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# A MIXED INTEGER PROGRAMMING MODEL FOR ROUTING CONTAINERSHIPS

Ching-Wu Chu, Tu-Cheng Kuo, and Jiing-Chang Shieh

Key words: containership routing, mixed integer programming.

## ABSTRACT

In this paper, we formulate a mixed integer programming model for routing containerships. Our model helps in evaluating the optimal sequence of port calls and the number of containers transported between port pairs given the trip cycle time. Some numerical examples and a real world application of the Trans Pacific route are presented. The computational results show that our model, which solve the mixed integer programming optimally, is quite efficient and applicable to real world problem.

## INTRODUCTION

A containership involves a major capital investment and the daily operating cost of a containership may amount to thousands of dollars. Proper selecting En route Ports and scheduling of containerships could yield great potential of improving their economic performance or cost savings.

Most liner shipping companies operating containerships assign a number of ships on a particular trade route fixed by two end ports and many en route ports.

The liner operation resembles a bus line-it publishes timetables and competes for cargo. Since the route of a containership, once determined, is difficult to change for a certain period of time, the routing decision should be made carefully after a thorough study. In order to adjust for changing environments, such as changes in freight rates, in cargo demand, in international regulations, or in operating cost, the liner shipping company should rearrange the route periodically. Based on the reasons mentioned above, it is worthwhile to formulate an analytic model solving the problem.

We conclude this section by presenting a survey

of relevant work in this area. Many useful vehicle routing problems have been studied. A comprehensive survey can be found in Bodin *et al.* [3]. In contrast to vehicle routing, little work has been done in ship routing and scheduling. In the past, there have been attempts to solve ship routing and scheduling problems. Dantzig and Fulkerson [6] considered the first linear programming model, which minimizes the number of tankers to meet a fixed schedule. Flood [8] dealt with the same problem by minimizing the total distance in ballast, given a fleet size. The above two models are the pioneers of Operations Research applications in ship routing and scheduling.

Further extension of the above two models are considered by Briskin [5], Bellmore *et al.* [2], McKay and Hartney [10]. Baker [1], Stott and Douglas [18], and Fisher and Rosenwein [7] developed an interactive ship scheduling systems. There have been some papers on optimization models in sea transportation, but the majority have been on industrial carriers, bulk carriers, and tankers.

As to liner management, Boffey *et al.* [4] developed a heuristic model and an interactive decision support system for scheduling containerships on the North Atlantic route. Olson *et al.* [11] used a simulation model to obtain the medium range scheduling for a freighter fleet. Rana and Vickson [14, 15] presented nonlinear models. They tried to maximize total profit by finding an optimal sequence of ports of call for each ship. For solution, they presented Lagrangean relaxation and decomposition methods.

Perakis and Jaramillo [12], and Jaramillo and Perakis [9] formulated a linear programming model to minimize total fleet operating and lay-up cost during a planning horizon. Powell and Perakis [13] extended the work developed by Jaramillo and Perakis. An integer programming model is developed to minimize the operating and lay-up cost for a fleet of liner ships operating on various routes.

A complete survey of literature on ship routing is presented in references [16] and [17]. The rest of this paper is organized as follows: The formal presentation

of the model is explained in Section 2. Section 3 provides some numerical examples and illustrates some of the capabilities of the model applied to the Trans Pacific route. The final section concludes this paper.

## MODEL FORMULATION

Most liner shipping companies operating containerships assign a number of ships on a particular trade route fixed by two end ports and many en route ports. Our model also restrict ourselves to this case.

### 1. Model Assumptions

The assumptions of our model are as the following:

1. It is not necessary to visit all the ports.
2. The ship changes direction at the starting (port 1) and the ending ports (port  $n$ ).
3. The operating costs at sea and at port are known.
4. The cargo is available at a uniform rate at each port.

