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# DESIGN OF FUZZY NONLINEAR ROBUST COMPENSATOR AND ITS APPLICATION ON SUBMARINES

Cheng-Neng Hwang* and Joe- Ming Yang**

Key words: composite controller, fuzzy-type pre-compensator, submarine.

## ABSTRACT

A robust nonlinear composite controller, which is simple in structure and has the characteristics of good tracking performance and guaranteed system stability, is designed in this study. A pre-compensator is proposed to eliminate uncertainties or disturbances so that the tracking error is bounded in a pre-specified ball. The composite control law proposed in this study guarantees system tracking precision and stability by the aid of a third-order differential equation that has been derived in this research. The control parameters in the proposed controller can be adjusted systematically. For unknown serious variation in system surrounding, a composite control law with fuzzy-type pre-compensator is proposed to guarantee the system stability and the tracking precision. The compensating terms of the proposed controller use the mean-values of uncertainties to represent the system time-varying uncertainties at the first trial and then instead of sticking to the mean values, the proposed controller utilizes the fuzzy logics to update its nonlinear compensating terms so that good tracking results can be maintained under the presence of disturbances or plant uncertainties. Finally a submarine is used as an example to demonstrate the feasibility of the proposed controller.

## INTRODUCTION

In recent years, a considerable amount of effort has been focused on developing theories for the control of nonlinear systems [1, 2, 3, 4]. But many of the available controllers are typically hard to construct and are complex in structures [4]. The robustness and performance achievement issues are also widely addressed in many publications by using various control techniques [5, 6, 7, 12]. In this paper, a robust nonlinear composite controller, which is simple in structure and has the characteristics of good tracking performance and guaranteed system stability, is designed. A pre-compensator is proposed to eliminate uncertainties or disturbances so that the tracking error is bounded in a pre-specified ball [3]. The composite control law proposed in this study guarantees system tracking precision and stability by the aid of a third-order differential equation that has been derived in this research. The control parameters in the proposed controller can be adjusted systematically. The proposed controller is relatively easy to construct and is simple in structure. Some of the benefits of using this method are the simplicity in the assumed system model and the proposed controller, the straight-forwardness of the basic philosophy, and the characteristic of the high tracking precision with low input energy requirements. A trade-off between accuracy of tracking and control effort will be identified in this research [3]. In general, in order to achieve accurate tracking of a desired trajectory, a suitable gain value should be chosen. A large gain value will speed up the rate of convergence and therefore will make the system more sensitive to position, velocity and integral tracking errors. However, too large a gain value can lead to instability and may cost high input energy [3]. The choice of gain values will be discussed in this paper. Finally, to illustrate the applicability of the control design, an example of submarine is used to demonstrate the feasibility and robustness of the proposed nonlinear composite controller.

## SYSTEM DESCRIPTION

A general 2$^{nd}$ order non-linear system can be modeled as [3]

$$D_i(x(t), x'(t), r(t), d(t), t)x''(t)$$

$$+ H_i(x(t), x'(t), r(t), d(t), t)$$

$$= U_i(x(t), x'(t), t)$$

and $x(t) = (x_1, x_2, \ldots x_n)$ (1)

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Where $i = 1, 2, n$.
If we let

$$(\bullet) = (x'(t), x(t), r(t), d(t), t)$$

$$(*) = (x'(t), x(t), t)$$

Then the above system can be written as $D_i(\bullet)x''(t) + H_i(\bullet) = U_i(*)$. Uncertainty $r(t)$ in the system is modeled by an unknown Lebesgue measurable function $r(t)$: $R \rightarrow \Omega$, where the set $\Omega$ is known and is compact. $d(t)$ is the disturbance terms. $D_i(.)$ and $H_i(.)$ represent coupling effects between states. Before proposing the controller $U_i(*)$, to ensure the existence and continuation of the feedback control system (1), some assumptions are made as follows:

1. $D_i(.)$, $i = 1, 2, n$ are continuous real positive functions and its inverse $[D_i(.)]^{-1}$ exists, where $(.) = (x'(t), x(t), r(t), d(t), t)$.
2. $H_i(.)$, $D_i(.)$ and $D_i^{-1}(.)$ are uniformly bounded with respect to time t and are continuous functions over $[0, T]$, $T$ is the working time.
3. The reference motions $(x_{ri}(t), x'_{ri}(t), x''_{ri}(t))$, $i = 1, 2, n$, are bounded and are absolutely continuous over $[0, T]$.

The uncertainty $r(t)$ and disturbance $d(t)$ are bounded as follows:

$$||r(t)|| \leq k_1(x, t)$$

$$||d(t)|| \leq k_2(x, t)$$

Where $k_1$, $k_2$ are continuous functions of $x(t)$ and $t$ and $k_1(x, t)$, $k_2(x, t)$ are continuous functions of $X$ and $t$||.|| is the sup-norm. Based on the above assumptions, we now proceed to design the controller to achieve good path tracking precision. By letting the tracking errors be

$$e_i(t) = x_i(t) - x_{ri}(t)$$

$$e'_i(t) = x'_i(t) - x'_{ri}(t)$$

$$e''_i(t) = x''_i(t) - x''_{ri}(t)$$

Obviously, the goal of the controller is to force errors to decay to zero asymptotically so that the desired motion can be tracked. With this goal in mind, we construct a controller that makes the error equation a stable third-order differential equation. Consequently, a robust nonlinear composite controller can then be proposed.

## DESIGN OF THE PROPOSED CONTROLLER

In order to track the desired motion, the controller is proposed as

$$U_i(*) = -D_i(*)[K_D e'_i(t) + K_P e_i(t) + K_I \int e_i(t)\,dt]$$

$$+ H_i(*) + D_i(*)\, x''_{ri}(t) \quad (2)$$

Where $K_D$, $K_P$ and $K_I$ are the gains and are to be chosen to make the closed system stable and robust. Since the uncertainty $r(t)$ and $d(t)$ are assumed to be bounded from the above constants, the mean of upper- and lower-bounds may be taken as reasonable estimates of $r(t)$ and $d(t)$ respectively so that the terms (*) in (2) can somewhat compensate the uncertain terms (.) in the left-hand side of the system's dynamic equation (1). Of course, extreme values of $r(t)$ and $d(t)$ may be used to yield more conservative results. Because local high gain can compensate not only the nonlinearities but also some errors in the estimate of uncertainty and disturbance, good tracking results may be obtained by using average values to approximate uncertainties and disturbances if the range of variation of parameters is relatively small. However, if the effect of time variation of such parameters (uncertainty and disturbance) is relatively large, a good choice for the estimated values of uncertainties and disturbance could be chosen as, instead of constant nominal values, a time-dependent function so that the compensation terms $H(*)$ and $D(*)$ become large as needed and stay small otherwise. The state variable description form of the system and the corresponding controller design will be shown in the next section.

