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A STABILITY CRITERION OF TIME-DELAY FUZZY SYSTEMS

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ABSTRACT

To guarantee the asymptotic stability, a stability criterion in terms of Lyapunov's direct method for multiple time-delay fuzzy interconnected systems is proposed in this paper. Each of these systems consists of a number of subsystems represented by Takagi-Sugeno fuzzy models with multiple time delays.

INTRODUCTION

The mathematical models of many physical and engineering systems are frequently of high dimension, or possessing interactive dynamic phenomena. The information processing and requirements to experiment with these models for control purposes are usually excessive. Moreover, the existence of time delays is frequently a source of instability in some way. Hence, the problem of stability analysis of time-delay systems has been one of the main concerns of researchers (see [1-3], for example) wishing to inspect the properties of such systems.

In this paper, we consider a multiple time-delay fuzzy interconnected system composed of J subsystems with interconnections and each subsystem is represented by the so-called Takagi-Sugeno (T-S) fuzzy model with multiple time delays. One critical property of control systems is stability and considerable reports have been issued in the literature on the stability problem of fuzzy dynamic systems (see [4-5] and the references therein). However, a literature survey indicates that the stability problem of fuzzy interconnected systems with multiple time delays has not yet been resolved. Thus, for the purpose of general application, a stability criterion in terms of Lyapunov's direct method is derived to guarantee the asymptotic stability of multiple time-delay fuzzy interconnected systems.

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SYSTEM DESCRIPTION AND STABILITY ANALYSIS

Consider an interconnected system $\boldsymbol{F}$ composed of J multiple time-delay subsystems F_j, $j = 1, 2, ..., J$. The jth subsystem F_j is described as follows:

$$\dot{x}_j(t) = f_j(x_j(t)) + \sum_{k=1}^{N_j} g_{kj}(x_j(t - \tau_{kj})) + \sum_{\substack{n=1 \\ n \neq j}}^{J} b_{nj} x_n(t), \qquad (1)$$

where f_j and g_{kj} are the nonlinear vector-valued functions; $x_j(t)$ denotes the state vector and $x_j^T(t) = [x_{1j}(t), x_{2j}(t), ..., x_{gj}(t)]$; τ_{kj}, the kth time delay of the jth subsystem, is a positive real number for $k = 1, 2, ..., N_j$; b_{nj} is the nonlinear interconnection matrix between the nth and jth subsystems.

In a little more than a decade ago, a fuzzy dynamical model had been developed primarily from the pioneering work of Takagi and Sugeno [6] to represent local linear input/output relations of nonlinear systems. Accordingly, the jth isolated subsystem (without interconnection) of $\boldsymbol{N}$ is approximated by a fuzzy model described by fuzzy IF-THEN rules. The ith rule of this fuzzy model for the nonlinear interconnected subsystem N_j is proposed as the following form:

IF $x_{1j}(t)$ is M_{i1j} and ... and $x_{gj}(t)$ is M_{igj}

$$\text{THEN} \quad \dot{x}_j(t) = A_{ij} x_j(t) + \sum_{k=1}^{N_j} A_{ikj} x_j(t - \tau_{kj}), \qquad (2)$$

where $x_j^T(t) = [x_{1j}(t), x_{2j}(t), ..., x_{gj}(t)]$, $i = 1, 2, ..., r_j$ and r_j is the number of IF-THEN rules of the jth subsystem; A_{ij} and A_{ikj} are constant matrices with appropriate dimensions, $x_j(t)$ is the state vector, τ_{kj} denotes the time delay, $M_{ipj}(p = 1, 2, ..., g)$ are the fuzzy sets, and $x_{1j}(t)$

~ $x_{gj}(t)$ are the premise variables. The final state of this fuzzy dynamic system is inferred as follows:

$$\dot{x}_j(t) = \sum_{i=1}^{r_j} w_{ij}(t)[A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj})] / \sum_{i=1}^{r_j} w_{ij}(t)$$
$$= \sum_{i=1}^{r_j} h_{ij}(t)[A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj})] \quad (3)$$

where $w_{ij}(t) = \prod_{p=1}^{g} M_{ipj}(x_{pj}(t))$, $h_{ij}(t) = w_{ij}(t) / \sum_{i=1}^{r_j} w_{ij}(t)$, $M_{ipj}(x_{pj}(t))$ is the grade of membership of $x_{pj}(t)$ in M_{ipj}. In this paper, it is assumed that $w_{ij}(t) \geq 0$, $i = 1, 2, ..., r_j$; $j = 1, 2, ..., J$ and $\sum_{i=1}^{r_j} w_{ij}(t)$ for all t. Therefore, $h_{ij}(t) \geq 0$ and $\sum_{i=1}^{r_j} h_{ij}(t)$ for all t. Therefore, from Eq. (1) and Eq. (3), we have

$$\dot{x}(t) = \sum_{i=1}^{r_j} h_{ij}(t)[A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj})$$
$$+ \sum_{\substack{n=1 \\ n \neq j}}^{J} b_{nj}x_n(t)] \quad (4)$$

In the following, a stability criterion is proposed to guarantee the asymptotic stability of the multiple time-delay fuzzy interconnected system $\boldsymbol{F}$ which consists of J fuzzy models F_j ($j = 1, 2, ..., J$) described in Eq. (3). Prior to examination of asymptotic stability of $\boldsymbol{F}$, a useful concept is given below.

