\omega^2 x_{2R} & 2\omega x_{2I} & x_{2R} & (x^2)_{2R} & (x^3)_{2R} & \cdots \\ -4\omega^2 x_{2I} & -2\omega x_{2R} & x_{2I} & (x^2)_{2I} & (x^3)_{3I} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix}$$

$$\{X\} = [m \;\; c \;\; k \;\; \alpha \;\; \beta \;\; \ldots]^T$$

$$\{F\} = [f_{0R} \;\; f_{1R} \;\; f_{1I} \;\; f_{2R} \;\; f_{2I} \;\; \ldots]^T \tag{4}$$

If the various values of ω are properly calculated, we can transfer equation (1) correctly into equation (3). We know that $\{x\}$ is an unknown vector, solved by the least-square estimator (LSE)[19]. The result of the LSE is then:

$$\{\hat{X}\} = ([A]^T[A])^{-1}[A]^T\{F\}, \tag{5}$$

From equation (5), we get the parameters m, c, k, α, β ... for the system. If necessary, the weighting matrix can be used to improve the accuracy. When the weighting matrix $[W]$ is used, equation (5) becomes:

$$\{\hat{X}\} = ([A]^T[W][A])^{-1}[A]^T[W]\{F\}, \tag{6}$$

The Apparatus and the Experiment Figure 1 and Figure 2 show a schematic diagram and a photo of the oil film rotating system. A proximitor assembly is used as the vibration probe along the horizontal direction(x-axis). The keyphasor probe is located at the shaft notch also in a horizontal direction. A rigid bearing near the motor side, and a lubricated cylindrical bearing on the other side support this rotor system. We put the unbalanced disk of the rotating shaft near the oil film bearing to reduce the degrees of freedom. Thus, the equation of motion in the rotor system can be simplified as a one-degree-of-freedom system. To establish a mathematical model of an oil film bearing system that analyzes the

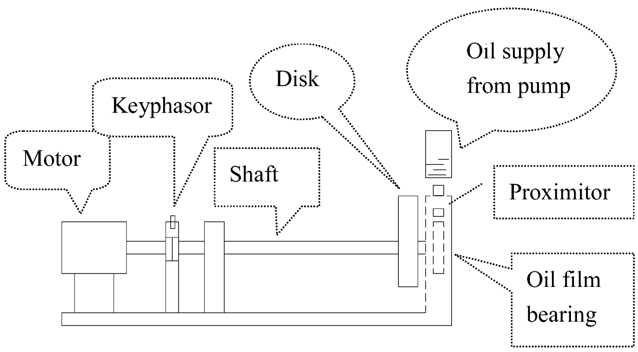

Fig. 1. Structure of the experimental oil-film bearing.

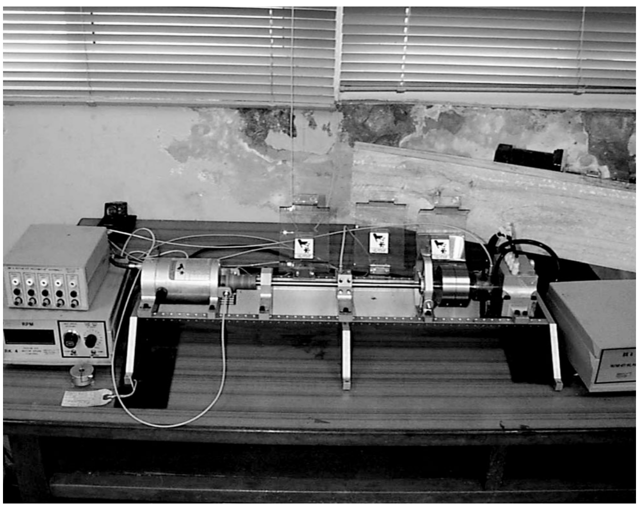

Fig. 2. Photograph of the experimental equipment.