### 2. Decision Variables and Parameters

The decision variables are defined as follows:

$X_{ij}$ : the number of containers transported from port  $i$  to port  $j$ .

$$Z_i = \begin{cases} 1 & \text{if port } i \text{ is visited in the outbound route} \\ 0 & \text{otherwise} \end{cases}$$

$$ZB_i = \begin{cases} 1 & \text{if port } i \text{ is visited in the inbound route} \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{ij} = \begin{cases} 1 & \text{if ports } i \text{ and } j \text{ are directly connected} \\ & \text{in the outbound route} \\ 0 & \text{otherwise} \end{cases}$$

$$YB_{ij} = \begin{cases} 1 & \text{if port } i \text{ and } j \text{ are directly connected} \\ & \text{in the inbound route} \\ 0 & \text{otherwise} \end{cases}$$

Other parameters are defined as follows:

- $n$ : total number of ports in the network.  
 $P_{ij}$ : revenue charged per container for a port pair  $(i, j)$ .  
 $K_i$ : the operating costs at port  $i$ .  
 $C_{ij}$ : the operating costs at sea from port  $i$  to port  $j$ .  
 $PD_{ij}$ : expected number of containers available to be transported from port  $i$  to  $j$ , over the planning period.  
 $T_{ij}$ : travel time for a ship from port  $i$  to port  $j$ . It is

equal to the distance between port  $i$  and port  $j$ , divided by the average ship's speed from port  $i$  to port  $j$ .

$SPT_i$ : time stayed in port  $i$  (including time required for the ship to load and unload at port  $i$ , plus pilotage time while leaving and entering harbour  $i$ ).

$W$ : the number of weeks allowed of a round trip (an integer number).

$SP\_LOAD$ : the shipload of a containership.

### 3. Objective Function

$$\begin{aligned} \text{Max } Z = & \sum_i \sum_j P_{ij} X_{ij} - \sum_i K_i Z_i - \sum_i K_i ZB_i \\ & - \sum_i \sum_j C_{ij} Y_{ij} - \sum_i \sum_j C_{ij} YB_{ij} \end{aligned}$$

The objective function maximizes the profit (revenue-operating costs). The constraints of the model are listed below:

### 4. Departure Constraints

$$\sum_{j=i+1}^n Y_{ij} - Z_i = 0 \quad (i = 1, \dots, n-1) \quad (1)$$

$$\sum_{j=1}^{i-1} YB_{ji} - ZB_i = 0 \quad (i = 2, \dots, n) \quad (2)$$

If a ship is not visiting the specific port  $i$ , the cargo cannot be transported. Constraint (1) and (2) ensure all possible departures from port  $i$  for outbound trip and inbound trip, respectively.

$$\sum_{i=j+1}^n Y_{ij} = 0 \quad (j = 1, \dots, n-1) \quad (3)$$

$$\sum_{i=j+1}^n YB_{ij} = 0 \quad (j = 1, \dots, n-1) \quad (4)$$

Constraint (3) and (4) prevent the ship from traveling in different direction for outbound trip and inbound trip, respectively.

### 5. Arrival Constraints

Constraint (5) and (6) enumerate all possible arrivals at port  $i$  for outbound trip and inbound trip, respectively.

$$\sum_{j=1}^{i-1} Y_{ji} - Z_i = 0 \quad (i = 2, \dots, n) \quad (5)$$

$$\sum_{j=i+1}^n YB_{ij} - ZB_i = 0 \quad (i = 1, \dots, n-1) \quad (6)$$

### 6. Capacity Constraints

$$X_{ij} \leq PD_{ij} Y_{ij} \quad (i = 1, \dots, n-1; j = i+1, \dots, n) \quad (7)$$

$$X_{ij} \leq PD_{ij} Y_{B_{ij}} (i = 1, \dots, n - 1; j = i + 1, \dots, n) \quad (8)$$

$$\sum_{\text{possible } i, j} \sum X_{ij} \leq SP\_LOAD$$

$$(i = 1, \dots, n; j = 1, \dots, n) \quad (9)$$

In constraint (7), if  $Y_{ij}$  equals to one, constraint (7) will be equal to  $X_{ij} \leq PD_{ij}$ . If  $Y_{ij}$  is equal to zero, constraint (7) will be equal to  $X_{ij} \leq 0$ . By coupling two different cases together, we can ensure that the number of containers transported from port  $i$  to port  $j$  will be equal to zero or less than or equal to the expected number of containers available to be transported for outbound trip. With similar reasoning, constraint (8) imposes a similar restriction on  $X_{ij}$  for inbound trip.

Constraint (9), applied to all port pairs, ensures that the total number of containers transported from any port  $i$  to port  $j$  ( $i < j$  for outbound trip;  $i > j$  for inbound trip) will not exceed the ship's capacity.