## STATE EQUATION FORM AND CORRESPONDING CONTROLLER

For convenience, we can rewrite the general nonlinear system (1) in the state equation form. By letting $x_{i1} = x_i$ and $x_{i2} = x'_i$, the system (l) can be written as

$$x'_{i1} = x_{i2}$$

$$x'_{i2} = -\frac{H_i(\cdot)}{D_i(\cdot)} + \frac{1}{D_i(\cdot)} U_i(*)$$

$$i = 1, 2, 3 \ldots n$$

Where $n$ represents the $n$ generalized coordinates. Thus, in general, we obtain $2n$ equations.

$$x'_{i1} = x_{i2} \qquad C_i(\cdot) = \frac{H_i(\cdot)}{D_i(\cdot)} \quad E_i(\cdot) = \frac{1}{D_i(\cdot)}$$

$x'_{i2} = -C_i(.) + E_i(.)U_i(*)$,

where $(.) = (x(t), x'(t), r(t), d(t), t)$

$i = 1, 2, 3 \ldots n, \qquad (*) = (x'(t), x(t), t)$

$x^T = [x_{11}, x_{12}, x_{21}, x_{22}, \ldots x_{n1}, x_{n2}]$

By defining the tracking error as

$$e_i(t) = x_i(t) - x_{ri}(t) = x_{i1} - x_{ri1}$$

$$e'_i(t) = x'_i(t) - x'_{ri}(t) = x_{i2} - x_{ri2}$$

We can rearrange the proposed controller in the state space form:

$$U_i(*) = \frac{1}{E_i(*)}[-(K_D(x_{i2} - x_{ri2}) + K_P(x_{i1} - x_{ri1}) + K_I\int e_i(t)\,dt - C_i(*) + x'_{ri2} \tag{3}$$

Where $x_{ri1}$, $x_{ri2}$ and $x'_{ri2}$ are the desired motions of the corresponding states.

**Proof of the Validity of the Proposed Controller**

First, let us investigate the situation in which the variations of uncertainty and disturbance are negligible in the system. The validity of the controller in (2) can be shown by simply putting (2) into (1), which yields

$$e'''_i + K_D e''_i + K_P e'_i + K_I e_i = 0$$

$$\Rightarrow m^3 + K_D m^2 + K_P m + K_I$$

$$\equiv (m + a)(m^2 + bm + c)$$

$$= m^3 + (a + b)m^2 + (ab + c)m + ac = 0 \tag{4}$$

For $(m + a)(m^2 + bm + c) = 0$

$$\Rightarrow m = -a, -b \pm \sqrt{b^2 - 4c}$$

For $b^2 \geq 4c$,

$$e_i(t) = Ae^{-at} + B_1 e^{(-b + \sqrt{b^2 - 4c})t} + B_2 e^{(-b - \sqrt{b^2 - 4c})t}$$

For $b^2 < 4c$,

$$e_i(t) = Ae^{-at} + Be^{-bt}\sin(\sqrt{4c - b^2}\,t - \phi)$$

The parameters $a$, $b$ and $c$ can now be selected to make the tracking errors decay to zero at desired rate so that the corresponding gains can be chosen to make the closed-loop system asymptotically stable. In other words, the desired motion can be tracked in finite time for this special case. However, the variation of loads or disturbance is not negligible in general. If the variations of uncertain load and/or disturbance are large, the difference equation will have the characteristics of a forced vibratory motion instead of free vibration as in (4) because the estimated nominal values of uncertainties and disturbance may have a larger deviation from actual values [3]. In this situation, the difference equation can be obtained by a similar computation.

By putting (2) into (1), we get

$$D_i(*)x''_i + H_i(*) = U_i(*)$$

$$= D_i(*)x''_{ri} + D_i(*)[-(K_D e'_i(t) + K_P e_i(t) + K_I\int e_i(t)\,dt] + H_i(*)$$

*and letting*

$$k_3(t) = (D_i(\cdot) - D_i(*))x''_i(t)$$

$$k_4(t) = H_i(.) - H_i(*)$$

$$k(t) = -D_i^{-1}(*)(k_3(t) + k_4(t)) \tag{5}$$

The equation (5) can be expressed as

$$D_i(*)[e''_i(t) + K_D e'_i(t) + K_P e_i(t) + K_I\int e_i(t)\,dt] + k_3 + k_4 = 0$$

Which can rewritten as

$$e''_i + K_D e'_i + K_I\int e_i dt$$

$$= -D_i^{-1}(*)(k_3(t) + k_4(t))$$

$$= k(t) \tag{6}$$

Hence the resulting differential equation can be viewed as a typical forced vibration with $k(t)$ as its input force. From previous assumptions, both of $k_3(t)$ and $k_4(t)$ are bounded. Thus, the value of $k(t)$ must be bounded by a certain constant [3], say, $k$, i.e.

$$||k(t)|| < k$$

Note that since the value of $k(t)$, caused by estimated errors, can be minimized to make it small by using the fuzzy nonlinear compensating terms shown in the proposed controller, the upper-bound of $k$ is also a small constant. Now we can investigate the differential equation with constant input $k$ i.e.

$$e_i''' + K_D e_i'' + K_P e_i' + K_I e_i = \frac{d}{dt} k(t)$$

By using a similar notation in equation (4), we get: For $b^2 \geq 4c$ case, the homogeneous solution is

$$e_i(t) = Ae^{-at} + B_1 e^{(-b + \sqrt{b^2 - 4c})t} + B_2 e^{(-b + \sqrt{b^2 - 4c})t}$$

And the complete solution can be expressed as $e_i(t) = Ae^{-at} + Be^{(-b \pm \sqrt{b^2 - 4c})t} + \varepsilon_i k$, where $\varepsilon_i k$ represents the particular solution of error equation.

For $b^2 < 4c$ case, the homogeneous and complete solution will be

$$e_i(t) = Ae^{-at} + Be^{-bt}\sin(\sqrt{4c - b^2}\, t - \phi)$$

And the complete solution can be expressed as

$$e_i(t) = Ae^{-at} + Be^{-bt}\sin(\sqrt{4c - b^2}\, t - \phi) + \varepsilon_i k$$

Thus, the gains can readily be chosen to make a finite steady state error within a value $\beta = \varepsilon_i k$

Now, if the input is a sinusoidal input with amplitude $F_{oi}$, the error dynamic equation will be

$$e_i''' + K_D e_i'' + K_P e_i' + K_I e_i = \frac{dF_{oi}\sin \omega t}{dt}$$

Or $e_i''' + K_D e_i'' + K_P e_i' + K_I e_i = \omega F_{oi} \cos \omega t$ (7)

Then, for the cases of $b^2 - 4c < 0$ and $b^2 - 4c \geq 0$, the tracking errors can respectively expressed as [3]:

$$e_i(t) = \frac{F_{oi}}{\sqrt{(ac(\omega - \omega^3(a + b))^2 + (\omega^4 - (ab + c)\,\omega^2)^2}} \cos(\omega t - \phi_{i2}) + Ae^{-at} + Be^{-bt}\sin(\sqrt{4c - b^2}\, t - \phi_{i1}) \quad (8)$$

And

$$e_i(t) = \frac{F_{oi}}{\sqrt{(ac\omega - (a + b)\,\omega^3)^2 + (\omega^4 - (ab + c)\,\omega^2)^2}} \cos(\omega t - \phi_i) + Ae^{-at} + B_1 e^{(-b + \sqrt{b^2 - 4c})t} + B_2 e^{(-b - \sqrt{b^2 - 4c})t} \quad (9)$$

Where $F_{oi}$ is a positive real number, $\phi_{i1}$, $\phi_{i2}$ and $\phi_{i1}$ are lag angles; $A$, B, $A$, $B_1$ and $B_2$ are determined by the initial conditions.

The above solutions reveal that the error will finally be bounded within accuracy $\beta$, where $\beta$ is defined as

$$\beta_i = \left| \frac{F_{oi}}{\sqrt{(ac\omega - (a + b)\,\omega^3)^2 + (\omega^4 - (ab + c)\,\omega^2)^2}} \right| \quad (10)$$

Since the value of $F_{oi}$ contributes part of the amplitude of a sinusoidal function in (7), which results from can be minimized to make it small by using the proposed fuzzy nonlinear compensator proposed in this paper, we can conclude that the proposed controller guarantees that the desired motion can be tracked to within calculable accuracy $\beta$, and tracking with arbitrarily small error can be achieved.