Table 1. Experimental data

Channel Name: Channel 1		Machine Name: Rotor Kit				Amplitude Units: μm	
Sample	Date	Time	Speed	Direct	Gap	Amplitude	Phase
71	24APR2001	19:10:05	1820	45.46	-8.91	42.42	151
72	24APR2001	19:10:05	1830	47.50	-8.92	43.18	151
73	24APR2001	19:10:06	1840	45.72	-8.91	42.67	152
74	24APR2001	19:10:06	1850	45.47	-8.92	42.93	153
75	24APR2001	19:10:06	1860	45.72	-8.92	42.93	153
76	24APR2001	19:10:07	1870	46.23	-8.92	42.93	153
77	24APR2001	19:10:07	1880	46.48	-8.92	42.67	154
78	24APR2001	19:10:07	1890	50.55	-8.92	42.67	154
79	24APR2001	19:10:08	1900	58.42	-8.92	42.67	154
80	24APR2001	19:10:08	1907	59.94	-8.92	42.93	153

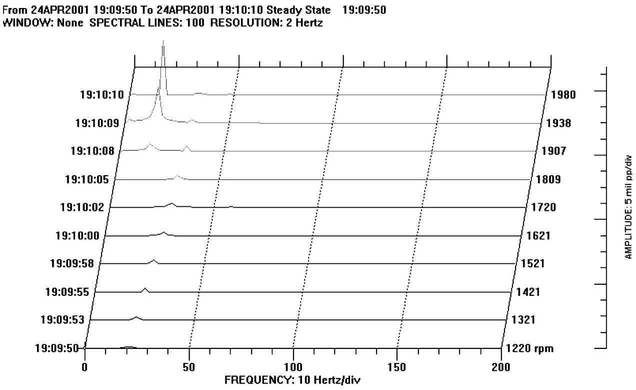

Fig. 3. Waterfall diagram of the oil film bearing system.

vibrational phenomena in the oil film bearing, is the main object of this study. It is helpful to observe the oil whirl phenomenon by running an experimental rotor kit set-up before the mathematical model is found. When the oil film bearing system, rotating in its fluid lubricated cylindrical bearings, has a lightly loaded, slightly unbalanced symmetric rotor, the dynamic phenomena is quite apparent. The vibration data for our experimental oil film bearing system are shown as Figure 3. The inertia forces of the unbalanced rotor mass may cause the vibrations at low operating speeds. At low rotational speeds, these vibrations are stable, and we can neglect the impulse perturbation of the rotor that causes a short term transient vibrational process. When the rotation speed increases, the synchronous vibration is not the only system motion. Along with first harmonic vibrations, an oil whirl appears, cause by the rotor's lateral forward subharmonic vibration around the bearing center at a frequency close to half the rotation speed (usually smaller than half). The amplitudes of the oil whirl are usually much higher than those of the synchronous vibrations, because the oil whirl occurs due the nonlinear fluid forces. At 1900 rpm shown in Table 1, the vibration of the system becomes unstable and the bearings fluid force becomes the main factor which influences the amplitude.

According to the dynamic phenomena described above, there are two causes of system vibration. One is the synchronous vibration caused by an unbalanced force, and the other is self-excitation caused by the nonlinear fluid force. When the vibration frequency grows, the nonlinear factors influence the system more. In order to perform identification, we need an external force to excite the oil film rotational system. Without using additional excitation equipment, the unbalanced mass already existing in the system may be used as the excitation force. Suppose that the unbalanced mass is m, the distance between the unbalanced mass and the center of the disk is r, the rotational speed of the system is ω, and the phase angle between the input and the output in x-axis is θ_x. The external force caused by an unbalanced mass is $F_{unbalance} = mr\omega^2$.

Since the systems input force can be estimated from the unbalanced mass, only the systems output can be measured during its operation. However, for performing system identification the systems input and output should be recorded simultaneously. To find the phase angle is the key to performing identification using an unbalanced mass as the excitation force. The system phase angle is the phase lag between the systems input and output. The calculation of the system phase angle is shown as Figure 4 and is described as follows.

