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A Case Study of S-Curve Regression Method to Project Control of Construction Management via T-S Fuzzy Model

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Short Paper

A CASE STUDY OF *S*-CURVE REGRESSION METHOD TO PROJECT CONTROL OF CONSTRUCTION MANAGEMENT VIA *T-S* FUZZY MODEL

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Key words: fuzzy *S*-curve regression, *T–S* fuzzy model, working capital management.

ABSTRACT

In the contractual business, construction firms are generally more concerned with short-term financial strategies than the long-term ones. Working capital management is the central issue of all short-term financial concerns. Thus, it's urgent to study the cash portion of working capital management to rationalize the amount of cash and current assets possessed in certain time. The *S*-curve is quite suitable to represent the relationship between project duration and complete progress in practical usage of construction management. Based on the technique of Takagi-Sugeno (*T–S*) fuzzy model, the fuzzy regression model is constructed for curve fitting problems. According to the cash flow of the example projects, this paper develops a practical S-curve regression model demonstrated and given tentative conclusions.

INTRODUCTION

The research of complex systems nowadays, such as engineering technology, environment and social economy, becomes so large in dimension and complex that the exact numerical data can not be obtained. Solving the problems caused by complex systems becomes very inefficient or even impossible if using the traditional mathematical tools not constructed for dealing with high dimensionality models. Similarly, the traditional least square regression may not be applicable when dealing with curve fitting problems. In the past twenty years, some approaches containing fuzzy information have been noticed, as proposed in the literature [2, 6, 7, 9].

Tanaka *et al*. [9] developed a fuzzy linear regression model using linear programming techniques in 1982. In 1988, Diamond [2] resembled traditional least squares regression to establish fuzzy linear least squares models. Ruoning [6, 7] considered the rationality of metric definition, discussed the problem for least squares fitting of fuzzy-value data expressed as fuzzy numbers, and developed an *S*-shaped curve regression model for fitting this type of data.

However, being large-scaled, long duration, high cost, and complex-technical, the large public construction exists many uncertain factors. Because of these factors, to perform this kind of project is difficult, especially for the dispatch of working capital. In order to overcome the difficulties of controlling projects, the *S*-curves are widely used. They are valuable to project management in reporting current status and to predict the future of projects. Consequently, the *S*-type distribution is believed to be suitable in regression on construction management, social economy and so on. However, as far as we know, the fuzzy *S*-curve regression for large public constructions via Takagi-Sugeno fuzzy model remains an open area.

This study is discussed as follows. First, classic *S*-curve theory is recalled. Then, based on fuzzy set theory and fuzzy inference engine as well as center of gravity defuzzification, a *T–S* type fuzzy *S*-curve is obtained for curve fitting problems. Finally, a numeri-

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cal example with simulations is given to demonstrate the methodology, and the conclusions are drawn.

CLASSIC *S*-CURVE THEORY

In biology and social economy, an *S*-shaped curve is often used to reflect the phenomena. It means that the trend of growth gets slow first and finally saturation rapidly. In practical problem of constructions, contractors' budgets are often performed on an overall basis. Changes in strategies and mix of contracts are very difficult to evaluate on such a basis [3]. Therefore, the principle of simulation with tools of computer was proposed to generate possible scenarios based on the specified strategies and the expected environment. The relationship between budgets and time limit for a project can be represented via *S*-curve fitting. A typical *S*-curve figure is shown in Fig. 1. The *x*-axis and y-axis denote project duration and complete progress, respectively.

Miskawi [5] proposed an *S*-curve equation which can be used in a variety of applications related to project control. The *S*-curve model is of the following form:

$$P = \frac{3^T}{2}\sin\left[\frac{\pi(1-T)}{2}\right]\sin(\pi T)\log\left(\frac{T+(1.5-T_p)}{T_p+T}\right) - 2T^3 + 3T^2 \qquad (1)$$

where P denotes percentage completion of a project or an activity; T denotes time at any point of the duration of a project or an activity; T_P is shape factor.

Fig. 2 is plotted with various values of T_P between $T = 0$ and $T = 100\%$ duration and the envelope of curves for $T_P = 5\%$ and $T_P = 95\%$ in Eq. (1).

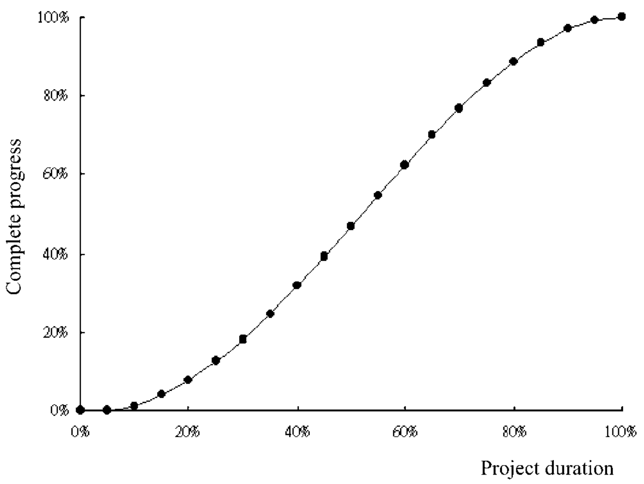

Fig. 1. Type *S*-curve figure.

Here we suppose we can exactly get all observed data taking part in the problems, but, actually, we may not know exact values, rather some approximation [7]. For this reason, the traditional fitting method may not be quite suitable. Before introducing fuzzy *S*-curve regression, we give some relative definitions and conclusions in the following.

FUZZY SET THEORY

Definition 1 [7]: Let R is a real number set. A fuzzy set $\tilde{A}$ on R is said to be a fuzzy number if the following conditions are satisfied:

(1) $\exists x_0 \in R$, such that $\mu_{\tilde{A}}(x_0) = 1$; and membership function $\mu_{\tilde{A}}(x)$ is piecewise continuous; and

(2) $\forall \alpha \in (0, 1]$, $A_\alpha \equiv \{x \mid \mu_{\tilde{A}}(x) \geq \alpha, x \in R\}$ is a convex set on R,

where x_0 is the mean value of $\tilde{A}$ and A_α is a crisp set. The convex set means that $\forall x \in [x_1, x_2]$, $\mu(x) \geq \min(\mu(x_1), \mu(x_2))$.