As for the choice of gain values, the controller gains can readily be selected by following the above formulation. For example, from equation (4), the controller gains can be selected as $K_D = a + b$, $K_P = ab + c$ and $K_I = ac$ respectively. The pre-specified tracking precision can then be achieved with the above predictable bounds.

## DESIGN OF FUZZY NONLINEAR ROBUST COMPENSATOR

Before proposing a fuzzy-typed controller to make the proposed controller of equation (2) more flexible, we would like to mention some fuzzy logics and theorems to be used in our controller- designed process later. During the past decade, fuzzy logic control has emerged as one of the most active and fruitful areas for research the compensating errors of the plant nonlinear terms, it in the application of fuzzy set theory, fuzzy logic and fuzzy reasoning [8, 9, 12]. Since fuzzy reasoning can be done in linguistic ways, which can effectively simply the complexity in compensating system dynamics to make the robust controller design process easier, especially for nonlinear and ill-defined systems like submarines, we will use the fuzzy logic to compensate the uncertain variation, which may be caused by the variation of uncertain system parameters or un-modeled dynamics, appearing in the nonlinear terms of a nonlinear system to form a robust controller for nonlinear systems in the paper. The basic operation of a fuzzy set can be illustrated as follows [8]:

(a) Fuzzy set:

A fuzzy set A can be expressed as: [8]
when $U$ (the universe of discourse) is discrete, a fuzzy set $A$ can be represented as

$$A = \frac{\mu_A(\chi_1)}{\chi_1} + \frac{\mu_A(\chi_2)}{\chi_2} + \cdots + \frac{\mu_A(\chi_n)}{\chi_n} \quad (11)$$

Where $\frac{\mu_A(\chi_i)}{\chi_i}$ represents the relationship between the generic element $\chi_i$ of $U$ and its grade of membership $\mu_A(\chi_i)$.

(b) Fuzzy Intersection:

The membership function $\mu_c(x)$ of the intersection $A \cap B$ is defined for all $\mu \in U$ by

$$\mu_C(\chi) = \min\{\mu_A(\chi), \mu_B(\chi)\} = \mu_A(\chi) \wedge \mu_B(\chi) \quad (12)$$

(c) Fuzzy Union:

The membership function $\mu_c(x)$ of the union $A \cup B$ is defined for all  by

$$\mu_C(\chi) = \max\{\mu_A(\chi), \mu_B(\chi)\} = \mu_A(\chi) \vee \mu_B(\chi) \quad (13)$$

(d) Fuzzy Complement:

The membership function $\mu_{\overline{A}}(x)$ of the complement of a fuzzy set $A$ is defined for all $\mu \in U$ by

$$U_{\overline{A}}(x) = 1 - \mu_A(\chi) \quad (14)$$

(e) Fuzzy Relation:

If $A$ and $B$ are fuzzy relation in $X * Y$ and $Y * Z$, respectively, the composition of $A$ and $B$ is a fuzzy relation denoted by $A \circ B$ and the membership function $\mu_c(x, z)$ of the composition $A$ and $B$ is defined by

$$\mu_C(x, z) = \mu_{A \circ B}(x, z) = \sup\{\min[\mu_A(x, y), \mu_B(y, z)]\}$$

$$\text{Or } c_{ij} = \vee_k\{a_{ik} \wedge b_{kj}\} \quad (15)$$

Based on the above fuzzy operation concepts, the basic configuration of a fuzzy logic controller (FLC) is proposed and shown in Figure 1, which comprises four principal components: A fuzzification interface, a knowledge base, an inference engine and a defuzzification interface. The main functions of these four components can be described as follows:

(1) The fuzzification interface involves the following functions:
  (a) It receives the state variables from the plant.
  (b) It transfers the range of values of input variables into corresponding universes of discourse.
  (c) It performs the function of fuzzification that converts input data into suitable linguistic values.

(2) The knowledge base consists of a “data base” and a “linguistic control rule  base”:
  1. The database provides necessary definitions, which are used to define linguistic control rules and fuzzy data manipulation in an FLC.
  2. The rule base characterizes the control policy and control goals of the domain experts by means of a set of linguistic control rules
  3. The inference engine is the most important kernel and it is the decision-making center of a FLC, which is designed by simulating human thinking model

(3) The defuzzification interface performs the following functions:
  (a) It yields a non-fuzzy control action from an inferred fuzzy control action.
  (b) It converts the range of values of output variables into corresponding universes of discourse.

Fuzzification is related to the vagueness and imprecision in a natural language.  It is a subjective valuation to transform measurement data into valuation of a subjective value.  Hence it can be defined as a mapping from an observed input apace to labels of fuzzy sets in a specified input universe of discourse.  Since the data manipulation in a FLC is based on fuzzy set theory, fuzzification is necessary and desirable at an early stage.  In fuzzy control applications, the observed data are usually crisp.  A natural and simple fuzzification approach is to convert a crisp value $X_0$ into a fuzzy singleton $A$ within the specified universe of discourse. That is, the membership function $\mu_A(x)$ of $A$ is equal to 1 at the point $X_0$ as zero at other places.

A fuzzy system is characterized by a set of linguistic statements based on expert knowledge.  The expert knowledge is usually as “if-then” rules, which are easily implemented by fuzzy conditional statements in fuzzy logic.  Fuzzy control rules have the form of fuzzy conditional statements that relate the state variables in the antecedent and process control variables in the consequence.  Many experts have found that fuzzy control rules provide a convenient way to express their domain knowledge.  This explains why most FLC are based on the knowledge and experience that are expressed in the language of fuzzy “if- then” rules.  The general form of the fuzzy control rules in the case of two-input single-output systems is:

IF $x$ is $A_1$ and $y$ is $B_1$ THEN $z$ is $C_1$
IF $x$ is $A_2$ and $y$ is $B_2$ THEN $z$ is $C_2$
…
IF $x$ is $A_n$ and $y$ is $B_n$ THEN $z$ is $C_n$ (16)

Where $x$, $y$ and $z$ are linguistic variables representing the process state variable and control variable, respectively?  $A_n$, $B_n$ and $C_n$ are the linguistic values of the linguistic variables $x$, $y$ and $z$ in the universe of discourse $U$, $V$, and $W$.  In what follows, we consider some useful properties of the FLC inference engine [8, 9, 12].

