Journal of
Marine Science and Technology

Volume 19 | Issue 4 Article 2

AN INTEGRATED FUZZY TOPSIS METHOD FOR RANKING ALTERNATIVES AND ITS APPLICATION

Ji-Feng Ding
Department of Aviation and Maritime Transportation Management, Chang Jung Christian University, Gui-Ren, Tainan County 711, Taiwan, R.O.C., jfding@mail.cjcu.edu.tw

AN INTEGRATED FUZZY TOPSIS METHOD FOR RANKING ALTERNATIVES AND ITS APPLICATION

Acknowledgements

The author would like to thank two anonymous referees for their excellent comments and valuable advice in this paper.

AN INTEGRATED FUZZY TOPSIS METHOD FOR RANKING ALTERNATIVES AND ITS APPLICATION

Ji-Feng Ding*

Key words: fuzzy TOPSIS, ranking alternatives, fuzzy numbers.

ABSTRACT

The main purpose of this paper is to develop an integrated fuzzy technique for order preference by similarity to ideal solution (TOPSIS) method to improve the quality of decision-making for ranking alternatives. The proposed fuzzy TOPSIS method mainly accounts for the classification of criteria, the integrated weights of criteria and sub-criteria, and the performance values of decision matrix. In this model, the criteria are classified into subjective criteria and objective ones. The fuzzy analytic hierarchy process approach and the entropy weighting method are used to solve the subjective weights and objective ones. In addition, the adjusted integration weights are measured by combining these two methods. The performance values of subjective criteria and of objective ones will be obtained by linguistic expressions and objective evaluation values, respectively. Furthermore, the graded mean integration representation method and the modified distance method are employed to the integrated fuzzy TOPSIS method. Finally, a hypothetical example of partner selection of a shipping company is designed to demonstrate the computational process of this fuzzy TOPSIS algorithm.

I. INTRODUCTION

Many ranking methods based on the fuzzy concepts have been proposed to solve the multiple criteria decision-making (MCDM) problems, e.g. Ballı and Korukoğlu [1], Büyüközkan *et al*. [3], Chen [6], Chou [8], Chou and Liang [9], Ding [11], Ertuğrul and Karakaşoğlu [13], Lee and Chou [18], Liang [19], Tsaur *et al*. [22], Valls and Vicenc [23], Wang *et al*. [25], Wang and Lee [24], etc. One of the well known ranking methods for MCDM, named the technique for order preference by similarity to ideal solution (TOPSIS), is firstly proposed by Hwang and Yoon [16]. The logic of the TOPSIS approach is to define the ideal and anti-ideal solutions [19], which are based on the concept of relative closeness in compliance with the shorter (longer) the distance of alternative i to ideal (anti-ideal), the higher the priority can be ranked [28]. However, to efficiently resolve the ambiguity frequently arising in available information and do more justice to the essential fuzziness in human judgment and preference, the fuzzy set theory [26] has been used to establish a fuzzy TOPSIS problem [1, 3, 5, 6, 13, 19, 24].

The decision for the problem of determination and selection poses a multiple criteria problem that changes with time. The goal of the MCDM method is to aid decision-makers (DMs) in integrating objective measurements with value judgments that are based not on individual opinions but on collective group ideas [2]. Further, there are situations in which information is incomplete or imprecise or views that are subjective or endowed with linguistic characteristics creating a fuzzy decision-making environment. Therefore, a fuzzy MCDM problem with group decision accounts for raising some evaluation points, which are evaluation criteria/sub-criteria, feasible alternatives, DMs, and decision ranking rules. We can describe in detail that multiple DMs will be usually discussed to apply the fuzzy problem involving the compromise solutions or trade-off solutions, which in the process of decision-making have the characteristics or properties of bargaining. Then, a set of alternatives is both feasible to the DMs and known during the decision process. The feasibility of an alternative is defined by a variety of constraints such as physical availability, monetary resources, information constraints, and so on. Later, the evaluation criteria of every available alternative should be found out to evaluate the attractiveness of alternatives in terms of criteria values or performance value. The performance values of each alternative A_i (i = 1, 2, …, m) for each criterion C_j (j = 1, 2, …, n) can be expressed as a evaluation matrix or decision matrix, which can be obtained as $D = [x_{ij}]_{m\times n}$, i = 1, 2, …, m; j = 1, 2, …, n. Finally, a choice from two or more alternatives requires a decision rule or ranking rule in which the DMs can obtain the information available to make a best choice. In this paper, the ranking rule based on the fuzzy TOPSIS method will be described in the following context.

It might be noted that the criteria are measures, rules, and

Paper submitted 08/06/09; revised 03/09/10; accepted 03/18/10. Author for correspondence: Ji-Feng Ding (e-mail: jfding@mail.cjcu.edu.tw).
**Department of Aviation and Maritime Transportation Management, Chang Jung Christian University, Gui-Ren, Tainan County 711, Taiwan, R.O.C.*

standards which can assist decision-making. There are three issues needing to describe in terms of classification, and weights of the criteria, and the decision matrix, respectively.

At first, it is well known that criteria are described to classify into two categories: (1) subjective criteria, which have linguistic/qualitative definition; (2) objective criteria, which are defined in monetary/quantitative terms. However, this categorization does not mean there exists subjective criteria and objective ones simultaneously. This depends on the characteristic of the problem. In this paper, the criteria of these two categories will be applied to the algorithm and numerical study.

Secondly, the weights of these criteria are greatly influenced the final selection of fuzzy MCDM problem. Deng *et al.* [10] had referred that the criteria weights can be obtained by many methods. The weights of criteria reflected the DM's subjective preference and it is traditionally obtained by using a preference elicitation technique, e.g. the analytic hierarchy process (AHP) approach, which was proposed by Saaty [21]. However, the weights of objective criteria above the alternatives level not only can express the explanation ability and reliability of the decision-making problem but also can represent actual conditions of decision-making and improve the quality of decision-making. It is usually obtained by using the entropy weighting method [28], which can effectively measure the average essence of information quantity, and the larger the entropy value, the lower the information express quantity [28]. In this paper, the weights of the subjective and objective criteria, using the fuzzy AHP approach and the entropy weighting method, will be applied to the generalized algorithm.

Thirdly, another key point greatly influencing the final selection of fuzzy MCDM problem is the performance values embedded in the decision matrix. The decision matrix $D = [x_{ij}]_{m\times n}$, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$. represents the performance rating or evaluation score x_{ij} of each alternative A_i with regard to each criterion C_j. Many authors have tackled this aspect in the MCDM problem and the papers are too numerous to enumerate. However, the proposed model involves with different subjective and objective criteria. The performance values of these different criteria have different units of measurement, which should have a normalized or standardized method to eliminate the impacts of different measure units of different criteria. In this paper, the performance values of subjective criteria and of objective ones will be obtained by linguistic expressions and objective evaluation values, respectively. Besides, a normalized method will be drawn, too.

In summary, experience has shown that the problem of ranking alternatives is no easy matter. It involves a multiplicity of complex considerations. And yet, particularly with regard to linguistic terms are difficult to evaluate. The fuzzy set theory is ideal for sorting through the maze of vague and at times conflicting information. The main purpose of this paper is to develop a fuzzy model - authentically speaking, an integrated fuzzy TOPSIS method involves in fuzzy MCDM problem with group decision - to improve the quality of decision-making for ranking alternatives. The framework of this paper is arranged in six sections of this paper. The research methodologies are presented in Section II. The integrated weights of all criteria using the fuzzy AHP approach and entropy weighting method are proposed in Section III. The model based on the fuzzy TOPSIS method is constructed in Section IV. A numerical example is studied in Section V. Finally, conclusions are made in the last section.

II. RESEARCH METHODOLOGIES

In this section, some concepts and methods used in this paper are briefly introduced.

1. Fuzzy Set Theory

The fuzzy set theory [26] is designed to deal with the extraction of the primary possible outcome from a multiplicity of information that is expressed in vague and imprecise terms. Fuzzy set theory treats vague data as probability distributions in terms of set memberships. Once determined and defined, sets of memberships in probability distributions can be effectively used in logical reasoning.

2. Triangular Fuzzy Numbers and the Algebraic Operations

In a universe of discourse X, a fuzzy subset A of X is defined by a membership function $f_A(x)$, which maps each element x in X to a real number in the interval [0, 1]. The function value $f_A(x)$ represents the grade of membership of x in A.

A fuzzy number A [12] in real line $\Re$ is a triangular fuzzy number if its membership function $f_A : \Re \to [0, 1]$ is

$$f_A(x) = \begin{cases} (x-c)/(a-c), & c \le x \le a \\ (x-b)/(a-b), & a \le x \le b \\ 0, & otherwise \end{cases} \tag{1}$$

with $-\infty < c \le a \le b < \infty$. The triangular fuzzy number can be denoted by (c, a, b).