Definition 2: A fuzzy number $\tilde{A}$ is said to be bounded if $(\tilde{A}) \equiv \{x \mid \mu_{\tilde{A}}(x) > 0\}$ is a bounded set, where supp($\tilde{A}$) is a crisp set.

Evidently for any $\forall \alpha \in (0, \alpha]$ the α-level set, $\tilde{A}_\alpha$, will be expressed as a closed interval $[p, q]$. Based on the fuzzy extension principle [12], linear operations about closed intervals are obtained as follows:

Lemma 1 [4]: Let $[a, b]$, $[d, e]$ be closed intervals of real number. Then

$$[a, b] + [d, e] = [a + d, b + e]; [a, b] - [d, e] = [a - e, b - d], \qquad (2)$$

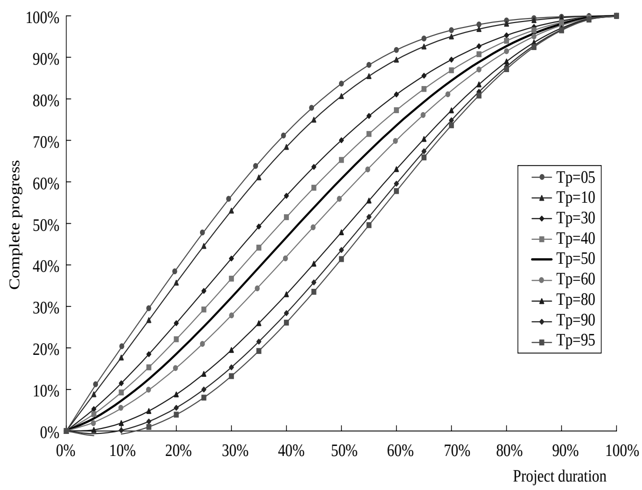

Fig. 2. Miskawi *S*-curve model.

$$[a, b] \bullet [d, e] = [\min(dd, ae, bd, be), \max(ad, ae, bd, be)]; \tag{3}$$

$$[a, b]/[d, e] = [a, b] \bullet [1/e, 1/d] = [\min(a/d, a/e, b/d, b/e), \max(a/d, a/e, b/d, b/e). \tag{4}$$

Remark 1: Given any operations which have commutative and associative characteristics, the operations of extension still have these characters.

From the theory of α-level described above and decomposition theorem [4], we have

$$(A * B)_\alpha \equiv A_\alpha * B_\alpha \tag{5}$$

$$A * B \equiv \bigcup_{\alpha \in (0,1]} (A * B)_\alpha \tag{6}$$

where * denotes any arithmetic operation; A and B are fuzzy numbers and $A * B$ will be a fuzzy number.

Remark 2: Wang and Chiu [11] proposed that the resultant fuzzy number is the same type as the original fuzzy numbers after the operation of addition, subtraction or multiplication. Namely, If A and B are the fuzzy numbers with the same type of membership function, then $A + B$, $A - B$ and $K \bullet A$, $K \in R$, are also the same type as A and B.

Because the parameterizable membership function most commonly used in practice is triangular membership function, a useful concept described below is given. In which the membership function has three parameters and in general we assume that the peak of the membership function is 1. Fig. 3 is an example of triangular fuzzy set to represent the fuzzy number with three crisp parameters.

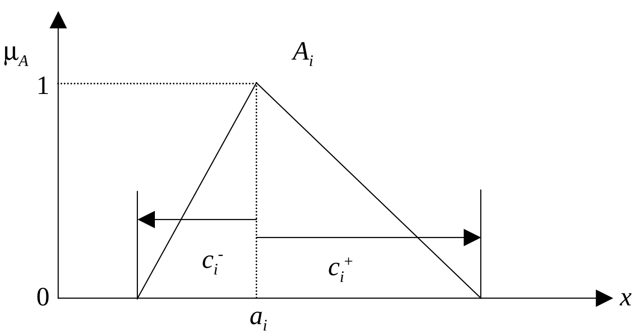

Fig. 3. Triangular fuzzy set.

Definition 3:

A fuzzy number $\tilde{A}$ is LR-type, if there exists positive constants $\beta > 0$, $\gamma > 0$ and

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{m-x}{\beta}\right) & \text{for } x \le m \\ R\left(\frac{x-m}{\gamma}\right) & \text{for } x \ge m \end{cases} \tag{7}$$

where m, a real number, is mean value of $\tilde{A}$; β, γ denote left spread and right spread, respectively and L and R are strictly nondecreasing continuous functions from [0, 1] to [0, 1] such that $L(0) = R(0) = 1$ and $L(1) = R(1) = 0$; moreover, $\tilde{A}$ could be represented as $(m, \beta, \gamma)_{LR}$. If $L = R$ and $\beta = \gamma$, then the symmetric $L - L$ fuzzy number is denoted as $(m, \beta)_L$.

Lemma 2: Given two LR-type fuzzy numbers $\tilde{A}$ and $\tilde{B}$ of the same type, we have

$$(m, \beta, \gamma)_{LR} + (n, \delta, \eta)_{LR} = (m + n, \beta + \delta, \gamma + \eta)_{LR}, \tag{8}$$

$$(m, \beta, \gamma)_{LR} - (n, \delta, \eta)_{LR} = (m - n, \beta + \eta, \gamma + \delta)_{LR}, \tag{9}$$

if $\tilde{A}$ and $\tilde{B}$ are not the same type, equations (8) and (9) would be inadequate.

In the next section, the concept of a so-called Takagi-Sugeno fuzzy model is utilized in fuzzy inference engine to establish a fuzzy *S*-curve regression model. Based on this regression model, an example in which the observed data are fuzzy numbers is given to demonstrate the proposed methodology.