**[Theorem 1]**

$$(A', B') \circ \bigcup_{i=1}^{n} R_i \Rightarrow \mu_{C'}(z) = (\mu_{A'}(x), \mu_{B'}(y))$$

$$\circ \max_{x,y,z}(\mu_{R_1}(x,y,z), \cdots, \mu_{R_n}(x,y,z))$$

$$= \sup_{x,y}\max_{x,y,z}\{\min\left[(\mu_{A'}(x), \mu_{B'}(y)), \mu_{R_1}(x,y,z)\right], \cdots,$$

$$\min\left[(\mu_{A'}(x), \mu_{B'}(y)), \mu_{R_n}(x,y,z)\right]\}$$

$$= \max_{x,y,z}\{[(\mu_{A'}(x), \mu_{B'}(y)) \circ \mu_{R_1}(x,y,z)], \cdots,$$

$$[(\mu_{A'}(x), \mu_{B'}(y)) \circ \mu_{R_n}(x,y,z)]\}$$

$$\Rightarrow = \bigcup_{i=1}^{n} C'_i = \bigcup_{i=1}^{n} (A', B') \circ R_i \tag{17}$$

**[Theorem 2]**

For the intersection operation of fuzzy sets, the minimum and the product methods are formulated as follows:

If $\mu_{A_i \times B_i} = \mu_{A_i} \wedge \mu_{B_i}$ then

$$(A', B') \circ (A_i \text{ and } B_i \Rightarrow C_i) = [A' \circ (A_i) \Rightarrow C_i]$$

$$\cap\ [B' \circ (B_i \Rightarrow C_i)]$$

If $\mu_{A_i \times B_i} = \mu_{A_i} \wedge \mu_{B_i}$ then

$$(A', B') \circ (A_i \text{ and } B_i \Rightarrow C_i) = [A' \circ (A_i \Rightarrow C_i]$$

$$\times\ [B' \circ (B_i \Rightarrow C_i)] \tag{18}$$

The above two formulas implies that we need to make a combination of the membership function operation and the logic operation. Because $A_i$ and $B_i \Rightarrow C_i$ is not easy to be operated, we partition it into two parts and evaluate them separately.

**[Theorem 3]**

If the inputs are fuzzy singletons, namely, $A' = x_0$, $B' = y_0$, based on the minimum operation and the product operation rules, we have the following four different operations:

$$\alpha_i^{\wedge} \wedge \mu C_i(z)$$

$$\alpha_i^{\wedge} \bullet \mu C_i(z) \qquad \alpha_i^{\wedge} = \mu_{A_i}(x_0) \wedge \mu_{B_i}(y_0)$$

Where (19)

$$\alpha_i^{\bullet} \wedge \mu C_i(z) \qquad \alpha_i^{\bullet} = \mu_{A_i}(x_0) \bullet \mu_{B_i}(y_0)$$

$$\alpha_i^{\bullet} \bullet \mu C_i(z)$$

The above theorems explain the process of fuzzy inference. Fig. 2 gives a graphic interpretation of theorem 3 in terms of minimum operation rule, while Fig. 3 offers a graphic interpretation of theorem 3 in terms of product operation rule.

Basically, defuzzificaton is a mapping from a space of fuzzy control actions defined over an output universe of discourse into a space of non-fuzzy control actions. It is employed because a crisp control action is required in many practical applications. At present, the commonly

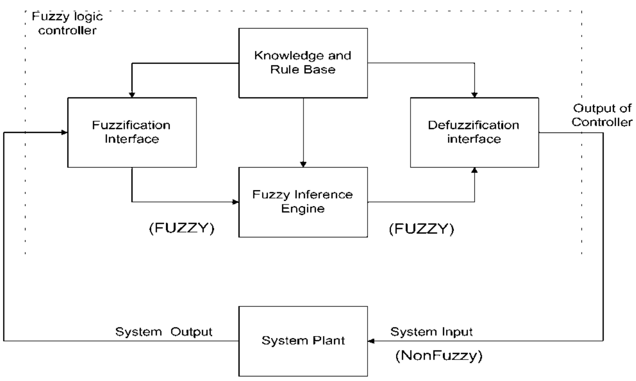


**Fig. 1. Basic configuration of fuzzy logic controller.**

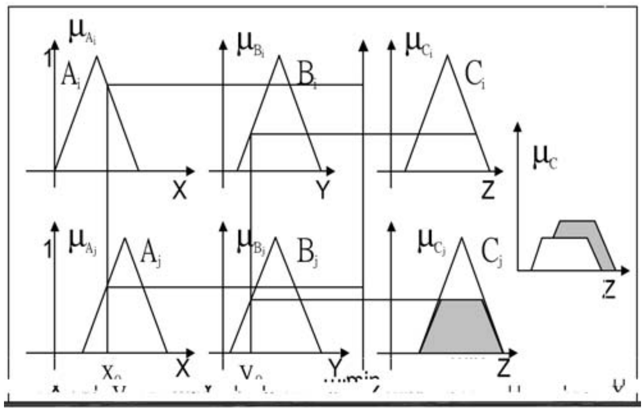

**Fig. 2. Graphical interpretation of fuzzy inference under minimum rule.**

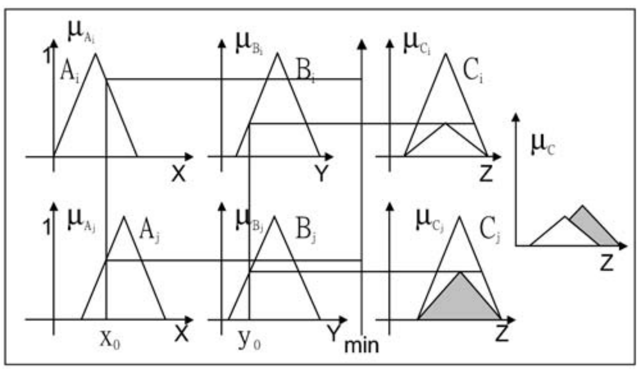

**Fig. 3. Graphical interpretation of fuzzy inference under product rule.**

used defuzzification strategies may be described by the method of the center of area or the mean of maximum [10]. The graphic interpretation of defuzzification on the general output of fuzzy controller is described in Fig. 4.

**Construction of Fuzzy Nonlinear Robust Compensator:**

For a nonlinear system shown in equation (1), the uncertainty $r$(t) and $d(t)$ are assumed to be bounded by some known constants, the mean of upper- and lower-bounds are then taken as reasonable estimates of $r(t)$ and $d(t)$ respectively so that the terms (*) in controller $U$ can then somewhat compensate the uncertain terms (.) in the left-hand side of the system's dynamic equation (1). Thus, the above controller is a nominal value compensated controller and normally good tracking results can be obtained by using average values to approximate uncertainties and disturbances if the range of variation of parameters is relatively small. However, if the effect of time variation of such parameters (uncertainty and disturbance) is relatively large, a fuzzy-based compensation term can be designed to estimate the values of uncertainties and disturbance. That is, instead of constant nominal values in the compensation terms, a fuzzy time-dependent function is used to make the compensation terms $H(*)$ and $D(*)$ become large as needed and stay small otherwise. Then following the fuzzification, fuzzy inference engine and defuzzification logic operations discussed in the previous section, we can now revise the controller proposed in equation (2) to the following form:

$$U(*) = -[fuzzy\ D_i(*)]\,(K_D e_i'(t) + K_P e_i(t) + K_I \int e_i(t)\,dt + [fuzzy\ H_i(*)] + [fuzzy\ D_i(*)]\,x_{ri}'' \tag{20}$$

Where the terms [*fuzzy* $D_i(*)$] and [*fuzzy* $H_i(*)$] denote the fuzzy-adjusted compensated terms, which are the corrected values of the compensated terms $D_i(*)$ and $H_i(*)$ of the controller proposed in equation (2) and the corrections are obtained from the fuzzification, fuzzy inference engine and defuzzification processes of the matching errors between $H_i(*)$ and $H_i(.)$ as well as between $D_i(*)$ and $D_i(.)$ as shown in equation (1) respectively. The fuzzy processes have been discussed in the previous section. The above fuzzy nonlinear controller is formulated in the form to apply the fuzzy logic to make the compensated terms of the proposed controller more flexible. The design procedure can be described as follows:

**Step 1:** Obtain the model of the nonlinear uncertain plant.
**Step 2:** Check the matching conditions on assumptions.
**Step 3:** Use the mean-values of uncertainties to form the compensation terms $H(*)$ and $D(*)$, which are set as the initial values of the terms $D_i(*)$ and $D_i(.)$ in the controller.
**Step 4:** Evaluate the matching errors between $H_i(*)$ and $H_i(.)$ as well as those between [*fuzzy* $D_i(*)$] and [*fuzzy* $H_i(*)$]. These errors are treated as fuzzy sets, which are operated by the fuzzification, fuzzy inference engine and defuzzification processes described in equations (11)-(19).
**Step 5:** Obtain the fuzzy - adjusted compensation terms, and , which are the modified values attained by compensating the matching errors.
**Step 6:** If the desired tracking result is achieved, go to step 7, or else repeat steps 4-5.
**Step 7:** Form the nonlinear composite compensator of the system.