Let $A_1 = (c_1, a_1, b_1)$ and $A_2 = (c_2, a_2, b_2)$ be fuzzy numbers. According to the extension principle [26], the algebraic operations of any two fuzzy numbers A_1 and A_2 can be expressed as

- Fuzzy addition, $\oplus$:
  $A_1 \oplus A_2 = (c_1 + c_2, a_1 + a_2, b_1 + b_2)$,
- Fuzzy subtraction, $\ominus$:
  $A_1 \ominus A_2 = (c_1 + b_2, a_1 + a_2, b_1 + c_2)$,
- Fuzzy multiplication, $\otimes$:
  $k \otimes A_2 = (kc_2, ka_2, kb_2)$, $k \in \Re$, $k \ge 0$, ,
  $A_1 \otimes A_2 \cong (c_1c_2, a_1a_2, b_1b_2)$ $c_1 \ge 0$, $c_2 \ge 0$,,
- Fuzzy division, $\varnothing$:
  $(A_1)^{-1} = (c_1, a_1, b_1)^{-1} \cong (1/b_1, 1/a_1, 1/c_1)$, $c_1 > 0$,
  $A_1 \varnothing A_2 \cong (c_1/b_2, a_1/a_2, b_1/c_2)$, $c_1 \ge 0$, $c_2 > 0$.

3. Linguistic Values

In fuzzy decision environments, two preference ratings can be used. They are fuzzy numbers and linguistic values characterized by fuzzy numbers [27]. Depending on practical needs, DMs may apply one or both of them. In this paper, the rating set is used to analytically express the linguistic value and describe how good of the alternatives against various criteria above the alternative level is. The rating set is defined as $S = \{VP, P, F, G, VG\}$; where VP = Very Poor, P = Poor, F = Fair, G = Good, and VG = Very Good. Here, we define the linguistic values [4] of $VP = (0, 0, 0.25)$, $P = (0, 0.25, 0.5)$, $F = (0.25, 0.5, 0.75)$, $G = (0.5, 0.75, 1)$, and $VG = (0.75, 1, 1)$, respectively.

4. Defuzzification of Triangular Fuzzy Numbers

For solving the problem of defuzzification powerfully, the graded mean integration representation (GMIR) method, proposed by Chen and Hsieh [7], is used to defuzzify the triangular fuzzy numbers.

Let $A_i = (c_i, a_i, b_i)$, $i = 1, 2, ..., n$ be n triangular fuzzy numbers. By the GMIR method, the GMIR $R(A_i)$ of A_i is

$$R(A_i) = \frac{c_i + 4a_i + b_i}{6} \tag{2}$$

Suppose $R(A_i)$ and $R(A_j)$ are the GMIR of the triangular fuzzy numbers A_i and A_j, respectively. We define:

(1) $A_i > A_j \Leftrightarrow R(A_i) > R(A_j)$,
(2) $A_i < A_j \Leftrightarrow R(A_i) < R(A_j)$,
(3) $A_i = A_j \Leftrightarrow R(A_i) = R(A_j)$.

5. Distance Measure Approach

Two famous distance measure approaches between two fuzzy numbers, i.e. mean and geometrical distance measures, were introduced by Heilpern [14] in 1997. However, Heilpern's method cannot satisfy some special cases between two fuzzy numbers. Hsieh and Chen [15] had proposed the modified geometrical distance approach to improve the drawback. For matching the fuzzy TOPSIS algorithm in this paper, this modified distance approach is used to measure the distance of two fuzzy numbers.

Let $A_i = (c_i, a_i, b_i)$ and $A_j = (c_j, a_j, b_j)$ be fuzzy numbers. Then, the Hsieh and Chen's modified distance can be denoted by

$$\delta_M(A_i, A_j) = \left\{ \frac{1}{4} \left[(c_i - c_j)^2 + 2(a_i - a_j)^2 + (b_i - b_j)^2 \right] \right\}^{1/2} \tag{3}$$

III. THE WEIGHTS OF CRITERIA

The weights of the subjective and objective criteria will be obtained by using the fuzzy AHP approach and the entropy weighting method. Finally, the integrated weights of all cri-

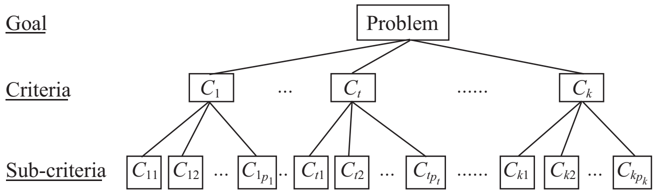

Fig. 1. Hierarchical structure.

teria above the alternatives layer can be computed by combining the subjective weights and objective ones.

1. Fuzzy AHP Approach

A fuzzy AHP approach is used to measure relative weights for evaluating subjective criteria. The systematic steps for evaluating relative weights using fuzzy AHP to be taken are described below.

Step 1: Develop a Hierarchical Structure

A hierarchy structure is the framework of system structure. We can skeletonize a hierarchy to evaluate research problems and benefit the context. It is not only useful in studying the interaction amongst the elements involved in each level, but it can also help decision-makers to explore the impact of different elements on the evaluated system. Fig. 1 is an incomplete hierarchical structure with k criteria, and $p_1 + \ldots + p_t + \ldots + p_k$ sub-criteria.

Step 2: Build Fuzzy Pair-wise Comparison Matrices

Collecting pair-wise comparison matrices of each layer to represent the relative importance is an important step in fuzzy AHP method. Consequently, these relative importance are evaluated by experts, and these data are transformed into triangular fuzzy numbers using the geometric mean approach [21] to convey the opinions of all experts.

The generalized means is a typical representation of many well-known averaging operations [17], e.g., min, max, geometric mean, arithmetic mean, harmonic mean, etc. The min and max are the lower bound and upper bound of generalized means, respectively. Besides, the geometric mean is more effective in representing the multiple decision-makers' consensus opinions [21]. To aggregate all information generated by different averaging operations, we use the grade of membership to demonstrate their strength after considering all approaches. For the above-mentioned reasons, the triangular fuzzy numbers characterized by using the min, max and geometric mean operations are used to convey the opinions of all experts.

That is, let $x_{ij}^h \in \{\frac{1}{9}, \frac{1}{8}, \ldots, \frac{1}{2}, 1\} \cup \{1, 2, \ldots, 8, 9\}$ ($h = 1, 2, \ldots, n$, $\forall i, j = 1, 2, \ldots, k$) be the relative importance given to i^{th} criterion to j^{th} criterion by h^{th} expert on the criteria layer in Fig. 1. Then, the pair-wise comparison matrix is defined as $[x_{ij}^h]_{k \times k}$. After integrating the opinions of all n experts, the triangular fuzzy numbers can be denoted by $\tilde{A}_{ij}^{C} = (c_{ij}, a_{ij}, b_{ij})$, where $c_{ij} = \min\{x_{ij}^1, x_{ij}^2, \ldots, x_{ij}^n\}$, $a_{ij} = \left(\prod_{h=1}^{n} x_{ij}^h\right)^{1/n}$, $b_{ij} = \max\{x_{ij}^1, x_{ij}^2, \ldots, x_{ij}^n\}$.

We use the integrated triangular fuzzy numbers to build a fuzzy pair-wise comparison matrix (given to i^{th} criterion to j^{th} criterion). For the criteria layer, the fuzzy pair-wise compareson matrix can be denoted by

$$A_k^C = \left[\tilde{A}_{ij}^C\right]_{k\times k} = \begin{bmatrix} \tilde{1} & \tilde{A}_{12}^C & \cdots & \tilde{A}_{1k}^C \\ 1/\tilde{A}_{12}^C & \tilde{1} & \cdots & \tilde{A}_{2k}^C \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{A}_{1k}^C & 1/\tilde{A}_{2k}^C & \cdots & \tilde{1} \end{bmatrix},$$

where $\tilde{A}_{ij}^C \otimes \tilde{A}_{ji}^C \cong 1$, $\forall i, j = 1, 2, \ldots, k$.