FUZZY *S*-CURVE VIA *T–S* FUZZY MODEL

The *T–S* fuzzy model was developed primarily from the pioneering work of Takagi and Sugeno [8], to represent the nonlinear relation of multiple input and output data, according to the format of fuzzy reasoning. Namely, the resulting overall fuzzy regression model, nonlinear in general, is achieved by fuzzy blending of each individual input-output realization (for more detail, please see [10]). Therefore, the ith rule of fuzzy inference is described by a set of fuzzy IF-THEN rules in the following form:

R^i: IF x_1 is $\tilde{A}_1^i$, y_1 is $\tilde{B}_1^i$ and ... and x_n is $\tilde{A}_n^i$ is $\tilde{B}_n^i$

$$\text{THEN } Y = a_{ik} x_i^k + b_{ik} \tag{10}$$

where n points $(x_1, y_1) \sim (x_n, y_n)$ and k order curve fitting is adopted, R^i denotes the ith fuzzy inference rule and r

is the number of IF-THEN rules for $i = 1, 2, ..., r$; $\tilde{A}_p^i$ and $\tilde{B}_p^i$ ($p = 1, 2, ..., n$) are the LR-type fuzzy sets, and $x_1 \sim x_n$ as well as $y_1 \sim y_n$ are the premise variables. Using the center of gravity defuzzification, product inference, and single fuzzifier, the final output is inferred as follows:

$$Y = \frac{\sum_{i=1}^{r} w_i \, [a_{ak} x_i^k + b_{ik}]}{\sum_{i=1}^{r} w_i} = \sum_{i=1}^{r} h_i (a_{ik} x_i^k + b_{ik}) \qquad (11)$$

with

$$w_i \equiv \prod_{g=1}^{n} (A_g^i(x_g), B_g^i(y_g)), \; h_i \equiv w_i / \sum_{i=1}^{r} w_i \qquad (12)$$

in which $A_g^i(x_g)$ and $B_g^i(y_g)$ are the grade of membership of x_g and y_g in A_g^i and B_g^i. In this paper, it is assumed that $w_i \geq 0$, $i = 1, 2, ..., r$; $\sum_{i=1}^{r} w_i > 0$. Therefore, $h_i \geq 0$ and $\sum_{i=1}^{r} h_i > 1$.

Remark 3: The order k of the consequence part can be determined by the method of added variable plots to find a suitable regression model (for more details, see [1]). In this paper, we only consider the factor of correlation coefficient. Therefore, the order k is assigned hereafter to have a higher value of correlation coefficient.

EXAMPLE

To illustrate the procedure of this fuzzy regression model, refer to the following example project taken from Department of Rapid Transit Systems, Taipei City Government. The mean scale and duration of six metro bids data are 2.7 billions and 6 years or so. The data are normalized and transformed into the rate of percentage shown as Table 1. The first time of evaluation is 4.5% of total duration.

According to the data above, Figs. 3-5 will be discussed as follows. Fig. 3 shows the triangular fuzzy set to represent the fuzzy number with three crisp parameters. Hence, the expression $A_i = (a_i, c_i^+, c_i^-)$ stands for a triangular fuzzy number hereafter. In Table 1, the data X_i, Y_i are the model value $\tilde{X}$, $\tilde{Y}$ where $\tilde{X} = (X_i, u_i, v_i)$ and $\tilde{Y} = (Y_i, r_i, s_i)$ are all triangular fuzzy numbers and $u_i = 10$ %, $v_i = 10$ %, $r_i = 10$ %, and $s_i = 10$% are the left and right spreads, respectively. Fig. 4 is the valuation data of six metro bids and Fig. 5 is plotted by the technique of the proposed fuzzy regression method in Eqs. (10-11). According to Fig. 5, the simulation shows the results: $y = -1293x^6 + 4207x^5 - 49.49x^4 + 23.07x^3 - 1.98x^2 + 0.25x$ [x denotes the completive percentage (%), and y is the time of duration (%)] and it shows the square of the correlation coefficient, R^2 is 0.94.

CONCLUSIONS

The least-squares method can usually be applied to the problems of curve fitting, but when the data are not obtained exactly, it may not be suitable. Therefore, we propose here an *S*-curve regression method for a better understanding of the issues involved. The aim is to develop a practical model for construction firms in Taiwan to rationalize the amount of cash and current assets possessed in certain time of duration.

Furthermore, Fig. 4 shows the data is under a

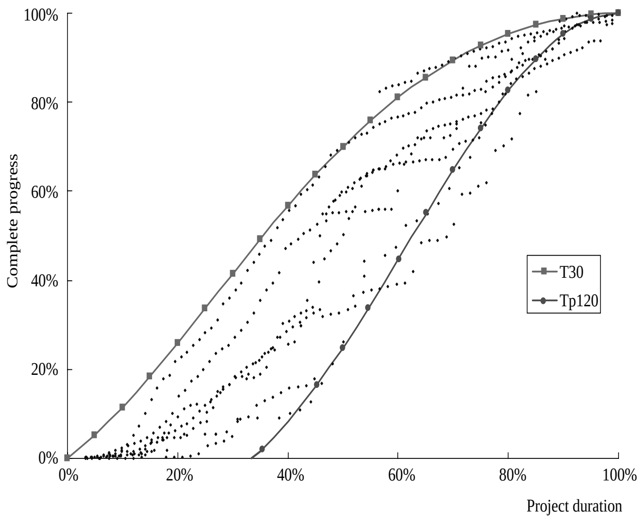

Fig. 4. The valuation data of six metro bids.