**7. Connected Constraints**

$$Z_i + Z_j - Y_{ij} \geq 0 (i = 1, \dots, n - 1; j = i + 1, \dots, n) \quad (10)$$

$$Z_i + Z_j - 2Y_{ij} \geq 0 (i = 1, \dots, n - 1; j = i + 1, \dots, n) \quad (11)$$

$$ZB_i + ZB_j - Y_{B_{ij}} \geq 0$$

$$(i = j + 1, \dots, n; j = 1, \dots, n - 1) \quad (12)$$

$$ZB_i + ZB_j - 2Y_{B_{ij}} \geq 0$$

$$(i = j + 1, \dots, n; j = 1, \dots, n - 1) \quad (13)$$

The inequalities in constraints (10) and (11) stand for the relationship between the port pair ( $i, j$ ) for outbound trip. By combining constraints (10) and (11), we can ensure that  $Y_{ij}$  represents whether port  $i$  and  $j$  are directly connected. Constraint (10) would be equal to one only if port  $i$  and  $j$  are directly connected. If port  $i$  and  $j$  are not directly connected, constraint (11) would force  $Y_{ij}$  to equal zero.

The constraints (12) and (13) impose a similar relationship between port pair ( $i, j$ ) for inbound trip.

**8. Route Time Constraints**

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^n T_{ij} Y_{ij} + \sum_{i=j+1}^n \sum_{j=1}^{n-1} T_{ij} Y_{B_{ij}} + \sum_{i=1}^n SPT_i Z_i$$

$$+ \sum_{i=1}^n SPT_i ZB_i - SPT_n ZB_n \leq 168W$$

$$(i = 1, \dots, n; j = 1, \dots, n) \quad (14)$$

There are 168 hours in one week and  $W$  is an integer number. 168W specifies the cycle time voyage in hours and the left hand side of constraint (14) ensures that the total time spend by a ship at sea and in harbor do not exceed the cycle time.

In general, the liner serves its customers based on weekly service. Once the cycle time is determined; the required vessels for operating will be known automatically. For example, if the cycle time is one week, one week trip needs only one vessel. If the cycle time is two weeks, a two-week trip needs two vessels.

**NUMERICAL EXAMPLE**

In order to demonstrate the applicability of our model, we first use an artificial data to provide some numerical examples and sensitivity analysis. Then, a real world application of the Trans Pacific route from Yang Mine Line in Taiwan is also presented. The detailed information of the artificial data (Tables 1-5) is listed as follows:

Both cycle time and shipload are main factors in route planning. First, we investigate the effect on the selection of En route ports by varying cycle time. The optimization software LINDO version 6.1 is used to solve our model. Figures 1-4 show the service routes under different cycle time. From Figure 1, we can see that all ports are visited. In this case, the cycle time is 7 weeks so that the liner shipping company needs 7 vessels for weekly service. By reducing the cycle time one week at a time, we can see that the number of ports visited and the total profit of each scenario (Table 6) with respect to cycle time are decreasing (Figures 2-4). It is clear that when the cycle time is not long enough (the company does not own enough containerships and does not intend to charter additional ships), the containership cannot visit all ports. From the above observations, we know that our model can help in selecting the optimal sequence of port calls.

**Table 1. Stay time and operating cost at port (hour, \$)**

	1	2	3	4	5	6	7	8
Stay time	16	12	16	12	13	13	17	18
Port charges	1500	1000	1700	1100	1150	1400	1800	1950

**Table 2. Revenue charged per container between port pairs (\$/TEU)**

	1	2	3	4	5	6	7	8
1	0	35	75	100	230	220	255	270
2	30	0	50	100	198	170	215	280
3	80	65	0	75	160	140	205	265
4	115	105	65	0	45	120	260	300
5	115	113	65	50	0	150	220	245
6	220	195	130	120	90	0	70	90
7	180	150	150	140	160	80	0	120
8	200	210	185	210	165	90	50	0

**Table 3. Expected number of containers available to be transported between port pairs (TEU)**

	1	2	3	4	5	6	7	8
1	0	95	125	150	190	250	260	170
2	110	0	80	130	160	210	190	210
3	140	70	0	110	110	90	150	200
4	190	140	50	0	85	90	120	170
5	250	90	115	50	0	160	200	180
6	220	95	140	130	210	0	80	95
7	180	150	120	75	140	60	0	80
8	120	140	120	110	105	80	40	60

**Table 4. The operating costs at sea between port pairs (\$)**

	1	2	3	4	5	6	7	8
1	0	36	100	144	168	200	258	286
2	42	0	72	136	160	190	215	238
3	108	80	0	80	100	106	156	192
4	152	148	90	0	24	60	173	154
5	180	172	112	30	0	42	141	174
6	216	204	118	70	48	0	73	51
7	250	207	151	168	135	67	0	49
8	274	225	185	148	164	55	47	0

