PENETRATION LAG OF CHLORIDE DIFFUSION THROUGH CONCRETE PLATE BASED ON ADVANCING MODEL

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Key words: chloride, concrete, diffusion, penetration lag.

ABSTRACT

The solution, based on Neumann's theory, for the content distribution of a diffusion material in a semi-infinite medium is applicable for the case in which a diffusion material diffuses from one side of a plate medium to the other side. After the diffusion material reaches the other side, the penetration rate of the diffusion material is solved using Fick's diffusion law. In this study, concrete plates are manufactured and submerged individually in special containers with saltwater on one side and freshwater on the other side, and no electric charge is applied. The amount of chloride penetrating the plate is monitored. Data are used to derive the diffusivity and Neumann's constant, which are used to characterize penetration lag and rate. The value of Neumann's constant is derived for the first time. This advancing model for a plate-shaped medium is also novel, and makes calculation of the penetration lag and diffusion depth possible.

I. INTRODUCTION

The penetration lag of chloride through a concrete plate is as important as the diffusivity of chloride diffusion in concrete. Penetration lag and diffusivity determine the service life of reinforced concrete structures in salty environments. Fick's diffusion law [6] is conventionally applied to derive the chloride penetration rate through concrete plates [8]. Penetration is caused by natural diffusion, which is a prolonged process. Methods for rapid chloride diffusion applied an external electrical field. Rapid Chloride Permeability Test (RCPT) method

(ASTM C1202-97 [1]) developed by Whiting [14] has been widely applied in recent years. However, McCarter *et al.* [9] reported that the measured conductivity is adversely affected by the raised temperature in RCPT. Many researchers improved the RCPT, such as Tang and Nilsson [11], Meck and Sirivivatnanon [10], and others [3, 7, 12, 13, 17, 18, 20]. Another technique, called Accelerated Chloride Migration Test (ACMT) was developed to improve the accuracy for the measurement of the chloride migration rate in steady state [16].

The problem caused by the raised temperature in an accelerated test by an electrical charge has not been fully solved, and the effect of temperature on diffusion is material-dependent, which is not easy to characterize. Another significant problem is that the decomposition of concrete under an electrical charge, which totally changes the micro-structure of concrete. To prevent these problems, natural diffusion is applied in this study; natural diffusion is very time consuming but it is much closer to real-world situations than the accelerated ones.

Based on Fick's diffusion law, the chloride content in concrete induced by a salty environment is derived. The solution for content of a diffusing material encounters a serious problem for the case of a semi-infinite medium domain. The solution indicates that the content of a diffusion material cannot be zero, even at the deepest part of the medium and at the very beginning of diffusion. In reality, molecules of a diffusion material need time to travel from the surface contacting the diffusion material to a deep location in the medium. An advancing diffusion front tip indicating diffusion depth should exist. A no-zero solution for content of a diffusion material everywhere inside a medium at the start of the diffusion process does not make sense. The solution of diffusion material content in medium was modified by Carslaw and Jaeger based on Neumann's algorithm for heat conduction [2]. The modified solution yields diffusion depth as a function of diffusion duration.

For the case in that a molecule in the diffusion material starts its diffusion trip from one surface of a plate medium toward the other surface, before the molecule reaches the other surface, this case is exactly the same as that in a semi-infinite medium domain, and the solution for the content of the diffusion material for a semi-infinite medium domain is applied.

However, the problem must be solved separately after that to derive the content [4, 5, 15, 19].

In the experiment of this study, round concrete plates are manufactured and connected individually to two angled polyvinylchloride (PVC) pipes on their both sides to form a U-type container. The pipes are filled with saltwater and freshwater. Freshwater means water with undetectable chloride content which is smaller than 0.0005% by weigh for our equipment. The total amount of chloride appear in the freshwater side is analyzed based on the proposed algorithm to derive the diffusivity and Neumann constant [15, 19]. Results are utilized to quantify the chloride content in concrete submerged in saltwater. This process is time consuming, but closely mirrors real cases in nature and does not use electric charge, which always causes problems.

II. THEORY

The bit stream to be transmitted is organized in frames as shown in Fig. 1. The preamble and the end code indicate the start and the end of a message frame, respectively. They are used by the receiver for the frame synchronization. The training sequence is used by the adaptive equalizer in the receiver for optimizing the equalizer coefficients. The information-bearing data follow the training sequence. The gap between the preamble and the training sequence is for ensuring the completion of the frame synchronization in the receiver before the reception of the training sequence.