Lemma 1 [7]: For any matrices X and Y with appropriate dimensions, we have

$$X^TY + Y^TX \leq \kappa X^TX + \kappa^{-1}Y^TY$$

where κ is a positive constant.

Theorem 1: The multiple time-delay fuzzy interconnected system $\boldsymbol{F}$ is asymptotically stable, if there exist positive definite matrices $P_j > 0$, $R_{kj} > 0$ and positive constants $\alpha_j > 0$, $\eta > 0$ such that the following inequalities hold:

$$\hat{\lambda}_{ij} = \lambda_M(Q_{ij}) < 0\,; \quad \hat{\lambda}_{ikj} = \lambda_M(Q_{ikj}) < 0 \quad (5)$$

for $i = 1, 2, ..., r_j$, $j = 1, 2, ..., J$, $k = 1, 2, ..., N_j$

where

$$Q_{ij} = \{A_{ij}^TP_j + P_jA_{ij} + \sum_{k=1}^{N_j} R_{kj} + \sum_{k=1}^{N_j} \alpha_jP_jA_{ikj}A_{ikj}^TP_j$$
$$+ \eta(J-1)I + \eta^{-1}J(P_jb_{nj}b_{nj}^TP)\}\,, \quad (6)$$

$$Q_{ikj} = \alpha_j^{-1}I - R_{kj}\,. \quad (7)$$

with $P_j = P_j^T$, $R_{kj} = R_{kj}^T$, and $\lambda_M(Q_{ij})$ as well as $\lambda_M(Q_{ikj})$ denote the maximum eigenvalues of Q_{ij} and Q_{ikj}, respectively.

PROOF OF THEOREM 1

Let the Lyapunov function for the multiple time-delay fuzzy interconnected system $\boldsymbol{F}$ be defined as

$$V(t) = \sum_{j=1}^{J} v_j(t) = \sum_{j=1}^{J} \{\dot{x}_j^T(t)P_jx_j(t)$$
$$+ \sum_{k=1}^{N_j} \int_0^{\tau_{kj}} x_j^T(t-\tau)P_{kj}x_j(t-\tau)d\tau\} \quad (8)$$

where $P_j = P_j^T > 0$ and the weighting matrix $R_{kj} = R_{kj}^T > 0$. We then evaluate the time derivative of V on the trajectories of Eq. (4) to get

$$\dot{V} = \sum_{j=1}^{J} \dot{v}_j(t) = \sum_{j=1}^{J} [\dot{x}_j^T(t)P_jx_j(t) + x_j^T(t)P_j\dot{x}_j(t)]$$
$$+ \sum_{j=1}^{J} \sum_{k=1}^{N_j} [x_j^T(t)R_{kj}x_j(t) - x_j^T(t-\tau_{kj})R_{kj}x_j(t-\tau_{kj})]$$
$$= \sum_{j=1}^{J} \{[\sum_{i=1}^{r_j} h_{ij}(t)[A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj})$$
$$+ \sum_{\substack{n=1 \\ n \neq j}}^{J} b_{nj}x_n(t))]^TP_jx_j(t) + x_j^T(t)P_j[\sum_{i=1}^{r_j} h_{ij}(t)(A_{ij}x_j(t)$$
$$+ \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj}) + \sum_{\substack{n=1 \\ n \neq j}}^{J} b_{nj}x_n(t))]\}$$
$$+ \sum_{j=1}^{J} \sum_{i=1}^{r_j} \sum_{k=1}^{N_j} h_{ij}[(x_j^T(t)R_{kj}x_j(t)$$
$$- x_j^T(t-\tau_{kj})R_{kj}x_j(t-\tau_{kj}))]$$
$$= \sum_{j=1}^{J} \sum_{i=1}^{r_j} h_{ij}(t)x_j^T(t)(A_{ij}^TP_j + P_jA_{ij} + \sum_{k=1}^{N_j} R_{kj})x_j(t)$$
$$+ \sum_{j=1}^{J} \sum_{i=1}^{r_j} h_{ij}(t) \sum_{k=1}^{N_j} [x_j^T(t-\tau_{kj})A_{ikj}^TP_jx_j(t)$$
$$+ x_j^T(t)P_jA_{ikj}x_j(t-\tau_{kj})] + \sum_{j=1}^{J} \sum_{\substack{n=1 \\ n \neq j}}^{J} [x_n^T(t)b_{nj}^TP_jx_j(t)$$
$$+ x_j^T(t)P_jb_{nj}x_n(t)]$$
$$+ \sum_{j=1}^{J} \sum_{i=1}^{r_j} \sum_{k=1}^{N_j} h_{ij}(t)[-x_j^T(t-\tau_{kj})R_{dj}x_j(t-\tau_{kj})] \quad (9)$$