If the speed of rotation is far below the first critical speed of the system, the phase angle is zero. In this case the heavy spot (the unbalanced mass) can be located by observing the angle θ_1 (as indicated in Figure 4a) from the keyphasor dot on the time response to the next positive peak on the time response. Thus, if the machine is stopped and the notch on the shaft lined up with the keyphasor probe, the heavy spot will be located at the observed angle θ as indicated in Figure 4(b), away from the vibration probe. For the case that the speed of rotation is far below the first critical, then θ_1 is equal to θ and the phase angle θ_x between input response and output response is zero. Since the location of the heavy spot in the oil-film rotating system is already known, the phase angle θ_x for any rotational speed can be obtained by calculating the difference between the angle θ of the heavy spot and the vibration probe after the keyphasor probe is lined up with the notch on the shaft, and the angle θ_2 of the keyphasor dot and the next peak on the time response as indicated in Figure 4(c). The calculation of the phase θ_x is expressed as $\theta_x = \theta_2 - \theta_1$. After obtaining the phase angle, the system input corresponds with the output and we can perform identification experimentally.

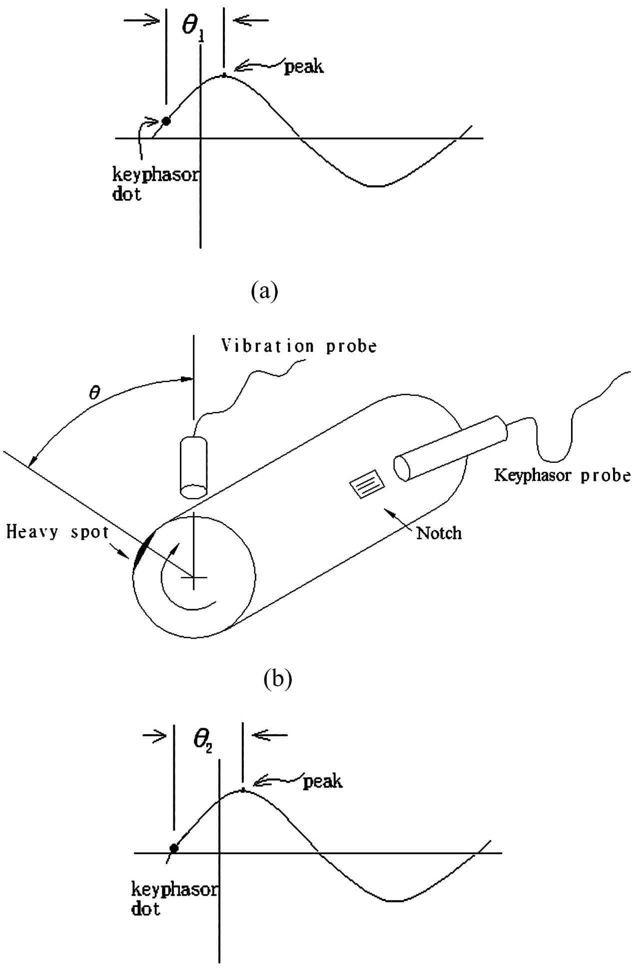

Fig. 4. The relationship of system phase angle.

EXPERIMENT WITH OIL FILM BEARINGS AND IDENTIFICATION RESULTS

In this experiment, we adopt a linear approach to identifing the oil-film rotating system. The linearized simplified system model is assumed to be:

$$m_x\ddot{x} + c_x\dot{x} + k_x x = m{*}r{*}\omega^2{*}\cos(\omega t)\,, \tag{7}$$

It should be noted that the parameters of the inertial, damping, and stiffness terms of the oil-film system are a function of the rotational speed. Thus, the parameters of the linearized system are dependent on the rotational speed. The parameters of the linearized system vary as the rotational speed varies. By performing stability analysis on the linearized system, we can predict at which rotational speeds the linearized system will become unstable and when an oil whirl will occur. We identify the parameters of the one-dimensional linearized oil-film bearing system in the x-axis. Based on the identification method used in section 2, the system is identified from 1400 rpm to 1880 rpm. The results and the eigenvalues for the linearized system are shown in Table 2. It should be noted that since the equivalent mass, damping, and stiffness of the system are a function of the rotational frequency, the parameter values of m, c, k will be varied with this frequency.