By the same concept, let $x_{uv}^{sh} \in \{\frac{1}{9}, \frac{1}{8}, \ldots, \frac{1}{2}, 1\} \cup \{1, 2, \ldots, 8, 9\}$ ($h = 1, 2, \ldots, n$, $\forall u, v = 1, \ldots, p_1$; $\forall u, v = 1, \ldots, p_t$; …; $\forall u, v = 1, \ldots, p_k$) be the relative importance given to u^{th} sub-criterion to v^{th} sub-criterion by h^{th} expert on the sub-criteria layer in Fig. 1. Then, the pair-wise comparison matrices are defined as $[x_{uv}^{sh}]_{p_1\times p_1}, \ldots, [x_{uv}^{sh}]_{p_t\times p_t}, \ldots, [x_{uv}^{sh}]_{p_k\times p_k}$. Therefore, we can integrate the opinions of all n experts given to sub-criterion u to sub-criterion v on the sub-criteria layer, the triangular fuzzy numbers can be denoted by $\tilde{A}_{uv}^{SC} = (c_{uv}, a_{uv}, b_{uv})$, $\forall u, v = 1, \ldots, p_1$; $\forall u, v = 1, \ldots, p_t$; …; $\forall u, v = 1, \ldots, p_k$ where $c_{uv} = \min\{x_{uv}^{s1}, x_{uv}^{s2}, \ldots, x_{uv}^{sn}\}$, $a_{uv} = \left(\prod_{h=1}^{n} x_{uv}^{sh}\right)^{1/n}$, $b_{uv} = \max\{x_{uv}^{s1}, x_{uv}^{s2}, \ldots, x_{uv}^{sn}\}$.

We use the integrated triangular fuzzy numbers to build the fuzzy pair-wise comparison matrices for the sub-criteria layer can be denoted by

$$A_{p_1}^{SC} = \left[\tilde{A}_{uv}^{SC}\right]_{p_1\times p_1} = \begin{bmatrix} \tilde{1} & \tilde{A}_{12}^{SC} & \cdots & \tilde{A}_{1p_1}^{SC} \\ 1/\tilde{A}_{12}^{SC} & \tilde{1} & \cdots & \tilde{A}_{2p_1}^{SC} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{A}_{1p_1}^{SC} & 1/\tilde{A}_{2p_1}^{SC} & \cdots & \tilde{1} \end{bmatrix},$$

where $\tilde{A}_{uv}^{SC} \otimes \tilde{A}_{vu}^{SC} \cong 1$, $\forall u, v = 1, 2, \ldots, p_1$.

, ……,

$$A_{p_t}^{SC} = \left[\tilde{A}_{uv}^{SC}\right]_{p_t\times p_t} = \begin{bmatrix} \tilde{1} & \tilde{A}_{12}^{SC} & \cdots & \tilde{A}_{1p_t}^{SC} \\ 1/\tilde{A}_{12}^{SC} & \tilde{1} & \cdots & \tilde{A}_{2p_t}^{SC} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{A}_{1p_t}^{SC} & 1/\tilde{A}_{2p_t}^{SC} & \cdots & \tilde{1} \end{bmatrix},$$

where $\tilde{A}_{uv}^{SC} \otimes \tilde{A}_{vu}^{SC} \cong 1$, $\forall u, v = 1, 2, \ldots, p_t$.

, ……, and

$$A_{p_k}^{SC} = \left[\tilde{A}_{uv}^{SC}\right]_{p_k\times p_k} = \begin{bmatrix} \tilde{1} & \tilde{A}_{12}^{SC} & \cdots & \tilde{A}_{1p_k}^{SC} \\ 1/\tilde{A}_{12}^{SC} & \tilde{1} & \cdots & \tilde{A}_{2p_k}^{SC} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{A}_{1p_k}^{SC} & 1/\tilde{A}_{2p_k}^{SC} & \cdots & \tilde{1} \end{bmatrix},$$

where $\tilde{A}_{uv}^{SC} \otimes \tilde{A}_{vu}^{SC} \cong 1$, $\forall u, v = 1, 2, \ldots, p_k$.

Step 3: Calculate the Fuzzy Weights of the Fuzzy Pair-wise Comparison Matrices

Let $\tilde{Z}_i^C = (\tilde{A}_{i1}^C \otimes \tilde{A}_{i2}^C \otimes \cdots \otimes \tilde{A}_{ik}^C)^{1/k}$ ($\forall i = 1, 2, \ldots, k$) be the geometric mean of triangular fuzzy number of i^{th} criterion on the criteria layer. Then, the fuzzy weight of i^{th} criterion can be denoted by $\tilde{W}_i^C = \tilde{Z}_i^C \otimes (\tilde{Z}_1^C \oplus \tilde{Z}_2^C \oplus \cdots \oplus \tilde{Z}_k^C)^{-1}$. For being convenient, the fuzzy weight is denoted by $\tilde{W}_i^C \cong (w_{ic}, w_{ia}, w_{ib})$.

By the same concept, let $\tilde{Z}_u^{SC} = (\tilde{A}_{u1}^{SC} \otimes \tilde{A}_{u2}^{SC} \otimes \cdots \otimes \tilde{A}_{up_1}^{SC})^{1/p_1}$ ($\forall u = 1, 2, \ldots, p_1$) be the geometric mean of triangular fuzzy number of u^{th} sub-criterion on the sub-criteria layer. Then, the fuzzy weight of u^{th} sub-criterion can be denoted by $\tilde{W}_u^{SC} = \tilde{Z}_u^{SC} \otimes (\tilde{Z}_1^{SC} \oplus \tilde{Z}_2^{SC} \oplus \cdots \oplus \tilde{Z}_{p_1}^{SC})^{-1}$, where the fuzzy weight is denoted by $\tilde{W}_u^{SC} \cong (w_{uc}, w_{ua}, w_{ub})$, $\forall u = 1, 2, \ldots, p_1$. For saving space, the fuzzy weights of $[(p_1 + \ldots + p_t + \ldots + p_k) - p_1]$ sub-criteria can be obtained by the above-mentioned method.

Step 4: Defuzzify the Fuzzy Weights to Crisp Weights

For solving the problem of defuzzification powerfully, the GMIR method is used to defuzzify the fuzzy weights. Let $\tilde{W}_i^C \cong (w_{ic}, w_{ia}, w_{ib})$ ($\forall i = 1, 2, \ldots, k$) be k triangular fuzzy numbers. By the powerful method, the GMIR of crisp weights k can be denoted by $W_i^C = \frac{w_{ic} + 4w_{ia} + w_{ib}}{6}$, $\forall i = 1, 2, \ldots, k$.

For saving space, the defuzzifications of fuzzy weights are omitted to reason by analogy on the sub-criteria layer.

Step 5: Calculate and Normalize the Weight Vector of Each Layer

For being convenient to compare the relative importance between each layer, these crisp weights are normalized and denoted by $NW_i^C = W_i^C \Big/ \sum_{i=1}^{k} W_i^C$.

Let NW_i^C and NW_u^{SC} be the normalized crisp weights on the criteria and sub-criteria layers, respectively. Then,

(1) The integrated weight of each criterion on the criteria layer is

$$IW_i^C = NW_i^C,\ \forall i = 1, 2, \ldots, k. \tag{4}$$

(2) The integrated weight of each sub-criterion on the sub-criteria layer is

$$IW_u^{SC} = NW_i^C \times NW_u^{SC},\ \forall i = 1, 2, \ldots, k;\ \forall u = 1, \ldots, p_1;$$
$$\forall u = 1, \ldots, p_t;\ \cdots;\ \forall u = 1, \ldots, p_k. \tag{5}$$

2. Entropy Weighting Method

This section tries to solve the objective weight of objective sub-criteria above the alternative level using the entropy weighting method. Thus, the steps can be summarized as follows.

Step 1: Construct a Decision Matrix

Here, let m and q respectively denote the numbers of alternatives and the objective sub-criteria above the alternatives layer. Allow $\tilde{X}_{ij}$, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, q$, to be the triangular fuzzy number of original evaluation value of i^{th} alternative under j^{th} sub-criterion. Then, the decision matrix $D = [\tilde{X}_{ij}]_{m\times q}$, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, q$, can be obtained.

To ensure compatibility between the positive sub-criterion j (the criterion that has positive contribution to the objective, e.g., benefit criterion) and the negative one (the criterion that has negative contribution to the objective, e.g., cost criterion), the original evaluation value must convert to dimensionless index. Let d_{ij} ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, q$) denote the normalized evaluation value of i^{th} alternative under j^{th} sub-criterion. By using the equation (2) of GMIR method mentioned in subsection 4 of Section II, the representation value of $\tilde{X}_{ij}$ can be express as $R(\tilde{X}_{ij})$. The fuzzy positive value $\tilde{X}_j^P$ and fuzzy negative value $\tilde{X}_j^N$ of each criterion above the alternatives layer can be judged and determined by comparing with these representation values $R(\tilde{X}_{ij})$. Then

(1) For the positive sub-criterion j:
$d_{ij} = R(\tilde{X}_{ij}) / R(\tilde{X}_j^P)$, where $\tilde{X}_j^P = \max_i\{\tilde{X}_{ij}\}$ and $0 \le d_{ij} \le 1$.