A tip for designing the proposed fuzzy-based nonlinear controller containing a fuzzy-based compensated term is described as follows: The compensating terms of the above controller use the mean-values of uncertainties to represent the system time-varying uncertainties at the first trial and then instead of sticking to the mean values, the proposed controller utilizes the fuzzy logics for checking the matching errors to update its nonlinear compensating terms so that good tracking results can be maintained under the presence of disturbances or plant uncertainties. Since the uncertainty $r(t)$ and $d(t)$ are assumed to be bounded from some constraints, the mean of upper and lower bounds may be taken as reasonable estimates of $r(t)$ and $d(t)$ respectively so that the fuzzy nonlinear terms of the controller can somewhat compensate the corresponding terms of the nonlinear systems when $r(t)$ and $d(t)$ varies from their mean values in a real system. Because local high gains can compensate not only nonlinearities but also some errors in the estimation of uncertainties and disturbances, the con-

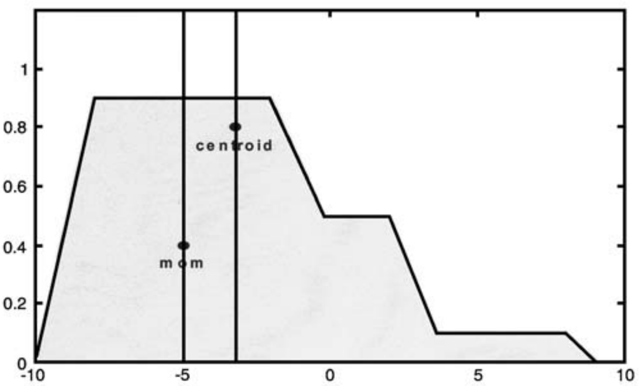


**Fig. 4. The general output of fuzzy controller.**

troller gains can be chosen to be higher to achieve better system robustness as long as the saturation condition of control energy is not violated.

## COMPUTER SIMULATION

To illustrate the application of the proposed non-linear controller, we would like to consider one of the water vehicles whose models are available in literatures [1, 2, 18] as an example and let us consider the submarine shown in Figure 5, which has the following dynamics [1]:

$$\dot{w}(t) = \frac{Z'_w U}{Lm'_3} w(t) + \frac{1}{m'_3}(Z'_{\dot\theta} + m')U\dot\theta(t) + \frac{Z'_{\ddot\theta}}{m'_3}\ddot\theta(t) + \frac{Z'_{\delta S}U^2}{m'_3 L}\delta B(t) + \frac{Z'_{\delta S}U^2}{m'_3 L}\delta S(t) + \frac{2}{\rho L^3 m'_3} Z_{wave}(t) + W_e(t)\cos\theta\cos\phi\frac{2}{\rho L^3 m'_3}$$

$$\ddot\theta(t) = \frac{M'_{\dot w}}{LI'_2}\dot w(t) + \frac{M'_w U}{L^2 I'_2} w(t) + \frac{M'_{\dot\theta}}{LI'_2}\dot\theta(t) + \frac{M'_{\delta B}U^2}{L^2 I'_2}\delta B(t) + \frac{M'_{\delta S}U^2}{L^2 I'_2}\delta S(t) + \frac{2mg(z_G - z_B)}{\rho L^5 I'_2}\theta(t) + \frac{M_{wave}}{\rho/_{2L^5 I'_2}}$$

Where $I'_2 = I_y - M'_{\ddot\theta}$ , $m'_3 = m' - Z'_{\dot w}$ and $m' = 2m / \rho L^3$. The numerical values of the above parameters are: [1]

$Z'_w = -0.0110$ $Z'_{\dot w} = -0.0075$ $Z'_{\dot\theta} = -0.0045$

$Z'_{\ddot\theta} = -0.0002$ $Z'_{\delta B} = -0.0025$

$M'_w = 0.0030$ $M'_{\dot w} = -0.0002$ $M'_{\dot\theta} = -0.0025$

$M'_{\ddot\theta} = -0.0004$ $M'_{\delta B} = 0.0005$

$Z'_{\delta S} = -0.0050$ $M'_{\delta S} = -0.0025$ $L = 286ft$

$C_{M1} = 0.35$ $C_{Z2} = 077$

$C_{Z1} = 1.28$ $U = 8.43 ft/\sec$ $\rho = 2.0^{slugs}/_{ft^3}$

$m = 1.52 \times 10^5$ $I_y = 5.68677 \times 10^{-4}$ $\nabla = 7.6 \times 10^4$

$z_B = 0.005$ $z_G = 0.0025$

The system dynamics of the submarine can then be expressed as [1]

$$\dot\theta(t) = 3.372 \times 10^{-4} w(t) - 7.713 \times 10^{-2}\dot\theta(t) + 4.48182 \times 10^{-2}\delta B(t) - 7.9592 \times 10^{-2}\delta S(t) - 2.7 \times 10^{-5}\theta(t) - 2.2 \times 10^{-9} M_{wave}(t) + 3.06 \times 10^{-6} Z_{wave} + 9.8 \times 10^{-5} Me(t)\cos\theta(t)\cos\phi$$

$$\dot\theta(t) = 3.372 \times 10^{-4} w(t) - 7.713 \times 10^{-2}\dot\theta(t) + 4.48182 \times 10^{-4}\delta B(t) - 2.1816 \times 10^{-3}\delta S(t) - 6.625 \times 10^{-6}\theta(t) - 7.14 \times 10^{-8} Me(t) + 5.42 \times 10^{-10} M_{wave}(t) - 2.2 \times 10^{-9} Z_{wave}(t)$$

To construct the state equation, we define the state variables as follows: $x_1(t) = r(t)$, $\dot x_1(t) = x_3(t) = w(t)$, $x_2(t) = \theta(t)$ and $\dot x_2(t) = x_4(t)$. The controller becomes

$$u_1(t) = -4.61 \times 10^{-2}\delta B(t) - 7.952 \times 10^{-2}\delta S(t)$$

$$u_2(t) = -4.8182 \times 10^{-4}\delta B(t) - 2.1816 \times 10^{-3}\delta S(t).$$

Where the hydroplane angles, are shown in figure 1, in which the notations ($\delta B(t)$ and $\delta S(t)$) denote the hydroplane angles, $\theta(t)$ represents the pitch angle, $h(t)$ is the depth variation, $H$ gives the ordered depth, $U(t)$, $W(t)$ and $V(t)$ represent the speed along the ship $x$-axis, the ship $z$-axis and the sea surface elevation respectively. The relation between the hydroplane angles and torques are: [1]