In Table 2, we find that the damping coefficient becomes small when the frequency is increased, and that the eigenvalues are close to the image axis. Figures 5 and 6 show the real time responses at 1860 rpm and 1880 rpm. Figure 7 shows the root locus of the linearized oil film bearing system. This means that when the frequency is growing and the damping coefficient is getting smaller, the system becomes unstable. In this experiment, the system becomes unstable at the frequency of 1900 rpm. In table 2, we see that the eigenvalues change very fast. Although at 1800 rpm they are not very close to the image axis, we can expect that the eigenvalues will cross the image axis if the frequency is increased slightly.

The experimental results show that the linear model can only predict until the speed where the system become unstable. Theoretically it can not show what happens after a system becomes unstable. However, the experimental results show that the oil whirl occurs after the rotational speed that the identified linear system becomes unstable. In order to characterize the systems behavior due to nonlinear effects we adopt a nonlinear approach to the identification of oil film rotating system. We assume that the nonlinear term is $N = \alpha x^2 + \beta x^3 + \ldots$. Then the system model is

$$m_x\ddot{x} + c_x\dot{x} + k_x x + \beta x^3 = m{*}r{*}\omega^2{*}\cos(\omega t)\,, \tag{8}$$

Table 2. ID Results and eigenvalues for a linearized system on the *x*-axis

Frequency (rpm)	m_x (kg)	c_x (Kg/sec)	k_x (Kg/sec^2)	Eigenvalues
1400	0.0968	98.854	15489.837	-827.714, -193.285
1500	0.0726	74.887	13146.133	-807.358, -224.323
1600	0.0742	77.936	14976.191	-797.529, -253.158
1700	0.0733	64.247	17117.222	-438.31±203.57j
1800	0.0661	48.376	16680.106	-365.91±344.16j
1820	0.0628	45.387	16515.354	-361.20±363.87j
1840	0.0593	37.445	16413.779	-315.67±420.83j
1860	0.0615	31.773	17481.478	-258.23±466.34j
1880	0.0665	21.281	18420.124	-160.00±501.38j

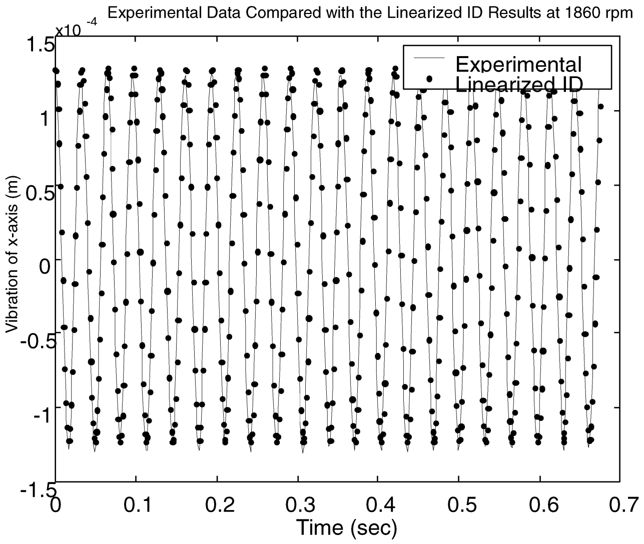

Fig. 5. Experimental data compared with the linearized ID results at 1860rpm.