The governing equation of Fick's diffusion law for one-dimensional diffusion is

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} \tag{1}$$

where c is the content of the diffusion material defined as the weight of the diffusion material per unit volume of medium, t is diffusion duration, D is diffusivity, and x is the coordinate in the diffusion direction. Notably, c is dependent on both t and x. For a semi-infinite medium (Fig. 1), when the diffusion material crosses the surface of the medium contacting the diffusion material and diffuses in the medium where initially has no diffusion material, the content of the diffusion material in the medium is

$$c(t;x) = c_\infty [1 - \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} (-1)^n \frac{(\frac{x}{2\sqrt{Dt}})^{2n+1}}{n!(2n+1)}] \tag{2}$$

where c_∞ is the saturated c [6]. The boundary of the medium is assumed saturated when it contacts the diffusion material, meaning that the medium surface where $x = 0$ (Fig. 1) is c_∞ from the very beginning and remains constant throughout the whole process. Eq. (2) ensures that the boundary condition is $c = c_\infty$ at $x = 0$. The initial condition is $c = 0$ in the medium
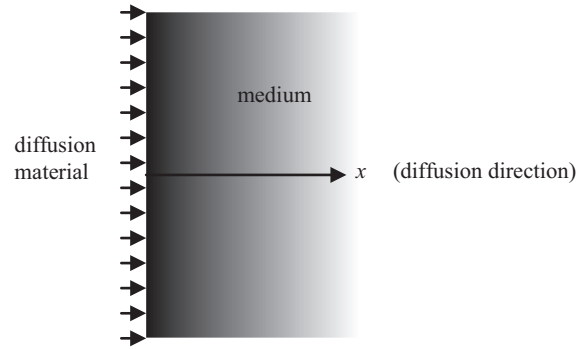

Fig. 1. Diffusion material penetrating the surface of a semi-infinite medium and diffusing toward the deeper inside of the medium.

when t approaches 0, and the final condition is $c = c_\infty$ when t approaches ∞. However, a problem occurs in that c cannot be zero regardless of the depth of x and how short t is as long as t is not zero. Molecules of the diffusing material cannot reach everywhere in the medium instantly; that is a reasonable amount of time is required.

Neumann modified Eq. (2) for heat conduction, and Carslaw and Jaeger applied the modification to material diffusion [4, 19], such that Eq. (2) becomes

$$c(t;x) = \begin{cases} c_\infty [1 - \dfrac{\displaystyle\sum_{m=0}^{\infty} \dfrac{(-1)^m (\frac{x}{2\sqrt{Dt}})^{2m+1}}{m!(2m+1)}}{\displaystyle\sum_{m=0}^{\infty} \dfrac{(-1)^m v^{2m+1}}{m!(2m+1)}}], & \text{for } x \leq 2v\sqrt{Dt} \\ 0, & \text{for } x \geq 2v\sqrt{Dt} \end{cases} \tag{3}$$

In Eq. (3), v is called Neumann's constant. The first equation on the right side of Eq. (3) satisfies Eq. (1); however, x must not exceed $2v\sqrt{Dt}$, or it becomes negative, which does not make sense. When x exceeds $2v\sqrt{Dt}$, $c(t; x)$ is zero for the undisturbed area. As a mater of fact, $2v\sqrt{Dt}$ is the location of the diffusion front, which is actually diffusion depth. Eq. (3) is called the advancing model for Fick's diffusion law.

For a different case, the diffusion material crosses the left surface of an h-thick plate-shaped medium, flows into the medium, diffuses through the medium toward the right surface and flees into the "free" space on the right side (Fig. 2). Before the diffusion material reaches the right surface, the right surface has no affect on the diffusion, and this case at this stage is exactly the same as diffusion in a semi-infinite medium. The critical moment when diffusion material reaches the right surface is $t_c = \dfrac{h^2}{4v^2 D}$, which is derived by letting h to equal $2v\sqrt{Dt}$. The assumption that the left boundary of the medium becomes saturated instantly when it contacts the diffusion material still holds.
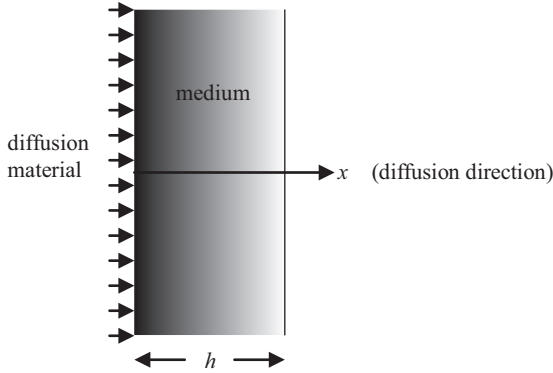
Fig. 2. Diffusion material penetrating the left surface of a plate shape medium and diffusing toward the right surface of the medium.