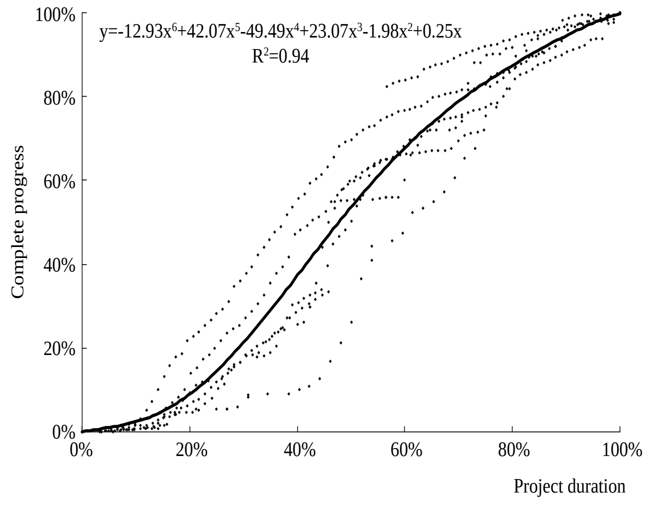

Fig. 5. An *S*-curve by fuzzy regression method.

Table 1a. The observed data of the first metro bid

Valuation times	Valuation month	Accumulative valuation total	X1	Y1
To begin construction	80/5	0	0.00%	0.00%
1	80/12	74,144,882	7.57%	1.39%
2	81/1	98,654,310	8.73%	1.85%
3	81/2	127,656,501	9.90%	2.39%
4	81/3	152,724,294	10.98%	2.86%
5	81/4	178,169,197	12.14%	3.33%
6	81/5	213,580,145	13.27%	4.00%
7	81/6	255,800,965	14.43%	4.79%
8	81/7	309,414,337	15.55%	5.79%
9	81/8	378,460,468	16.72%	7.08%
10	81/9	442,320,349	17.88%	8.28%
11	81/10	536,571,926	19.00%	10.04%
12	81/11	742,336,497	20.16%	13.89%
13	81/12	822,436,935	21.29%	15.39%
14	82/1	931,406,994	22.45%	17.43%
15	82/2	987,661,260	23.61%	18.48%
16	82/3	1,067,296,113	24.66%	19.97%
17	82/4	1,162,841,603	25.82%	21.76%
18	82/5	1,257,354,425	26.95%	23.53%
19	82/6	1,320,772,152	28.11%	24.71%
20	82/7	1,361,222,085	29.24%	25.47%
21	82/8	1,447,318,620	30.40%	27.08%
22	82/9	1,541,962,339	31.56%	28.85%
23	82/10	1,629,858,563	32.68%	30.49%
24	82/11	1,750,420,395	33.85%	32.75%
25	82/12	1,898,633,071	34.97%	35.52%
26	83/1	2,024,377,919	36.13%	37.88%
27	83/2	2,103,877,262	37.29%	39.36%
28	83/3	2,232,694,031	38.34%	41.77%
29	83/4	2,519,705,280	39.51%	47.14%
30	83/5	2,571,581,797	40.63%	48.11%
31	83/6	2,629,026,261	41.79%	49.19%
32	83/7	2,700,041,284	42.92%	50.52%
33	83/8	2,747,993,528	44.08%	51.42%
34	83/9	2,805,024,504	45.24%	52.48%
35	83/10	2,935,567,858	46.36%	54.92%
36	83/11	3,024,292,651	47.53%	56.58%
37	83/12	3,107,403,233	48.65%	58.14%
38	84/1	3,199,752,852	49.81%	59.87%
39	84/2	3,258,272,603	50.97%	60.96%
40	84/3	3,310,923,072	52.02%	61.95%
41	84/4	3,362,164,118	53.19%	62.91%
42	84/5	3,397,079,648	54.31%	63.56%
43	84/6	3,460,527,952	55.47%	64.75%
44	84/7	4,403,859,399	56.60%	82.40%
45	84/8	4,446,190,555	57.76%	83.19%
46	84/9	4,467,998,609	58.92%	83.60%
47	84/10	4,491,507,050	60.04%	84.04%
48	84/11	4,515,791,597	61.21%	84.49%
49	84/12	4,533,959,483	62.33%	84.83%
50	85/1	4,619,987,215	63.49%	86.44%
51	85/2	4,657,660,695	64.66%	87.15%
52	85/3	4,674,670,042	65.74%	87.46%
53	85/4	4,692,964,462	66.90%	87.81%
54	85/5	4,720,798,078	68.03%	88.33%
55	85/6	4,761,901,206	69.19%	89.10%
56	85/7	4,799,763,766	70.31%	89.80%
57	85/8	4,831,040,196	71.48%	90.39%
58	85/9	4,856,949,225	72.64%	90.87%
59	85/10	4,893,195,670	73.76%	91.55%
60	85/11	4,915,114,734	74.93%	91.96%
61	85/12	4,931,559,544	76.05%	92.27%
62	86/1	4,946,721,205	77.21%	92.55%
63	86/2	4,980,138,304	78.37%	93.18%
64	86/3	5,001,588,959	79.42%	93.58%
65	86/4	5,034,731,706	80.58%	94.20%
66	86/5	5,064,029,324	81.71%	94.75%
67	86/6	5,080,106,114	82.87%	95.05%
68	86/7	5,094,461,369	84.00%	95.32%
69	86/8	5,104,146,881	85.16%	95.50%
70	86/9	5,127,692,651	86.32%	95.94%
71	86/10	5,130,617,000	87.44%	95.99%
72	86/11	5,146,615,889	88.61%	96.29%
73	86/12	5,167,787,282	89.73%	96.69%
74	87/1	5,182,884,070	90.89%	96.97%
75	87/2	5,195,927,731	92.05%	97.22%
76	87/3	5,198,499,181	93.10%	97.26%
77	87/4	5,229,133,485	94.27%	97.84%
78	87/5	5,230,091,831	95.39%	97.86%
79	87/6	5,233,925,940	96.55%	97.93%
80	87/7	5,244,739,340	97.68%	98.13%
81	87/8	5,261,551,354	98.84%	98.44%
82	87/9	5,344,703,689	100.00%	100.00%