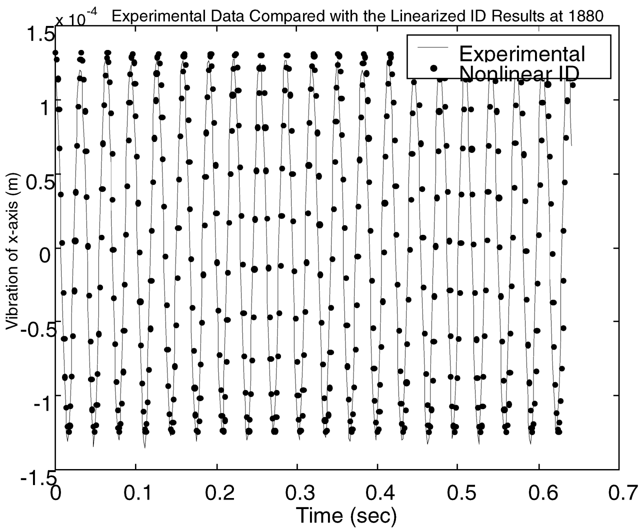

Fig. 6. Experimental data compared with the linearized ID results at 1880rpm.

Table 3 shows the results by of system identification from 1400 rpm to 1880 rpm. We also see that the damping coefficients are going to become small when the frequency gets large. It is noted that since the value of α is very small compared to that of β, so α is neglected. Figures 8 and 9 show the time responses experimental data, at 1860 rpm and 1880 rpm along the x-axis compared with the identification results for the oil film bearing system along the x-axis. To analyze the stability of the results in a nonlinear system, we need to calculate the eigenvalues of the Poincaré map. Because these cannot be computed directly in a nonlinear system model, we apply the method presented by Friedmann *et al.*[20]. The improved integration scheme for evaluating the transition matrix $[\Phi_A(T, 0)]$ is based upon the fourth order Runge-Kutta scheme with Gill coefficients. The most straightforward method for dealing with the stability of a periodic system consists of applying the

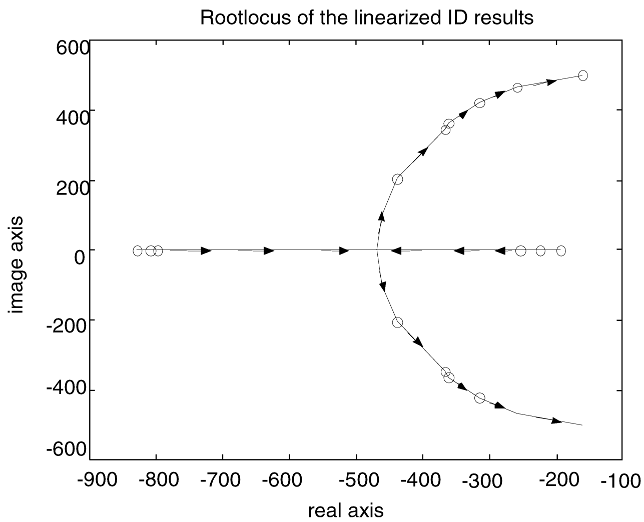

Fig. 7. Root locus of the linearized system.

Table 3. ID results for a nonlinear system on the *x*-axis

Frequency (rpm)	m_x (Kg)	c_x (Kg/sec)	k_x (Kg/sec^2)	β
1400	0.0693	98.777	19135.400	$-169.970*10^{10}$
1500	0.0642	74.823	14523.634	$-32.822*10^{10}$
1600	0.0655	77.882	16682.132	$-39.049*10^{10}$
1700	0.0613	64.212	18621.703	$-28.6371*10^{10}$
1800	0.0586	48.344	17597.422	$-12.728*10^{10}$
1820	0.0525	45.370	17173.198	$-9.926*10^{10}$
1840	0.0508	37.433	16734.656	$-5.528*10^{10}$
1860	0.0529	31.762	17707.601	$-4.656*10^{10}$
1880	0.0550	21.277	18710.396	$-6.083*10^{10}$

Table 4. Eigenvalues of the transition matrix [$\Phi_A(T, 0)$] in a nonlinear system

Frequency (rpm)	Eigenvalues
1400	$7.313*10^{-3}$, $1.952*10^{-18}$
1500	$8.262*10^{-4}$, $7.617*10^{-18}$
1600	$6.233*10^{-4}$, $6.847*10^{-17}$
1700	$4.483*10^{-7}$, $1.078*10^{-9}$
1800	$-1.030*10^{-6}\pm1.906*10^{-7}j$
1820	$-4.697*10^{-7}\pm4.468*10^{-7}j$
1840	$-3.442*10^{-6}\pm4.925*10^{-6}j$
1860	$-5.454*10^{-5}\pm2.836*10^{-5}j$
1880	$-1.049*10^{-3}\pm1.755*10^{-3}j$