When the diffusion material reaches the right surface of the plate-shaped medium, this case is no longer the same as a semi-infinite case. The limitation of plate thickness causes the content solution to differ from Eq. (3) [4]. Let the critical moment be the initial moment of the following step, which considers the right boundary. At this time, the content of the diffusion material is

$$c(t_c;x) = c_\infty[1 - \frac{\sum_{m=0}^{\infty} \dfrac{(-1)^m (\frac{x}{h})^{2m+1}}{m!(2m+1)}}{\sum_{m=0}^{\infty} \dfrac{(-1)^m v^{2m+1}}{m!(2m+1)}}], \; for \; 0 \le x \le h \qquad (4)$$

The right boundary is assumed "free," meaning $c(t; h) = 0$, which is satisfied by Eq. (4).

Solve Eq. (1) for this plate medium case. Adopt the separation variable technique and let

$$c(t;x) = X(x)T(t) \qquad (5)$$

where X is a function of x, and T is a function of t. Based on Eq. (5), Eq. (1) is rearranged into

$$\frac{\dot{T}}{T} = D\frac{X''}{X} \qquad (6)$$

In Eq. (6), D is a constant, the left side contents functions of t only, and the right side contents functions of x only. Both sides should be constant to make Eq. (6) valid. Let

$$\frac{\dot{T}}{T} = D\frac{X''}{X} = -k, \quad k : constant \qquad (7)$$

Thus,

$$\begin{cases} \dot{T} + kT = 0 \\ X'' + \dfrac{k}{D}X = 0 \end{cases} \qquad (8)$$

For $k = 0$, the particular solutions of T and X are

$$\begin{cases} T = K_1, \quad K_1 : constant \\ X = K_2 + K_3x, \quad K_2, K_3 : constants \end{cases} \qquad (9)$$

For $k \ne 0$, the harmonic solutions of T and X are

$$\begin{cases} T = K_4 e^{-kDt}, \quad K_4 : constant \\ X = K_5\cos(\sqrt{k}x) + K_6\sin(\sqrt{k}x), \quad K_5, K_6 : constants \end{cases} \qquad (10)$$

To prevent T from becoming ∞ when $t \to \infty$, k must be positive for a positive D. For the multi-values of k, kn, $n = 1$ to ∞, Eq. (10) becomes

$$\begin{cases} T = K_{4n} e^{-k_n Dt}, \quad k_n, K_{4n} : constants \\ X = K_{5n}\cos(\sqrt{k_n}x) + K_{6n}\sin(\sqrt{k_n}x), \quad K_{5n}, K_{6n} : constants \end{cases} \qquad (11)$$

The general form of c is the combination of the particular solution and the harmonic solution.

$$\begin{aligned} c(t;x) &= K_1K_2 + K_1K_3x + \sum_{n=1}^{\infty}[K_{4n}K_{5n}e^{-k_n Dt}\cos(\sqrt{k_n}x) \\ &\quad + K_{4n} K_{6n}e^{-k_n Dt}\sin(\sqrt{k_n}x)] \\ &= \alpha_0 + \alpha_1 x + \sum_{n=1}^{\infty}[\alpha_{2n}e^{-k_n Dt}\cos(\sqrt{k_n}x) \\ &\quad + \alpha_{3n}e^{-k_n Dt}\sin(\sqrt{k_n}x)] \\ & for \; 0 \le x \le h, \quad \alpha_0, \alpha_1, \alpha_{2n}, \alpha_{3n} : constants \end{aligned} \qquad (12)$$