Table 1b. The observed data of the second metro bid

Valuation times	Valuation month	Accumulative valuation total	X2	Y2
To begin construction	80/10	0	0.00%	0.00%
1	81/1	1,156,545	3.64%	0.04%
2	81/2	3,728,523	4.87%	0.14%
3	81/3	7,053,156	6.02%	0.27%
4	81/4	11,640,351	7.24%	0.45%
5	81/5	14,165,122	8.43%	0.55%
6	81/6	16,202,393	9.66%	0.63%
7	81/7	19,325,192	10.84%	0.75%
8	81/8	26,313,197	12.07%	1.02%
9	81/9	33,874,345	13.30%	1.31%
10	81/10	40,747,052	14.48%	1.58%
11	81/11	45,312,485	15.71%	1.75%
12	82/1	119,181,601	18.12%	4.61%
3	82/2	120,818,877	19.35%	4.67%
14	82/3	122,879,242	20.46%	4.75%
15	82/4	133,756,647	21.69%	5.17%
16	82/5	172,640,019	22.87%	6.68%
17	82/6	210,552,786	24.10%	8.14%
18	82/7	265,195,305	25.29%	10.25%
19	82/8	294,185,718	26.51%	11.37%
20	82/9	380,119,266	27.74%	14.70%
21	83/1	459,243,280	32.57%	17.76%
22	83/2	469,477,077	33.80%	18.15%
23	83/3	487,586,298	34.90%	18.85%
24	83/4	532,094,836	36.13%	20.57%
25	83/5	644,916,728	37.32%	24.94%
26	83/6	700,195,457	38.54%	27.07%
27	83/7	738,333,537	39.73%	28.55%
28	83/8	766,818,033	40.96%	29.65%
29	83/9	792,154,251	42.18%	30.63%
30	83/10	815,399,237	43.37%	31.53%
31	83/11	846,562,020	44.60%	32.73%
32	83/12	865,807,268	45.79%	33.48%
33	84/1	1,380,442,059	47.01%	53.37%
34	84/2	1,497,462,305	48.24%	57.90%
35	84/3	1,525,953,929	49.35%	59.00%
36	84/4	1,546,538,864	50.57%	59.80%
37	84/5	1,570,538,279	51.76%	60.72%
38	84/6	1,623,168,190	52.99%	62.76%
39	84/7	1,644,431,305	54.17%	63.58%
40	84/8	1,658,358,605	55.40%	64.12%
41	84/9	1,684,287,284	56.63%	65.12%
42	84/10	1,697,113,677	57.82%	65.62%
43	84/11	1,711,041,422	59.04%	66.16%
44	84/12	1,712,435,049	60.23%	66.21%
45	85/1	1,719,528,978	61.46%	66.48%
46	85/2	1,723,221,308	62.68%	66.63%
47	85/3	1,727,736,424	63.83%	66.80%
48	85/4	1,732,329,343	65.06%	66.98%
49	85/5	1,737,137,366	66.24%	67.17%
50	85/6	1,738,489,060	67.47%	67.22%
51	85/7	1,749,472,473	68.66%	67.64%
52	85/8	1,793,540,247	69.89%	69.35%
53	85/9	1,827,170,765	71.11%	70.65%
54	85/10	1,840,254,656	72.30%	71.15%
55	85/11	1,850,614,305	73.53%	71.55%
56	85/12	1,863,435,430	74.71%	72.05%
57	86/1	2,131,536,620	75.94%	82.41%
58	86/2	2,158,399,172	77.17%	83.45%
59	86/3	2,187,280,073	78.27%	84.57%
60	86/4	2,220,208,512	79.50%	85.84%
61	86/5	2,250,661,259	80.69%	87.02%
62	86/6	2,297,727,304	81.92%	88.84%
63	86/7	2,310,868,650	83.10%	89.35%
64	86/8	2,320,802,152	84.33%	89.73%
65	86/9	2,343,720,959	85.56%	90.62%
66	86/11	2,379,705,456	87.97%	92.01%
67	86/12	2,430,838,711	89.16%	93.99%
68	87/1	2,480,724,529	90.38%	95.92%
69	87/2	2,498,827,432	91.61%	96.62%
70	87/3	2,520,545,261	92.72%	97.46%
71	87/4	2,532,296,991	93.95%	97.91%
72	87/5	2,543,924,217	95.13%	98.36%
73	87/6	2,550,673,441	96.36%	98.62%
74	87/7	2,564,337,596	97.55%	99.15%
75	87/8	2,571,281,827	98.77%	99.42%
76	87/9	2,586,362,110	100.00%	100.00%