(2) For the negative sub-criterion j:
$d_{ij} = R(\tilde{X}_j^N) / R(\tilde{X}_{ij})$, where $\tilde{X}_j^N = \min_i\{\tilde{X}_{ij}\}$ and $0 \le d_{ij} \le 1$.

For example, assume three fuzzy numbers are denoted as $\tilde{X}_{11} = (5, 9, 11)$, $\tilde{X}_{21} = (6, 7, 10)$, and $\tilde{X}_{31} = (3, 5, 8)$, respectively. Using the equation (2), the GMIR values can be expressed as $R(\tilde{X}_{11}) = 8.67$, $R(\tilde{X}_{21}) = 7.33$ and $R(\tilde{X}_{31}) = 5.17$, respectively. Here, $R(\tilde{X}_{11}) > R(\tilde{X}_{21}) > R(\tilde{X}_{31})$, hence, the fuzzy positive value $\tilde{X}_1^P = (5, 9, 11)$ can be determined. Then, the normalized evaluation value of each positive criterion $d_{11} = 8.67/8.67 = 1$, $d_{21} = 7.33/8.67 = 0.845$, and $d_{11} = 5.17/8.67 = 0.596$ can be obtained.

Subsequently, the normalized decision matrix $D = [d_{ij}]_{m\times q}$, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, q$, can be determined. Here, we define $D_j = \sum_{i=1}^{m} d_{ij}$, $j = 1, 2, \ldots, q$.

Step 2: Calculate the Entropy Value of Each Criterion

The entropy value E_j of each objective evaluation sub-criterion j can be calculated by $E_j = -k\sum_{i=1}^{m}\frac{d_{ij}}{D_j}\ln\frac{d_{ij}}{D_j}$, where $k = \frac{1}{\ln m} > 0$, and $0 \le E_j \le 1$.

Step 3: Compute the Total Entropy Value

The total entropy value E can be computed as $E = \sum_{j=1}^{q} E_j$.

Step 4: Obtain the Objective Weight of Each Objective Criterion

The objective weight π_j of the j^{th} objective sub-criterion above the alternative level can be calculated by

$$\pi_j = \frac{1-E_j}{\sum_{j=1}^{q}(1-E_j)} = \frac{1-E_j}{q-E},\ 0 \le \pi_j \le 1,\ \sum_{j=1}^{q}\pi_j = 1 \tag{6}$$

3. The Integrated Weights

Here, we expand the incomplete hierarchical structure of Fig. 1 into a complete one, which has k criteria, $p_1 + \ldots + p_t + \ldots + p_k$ sub-criteria and m alternatives. The weights of subjective and objective sub-criteria above the alternatives layer can be obtained by using the fuzzy AHP approach and the entropy weighting method. The next step is computing the integration weights of all sub-criteria above the alternatives layer by combining the subjective weights and objective ones. Following will discuss the three cases appeared in the MCDM problems in terms of the criteria aspects.

Case I: If all the sub-criteria above the alternatives layer are subjective, then using the fuzzy AHP approach. The integrated weight of each subjective sub-criterion can be obtained by using the equation (5).

Case II: If all the sub-criteria above the alternatives layer are objective, then using the entropy weighting method. The integrated weight of each objective sub-criterion can be obtained by using the equation (6).

Case III: If some sub-criteria above the alternatives layer are subjective, and others are objective. Then, the adjusted integration weights of objective sub-criterion can be obtained by using the equation (7).

That is, let $O = \{o_1, \ldots, o_t, \ldots, o_q\}$ be the set of all q objective sub-criteria above the alternatives layer. Allow η_t, $t = 1, 2, \ldots, q$, to be the subjective integration weights of objective sub-criteria o_t, then go to the Case I. Using the same concept, let λ_t, $t = 1, 2, \ldots, q$, to be the objective weights of objective sub-criteria o_t, then go to the Case II. By combining the objective weights λ_t, and the subjective integrated weights η_t, the adjusted integration weights w_t^* of all q objective sub-criteria can be obtained:

$$w_t^* = \frac{\lambda_t \eta_t}{\sum_{t=1}^{q} \lambda_t \eta_t} \times \sum_{t=1}^{q} \eta_t, \quad t = 1, 2, \ldots, q. \tag{7}$$

IV. THE PROPOSED FUZZY TOPSIS METHOD

The systematic steps for ranking alternatives based on the proposed fuzzy TOPSIS method to be taken are described below.

1. Forming a committee of DMs to identify the appropriate alternatives and adopt the evaluation criteria and sub-criteria.
2. Classifying the sub-criteria above the alternatives layer into the subjective and objective categories.
3. Computing the subjective integration weights of all sub-criteria above the alternatives layer.
4. Estimating the superiority of alternatives versus all sub-criteria.
5. Utilizing the entropy weighting method to adjust the subjective integration weights of objective sub-criteria above the alternatives layer.
6. Calculating the fuzzy ideal solution and anti-ideal solution.
7. Computing the distance of different alternatives versus the fuzzy ideal solution and anti-ideal solution.
8. Calculating the relative approximation value of different alternatives versus ideal solution.
9. Ranking the alternatives to select the best one.

1. Estimating the Superiority of Alternatives versus All Sub-criteria

The sub-criteria above the alternatives layer are classified into the subjective and objective categories. Let $S = \{s_1, \ldots, s_r, \ldots, s_p\}$ and $O = \{o_1, \ldots, o_t, \ldots, o_q\}$ be the sets of all p subjective sub-criteria and q objective ones above the alternatives layer.

Case I: For the Subjective Sub-criteria

At first, the superiority of all alternatives versus all subjective sub-criteria above the alternatives layer can be obtained by using the preference ratings mentioned in the linguistic values of subsection 3 of Section II. For examples, assume that an expert evaluates the appropriateness ratings of alternatives A_1 and A_2 versus subjective sub-criterion C_{11} are 'Very Good' and 'Good,' respectively. Then, the fuzzy superiority of alternatives A_1 and A_2 are (0.75, 1, 1) and (0.5, 0.75, 1), respectively.

Subsequently, the arithmetic mean method is used to solve the average superiority of evaluation value for each alternative versus all subjective sub-criteria. Let $S_{ir}^h = (c_{ir}^h,\ a_{ir}^h,\ b_{ir}^h)$ ($i = 1, 2, \ldots, m$; $r = 1, 2, \ldots, p$; $h = 1, 2, \ldots, n$) be the fuzzy superiority of the i^{th} alternative versus the r^{th} subjecttive sub-criterion evaluated by the h^{th} expert. Then, the average fuzzy superiority value of the i^{th} alternative versus the r^{th} subjective sub-criterion can be expressed as $\left(\frac{\sum_{h=1}^{n} c_{ir}^h}{n}, \frac{\sum_{h=1}^{n} a_{ir}^h}{n}, \frac{\sum_{h=1}^{n} b_{ir}^h}{n}\right)$. For example, the appropriateness ratings of the alternative A_1 versus the subjective sub-criterion C_{11} evaluated by three experts are 'Very Good' (VG), 'Good' (G), and 'Very Good' (VG), respectively. These linguistic variables can be transformed into linguistic values characterized by fuzzy numbers, which are represented as (0.75, 1, 1), (0.5, 0.75, 1), and (0.75, 1, 1) respectively. Then, the average fuzzy superiority value of alternative A_1 versus the subjective sub-criterion C_{11} can be expressed as (0.667, 0.917, 1).

Case II: For the Objective Sub-criteria

The fuzzy ratings of all alternatives versus all objective sub-criteria above the alternatives layer can be tackled by the following method [20, 21].

(a) When the appropriateness rating of alternative can be estimated effectively in values, the triangular fuzzy numbers can be used directly. For example, if the return on investment (ROI) per year is about 10%, it can be subjectively expressed as (9.4%, 10%, 10.6%).

(b) If there are historical data, e.g. let $x_2, x_2, \ldots, x_k$ represent the ROI of past k periods, the fuzzy rating of the ROI can be used the geometric mean method to express as (L, M, U), where $L = \min_i \{x_i\}$, $M = \left(\prod_{i=1}^{k} x_i\right)^{1/k}$, $U = \max_i \{x_i\}$.

For example, if the current four historical data of the ROI of alternative A_1 are 6%, 9%, 3%, and 8%, then the evaluation value can be transformed into triangular fuzzy number as (3%, $\sqrt[4]{3 \times 6 \times 8 \times 9}$%, 9%) = (3%, 6%, 9%).