At the initial moment when $t = t_c$, Eq. (12) becomes

$$\begin{aligned} c(t_c;x) &= \alpha_0 + \alpha_1 x + \sum_{n=1}^{\infty}[\alpha_{2n}e^{-k_n Dt_c}\cos(\sqrt{k_n}x) \\ &\quad + \alpha_{3n}e^{-k_n Dt_c}\sin(\sqrt{k_n}x)] \end{aligned} \qquad (13)$$

and the boundary conditions are

$$c(t; x) = \begin{cases} c_\infty, x = 0 \\ 0, x = h \end{cases} \qquad (14)$$

The process employed herein is called the Two Sides Anti-symmetrical Images (TSAI) technique [4, 15]. From Eqs. (13, 14),

$$c(t_c;x) - \alpha_0 - \alpha_1 x = \begin{cases} c_\infty - \alpha_0 - \alpha_1 x, x = 0 \\ \sum_{n=1}^{\infty}[\alpha_{2n}e^{-k_n Dt_c}\cos(\sqrt{k_n}x) \\ \quad + \alpha_{3n}e^{-k_n Dt_c}\sin(\sqrt{k_n}x)], 0 < x < h \\ -\alpha_0 - \alpha_1 x, x = h \end{cases} \qquad (15)$$

According to Eq. (14), make both sides of Eq. (15) zero at $x = 0$ and h by letting $\alpha_0 = c_\infty$ and $\alpha_1 = \dfrac{-c_\infty}{h}$ to make the expression of Fourier series for Eq. (15) possible. Eq. (15) then becomes

$$c(t_c;x) - c_\infty + \frac{c_\infty}{h}x = \begin{cases} 0, & x = 0 \\ \displaystyle\sum_{n=1}^{\infty}[\alpha_{2n}e^{-k_nDt_c}\cos(\sqrt{k_n}\,x) \\ \quad + \alpha_{3n}e^{-k_nDt_c}\sin(\sqrt{k_n}\,x)], & 0 < x < h \\ 0, & x = h \end{cases} \tag{16}$$

Expand the right side of Eq. (16) into a periodic function $c_p(x)$ with a period of $2h$.

$$c_p(x) = \begin{cases} -[c(t_c;-x) - c_\infty + \dfrac{c_\infty}{h}(-x)], & -\dfrac{1}{2}h \leq x < 0 \\ 0, & x = 0 \\ c(t_c;x) - c_\infty + \dfrac{c_\infty}{h}x, & 0 < x < h \\ 0, & x = h \\ -[c(t_c;(2h-x)) - c_\infty + \dfrac{c_\infty}{h}(2h - x)], & h < x < \dfrac{3}{2}h \end{cases} \tag{17}$$

where $c_p(x)$ in the area of $h < x < 3h/2$ is the anti-symmetrical image of $c_p(x)$ in the area of $h/2 < x < h$, and $c_p(x)$ in the area of $-h/2 < x < 0$ is the anti-symmetrical image of $c_p(x)$ in the area of $0 < x < h/2$. Actually, $c_p(x)$ in the area of $-h/2 \leq x \leq 0$ and $h \leq x \leq 3h/2$ do not exist because they are not in the plate medium; That is, they are mathematical conveniences created to make $c_p(x)$ anti-symmetrical with respect to $x = 0$ and h to simplify the expression of Fourier series of Eq. (17). Under this delicate arrangement, $c_p(x)$ is exactly the same as $c(t_c;x) - c_\infty + (c_\infty/h)x$ in the area of $0 \leq x \leq h$ where the medium exists, and the solution for content of the diffusion material is interested. Expand Eq. (17) by Fourier series.

$$c_p(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos\frac{2n\pi x}{2h} + b_n\sin\frac{2n\pi x}{2h}\right) \tag{18}$$

where

$$a_0 = \frac{1}{2h}\int_{-h/2}^{3h/2}c_p(x)dx = 0 \tag{19}$$

$$a_n = \frac{2}{2h}\int_{-h/2}^{3h/2}c_p(x)\cos\frac{2n\pi x}{2h}dx = 0 \tag{20}$$

$$b_n = \frac{2}{2h}\int_{-h/2}^{3h/2}c_p(x)\sin\frac{2n\pi x}{2h}dx$$

$$= 2c_\infty\frac{\displaystyle\sum_{m=1}^{\infty}\sum_{r=1}^{m}\frac{(-1)^{n+m+r}(2m)!v^{2m+1}}{m!(2m+1-2r)!(n\pi)^{2r+1}}}{\displaystyle\sum_{m=0}^{\infty}\frac{(-1)^m v^{2m+1}}{m!(2m+1)}} \tag{21}$$