Table 1c. The observed data of the third metro bid

Valuation times	Valuation month	Accumulative valuation total	X3	Y3
To begin construction	81/7	0	0.00%	0.00%
1	81/10	770,150	3.29%	0.04%
2	81/11	3,660,218	4.39%	0.21%
3	81/12	6,468,329	5.46%	0.36%
4	82/1	8,744,528	6.57%	0.49%
5	82/2	9,540,080	7.68%	0.54%
6	82/3	10,466,539	8.68%	0.59%
7	82/4	30,587,616	9.79%	1.73%
8	82/5	57,283,475	10.86%	3.23%
9	82/6	90,958,299	11.96%	5.13%
10	82/7	128,780,447	13.04%	7.27%
11	82/8	178,058,830	14.14%	10.05%
12	82/9	233,101,158	15.25%	13.15%
13	82/10	279,934,914	16.32%	15.79%
14	82/11	315,939,427	17.43%	17.82%
15	82/12	331,823,791	18.50%	18.72%
16	83/1	384,135,189	19.61%	21.67%
17	83/2	404,360,405	20.71%	22.81%
18	83/3	420,393,448	21.71%	23.72%
19	83/4	448,483,677	22.82%	25.30%
20	83/5	471,861,908	23.89%	26.62%
21	83/6	500,240,139	25.00%	28.22%
22	83/7	516,704,749	26.07%	29.15%
23	83/8	552,712,519	27.18%	31.18%
24	83/9	616,268,555	28.29%	34.77%
25	83/10	638,781,109	29.36%	36.04%
26	83/11	670,837,936	30.46%	37.85%
27	83/12	696,804,627	31.54%	39.31%
28	84/1	747,886,459	32.64%	42.19%
29	84/2	780,734,148	33.75%	44.05%
30	84/3	811,864,699	34.75%	45.80%
31	84/4	845,871,058	35.86%	47.72%
32	84/5	867,506,050	36.93%	48.94%
33	84/6	917,912,126	38.04%	51.79%
34	84/7	952,643,465	39.11%	53.75%
35	84/8	987,249,602	40.21%	55.70%
36	84/9	1,006,968,362	41.32%	56.81%
37	84/10	1,050,964,004	42.39%	59.29%
38	84/11	1,068,425,625	43.50%	60.28%
39	84/12	1,087,530,687	44.57%	61.36%
40	85/1	1,120,856,728	45.68%	63.24%
41	85/2	1,163,552,902	46.79%	65.65%
42	85/3	1,205,617,726	47.82%	68.02%
43	85/4	1,226,324,572	48.93%	69.19%
44	85/5	1,236,342,045	50.00%	69.75%
45	85/6	1,256,566,420	51.11%	70.89%
46	85/7	1,277,850,992	52.18%	72.09%
47	85/8	1,290,013,052	53.29%	72.78%
48	85/9	1,295,975,943	54.39%	73.12%
49	85/10	1,315,728,497	55.46%	74.23%
50	85/11	1,330,760,753	56.57%	75.08%
51	85/12	1,341,051,277	57.64%	75.66%
52	86/1	1,352,369,641	58.75%	76.30%
53	86/2	1,358,357,473	59.86%	76.64%
54	86/3	1,362,707,210	60.86%	76.88%
55	86/4	1,372,404,716	61.96%	77.43%
56	86/5	1,378,980,471	63.04%	77.80%
57	86/6	1,395,853,072	64.14%	78.75%
58	86/7	1,412,018,013	65.21%	79.66%
59	86/8	1,418,774,059	66.32%	80.04%
60	86/9	1,428,797,828	67.43%	80.61%
61	86/10	1,432,738,639	68.50%	80.83%
62	86/11	1,438,702,459	69.61%	81.17%
63	86/12	1,444,497,877	70.68%	81.50%
64	87/1	1,446,168,246	71.79%	81.59%
65	87/2	1,449,373,445	72.89%	81.77%
66	87/3	1,463,579,040	73.89%	82.57%
67	87/4	1,468,590,317	75.00%	82.85%
68	87/5	1,500,100,500	76.07%	84.63%
69	87/6	1,515,408,037	77.18%	85.50%
70	87/7	1,521,528,896	78.25%	85.84%
71	87/8	1,527,702,009	79.36%	86.19%
72	87/9	1,540,236,294	80.46%	86.90%
73	87/10	1,556,406,732	81.54%	87.81%
74	87/11	1,566,018,131	82.64%	88.35%
75	87/12	1,587,020,792	83.71%	89.54%
76	88/1	1,598,887,531	84.82%	90.21%
77	88/2	1,603,173,509	85.93%	90.45%
78	88/3	1,618,830,901	86.93%	91.33%
79	88/4	1,628,781,452	88.04%	91.89%
80	88/5	1,654,153,614	89.11%	93.32%
81	88/6	1,723,126,058	90.21%	97.22%
82	88/8	1,724,596,923	92.39%	97.30%
83	88/10	1,756,578,360	94.57%	99.10%
84	89/1	1,762,491,332	97.86%	99.44%
85	89/3	1,772,482,708	100.00%	100.00%