2. Calculating the Fuzzy Ideal Solution and Anti-ideal Solution

The ideal and anti-ideal solutions [19] are based on the concept of relative closeness in compliance with the shorter (longer) the distance of alternative i to ideal (anti-ideal), the higher the priority can be ranked.

At first, let m and $p_1 + \ldots + p_t + \ldots + p_k = P_{sc}$ respectively denote the numbers of alternatives and the sub-criteria above the alternatives layer. Allow $X_{ij} = (c_{ij}, a_{ij}, b_{ij})$ ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, P_{sc}$) be the average fuzzy superiority value of i^{th}

alternative under j^{th} sub-criterion. To ensure compatibility between fuzzy ratings of objective criteria and linguistic ratings of subjective criteria, fuzzy superiority values must be converted to dimensionless indices. The fuzzy ideal values with minimum values in negative sub-criteria or maximum values in positive sub-criteria should have the maximum rating. Based on the principle stated as above, let $\alpha_j = \max_i\{b_{ij}\}$, $\beta_j = \min_i\{c_{ij}\}$, then the normalized fuzzy superiority value S_{ij} of i^{th} alternative under j^{th} sub-criterion can be defined as:

(1) For the positive sub-criterion j (the sub-criteria that have positive contribution to the objective, e.g., benefit sub-criterion):

$$S_{ij} = (p_{ij}, o_{ij}, q_{ij}) = (\frac{c_{ij}}{\alpha_j}, \frac{a_{ij}}{\alpha_j}, \frac{b_{ij}}{\alpha_j}) \tag{8}$$

(2) For the negative sub-criterion j (the sub-criteria that have negative contribution to the objective, e.g., cost sub-criterion):

$$S_{ij} = (p_{ij}, o_{ij}, q_{ij}) = (\frac{\beta_j}{b_{ij}}, \frac{\beta_j}{a_{ij}}, \frac{\beta_j}{c_{ij}}) \tag{9}$$

Subsequently, by using the GMIR method mentioned in subsection 4 of Section II, the GMIR value can be express as $R(S_{ij})$. The fuzzy ideal value S_j^+ and fuzzy anti-ideal value S_j^- of each sub-criterion above the alternatives layer can be judged and determined by comparing with these representation values $R(S_{ij})$. Then,

(1) if $R(S_{tj}) = \max_i R(S_{ij})$,

then the fuzzy ideal value $S_j^+ = S_{tj}$, (10)

(2) if $R(S_{kj}) = \min_i R(S_{ij})$,

then the fuzzy anti-ideal value $S_j^- = S_{kj}$. (11)

For example, assume that the average fuzzy superiority values of three alternatives (i.e. A_1, A_2, A_3, respectively) versus the positive subjective sub-criterion C_1 are denoted as $X_{11} = (0.667, 0.917, 1)$, $X_{21} = (0.417, 0.667, 0.917)$, and $X_{31} = (0.167, 0.25, 0.5)$, respectively. Using the equation (8), the normalized fuzzy superiority values of three alternatives versus C_1 are denoted as $S_{11} = (0.667, 0.917, 1)$, $S_{21} = (0.417, 0.667, 0.917)$, and $S_{31} = (0.167, 0.25, 0.5)$, respectively. Using the equation (2), the GMIR value can be expressed as $R(S_{11}) = 0.8892$, $R(S_{21}) = 0.6670$, and $R(S_{31}) = 0.2778$, respectively. Then, $R(S_{11}) > R(S_{21}) > R(S_{31})$, therefore, the fuzzy ideal value $S_1^+ = (0.667, 0.917, 1)$, and fuzzy anti-ideal value $S_1^- = (0.167, 0.25, 0.5)$ can be determined. Similarly, the average fuzzy superiority values of three alternatives versus the positive objective sub-criterion C_2 are denoted as $X_{12} = (15, 19.895, 25)$, $X_{22} = (16, 17.587, 20)$, and $X_{32} = (13, 15.845, 18)$, respectively. Then, the normalized fuzzy superiority values of three alternatives versus C_2 are denoted as $S_{12} = (0.6, 0.796, 1)$, $S_{22} = (0.64, 0.703, 0.8)$, and $S_{32} = (0.52, 0.634, 0.72)$, respectively. Then, $[R(S_{12}) = 0.7973] > [R(S_{22}) = 0.7087] > [R(S_{32}) = 0.6293]$, therefore, the fuzzy ideal value $S_2^+ = (0.6, 0.796, 1)$ and fuzzy anti-ideal value $S_2^- = (0.52, 0.634, 0.72)$ can be determined.

Finally, define the fuzzy ideal solution $I^+ = (S_1^+, S_2^+, \ldots, S_j^+, \ldots, S_{P_{sc}}^+)$ and fuzzy anti-ideal solution $AI^- = (S_1^-, S_2^-, \ldots, S_j^-, \ldots, S_{P_{sc}}^-)$, respectively.

3. Computing the Distance of Different Alternatives versus the Fuzzy Ideal Solution and Anti-ideal Solution

Let ρ_j^* ($j = 1, 2, \ldots, P_{sc}$) be the integrated weights of j^{th} sub-criterion above the alternatives layer (these integrated weights ρ_j^* can be obtained by using the criteria weights methods mentioned in Section III). Then, compute the distance of different alternatives versus I^+ and AI^- which were denoted by D_i^+ and D_i^-, respectively. Define

$$D_i^+ = \sqrt{\sum_{j=1}^{P_{sc}} \left[(\rho_j^*)^2 \times (\delta_M(S_j^+, S_{ij}))^2 \right]}, i = 1, 2, \ldots, m, \tag{12}$$

$$D_i^- = \sqrt{\sum_{j=1}^{P_{sc}} \left[(\rho_j^*)^2 \times (\delta_M(S_j^-, S_{ij}))^2 \right]}, i = 1, 2, \ldots, m, \tag{13}$$

where $\delta_M(\bullet)$ can be obtained by using the equation (3) of modified geometrical distance approach mentioned in subsection 5 of Section II.

4. Calculating the Relative Approximation Value of Different Alternatives versus Ideal Solution and Ranking the Alternatives

We calculate the relative approximation value of different alternatives A_i versus ideal solution I^+, denoted as RAV_i^*. Define

$$RAV_i^* = \frac{D_i^-}{D_i^+ + D_i^-}, i = 1, 2, \ldots, m, \tag{14}$$

It is obvious, $0 \leq RAV_i^* \leq 1$, $i = 1, 2, \ldots, m$. Suppose alternative A_i is an ideal solution (i.e. $D_i^+ = 0$), then $RAV_i^* = 1$; otherwise, if A_i is an anti-ideal solution (i.e. $D_i^- = 1$), then

$RAV_i^* = 0$. The nearer the value RAV_i^* close to 1 implies a closer alternative A_i approach to the ideal solution, i.e. the maximum value of RAV_i^*, then the optimal alternative can be ranked by a decision maker. Finally, the best alternative can be selected.

V. NUMERICAL EXAMPLE

In this section, a hypothetical example is designed to demonstrate the computational process of this fuzzy TOPSIS algorithm proposed herein.

Step 1: Assume that a shipping company needs to choose a partner to enlarge her business. Three candidates *X*, *Y*, and *Z* are chosen after a preliminary screening for further evaluation. A committee of three experts in the company, i.e., *A*, *B*, and *C*, has been formed to determine the most appropriate partner. In our simple case, four criteria and seventeen sub-criteria have been chosen and the code names of these ones are shown in parentheses. The sub-criteria above the alternative layer are classified into two groups. Seven objective sub-criteria, i.e. C_{12}, C_{13}, C_{31}, C_{32}, C_{33}, C_{41}, and C_{42} and the other ten sub-criteria are subjective. All sub-criteria are positive.

1. Complementary capabilities (C_1). This criterion includes four sub-criteria, that is, wider and deeper geographical scope (C_{11}), service channels or places (C_{12}), increase in frequency of service (C_{13}), and increase in local or regional market access (C_{14}).
2. Deeper contents and forms of collaboration (C_2). This criterion includes five sub-criteria, that is, ships fitting with the cooperative routes (C_{21}), using dedicated terminals together (C_{22}), extending interests in the integrated hinterland transport service (C_{23}), business-supported activities (C_{24}), and co-ordination of sales and marketing activities (C_{25}).
3. Financial health (C_3). This criterion includes three sub-criteria, that is, return on stockholders' equity (C_{31}), return on assets (C_{32}), and return on investment (C_{33}).
4. Adequate physical and intangible resources (C_4). This criterion includes five sub-criteria, that is, the amount of handling equipment (C_{41}), terminal hectares (C_{42}), information sharing system (C_{43}), brand and firm reputation (C_{44}), and experience sharing (C_{45}).