The TSAI technique removes the constant a_0 and coefficients of cosine terms a_n, leaving only the coefficients of sine terms b_n. The surviving sine terms are set to fit the boundary conditions such that $c_p(x) = c(t_c;x) - c_\infty + (c_\infty/h)x$ is zero at $x = 0$ and h. Comparing Eq. (16) and Eq. (18) in the area of $0 \leq x \leq h$ where the medium exists obtains

$$k_n = \left(\frac{n\pi}{h}\right)^2 \tag{22}$$

$$\alpha_{2n}e^{-\frac{k_n h^2}{4v^2}} = a_n = 0 \tag{23}$$

$$\alpha_{3n}e^{-\frac{k_n h^2}{4v^2}} = b_n = 2c_\infty\frac{\displaystyle\sum_{m=1}^{\infty}\sum_{r=1}^{m}\frac{(-1)^{n+m+r}(2m)!v^{2m+1}}{m!(2m+1-2r)!(n\pi)^{2r+1}}}{\displaystyle\sum_{m=0}^{\infty}\frac{(-1)^m v^{2m+1}}{m!(2m+1)}} \tag{24}$$

Eq. (12) then becomes

$$c(t;x) = c_\infty\Bigg\{1 - \frac{x}{h} + 2\sum_{n=1}^{\infty}\Bigg[e^{-\left(\frac{n\pi}{h}\right)^2 D\left(t - \frac{h^2}{4v^2D}\right)}$$

$$\frac{\displaystyle\sum_{m=1}^{\infty}\sum_{r=1}^{m}\frac{(-1)^{n+m+r}(2m)!v^{2m+1}}{m!(2m+1-2r)!(n\pi)^{2r+1}}}{\displaystyle\sum_{m=0}^{\infty}\frac{(-1)^m v^{2m+1}}{m!(2m+1)}}\sin\frac{n\pi x}{h}\Bigg]\Bigg\}$$

$$\text{for } 0 \leq x \leq h \text{ and } t \geq t_c = \frac{h^2}{4v^2D} \tag{25}$$

where t_c is the critical diffusion duration when the diffusion material reaches the right surface of the plate medium (Fig. 2), which is called the penetration lag which is the duration required for the diffusion material to penetrate the plate-shaped medium with thickness h. Characterization of the penetration lag according to advancing model is novel and is the major contribution of this study. Next, the diffusion material starts flowing into the free space on the right side of the plate medium. Based on Fick's first diffusion law, flow rate at $x = h$ is

$$\frac{\partial w}{\partial t} = -AD\frac{\partial c}{\partial x}\Big|_{x=h} = AD\frac{c_\infty}{h}\Bigg\{1 - 2\sum_{n=1}^{\infty}\Bigg[e^{-\left(\frac{n\pi}{h}\right)^2 D\left(t - \frac{h^2}{4v^2D}\right)}$$

$$\frac{\displaystyle\sum_{m=1}^{\infty}\sum_{r=1}^{m}\frac{(-1)^{m+r}(2m)!v^{2m+1}}{m!(2m+1-2r)!(n\pi)^{2r}}}{\displaystyle\sum_{m=0}^{\infty}\frac{(-1)^m v^{2m+1}}{m!(2m+1)}}\Bigg]\Bigg\} \text{ for } t \geq t_c = \frac{h^2}{4v^2D} \tag{26}$$
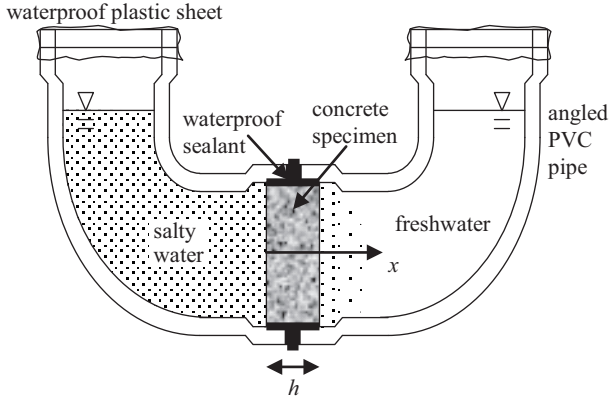
Fig. 3. U shape diffusion fixture for diffusion test using.