Table 1d. The observed data of the 4th metro bid

Valuation times	Valuation month	Accumulative valuation total	X4	Y4
To begin construction	81/1	0	0.00%	0.00%
1	81/4	9,717,588	3.25%	0.25%
2	81/5	14,452,656	4.32%	0.38%
3	81/6	20,292,030	5.43%	0.53%
4	81/7	33,230,588	6.50%	0.87%
5	81/8	36,803,660	7.61%	0.96%
6	81/9	39,030,540	8.71%	1.02%
7	81/10	58,392,258	9.79%	1.52%
8	81/11	61,404,357	10.89%	1.60%
9	81/12	63,700,019	11.96%	1.66%
10	82/1	76,507,432	13.07%	2.00%
11	82/2	80,702,452	14.18%	2.11%
12	82/3	124,158,031	15.18%	3.24%
13	82/4	141,275,103	16.29%	3.69%
14	82/5	158,256,672	17.36%	4.13%
15	82/6	214,734,109	18.46%	5.61%
16	82/7	241,802,878	19.54%	6.31%
17	82/8	277,317,105	20.64%	7.24%
18	82/9	301,646,427	21.75%	7.88%
19	82/10	343,260,841	22.82%	8.96%
20	82/11	411,486,177	23.93%	10.75%
21	82/12	458,279,034	25.00%	11.97%
22	83/1	503,693,842	26.11%	13.15%
23	83/2	573,992,217	27.21%	14.99%
24	83/3	599,929,009	28.21%	15.67%
25	83/4	631,090,609	29.32%	16.48%
26	83/5	703,922,093	30.39%	18.38%
27	83/6	742,878,532	31.50%	19.40%
28	83/7	780,853,820	32.57%	20.39%
29	83/8	817,428,123	33.68%	21.35%
30	83/9	840,862,895	34.79%	21.96%
31	83/10	898,536,513	35.86%	23.47%
32	83/11	942,509,760	36.96%	24.61%
33	83/12	1,044,902,352	38.04%	27.29%
34	84/1	1,158,880,592	39.14%	30.27%
35	84/2	1,184,351,532	40.25%	30.93%
36	84/3	1,218,684,512	41.25%	31.83%
37	84/4	1,245,689,628	42.36%	32.53%
38	84/5	1,271,673,941	43.43%	33.21%
39	84/6	1,294,753,195	44.54%	33.81%
40	84/7	1,518,131,791	45.61%	39.65%
41	84/8	1,714,198,207	46.71%	44.77%
42	84/9	1,783,621,592	47.82%	46.58%
43	84/10	1,845,495,018	48.89%	48.20%
44	84/11	1,924,914,284	50.00%	50.27%
45	84/12	2,059,183,278	51.07%	53.78%
46	85/1	2,160,490,338	52.18%	56.42%
47	85/2	2,340,027,081	53.29%	61.11%
48	85/3	2,446,181,017	54.32%	63.89%
49	85/4	2,478,213,303	55.43%	64.72%
50	85/5	2,489,499,064	56.50%	65.02%
51	85/6	2,491,949,283	57.61%	65.08%
52	85/7	2,560,875,043	58.68%	66.88%
53	85/8	2,609,003,087	59.79%	68.14%
54	85/9	2,666,691,388	60.89%	69.64%
55	85/10	2,684,622,178	61.96%	70.11%
56	85/11	2,700,243,113	63.07%	70.52%
57	85/12	2,751,078,967	64.14%	71.85%
58	86/1	2,817,796,220	65.25%	73.59%
59	86/2	2,836,173,261	66.36%	74.07%
60	86/3	2,859,819,153	67.36%	74.69%
61	86/4	2,868,127,716	68.46%	74.90%
62	86/5	2,877,930,823	69.54%	75.16%
63	86/6	2,894,348,761	70.64%	75.59%
64	86/7	2,914,840,708	71.71%	76.12%
65	86/8	2,938,418,345	72.82%	76.74%
66	86/9	2,948,724,619	73.93%	77.01%
67	86/10	2,963,782,956	75.00%	77.40%
68	86/11	2,991,148,829	76.11%	78.12%
69	86/12	3,003,984,537	77.18%	78.45%
70	87/1	3,064,820,124	78.29%	80.04%
71	87/2	3,136,645,130	79.39%	81.92%
72	87/3	3,222,054,828	80.39%	84.15%
73	87/4	3,266,282,921	81.50%	85.30%
74	87/5	3,283,787,821	82.57%	85.76%
75	87/6	3,317,376,633	83.68%	86.64%
76	87/7	3,349,582,153	84.75%	87.48%
77	87/8	3,373,093,925	85.86%	88.09%
78	87/9	3,393,205,437	86.96%	88.62%
79	87/10	3,417,732,181	88.04%	89.26%
80	87/11	3,440,252,746	89.14%	89.85%
81	87/12	3,476,282,372	90.21%	90.79%
82	88/1	3,493,605,664	91.32%	91.24%
83	88/2	3,512,035,658	92.43%	91.72%
84	88/3	3,535,261,109	93.43%	92.33%
85	88/4	3,585,095,640	94.54%	93.63%
86	88/5	3,586,666,772	95.61%	93.67%
87	88/6	3,591,453,546	96.71%	93.80%
88	88/7	3,733,326,575	97.79%	97.50%
89	88/8	3,735,184,113	98.89%	97.55%
90	88/9	3,829,030,861	100.00%	100.00%

Table 1e. The observed data of the 5th metro bid

Valuation times	Valuation month	Accumulative valuation total	X5	Y5
To begin construction	81/1	0	0.00%	0.00%
1	81/4	3,157,984	3.52%	0.17%
2	81/7	5,611,645	7.03%	0.30%
3	81/8	9,425,952	8.23%	0.51%
4	81/9	10,299,491	9.43%	0.56%
5	81/11	12,764,490	11.79%	0.69%
7	82/1	54,334,131	14.14%	2.94%
8	82/2	75,551,887	15.34%	4.08%
9	82/3	86,630,800	16.42%	4.68%
10	82/4	103,048,314	17.62%	5.57%
11	82/5	139,609,556	18.78%	7.55%
12	82/6	173,056,321	19.98%	9.35%
13	82/7	204,721,783	21.14%	11.07%
14	82/8	222,398,269	22.33%	12.02%
15	82/9	224,404,386	23.53%	12.13%
16	82/11	234,237,185	25.89%	12.66%
17	82/12	256,533,899	27.05%	13.87%
18	83/1	297,716,885	28.25%	16.09%
19	83/2	305,459,380	29.44%	16.51%
20	83/3	335,755,912	30.53%	18.15%
21	83/4	340,022,082	31.72%	18.38%
22	83/5	351,026,511	32.88%	18.97%
23	83/6	398,589,232	34.08%	21.54%
24	83/7	424,014,518	35.24%	22.92%
25	83/8	440,038,388	36.44%	23.79%
26	83/9	451,674,151	37.64%	24.41%
27	83/11	475,557,405	39.99%	25.71%
28	83/12	482,752,644	41.15%	26.09%
29	84/1	550,811,248	42.35%	29.77%
30	84/2	657,849,322	43.55%	35.56%
31	84/3	814,932,958	44.63%	44.05%
32	84/4	925,735,504	45.83%	50.04%
33	84/5	1,014,368,688	46.99%	54.83%
34	84/6	1,021,366,008	48.18%	55.21%
35	84/7	1,022,875,745	49.34%	55.29%
36	84/8	1,023,432,791	50.54%	55.32%
37	84/9	1,023,915,614	51.74%	55.35%
38	84/11	1,024,398,433	54.10%	55.37%
39	84/12	1,030,354,689	55.26%	55.69%
40	85/1	1,035,104,573	56.45%	55.95%
41	85/2	1,035,587,394	57.65%	55.98%
42	85/3	1,036,070,213	58.77%	56.00%
43	85/4	1,110,419,745	59.97%	60.02%
44	85/5	1,220,897,645	61.13%	65.99%
45	85/6	1,262,989,056	62.33%	68.27%
46	85/7	1,332,751,092	63.49%	72.04%
47	85/8	1,333,237,716	64.68%	72.07%
48	85/9	1,333,773,802	65.88%	72.09%
49	85/11	1,334,264,148	68.24%	72.12%
50	85/12	1,343,207,704	69.40%	72.60%
51	86/1	1,387,727,110	70.60%	75.01%
52	86/2	1,540,850,336	71.79%	83.29%
53	86/3	1,630,875,831	72.87%	88.15%
54	86/4	1,631,046,356	74.07%	88.16%
55	86/5	1,664,333,982	75.23%	89.96%
56	86/6	1,666,382,775	76.43%	90.07%
57	86/7	1,666,539,178	77.59%	90.08%
58	86/8	1,691,517,333	78.79%	91.43%
59	86/9	1,694,658,162	79.98%	91.60%
60	86/11	1,705,681,899	82.34%	92.20%
61	86/12	1,727,987,625	83.50%	93.40%
62	87/1	1,747,674,733	84.70%	94.47%
63	87/2	1,753,386,399	85.90%	94.78%
64	87/3	1,764,776,359	86.98%	95.39%
65	87/4	1,772,226,875	88.18%	95.79%
66	87/5	1,815,968,977	89.34%	98.16%
67	87/6	1,828,265,501	90.53%	98.82%
68	87/7	1,835,757,811	91.69%	99.23%
69	87/8	1,838,287,114	92.89%	99.37%
70	87/9	1,839,318,252	94.09%	99.42%
71	87/11	1,846,237,199	96.45%	99.79%
72	88/2	1,850,032,940	100.00%	100.00%