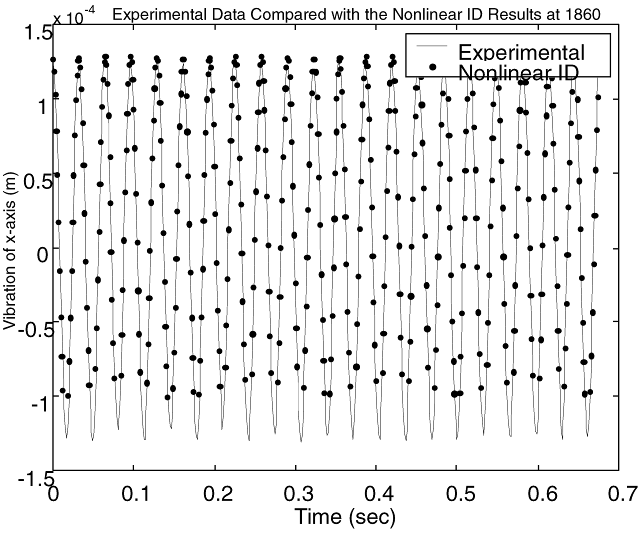

Fig. 8. Experimental data compared with the nonlinear ID results at 1860rpm.

Floquet-Liapunov theorem, that states that the knowledge of the state transition matrix over one period is sufficient in order to determine the stability of a periodic system. Thus, the approximate transition matrix at the end of a period can be obtained by one integration pass. Table 4 shows the eigenvalues of the transition matrix [$\Phi_A(T, 0)$] for our experimental system.

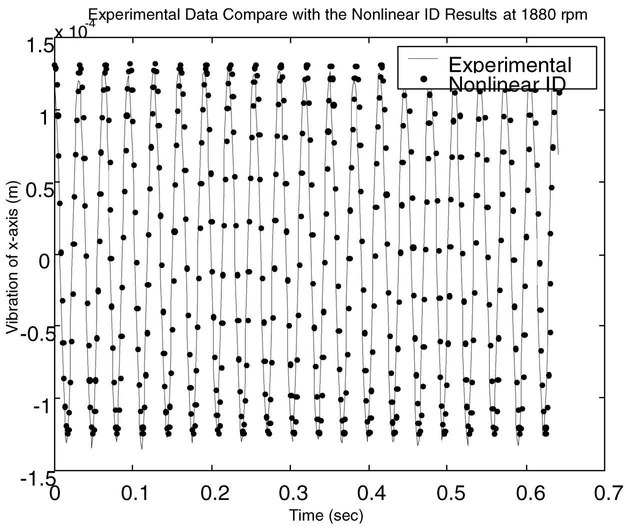

Fig. 9. Experimental data compared with the nonlinear ID results at 1880rpm.

In Table 4, we see that all the eigenvalues are not outside the unit circle, but the complex eigenvalues change very fast between 1800 rpm to 1880 rpm. From 1860 rpm to 1880 rpm they even differ by 2 orders. We expect that when the frequency is 1900 rpm, the eigenvalues may be over the unit circle. Comparing the results in Table 3 and Table 4, we can see that the tendency of the eigenvalues in both the linearized and the nonlinear models is similar. Figures 10, 11 and 12 show the root locus of the nonlinear system results on different scales. According to the bifurcation theory, if a pair of complex conjugate eigenvalue for a nonlinear system pass through the unit circle, then a Hopf bifurcation occurs, and a new vibrational structure appears. This actually predicts the occurrence and the behavior of the oil whirl.