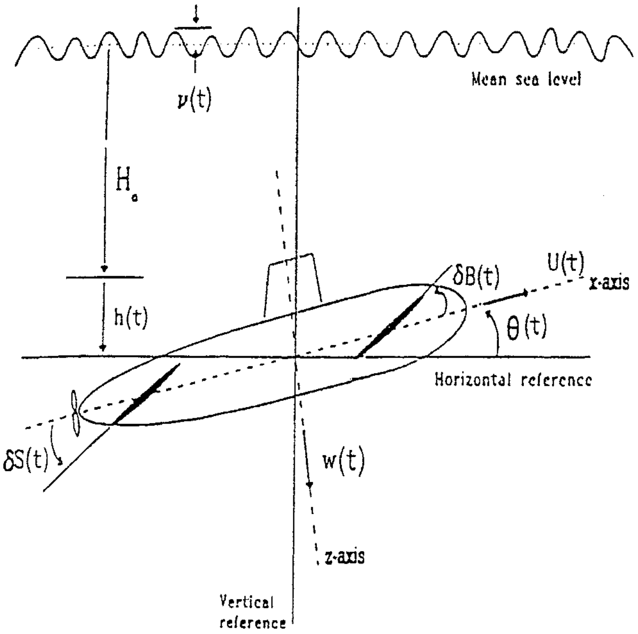


**Fig. 5. Direction and depth control of a submarine.**

$$T_1 = J_1 \frac{d^2\delta B}{dt^2} + B_1 \frac{d\delta B}{dt}$$

$$T_2 = J_2 \frac{d^2\delta B}{dt^2} + B_2 \frac{d\delta B}{dt}$$

where $J_1 = J_2 = 1000(lb - ft - sec^2)$

$B_1 = B_2 = 0.1(lb - ft / rad / sec)$

After rearrangement, the state variable description of the submarine system can be expressed as follows:

$$\dot{x}_1 = x_3(t)$$

$$\dot{x}_2 = x_4(t)$$

$$\begin{aligned}\dot{x}_3(t) &= -2.453 \times 10^{-2}x_3(t) + 1.5174x_4(t) - 2.7 \\ &\times 10^{-5}x_2(t) - 2.2 \times 10^{-9}M_{wave}(t) + 3.06 \\ &\times 10^{-6}Z_{wave}(t)\ 9.8 \times 10^{-3}\cos x_2(t) + u_1(t)\end{aligned}$$

$$\begin{aligned}\dot{x}_4(t) &= 3.372 \times 10^{-4}x_3(t) - 7.713 \times 10^{-2}x_4(t) \\ &- 6.625 \times 10^{-6}x_2(t) - 7.14 \times 10^{-6} + 5.42 \\ &\times 10^{-10}M_{wave}(t) - 2.2 \times 10^{-9}Z_{wave}(t) + u_2(t)\end{aligned}$$

Then, the proposed controller $U(*)$ in equation (20) is now in the form of $[u_1 \quad u_2]^T$, the proposed nonlinear controller is of the following form:

$$\begin{aligned}u_1(t) = &-[K_P(x_1(t) - x_{r1}(t)) + K_D(x_3(t) - \dot{x}_{r1}(t)) \\ &+ K_I \int (x_1(t) - x_{r1})\,dt] + 2.453 \times 10^{-2}x_3(t) \\ &- 1.5174x_4(t) + 2.7 \times 10^{-5}x_1(t) \\ &+ 2.2 \times 10^{-9} fuzzy(M^0_{wave}) - 3.06 \times 10^{-6} fuzzy(Z^0_{wave}) \\ &- 9.8 \times 10^{-3}\sin x_2(t) + \ddot{x}_{r1}\end{aligned}$$

$$\begin{aligned}u_2(t) = &-[K_P(x_2(t) - x_{r2}(t)) + K_D(x_4(t) - \dot{x}_{r2}(t)) \\ &+ K_I \int (x_2(t) - x_{r2})\,dt] - 3.372 \times 10^{-4}x_3(t) \\ &+ 7.713 \times 10^{-2}x_4(t) + 6.625 \times 10^{-6} \\ &+ 7.14 \times 10^{-6} - 5.42 \times 10^{-6} fuzzy(M^0_{wave}) \\ &2.2 \times 10^{-9} fuzzy(Z^0_{wave}) + \ddot{x}_{r2}(t)\end{aligned}$$

Where $M^0_{wave}$ and $Z^0_{wave}$ represent the nominal mean values of wave torque component and the wave force component respectively. The fuzzy compensating terms $(fuzzy(M^0_{wave}), fuzzy(Z^0_{wave}))$ in the proposed controller for system disturbances $(M_{wave}(t), Z_{wave}(t))$ are initialed at their nominal values, which are used as the initial values of the $U(*)$ and then updated by the proposed fuzzy control engine with time. The rule adopted here is the fuzzy inference under product rule as shown in Figure 3 and the membership functions used here are the triangular function as shown in Figures 4. The wave height is assumed to vary from 6 ft to 12 ft at sea. Now, if the desired reference path is specified as that in figure 6, the corresponding equation of the desired motion can then be expressed as

$$z(t) = -0.01t^2 + 7t - 100(ft) \quad t = 0 - 200\text{sec}$$

$$\theta(t) = 2.175 \times 10{-}6t^{-2} + 2.18 \times 10^{-3}t(\text{rad})$$

$$t = 0 - 200\text{sec}$$

Thus, to meet the performance specified in [18], the optimal controller parameters are selected as $KD$ = 16.6, $KP$ = 16.42 and $KI$ = 12.4 and the simulation results are shown in Figures 6-14. The matching of the actual submarine path and the reference path shown in Figure 6 reveals that the submarine can well track the desired path of motion with the proposed nonlinear controller. Under the uncertainties of wave height from

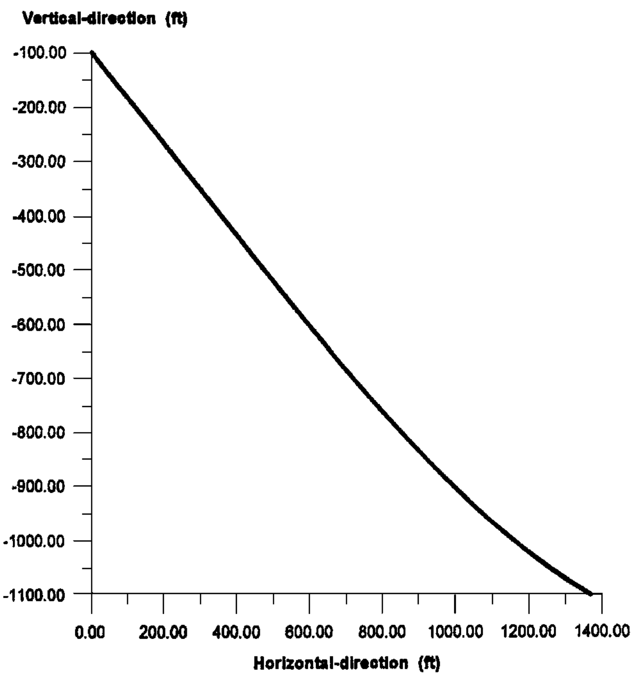


**Fig. 6. The matching of actual path and reference motion.**

6 ft to 12 ft, the submarine tracking errors, resulted from using the fuzzy-based nonlinear compensator, in horizontal and vertical directions are shown in Figure 7 and Figure 8 respectively, while on the other hand the tracking errors, followed as a result of utilizing the nominal-value compensated controller, are shown in Figures 9 and 10 for comparison. It is obvious that both the proposed fuzzy-based compensator and the nominal-value compensated controllers [3] are robust and the steady-state errors of the former one converge to a pre-specified ball of 0.02075 ft and 0.33023 ft in horizontal and in vertical directions respectively even that it encounters the mentioned large wave uncertainties of wave height from 6 ft to 12 ft. A comparison made in