Table 1f. The observed data of the 6th metro bid

Valuation times	Valuation month	Accumulative valuation total	X6	Y6
To begin construction	84/9	0	0.00%	0.00%
1	84/12	1,196,429	5.75%	0.11%
2	85/1	7,484,820	7.71%	0.66%
3	85/2	9,108,509	9.67%	0.81%
4	85/3	11,400,844	11.50%	1.01%
5	85/4	12,655,783	13.46%	1.12%
6	85/5	40,841,449	15.35%	3.62%
7	85/6	52,346,329	17.31%	4.63%
8	85/8	60,797,648	21.16%	5.38%
9	85/10	60,995,847	25.02%	5.40%
10	85/11	62,916,342	26.97%	5.57%
11	85/12	67,254,060	28.87%	5.95%
12	86/1	100,206,292	30.83%	8.87%
13	86/3	102,784,371	34.55%	9.10%
14	86/5	103,468,121	38.41%	9.16%
15	86/6	114,992,782	40.37%	10.18%
16	86/7	122,744,984	42.26%	10.87%
17	86/8	143,160,179	44.22%	12.67%
18	86/9	190,468,065	46.18%	16.86%
19	86/10	239,013,770	48.07%	21.16%
20	86/11	294,623,573	50.03%	26.08%
21	86/12	412,771,998	51.93%	36.54%
22	87/1	500,498,449	53.89%	44.31%
23	87/3	513,633,770	57.61%	45.47%
24	87/4	534,698,236	59.57%	47.34%
25	87/5	590,543,842	61.47%	52.28%
26	87/6	602,157,035	63.42%	53.31%
27	87/7	618,978,284	65.32%	54.80%
28	87/8	646,144,725	67.28%	57.20%
29	87/9	685,602,225	69.24%	60.69%
30	87/10	736,566,872	71.13%	65.21%
31	87/11	765,215,105	73.09%	67.74%
32	87/12	850,864,754	74.98%	75.33%
33	88/1	874,446,506	76.94%	77.41%
34	88/2	924,641,322	78.90%	81.86%
35	88/3	1,012,565,332	80.67%	89.64%
36	88/4	1,026,516,972	82.63%	90.88%
37	89/1	1,129,589,930	100.00%	100.00%

discrete and delayed situation. The process of the former 30% duration of the project is a bit slow. In addition, the first evaluation time of total duration is 4.5% when a contractor proposes. It implies that we must notice the delayed situation of cash flow and maintain the liquidity of cash portion to some degree to ensure the project can be finished smoothly and successfully.

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REFERENCES

1. Chatterjee, S. and Price, B., *Regression Analysis by Example*, John Wiley and Sons, Inc., New York (1991).
2. Diamond, P., "Fuzzy Least Squares," *Inform. Sci.*, Vol. 46, pp. 141-157 (1988).
3. Kaka, A.P., "Contractors' Financial Budgeting Using Computer Simulation," *Construc. Manag. Econ.*, Vol. 12, pp. 113-124 (1994).
4. Klir, G.J. and Yuan, B., *Fuzzy Sets and Fuzzy Logic Theory and Applications*, Prentic-Hall, Englewood

Cliffs, NJ (1995).
5. Miskawi, Z., “An *S*-curve Equation for Project Control,” *Construc. Manag. Econ.*, Vol. 7, pp. 115-124 (1989).
6. Ruoning, X., “A Linear Regression Model in Fuzzy Environment,” *Adv. Model. Simul.*, Vol. 27, pp. 31-40 (1991).
7. Ruoning, X., “*S*-curve Regression Model in Fuzzy Environment,” *Fuzzy Sets Syst.*, Vol. 90, pp. 317-326 (1997).
8. Takagi, T. and Sugeno, M., “Fuzzy Identification of Systems and Its Applications to Modeling and Control,” *IEEE Trans. Syst., Man, Cybernetics*, Vol. 15, pp. 116-132 (1985).
9. Tanaka, H., Uejima, S., and Asai, K. “Linear Regression Analysis with Fuzzy Model,” *IEEE Trans. Syst., Man, Cybernetics*, Vol. 12, pp. 903-907 (1982).
10. Wang, H.O., Tanaka, K., and Griffin, M.F., “An Approach to Fuzzy Control of Nonlinear Systems: Stability and Design Issues,” *IEEE Trans. Fuzzy Syst.*, Vol. 4, pp. 14-23 (1996).
11. Wang, W.J. and Chiu, C.H., “Entropy Variation on The Fuzzy Numbers with Arithmetic Operations,” *Fuzzy Sets Syst.*, Vol. 103, No. 3, pp. 443-456 (1999).
12. Zadeh, L.A., “Fuzzy Sets,” *Inform. Control*, Vol. 8, pp. 338-353 (1965).