**Table 5. The traveling time at sea between port pairs (hour)**

	1	2	3	4	5	6	7	8
1	0	36	100	144	268	300	358	386
2	42	0	72	136	260	290	315	338
3	108	80	0	80	200	206	256	292
4	152	148	90	0	224	260	273	254
5	180	172	112	30	0	42	51	74
6	216	204	118	70	48	0	35	51
7	350	307	251	268	48	67	0	27
8	374	325	285	248	64	51	27	0

Next, we test the effects on the selection of En route ports by changing shipload. 800 TEU and 1000 TEU, two different capacities of shipload are tested and the cycle time of 7 weeks is used. From Figure 5, we can see that both cases have the same sequence of port calls.

The main reason is that the liner shipping company wants to maximize its profit so all ports are visited while the cycle time is long enough. But if we look at Table 6 carefully, we can find that the numbers of containers transported between two cases are different. Based on

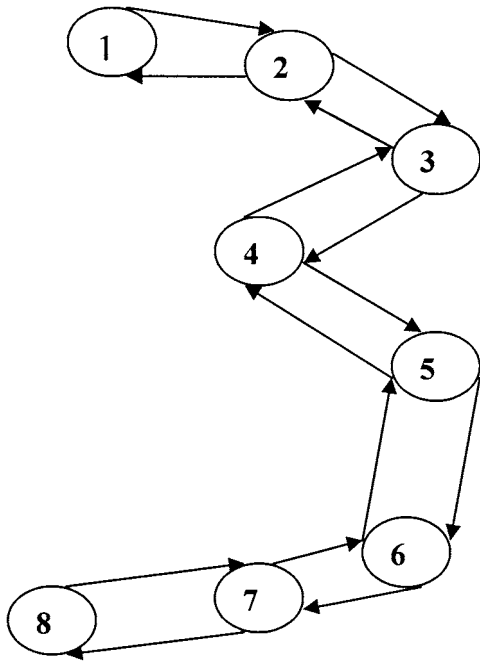


Fig. 1. Service route for cycle time 7 weeks and shipload 1500 TEU.

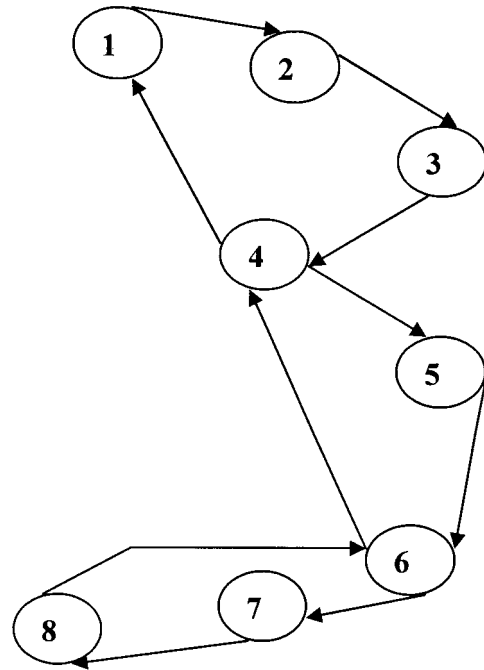


Fig. 3. Service route for cycle time 5 weeks and shipload 1500 TEU.

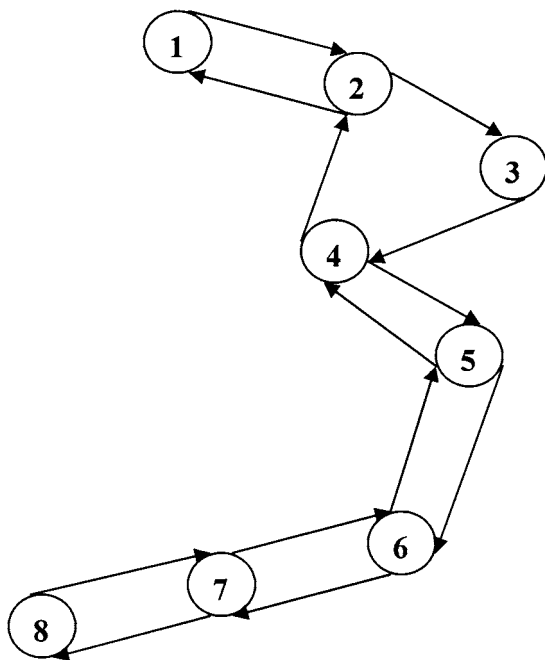


Fig. 2. Service route for cycle time 6 weeks and shipload 1500 TEU.