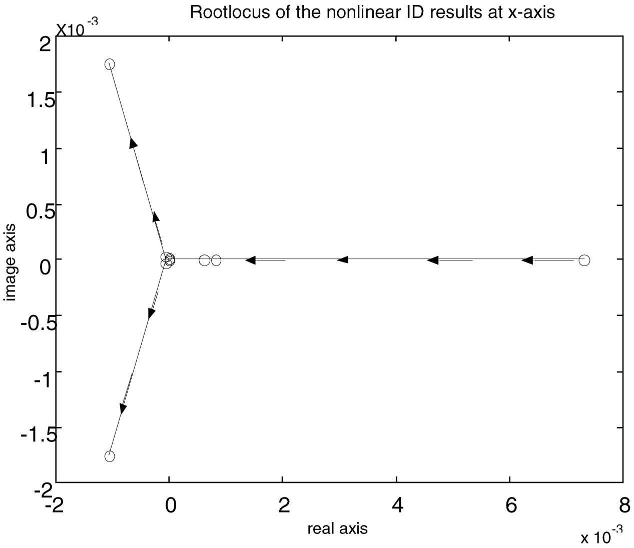

Fig. 10. Root locus of the nonlinear system.

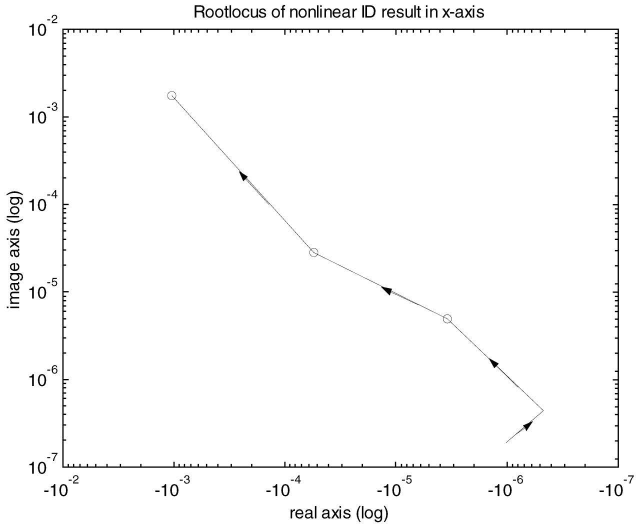

Fig. 12. Complex part of the eigenvalues using logarithmic scales for a nonlinear system.

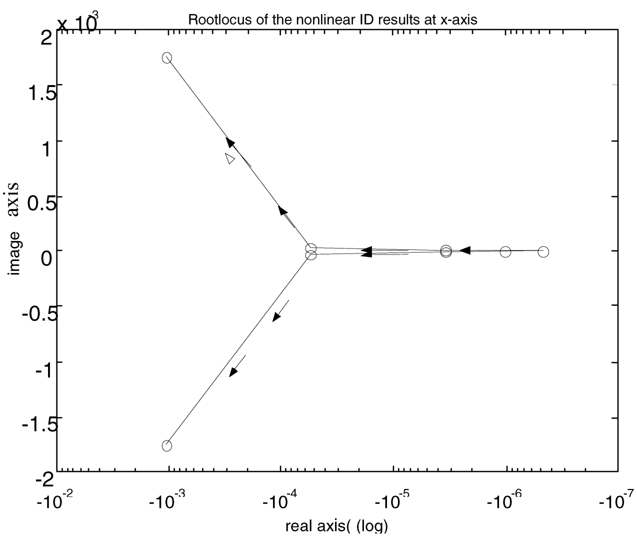

Fig. 11. Complex part of the eigenvalues using a logarithmic scale for a nonlinear system.

CONCLUSION

In practical applications, it is difficult to apply a persistent excitation for identification of an oil film system. In this paper we adapted an unbalanced force to excite the oil film system, and then developed a method to calculate the phase angle between the unbalanced force and the measured responses. Stability analysis and system performance are evaluated by the using root locus and Floquet-Liapunov theorem. Based on the identified model, the oil whirl in the journal bearing can be predicted. A comparison between the simulation results and the experimented data show the feasibility of the proposed approach.

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