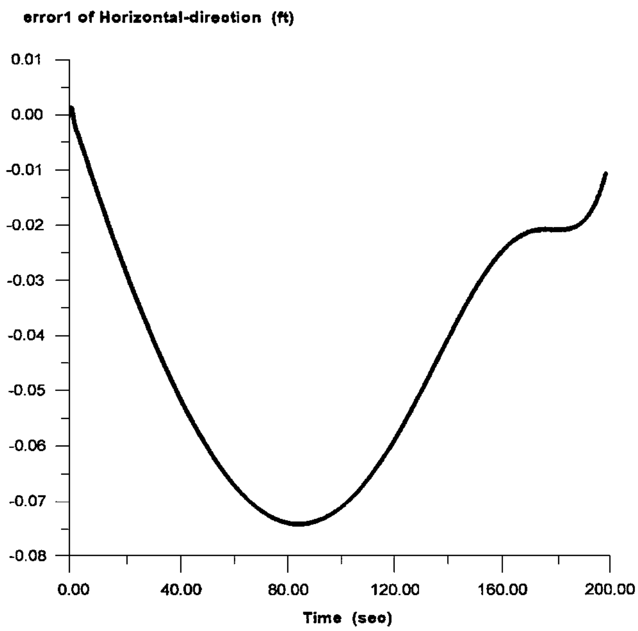


Fig. 7. Tracking errors in horizontal direction.

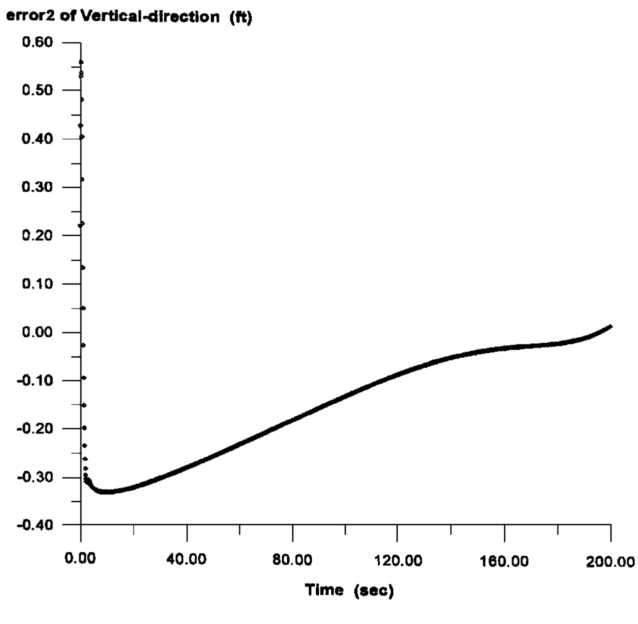


Fig. 8. Tracking error in vertical direction.

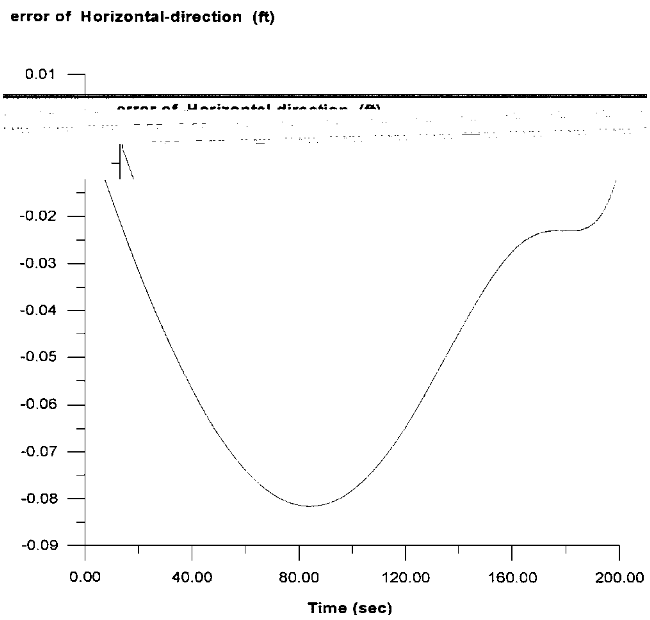


Fig. 9. Tracking error of a nominal value.

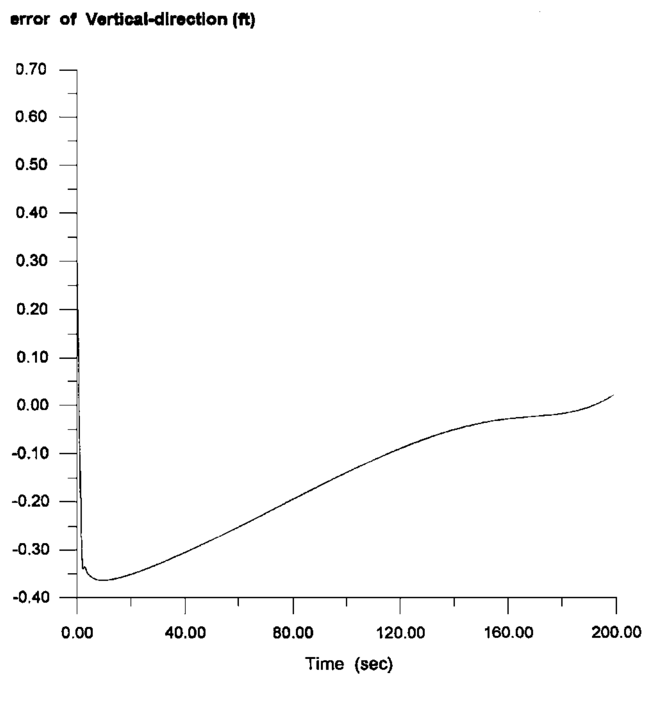


Fig. 10. Tracking error of a nominal value compensator in horizontal direction in vertical direction.

Figures 3, 5 and 4, 6 shows that maximum tracking errors made by the proposed controller are about 0.007 ft and 0.03 ft smaller than those made by the Hwang's controller [3] in horizontal and vertical directions respectively. Since the variation of the uncertainties of wave height from 6 ft to 12 ft is not large in comparison to its mean value 9 ft, the differences among these tracking errors are not quite apparent. Especially for the case having no sinusoidal disturbances and uncertainties, the two control forces will be exactly the same because there are no needs to change the initial mean-value settings in the nonlinear compensating terms of the proposed fuzzy-type compensator. Figures 11 and 12