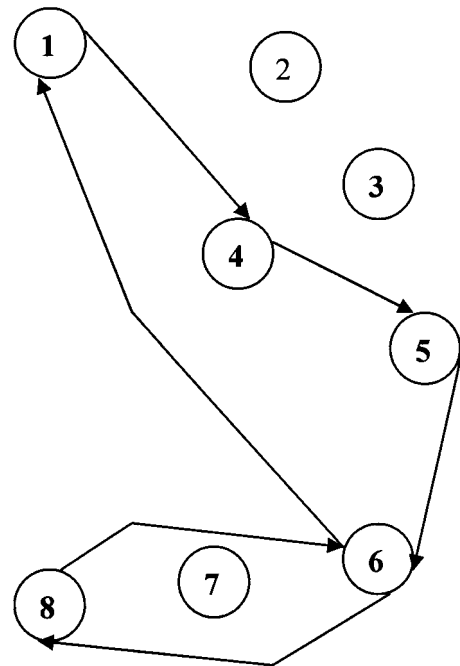


Fig. 4. Service route for cycle time 4 weeks and shipload 1500 TEU.

**Table 6.** The number of container transported between port pairs under different cycle time and shipload

	TEU $\leq$ 1500 Cycle $\leq$ 7 weeks	TEU $\leq$ 1500 Cycle $\leq$ 6 weeks	TEU $\leq$ 1500 Cycle $\leq$ 5 weeks	TEU $\leq$ 1500 Cycle $\leq$ 4 weeks	TEU $\leq$ 800 Cycle $\leq$ 7 weeks	TEU $\leq$ 1000 Cycle $\leq$ 7 weeks
1-2	95	95	95	0	0	0
1-3	125	125	125	0	0	0
1-4	150	150	150	150	0	130
1-5	190	190	190	190	120	190
1-6	200	200	200	200	200	200
1-7	260	260	260	0	260	260
1-8	170	170	170	170	170	170
2-3	80	80	80	0	80	80
2-4	130	130	130	0	130	130
2-5	160	160	160	0	160	160
2-6	110	110	110	0	110	110
2-7	190	190	190	0	190	190
2-8	210	210	210	0	210	210
3-4	110	110	110	0	110	110
3-5	110	110	110	0	110	110
3-6	90	90	90	0	90	90
3-7	150	150	150	0	150	150
3-8	200	200	200	0	200	200
4-5	85	85	85	85	85	85
4-6	90	90	90	90	90	90
4-7	120	120	120	0	120	120
4-8	170	170	170	170	170	170
5-6	160	160	160	160	160	160
5-7	200	200	200	0	200	200
5-8	180	180	180	180	180	180
6-7	80	80	80	0	80	80
6-8	95	95	95	95	95	95
7-8	80	80	80	0	80	80
8-1	120	120	120	120	120	120
8-2	140	140	0	0	140	140
8-3	120	0	0	0	120	120
8-4	110	110	110	0	110	110
8-5	105	105	0	0	105	105
8-6	80	80	80	80	80	80
8-7	40	40	0	0	40	40
7-1	180	180	0	0	180	180
7-2	150	150	0	0	150	150
7-3	120	0	0	0	120	120
7-4	75	75	0	0	75	75
7-5	140	140	0	0	140	140
7-6	60	60	0	0	60	60
6-1	220	220	220	220	220	220
6-2	95	95	0	0	95	95
6-3	140	0	0	0	140	140
6-4	130	130	130	0	130	130
6-5	210	210	0	0	210	210
5-4	50	50	0	0	50	50
5-3	115	0	0	0	115	115
5-2	90	80	0	0	90	90
5-1	250	40	0	0	250	250
4-3	50	0	0	0	50	50
4-2	140	140	0	0	140	140
4-1	190	190	190	0	190	190
3-2	70	0	0	0	70	70
3-1	140	0	0	0	140	140
2-1	110	110	0	0	110	110
PROFIT	1147467	1047740	812226	324717	1108320	1144920