Based on Lemma 1 and Eq. (9), we have

$$\dot{V} \leq \sum_{j=1}^{J} \sum_{i=1}^{r_j} h_{ij}(t)x_j^T(t)(A_{ij}^TP_j + P_jA_{ij} + \sum_{k=1}^{N_j} R_{kj})x_j(t)$$
$$+ \sum_{j=1}^{J} \sum_{i=1}^{r_j} h_{ij}(t) \sum_{k=1}^{N_j} [\alpha_jx_j^T(t)P_jA_{ikj}A_{ikj}^TP_jx_j(t)$$
$$+ \alpha_j^{-1}x_j^T(t-\tau_{kj})x_j(t-\tau_{kj})] + \sum_{j=1}^{J} \sum_{\substack{n=1 \\ n \neq j}}^{J} [\eta x_n^T(t)x_n(t)$$

$$+\eta^{-1}x_j^T(t)P_j b_{nj} b_{nj}^T P_j x_j(t)]$$

$$+\sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{k=1}^{N_j} h_{ij}(t)[-x_j^T(t-\tau_{kj})R_{kj}x_j(t-\tau_{kj})]$$

$$\dot{V}\le\sum_{j=1}^{J}\sum_{i=1}^{r_j} h_{ij}(t)x_j^T(t)(A_{ij}^T P_j + P_j A_{ij} + \sum_{k=1}^{N_j} R_{kj}$$

$$+\sum_{k=1}^{N_j}\alpha_j P_j A_{ikj}A_{ikj}^T P_j)x_j(t)$$

$$+\sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{k=1}^{N_j} h_{ij}(t)x_j^T(t-\tau_{kj})[\alpha_j^{-1}I - R_{kj}]x_j(t-\tau_{kj})$$

$$+\sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{j=1}^{J} h_{ij}(t)x_j^T(t)[\eta(\frac{J-1}{J})I$$

$$+\eta^{-1}P_j b_{nj} b_{nj}^T P_j]x_j(t)$$

$$=\sum_{j=1}^{J}\sum_{i=1}^{r_j} h_{ij}(t)x_j^T(t)\{A_{ij}^T P_j + P_j A_{ij} + \sum_{k=1}^{N_j} R_{kj}$$

$$+\sum_{k=1}^{N_j}\alpha_j P_j A_{ikj}A_{ikj}^T P_j + \eta(J-1)I$$

$$+\eta^{-1}J(P_j b_{nj} b_{nj}^T P)\}x_j(t)$$

$$+\sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{k=1}^{N_j} h_{ij}(t)x_j^T(t-\tau_{kj})[\alpha_j^{-1}I - R_{kj}]x_j(t-\tau_{kj})$$

$$=\sum_{j=1}^{J}\sum_{i=1}^{r_j} h_{ij}(t)x_j^T(t)Q_{ij}x_j$$

$$+\sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{k=1}^{N_j} h_{ij}(t)[x_j^T(t-\tau_{kj})Q_{ikj}x_j(t-\tau_{kj})]\,. \quad (10)$$

$$\dot{V}(t)\le\sum_{j=1}^{J}\{\sum_{i=1}^{r_j} h_{ij}(t)\hat{\lambda}_{ij}\}\left\|x_j(t)\right\|^2$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{N_j}\{\sum_{i=1}^{r_j} h_{ij}(t)\hat{\lambda}_{ikj}\}\left\|x_j(t-\tau_{kj})\right\|^2. \quad (11)$$

Based on Eq. (6) and Eq. (7), we have $\dot{V}<0$ and the proof is therefore completed.

CONCLUSIONS

This paper is concerned with the stability problem of the multiple time-delay fuzzy interconnected system which consists of a few interconnected subsystems. Each subsystem is represented by a T-S fuzzy models with multiple time delays. A stability criterion in terms of Lyapunov's direct method is proposed to guarantee the asymptotic stability of multiple time-delay fuzzy interconnected systems.

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時延模糊系統之穩定準則

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摘 要

本文利用李雅普諾夫直接法(Lyapunov's direct method)推導一穩定準則。此穩定準則可確保多時延相互交連之模糊系統達到漸近穩定。文中我們將使用T-S模糊模型(Takagi-Sugeno fuzzy models)的技巧來表示此系統，而此系統包含多個相互交連的多時延子系統。