Step 2: Calculate relative importance weights of criteria and sub-criteria. In our case, there are five pair-wise comparison matrices to collect. The author used the four criteria ($C_1 – C_4$) in the criteria layer as an example for illustrating the computational process of fuzzy AHP. As regards to other four pair-wise comparison matrices of sub-criteria on the sub-criteria layer, these are omitted to reason by analogy.

At first, the author used the data of the relative importance of three experts' questionnaires to collect pair-wise comparison matrix and then transformed these data into triangular fuzzy numbers using the geometric mean approach, as mentioned in the Step 2 of subsection 1 of Section III. The result of the fuzzy pair-wise comparison matrix ($A_k^C = \left[\tilde{A}_{ij}^C\right]_{4\times4}$) for the criteria layer ($C_1 – C_4$) is shown as Table 1.

Table 1. The fuzzy pair-wise comparison matrix of four criteria.

	C_1	C_2	C_3	C_4
C_1	(1, 1, 1)	(1, 1.260, 2)	(3, 3.915, 5)	(3, 3.557, 5)
C_2	(0.5, 0.794, 1)	(1, 1, 1)	(1,1.587, 2)	(1, 1.817, 3)
C_3	(0.2, 0.255, 0.333)	(0.5, 0.630, 1)	(1, 1, 1)	(1, 1.817, 3)
C_4	(0.2, 0.281, 0.333)	(0.333, 0.550, 1)	(0.333, 0.550, 1)	(1, 1, 1)

Secondly, using the equations in the Step 3 of subsection 1 of Section III, the geometric mean of triangular fuzzy numbers ($\tilde{Z}_i^C$) and the fuzzy weights ($\tilde{W}_i^C$) of five criteria can be shown as Table 2 and Table 3, respectively.

Table 2. The geometric mean of triangular fuzzy numbers ($\tilde{Z}_i^C$).

$\tilde{Z}_1^C$	$\tilde{Z}_2^C$	$\tilde{Z}_3^C$	$\tilde{Z}_4^C$
(1.732, 2.047, 2.659)	(0.841, 1.230, 1.565)	(0.562, 0.735, 1)	(0.386, 0.540, 0.760)

Table 3. The fuzzy weights ($\tilde{W}_i^C$).

$\tilde{W}_1^C$	$\tilde{W}_2^C$	$\tilde{W}_3^C$	$\tilde{W}_4^C$
(0.289, 0.450, 0.755)	(0.141, 0.270, 0.444)	(0.094, 0.161, 0.284)	(0.065, 0.119, 0.216)

Then, using the equations in the Step 4 and 5 of subsection 1 of Section III, the fuzzy weights can be defuzzified by the GMIR method to obtain the crisp weights, and then, to normalize these crisp ones. The results can be shown as Table 4.

Table 4. The defuzzified and normalized weights of four criteria.

	C_1	C_2	C_3	C_4
Defuzzified weights	0.474	0.278	0.170	0.126
Normalized weights	0.452	0.265	0.162	0.121

Finally, for saving space, the author used the same computational process of fuzzy AHP for each sub-criterion to obtain the normalized weights. Then, the results of the integrated weights of criteria and sub-criteria layers can be shown as Table 5.

Step 3: Evaluate the superiority of alternatives versus all sub-criteria. By using the method presented in subsection 1 of Section IV, the superiority of alternatives versus subjective

Table 5. The normalized weights and integrated weights of criteria and sub-criteria.

Criteria	Normalized/Integrated weights (A)	Sub-criteria	Normalized weights (B)	Integrated weights (C) = (A) * (B)
C_1	0.452	C_{11}	0.403	0.1822
		C_{12}	0.225	0.1017
		C_{13}	0.233	0.1053
		C_{14}	0.139	0.0628
C_2	0.265	C_{21}	0.162	0.0429
		C_{22}	0.159	0.0421
		C_{23}	0.186	0.0493
		C_{24}	0.254	0.0673
		C_{25}	0.239	0.0633
C_3	0.162	C_{31}	0.397	0.0643
		C_{32}	0.269	0.0436
		C_{33}	0.334	0.0541
C_4	0.121	C_{41}	0.235	0.0284
		C_{42}	0.242	0.0293
		C_{43}	0.131	0.0159
		C_{44}	0.202	0.0244
		C_{45}	0.190	0.0230

Table 6. The superiority of alternatives versus subjective sub-criteria.

Sub-criteria	DMs	Linguistic values			Fuzzy scores			Fuzzy ratings		
		X	*Y*	*Z*	*X*	*Y*	*Z*	*X*	*Y*	*Z*
C_{11}	A	*VG*	*G*	*G*	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.667, 0.917, 1)	(0.417, 0.667, 0.917)	(0.167, 0.25, 0.5)
	B	*G*	*F*	*VP*	(0.5, 0.75, 1)	(0.25, 0.5, 0.75)	(0, 0, 0.25)			
	C	*VG*	*G*	*VP*	(0.75, 1, 1)	(0.5, 0.75, 1)	(0, 0, 0.25)			
C_{14}	*A*	*VG*	*VG*	*G*	(0.75, 1, 1)	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.667, 0.917, 1)	(0.583, 0.833, 1)	(0.583, 0.833, 1)
	B	*G*	*G*	*G*	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.5, 0.75, 1)			
	C	*VG*	*G*	*VG*	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.75, 1, 1)			
C_{21}	*A*	*VG*	*VG*	*G*	(0.75, 1, 1)	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.667, 0.917, 1)	(0.667, 0.917, 1)	(0.5, 0.75, 0.917)
	B	*G*	*VG*	*F*	(0.5, 0.75, 1)	(0.75, 1, 1)	(0.25, 0.5, 0.75)			
	C	*VG*	*G*	*VG*	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.75, 1, 1)			
C_{22}	*A*	*G*	*VG*	*G*	(0.5, 0.75, 1)	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.417, 0.667, 0.917)	(0.5, 0.75, 0.917)	(0.583, 0.833, 1)
	B	*F*	*F*	*G*	(0.25, 0.5, 0.75)	(0.25, 0.5, 0.75)	(0.5, 0.75, 1)			
	C	*G*	*G*	*VG*	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.75, 1, 1)			
C_{23}	*A*	*VG*	*VG*	*G*	(0.75, 1, 1)	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.667, 0.917, 1)	(0.583, 0.833, 1)	(0.583, 0.833, 1)
	B	*G*	*G*	*G*	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.5, 0.75, 1)			
	C	*VG*	*G*	*VG*	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.75, 1, 1)			
C_{24}	*A*	*G*	*G*	*G*	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.417, 0.667, 0.917)	(0.417, 0.667, 0.917)	(0.583, 0.833, 1)
	B	*G*	*F*	*G*	(0.5, 0.75, 1)	(0.25, 0.5, 0.75)	(0.5, 0.75, 1)			
	C	*F*	*G*	*VG*	(0.25, 0.5, 0.75)	(0.5, 0.75, 1)	(0.75, 1, 1)			
C_{25}	*A*	*VG*	*G*	*G*	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.667, 0.917, 1)	(0.583, 0.833, 1)	(0.333, 0.583, 0.833)
	B	*G*	*G*	*F*	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.25, 0.5, 0.75)			
	C	*VG*	*VG*	*F*	(0.75, 1, 1)	(0.75, 1, 1)	(0.25, 0.5, 0.75)			
C_{43}	*A*	*VG*	*G*	*G*	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.5, 0.75, 0.917)	(0.583, 0.833, 1)	(0.417, 0.583, 0.833)
	B	*F*	*G*	*P*	(0.25, 0.5, 0.75)	(0.5, 0.75, 1)	(0, 0.25, 0.5)			
	C	*G*	*VG*	*VG*	(0.5, 0.75, 1)	(0.75, 1, 1)	(0.75, 1, 1)			
C_{44}	*A*	*G*	*G*	*G*	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.583, 0.833, 1)	(0.417, 0.667, 0.917)
	B	*G*	*G*	*F*	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.25, 0.5, 0.75)			
	C	*G*	*VG*	*G*	(0.5, 0.75, 1)	(0.75, 1, 1)	(0.5, 0.75, 1)			
C_{45}	*A*	*VG*	*G*	*VP*	(0.75, 1, 1)	(0.5, 0.75, 1)	(0, 0, 0.25)	(0.667, 0.917, 1)	(0.417, 0.667, 0.917)	(0, 0.083, 0.333)
	B	*G*	*G*	*VP*	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0, 0, 0.25)			
	C	*VG*	*F*	*P*	(0.75, 1, 1)	(0.25, 0.5, 0.75)	(0, 0.25, 0.5)			

Table 7. The superiority of alternatives versus objective sub-criteria.