Table 1. Percentages by weight of ingredients of concrete.

Ingredient	Water	Type I cement	Coarse Aggregate	Sand
Percentage by Weight	8.67%	12.51%	34.71%	42.58%

Table 2. Percentages by weight of each grain size in coarse aggregate and sand.

U.S. standard sieve size	37.5 mm	25 mm	19 mm	12.5 mm	#4	#8	#16	#30
coarse aggregate	0	0	20	60	9	9	1	1
U.S. standard sieve size	#4	#8	#16	#30	#50	#100	#200	pan
sand	0	16	28	21	16	12	5	2

where w is the weight of the diffusion material flowing out of the right surface of the plate medium, and A is the area of the surface penetrated. After a duration t, the accumulated weight W of the diffusion material collected from the right chamber (Fig. 3) is

$$W = -AD\int_{t_c}^{t} \frac{\partial c}{\partial x}\Big|_{x=h}\, d\tau\sqrt{a^2+b^2} = AD\frac{c_\infty}{h}\{t - \frac{h^2}{4v^2D} - 2\frac{h^2}{D}$$

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{r=1}^{m} \frac{[1-e^{-(\frac{n\pi}{h})^2 D(t-\frac{h^2}{4v^2D})}](-1)^{m+r}(2m)!v^{2m+1}}{m!(2m+1-2r)!(n\pi)^{2(r+1)}}$$
$$\overline{\sum_{m=0}^{\infty}\frac{(-1)^m v^{2m+1}}{m!(2m+1)}}\}$$

$$for\ t \ge t_c = \frac{h^2}{4v^2D} \qquad (27)$$

where τ is diffusion duration, and $t_c \le \tau \le t$. Measuring W experimentally and analyzing W versus t theoretically to obtain the parameters in Eq. (27) are major tasks in this study.

III. EXPERIMENT

In the study case, cylindrical concrete specimens 10 cm in diameter are sliced into 2.0 cm-thick, disk-shaped specimens. Type I cement is used. Mix design for the concrete and grain sizes of coarse aggregate and sand are shown in Tables 1, 2. The edge of each specimen is sealed with epoxy glue, rendering it waterproof. Half of the thickness on the left side of each specimen is squeezed into an angled PVC pipe with a 10cm inner diameter, and the other half on the other side is squeezed into another angled PVC pipe to form a U-shaped fixture with the disk-shaped concrete specimen blocked at its center. The angled PVC pipe on the left side is filled with 1000 c.c. of saltwater with 0.1 g/cm³ chloride. The other pipe is filled with freshwater of same volume (Fig. 3). The open tops of the fixture are covered with plastic sheets and sealed by rubber bands to prevent moisture vaporization.

Chloride dissolved in water is the diffusion material, and water-logged concrete is the medium. This experiment is conducted in conditional chamber, in which the temperature is maintained at $25 \pm 1°C$. The saltwater is replaced weekly to maintain the chloride content, and the water on the other side of the specimen is removed weekly and tested for its chloride content to calculate the total weight of chloride flowing out of the specimen. The pipe is refilled with freshwater every time the water is removed. Each experiment takes 14 weeks.

Eq. (27) has three parameters: c_∞, v and D. Originally, by fitting Eq. (27) to the data for W versus t, the three parameters can be derived using the least square method. However, the exponential term in Eq. (27) approaches zero so quickly that the penetration rate stabilizes in just a couple of days after chloride penetrates the concrete specimen (Fig. 4); thus, Eq. (27) becomes

$$W = AD\frac{c_\infty}{h}(t - \frac{h^2}{4v^2D}) \qquad (28)$$

Eq. (28) is a linear function with two parameters: a slope of ADc_∞/h, representing the accumulation rate; and an x-intercept of $h^2/(4v^2D)$ representing penetration lag. The transition period from Eq. (27) to Eq. (28) is much shorter than total experiment duration. The data for W with respect to t become liner instantly once chloride penetrates the specimen. Numerically, when the data of are almost liner, deriving three unknowns, c_∞, v and D, from the data generates solutions with unacceptable variation. One unknown must be obtained using another method.

Under the same condition, Eq. (25) becomes

$$c(t; x) = c_\infty(1 - \frac{x}{h}) \qquad (29)$$

Table 3. Saturated chloride content, diffusivity, neumann's constant and critical diffusion duration of concrete under 25 ± 1°C in salty water with 0.1 g/cm³ chloride content.