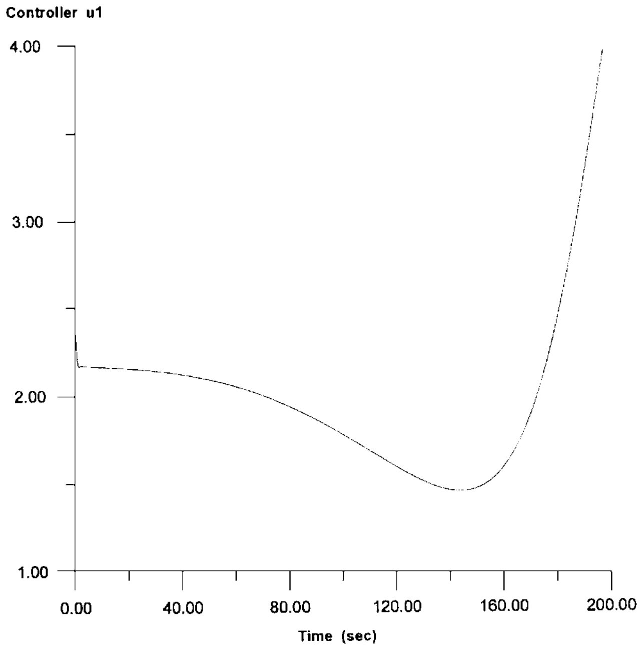


**Fig. 11. The control history of u1.**

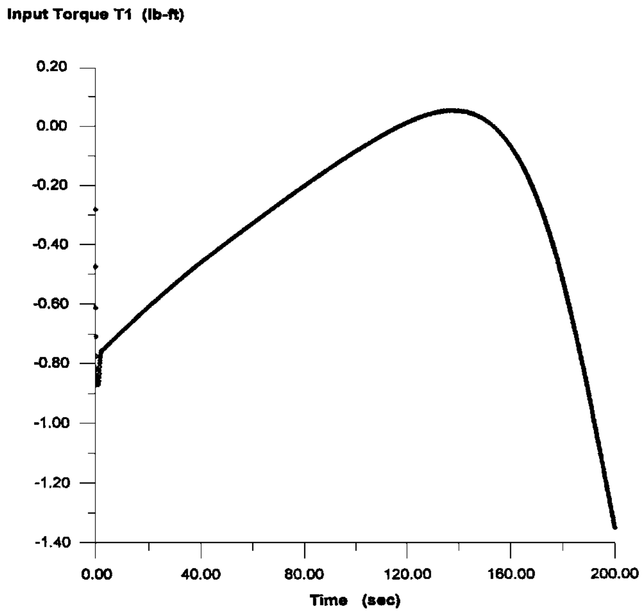


**Fig. 13. Energy history of torque T1.**

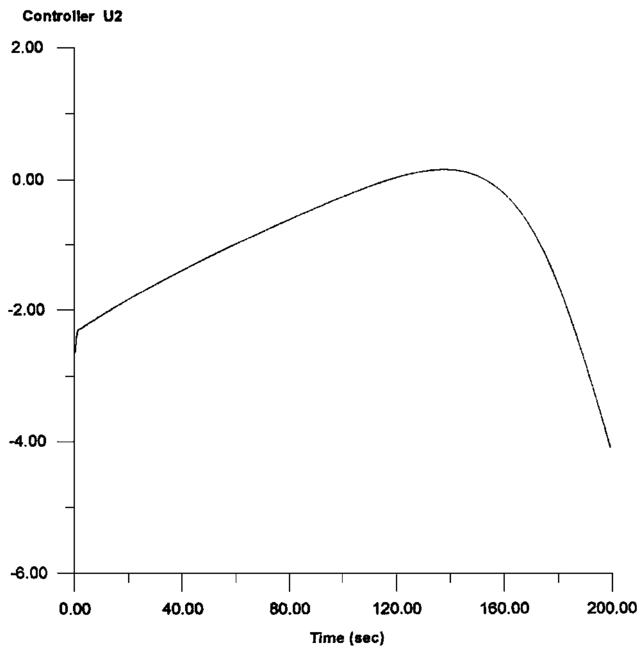


**Fig. 12. The control history of u2.**

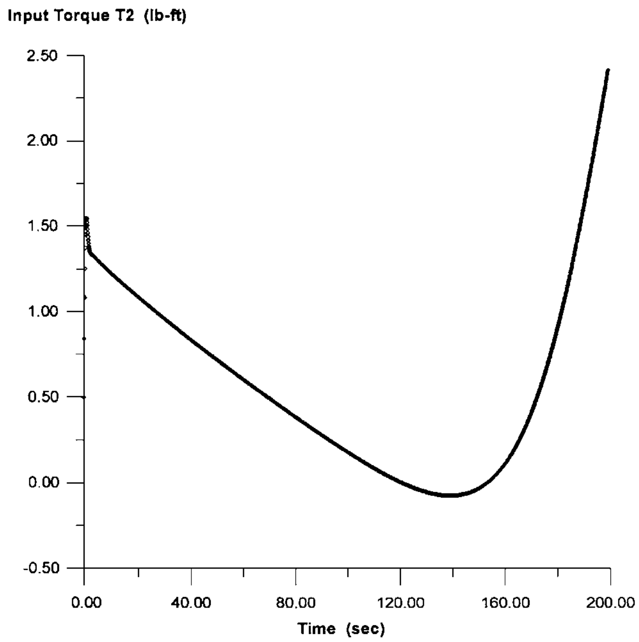


**Fig. 14. Energy history of torque T2.**

give the control histories and the actual input torques for the submarine are shown in Figures 13 and 14.

Note that the pre-compensated terms in the proposed controller are designed to compensate the corresponding nonlinear terms of the nonlinear systems to be controlled so that the tracking performance can be achieved as expected. Hence, the mean values of the uncertainties appeared in these nonlinear terms are usually used to replace the unknown yet bounded uncertain elements, e.g., disturbances, to form an explicit nonlinear controller [3]. Thus, the uncertain variation of uncertainties from the mean value must be relatively small so that the explicit controller with mean value of the unknown disturbance can perform it well. Obviously, in this case, the controller is not adjustable with similar control effort and therefore, the bounds of the uncertainties should be relatively small. However, for a real system, the uncertain disturbances may be bounded yet time-varying and hence the controller, containing mean-valued uncertainty in its nonlinear compensating term, can no longer work well. The merits of the nonlinear controller proposed in the paper is that the compensating terms of the proposed controller use the mean-values of uncertainties to represent those time-varying uncertainties in the first trial and then instead of sticking to the mean values, the proposed controller utilizes the fuzzy logics to update its nonlinear compensating terms so that good tracking results can be achieved. A comparison between the proposed controller and the nonlinear controller proposed in reference [3] are made for robot manipulator and the results shows that the tracking errors becomes smaller with similar control efforts and the same conditions of plant uncertainties. The results of the computer simulation of a submarine shown in this paper also reveal that tracking performance of the proposed controller is acceptable and the controller is feasible.

## CONCLUSIONS

In this paper, a robust nonlinear composite controller, which is simple in structure and has the characteristics of good tracking performance and guaranteed system stability, is designed. The pre-compensated terms in the proposed controller is designed to eliminate uncertainties or disturbances so that the tracking error is bounded in a pre-specified ball. If the variation in uncertainties is small, a nominal value compensated controller is suggested for its simplicity in structure, whereas a fuzzy-based time varying compensated term is needed in designing a robust controller when the uncertainty in plant is large. Therefore, a fuzzy-based nonlinear controller is simultaneously proposed in this research.

The feasibility of the proposed controllers is demonstrated by the simulation results. With the mentioned uncertainties in wave heights, the tracking precisions obtained from both the fuzzy-base compensated nonlinear controller and the nominal-compensated nonlinear controllers, are robust enough to meet the performance requirement for a submarine of 286 ft in length. However, the comparison between their simulation results also reveals that if the effect of time variation of such parameters (uncertainty and disturbance) is relatively large, a fuzzy-based nonlinear controller is a better alternative because the compensated term in the proposed fuzzy-based nonlinear controller, designed for estimating the values of uncertainties and disturbance, is time-dependent function. That is, the proposed fuzzy-based nonlinear controller can adjust the compensation terms $H(*)$ and $D(*)$ to make these terms become large as needed and stay small otherwise with the variation of uncertainties.

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