Sub-criteria	Original data				Fuzzy ratings		
	Year	X	Y	Z	X	Y	Z
C_{12}	2007	21	16	18	(15, 19.895, 25)	(16, 17.587, 20)	(13, 15.845, 18)
	2008	15	20	13			
	2009	25	17	17			
C_{13}	2007	18	16	17	(18, 22.030, 27)	(16, 19.730, 24)	(15, 16.921, 19)
	2008	27	24	19			
	2009	22	20	15			
C_{31}	2007	61.98%	83.51%	68.58%	(60.99%, 61.61%, 61.98%)	(55.28%, 73.97%, 83.51%)	(44.2%, 52.92%, 68.58%)
	2008	60.99%	55.28%	44.20%			
	2009	61.86%	87.67%	48.89%			
C_{32}	2007	16.09%	14.67%	4.90%	(11.68%, 14.33%, 16.09%)	(14.67%, 15.61%, 16.77%)	(4.9%, 12.46%, 17.04%)
	2008	11.68%	16.77%	17.04%			
	2009	15.66%	15.46%	23.17%			
C_{33}	2007	26.04%	25.33%	16.63%	(18.45%, 23.10%, 26.04%)	(22.13%, 23.66%, 25.33%)	(13.71%, 14.01%, 16.63%)
	2008	18.45%	22.13%	13.71%			
	2009	25.66%	23.63%	12.06%			
C_{41}	2007	70	90	60	(70, 82.426, 100)	(65, 74.259, 90)	(50, 59.439, 70)
	2008	100	70	50			
	2009	80	65	70			
C_{42}	2007	30500	27000	25000	(27800, 29079, 30500)	(26000, 26497, 27000)	(22000, 23300, 25000)
	2008	29000	26000	22000			
	2009	27800	26500	23000			

Table 8. The objective weights of seven objective sub-criteria.

Sub-criteria	Objective weights	Sub-criteria	Objective weights
C_{12}	0.0234	C_{33}	0.3996
C_{13}	0.1053	C_{41}	0.1657
C_{31}	0.1286	C_{42}	0.0702
C_{32}	0.1072		

sub-criteria and objective ones can be obtained, as shown in Table 6 and 7, respectively.

Step 4: Calculate the integration weights of all sub-criteria above the alternatives layer. At first, the subjective weights of ten subjective sub-criteria and seven objective ones are shown in Table 5. Secondly, by utilizing the equation (6) of subsection 2 of Section III, the objective weights of these seven objective sub-criteria can be obtained by using the data of Table 7. The results can be shown in Table 8. Finally, the adjusted integration weights of all sub-criteria above the alternatives layer w_t^* can be obtained using the method presented in subsection 3 of Section III, as shown in Table 9.

Step 5: Calculate the fuzzy ideal solution and anti-ideal solution. In our case, all sub-criteria are positive. By using the equation (8), the normalized fuzzy superiority values above the three alternatives can be obtained by using the data of Table 6 and 7. The results can be shown in Table 10.

Table 9. The adjusted integration weights of all sub-criteria.

Sub-criteria	Integrated weights	Sub-criteria	Integrated weights
C_{11}	0.1822	C_{31}	0.0644
C_{12}	0.0185	C_{32}	0.0364
C_{13}	0.0864	C_{33}	0.1684
C_{14}	0.0628	C_{41}	0.0366
C_{21}	0.0429	C_{42}	0.0160
C_{22}	0.0421	C_{43}	0.0159
C_{23}	0.0493	C_{44}	0.0244
C_{24}	0.0673	C_{45}	0.0230
C_{25}	0.0633		

By utilizing the methods in subsection 2 of Section IV, the fuzzy ideal value and fuzzy anti-ideal value of seventeen sub-criteria above the three alternatives can be determined, as shown in Table 11.

Subsequently, these fuzzy ideal and anti-ideal values can be transformed to the fuzzy ideal solution I^+ and fuzzy anti-ideal solution AI^-. That is

Table 10. The normalized fuzzy superiority values.

	X	Y	Z
C_{11}	(0.667, 0.917, 1)	(0.417, 0.667, 0.917)	(0.167, 0.25, 0.5)
C_{12}	(0.6, 0.796, 1)	(0.64, 0.703, 0.8)	(0.52, 0.634, 0.72)
C_{13}	(0.667, 0.816, 1)	(0.593, 0.731, 0.889)	(0.556, 0.627, 0.704)
C_{14}	(0.667, 0.917, 1)	(0.583, 0.833, 1)	(0.583, 0.833, 1)
C_{21}	(0.667, 0.917, 1)	(0.667, 0.917, 1)	(0.5, 0.75, 0.917)
C_{22}	(0.417, 0.667, 0.917)	(0.5, 0.75, 0.917)	(0.583, 0.833, 1)
C_{23}	(0.667, 0.917, 1)	(0.583, 0.833, 1)	(0.583, 0.833, 1)
C_{24}	(0.417, 0.667, 0.917)	(0.417, 0.667, 0.917)	(0.583, 0.833, 1)
C_{25}	(0.667, 0.917, 1)	(0.583, 0.833, 1)	(0.333, 0.583, 0.833)
C_{31}	(0.73, 0.738, 0.742)	(0.662, 0.886, 1)	(0.529, 0.634, 0.821)
C_{32}	(0.685, 0.841, 0.944)	(0.861, 0.916, 0.984)	(0.288, 0.731, 1)
C_{33}	(0.709, 0.887, 1)	(0.85, 0.909, 0.973)	(0.526, 0.538, 0.639)
C_{41}	(0.7, 0.824, 1)	(0.65, 0.743, 0.9)	(0.5, 0.594, 0.7)
C_{42}	(0.911, 0.953, 1)	(0.852, 0.869, 0.885)	(0.721, 0.764, 0.82)
C_{43}	(0.5, 0.75, 0.917)	(0.583, 0.833, 1)	(0.417, 0.583, 0.833)
C_{44}	(0.5, 0.75, 1)	(0.583, 0.833, 1)	(0.417, 0.667, 0.917)
C_{45}	(0.667, 0.917, 1)	(0.417, 0.667, 0.917)	(0, 0.083, 0.333)

I^+ = [(0.667, 0.917, 1), (0.6, 0.796, 1), (0.667, 0.816, 1), …, …, (0.583, 0.833, 1), (0.583, 0.833, 1), (0.667, 0.917, 1)].

AI^- = [(0.167, 0.25, 0.5), (0.52, 0.634, 0.72), (0.556, 0.627, 0.704), …, …, (0.417, 0.583, 0.833), (0.417, 0.667, 0.917), (0, 0.083, 0.333)].

Step 6: Compute the distance of three companies versus the fuzzy ideal solution and anti-ideal solution. In our case, by using the equations (3), (12), and (13), we can obtain the distance of three alternatives versus ideal and anti-ideal solutions, respectively. The results can be shown in Table 12.

Step 7: Calculate the relative approximation value of three alternatives versus ideal solution and rank the alternatives. By using the equation (14), the relative approximation value of three alternatives versus ideal solution can be obtained:

$$RAV_X^* = (0.1254)/(0.0208+0.1254) = 0.8577,$$

$$RAV_Y^* = (0.0962)/(0.0434+0.0962) = 0.6891,$$

$$RAV_Z^* = (0.0119)/(0.1285+0.0119) = 0.0848.$$

The ranking order of RAV_i^* for the three alternatives is X, Y, and Z, respectively. The optimal selection is obviously company X. Therefore, the committee should recommend that company X be the most appropriate partner based on the proposed fuzzy TOPSIS algorithm.

VI. CONCLUSIONS

In this paper, an integrated fuzzy TOPSIS method is proposed to improve the quality of decision-making for ranking alternatives. The proposed fuzzy TOPSIS method, involves in fuzzy MCDM problem with group decision, mainly accounts for some evaluation points, which are the classification of evaluation criteria and sub-criteria, the integrated weights of criteria and sub-criteria layers, and the performance values embedded in the decision matrix.

Table 11. Fuzzy ideal value and fuzzy anti-ideal value of seventeen sub-criteria.