Specimen	H (cm)	c_∞ (g/cm³)	D (cm²/day)	v	t_c (day)
#1	2.0	0.0232	0.00748	2.69	18.5
#2	2.0	0.0273	0.00767	2.62	19.0
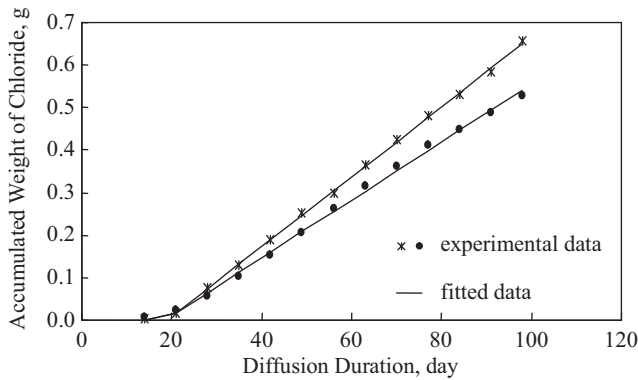
Fig. 4. Two typical data sets of accumulated weight of chloride penetrating 2.0 cm thick disk shape concrete specimen versus diffusion duration under 25°C.

The average content of chloride in the concrete disk is half c_∞, and 14 weeks is much more than enough for penetration rate to stabilize [15]. After each experiment, the specimen is removed from the fixture and crushed into pieces smaller than 0.5 cm in diameter, except for the coarse aggregate, which remains around the original size. The crushed pieces are submerged in freshwater for 24 hours to weigh the chloride extracted from the concrete. The weight of the chloride is divided by the specimen volume to obtain $c_\infty/2$ and then calculate c_∞.

After c_∞ is obtained, the data for W versus t are fitted by Eq. (27) using the least square method to derive D and v. Fig. 4 shows a typical set of data fitted by Eq. (27) for a 2.0 cm-thick. Table 3 shows the derived c_∞, D and v from two experiments. Convention solution of the W versus t can not be applied here, because it has no penetration lag at all.

IV. DISCUSSION AND CONCLUSIONS

The conventional solution for the content of a diffusion material in a semi-infinite medium derived by Eq. (2) can never give the diffusion depth, because the solution approaches zero asymptotically, but never becomes zero, regardless diffusion duration and location depth. A molecule of the diffusion material cannot reach deeply inside a medium instantly when the diffusion material contacts the medium. Neumann modified Eq. (2) ingeniously as Eq. (3) and solved this problem [6]. In Eq. (3), critical time $t_c = h^2/(4 v^2 D)$ does not exist in Eq. (2),

and is the time required for a diffusion material to travel a distance of h, and diffusion depth is $2v\sqrt{Dt}$. Applying Neumann's idea to a plate medium and solving the content of diffusion material are the most important contributions of this study.

Monitoring diffusion depth experimentally is extremely difficult, because the diffusion front tip does not have a clear cut. In this study, a novel experimental procedure is designed to collect data for accumulated weight of the diffusion material penetrating a plate medium and to obtain the saturated content, both of which are used to derive diffusion parameters. The novel experimental procedure makes theoretical analysis easy.

The value of Neumann's constant, v, is derived experimentally for the first time in diffusion mechanics. The advancing model for a plate-shaped medium is also novel, and makes calculating the penetration lag and diffusion depth of diffusion material penetrating a plate-shaped medium possible.

The experiment in this study models natural diffusion, which lasts months. Although the duration of this experiment is considerable, the experiment does not use an electrical charge, which does not exist in real cases.

An interesting topic is that how environmental conditions such as temperature effect the diffusivity, Neumann's constant and saturated content of the diffusing substance for different materials in different diffusion cases. This topic contents too much work to be included herein and too important to be neglected in the future.