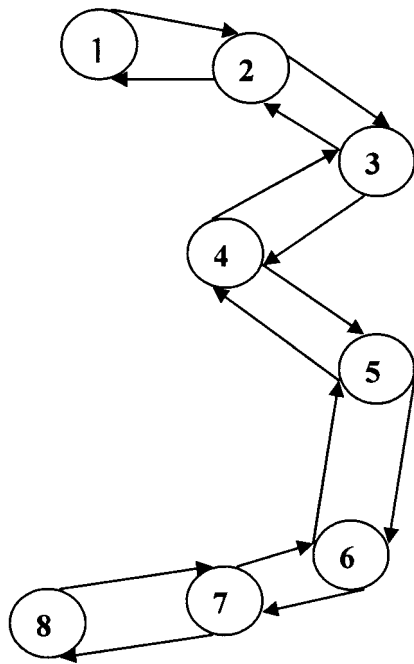


Fig. 5. Service route for cycle time 7 weeks and shipload 800TEU and 1000TEU.

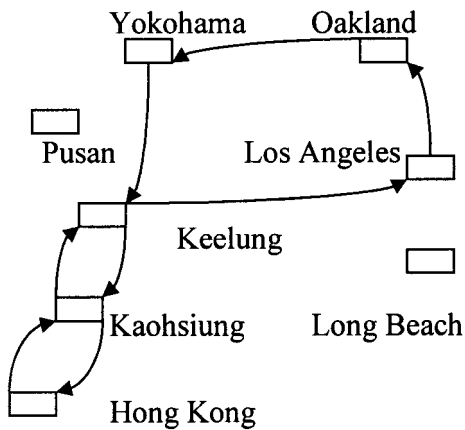


Fig. 6. Service route of Yang Ming Line.

the observations above, we know that our model can help in determining the optimal number of container transported between port pairs in different scenarios.

Finally, we use one of the Trans Pacific route provided by Yang Ming Line in Taiwan for testing our model. The service ports of Trans Pacific route in the United States include Los Angeles and Okaland. In the Asia ports, Hong Kong, Keelung, Kaohsiung and Yokohama are included (see Figure 6). Currently Yang Mine Line deploys 5 vessels with the capacities of 3500 TEU on this route for weekly service and the cycle

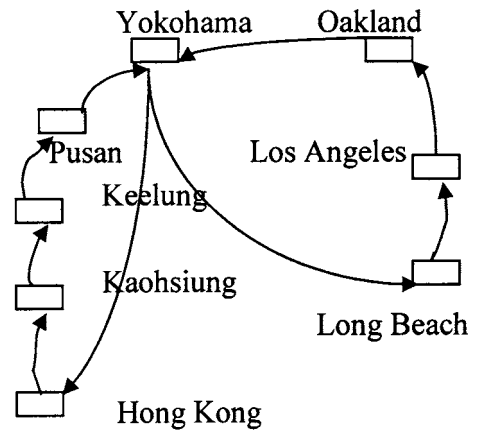


Fig. 7. Service route solved by our model.

time is 5 weeks.

We use the same cycle time to test the applicability of our model. The parameters are the same as those in Tables 1-5 and the real data are not shown here. Test result solved by our model is shown in Figure 7. By comparing two figures, we note that the ports called solved by our model are not identical to those by the Yang Ming Line.

Pusan, Yokohama and Long Beach are not included in the service ports of Trans Pacific route for outbound trip. After discussing with the route manager of the Yang Ming Line, we know that in order to provide better service level and increase competition ability, the Yang Ming Line opens three different routes to serve the customers in Asia and the United States of America. Pusan and Yokohama are currently served by another route.

The main reason for Long Beach is not visited is that the shipping company usually chooses only port to call in the neighboring area to cut down operating costs and shorten the cycle time. Since the distance between Long Beach and Los Angeles is not far away, containers can be transferred by land.

Though our model is applicable to real world problem and has taken main factors in routing into consideration, the route planning manager suggests that in a highly competition environment, a single route planning is not adequate for business decisions in the future. A more complicate network planning and even an expert system should be developed to facilitate in decision making.

### CONCLUSION

In this paper, we have formulated a mixed integer programming model for routing containerships. Our model helps in evaluating the optimal sequence of port



calls, the number of containers transported between port pairs given the trip cycle time. Our model is different from previous works in that our problem can be solved directly using optimization software and very quick in terms of time. Furthermore, once the cycle time is determined; the number of vessels needed for weekly service is known automatically. Based on this information, the liner shipping company can evaluate whether additional containerships should be chartered or not. An important topic for future research is an extension of single route planning to more complicate network planning.

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