Sub-criteria	Fuzzy ideal value	Fuzzy anti-ideal value
C_{11}	(0.667, 0.917, 1)	(0.167, 0.25, 0.5)
C_{12}	(0.6, 0.796, 1)	(0.52, 0.634, 0.72)
C_{13}	(0.667, 0.816, 1)	(0.556, 0.627, 0.704)
C_{14}	(0.667, 0.917, 1)	(0.583, 0.833, 1)
C_{21}	(0.667, 0.917, 1)	(0.5, 0.75, 0.917)
C_{22}	(0.583, 0.833, 1)	(0.417, 0.667, 0.917)
C_{23}	(0.667, 0.917, 1)	(0.583, 0.833, 1)
C_{24}	(0.583, 0.833, 1)	(0.417, 0.667, 0.917)
C_{25}	(0.667, 0.917, 1)	(0.333, 0.583, 0.833)
C_{31}	(0.662, 0.886, 1)	(0.529, 0.634, 0.821)
C_{32}	(0.861, 0.916, 0.984)	(0.288, 0.731, 1)
C_{33}	(0.85, 0.909, 0.973)	(0.526, 0.538, 0.639)
C_{41}	(0.7, 0.824, 1)	(0.5, 0.594, 0.7)
C_{42}	(0.911, 0.953, 1)	(0.721, 0.764, 0.82)
C_{43}	(0.583, 0.833, 1)	(0.417, 0.583, 0.833)
C_{44}	(0.583, 0.833, 1)	(0.417, 0.667, 0.917)
C_{45}	(0.667, 0.917, 1)	(0, 0.083, 0.333)

Table 12. Distance of three alternatives versus fuzzy ideal and anti-ideal solutions.

Alternatives	D_i^+	D_i^-
X	0.0208	0.1254
Y	0.0434	0.0962
Z	0.1285	0.0119

In the proposed fuzzy TOPSIS algorithm, the criteria are classified into subjective criteria and objective ones. The fuzzy AHP approach and the entropy weighting method are used to solve the subjective weights and objective ones. In addition, the adjusted integration weights are measured by combining these two methods. Due to the proposed model involves with different subjective and objective criteria, the performance values of these different criteria have different units of measurement, which should have a normalized or standardized method to eliminate the impacts of different measure units of different criteria. In this paper, the performance values of subjective criteria and of objective ones are obtained by linguistic expressions and objective evaluation values, respectively. Furthermore, a powerful defuzzification method - the Chen and Hsieh's GMIR method - and a modified distance method - the Hsieh and Chen's distance method - are employed to the integrated fuzzy TOPSIS method. Finally, a hypothetical example partner selection of a shipping company is designed to demonstrate the computational process of this fuzzy TOPSIS algorithm.

The merits of the integrated fuzzy TOPSIS model can be summarized as follows.

1. The subjective and objective criteria are simultaneously considered in the real life.
2. The subjective weights can be elicited the DM's subjective preference from using the fuzzy AHP approach.
3. The objective weights can be measured by utilizing the entropy weighting method, which can efficiently grasp the actual conditions of decision-making and express the ability and reliability of evaluation criteria/sub-criteria.
4. The GMIR method is used to sufficiently grasp the representation of fuzzy weights and fuzzy ratings and facilitate the procedure of decision-making.
5. The modified distance method can improve the quality of this integrated TOPSIS algorithm process.
6. The proposed model not only release the limitation of crisp values, but also facilitate its implementation as a computer-based decision support system for ranking alternatives in a fuzzy environment.

ACKNOWLEDGMENTS

The author would like to thank two anonymous referees for their excellent comments and valuable advice in this paper.

REFERENCES

1. Ballı, S. and Korukoğlu, S., "Operating system selection using fuzzy AHP and TOPSIS methods," *Mathematical and Computational Applications*, Vol. 14, No. 2, pp. 119-130 (2009).
2. Belton, V. and Stewart, T. J., *Multiple Criteria Decision Analysis: An Integrated Approach*, Kluwer Academic Publisher, Boston (2002).
3. Büyüközkan, G., Feyzioğlu, O., and Nebol, E., "Selection of the strategic alliance partner in logistics value chain," *International Journal of Production Economics*, Vol. 113, No. 1, pp. 148-158 (2008).
4. Chang, P. L. and Chen, Y. C., "A fuzzy multi-criteria decision making method for technology transfer strategy selection in biotechnology," *Fuzzy Sets and Systems*, Vol. 63, No. 2, pp. 131-139 (1994).
5. Chen, C. T., "Extensions of the TOPSIS for group decision-making under fuzzy environment," *Fuzzy Sets and Systems*, Vol. 114, No. 1, pp. 1-9 (2000).
6. Chen, C. T., "A fuzzy approach to select the location of the distribution center," *Fuzzy Sets and Systems*, Vol. 118, No. 1, pp. 65-73 (2001).
7. Chen, S. H. and Hsieh, C. H., "Representation, ranking, distance, and similarity of L-R type fuzzy number and application," *Australian Journal of Intelligent Information Processing Systems*, Vol. 6, No. 4, pp. 217-229 (2000).
8. Chou, C. C., "A fuzzy MADM method for solving A/R collection instruments selection problem," *Journal of Marine Science and Technology*, Vol. 15, No. 2, pp. 115-122 (2007).
9. Chou, T. Y. and Liang, G. S., "Application of a fuzzy multi-criteria decision-making model for shipping company performance evaluation," *Maritime Policy and Management*, Vol. 28, No. 4, pp. 375-392 (2001).
10. Deng, H., Yeh, C. H., and Willis, R. J., "Inter-company comparison using modified TOPSIS with objective weights," *Computers and Operations Research*, Vol. 27, No. 10, pp. 963-973 (2000).
11. Ding, J. F., "Partner selection of strategic alliance for a liner shipping company using extent analysis method of fuzzy AHP," *Journal of Marine Science and Technology*, Vol. 17, No. 2, pp. 97-105 (2009).
12. Dubois, D. and Prade, H., "Operations on fuzzy numbers," *The International Journal of Systems Science*, Vol. 9, No. 6, pp. 613-626 (1978).
13. Ertuğrul, İ. and Karakaşoğlu, N., "Comparison of fuzzy AHP and fuzzy TOPSIS methods for facility location selection," *The International Journal of Advanced Manufacturing Technology*, Vol. 39, No. 7-8, pp. 783-795 (2008).
14. Heilpern, S., "Representation and application of fuzzy numbers," *Fuzzy Sets and Systems*, Vol. 91, No. 2, pp. 259-268 (1997).
15. Hsieh, C. H. and Chen, S. H., "A model and algorithm of fuzzy product positioning," *Information Sciences*, Vol. 121, No. 1-2, pp. 61-82 (1999).
16. Hwang, C. L. and Yoon, K., *Multiple Attribute Decision Making: Methods and Application*, Springer, New York (1981).
17. Klir, G. L. and Yuan, B., *Fuzzy Sets and Fuzzy Logic Theory and Application*, Prentice-Hall, New York (1995).
18. Lee, H. S. and Chou, M. T., "A fuzzy multiple criteria decision making model for airline competitiveness evaluation," *Lecture Notes in Computer Science*, No. 4252, pp. 902-909 (2006).
19. Liang, G. S., "Fuzzy MCDM based on ideal and anti-ideal concepts," *European Journal of Operational Research*, Vol. 112, No. 3, pp. 682-691 (1999).
20. Liang, G. S. and Han, T. C., "Fuzzy critical path for project network," *International Journal of Information and Management Sciences*, Vol. 15, No. 4, pp. 29-40 (2004).
21. Saaty, T. L., *The Analytic Hierarchy Process*, McGraw-Hill, New York (1980).
22. Tsaur, S. H., Chang, T. Y., and Yen, C. H., "The evaluation of airline service quality by fuzzy MCDM," *Tourism Management*, Vol. 23, No. 2, pp. 107-115 (2002).
23. Valls, A. and Vicenc, T., "Using classification as an aggregation tool in MCDM," *Fuzzy Sets and Systems*, Vol. 115, No. 1, pp. 159-168 (2000).
24. Wang, Y. J. and Lee, H. S., "Generalizing TOPSIS for fuzzy multiple-criteria group decision-making," *Computers and Mathematics with Applications*, Vol. 53, No. 11, pp. 1762-1772 (2007).
25. Wang, Y. J., Lee, H. S., and Lin, K., "Fuzzy TOPSIS for multi-criteria decision-making," *International Mathematical Journal*, Vol. 3, No. 4, pp. 367-379 (2003).
26. Zadeh, L. A., "Fuzzy sets," *Information and Control*, Vol. 8, No. 3, pp. 338-353 (1965).
27. Zadeh, L. A., "The concept of a linguistic variable and its application to approximate reasoning, Part 1, 2 and 3," *Information Sciences*, Vol. 8, No. 3, pp. 199-249 (1975); Vol. 8, No. 4, pp. 301-357 (1975); Vol. 9, No. 1, pp. 43-80 (1976).
28. Zeleny, M., *Multiple Criteria Decision Making*, McGraw-Hill, New York (1982).