V. NOTATIONS

A = diffusion area of specimen
a_0, a_n, b_n = coefficients of Fouier series
c = content of diffusion material
$c_p(x)$ = periodical function of x
$c(t; x)$ = c as a function of t and x
c_∞ = saturated c
D = diffusivity
h = thickness of a medium or specimen
$K_{1\sim4}$, $K_{4n\sim6n}$, k, k_n = constants
m, n, r = indices
T = function of t
t = diffusion duration
t_c = critical t
X = function of x
W = accumulated weight of diffusion material penetrating through plate medium
w = weight of diffusion material penetrating a plate medium
x = coordinate in the diffusion direction
α_0, α_1, α_{2n}, α_{3n} = constants
v = Neumann's constant
τ = diffusion duration $t_c \le \tau \le t$

REFERENCES

1. ASTM Standard C1202-97, "Standard test method for electrical indica-

tion of concrete's ability to resist chloride ion penetration," *Annual Book of ASTM Standards, ASTM International*, West Conshohocken, PA (1997).

2. Carslaw, H. S. and Jaeger, J. C., *Condition of Heat in Solids*, Clarendon Press, Oxford, NY (1959).

3. Castellote, M., Andrade, C., and Alonso, C., "Modeling of the processes during Steady-State migration test: quantification of transference numbers," *Materials and Structures*, Vol. 32, No. 217, pp. 180-186 (1999).

4. Chang , T. W., Wang, C. H., and Tsai, C. L., "Advancing boundary model for moisture diffusion in a composite laminate," *Journal of Composite Materials*, Vol. 42, No. 10, pp. 957-974 (2008).

5. Cheng, M. C., *Hygric Characterization of Carbon/Epoxy Composites and Hygric Strain Measurement,* Ph.D. Dissertation, Graduate School of Engineering Science & Technology, National Yun-Lin University of Science & Engineering, Yun-Lin, Taiwan (2005).

6. Crank, J., *The Mathematics of Diffusion*, 2nd ed., Oxford University Press, Oxford, NY (1975).

7. Delagrave, A., Marchaud, J., and Samson, E., "Prediction of diffusion coefficients in cement-based materials on the basis of migration experiments," *Cement and Concrete Research*, Vol. 26, No. 12, pp. 1831-1842 (1996).

8. Li, Z., Peng, J., and Ma, B., "Investigation of chloride diffusion for high-performance concrete containing fly ash, microsilica and chemical admixtures," *ACI Materials Journal*, Vol. 96, No. 3, pp. 391-396 (1999).

9. McCarter, W. J., Starrs, G., and Chrisp, T. M., "Electrical conductivity, diffusion, and permeability of Portland cement-based mortars," *Cement and Concrete Research*, Vol. 30, No. 9, pp. 1395-1400 (2000).

10. Meck, E. and Sirivivatnanon, V., "Field indicator of chloride penetration depth," *Cement and Concrete Research*, Vol. 33, pp. 1113-1117 (2003).

11. Tang, L. and Nilsson, L. O., "Rapid determination of the chloride diffusivity in concrete by applying an electrical field," *ACI Materials Journal*, Vol. 89, pp. 49-53 (1992).

12. Tong, L. and Gjorv, O. E., "Chloride diffusivity based on migration testing," *Cement and Concrete Research*, Vol. 31, No. 7, pp. 973-982 (2001).

13. Truc, O., Ollivier, J. P., and Carcasses, M., "A new way for determining the chloride diffusion coefficient in concrete from Steady-State migration test," *Cement and Concrete Research*, Vol. 30, No. 2, pp. 217-226 (2003).

14. Whiting, D., "Rapid measurements of chloride permeability of concrete," *Public Works*, Vol. 45, No. 3, pp. 101-112 (1981).

15. Wu, C. L., *Experimental Analysis of Chloride Diffusing through Concrete Based on Advancing Mode*, Master Dissertation, Department of Construction Engineering, National Yunlin University of Science & Technology, Yun-Lin, Taiwan (2009).

16. Yang, C. C. and Chiang, S. C., "A rapid method for determination of the chloride migration coefficient in concrete using electrical field," *Journal of ASTM International*, Vol. 5, No. 4, Paper ID JAI101409 (NSC 96-2628-E-019-MY3) (2008).

17. Yang, C. C. and Cho, S. W., "The relationship between chloride migration rate for concrete and electrical current in Steady State using the accelerated chloride migration test," *Materials and Structures*, Vol. 37, No. 271, pp. 456-463 (2004).

18. Yang, C. C., Cho, S. W., and Hung, R., "The relationship between charge passed and the chloride-ion concentration in concrete using Steady-State chloride migration test," *Cement and Concrete Research*, Vol. 32, No. 2, pp. 217-222 (2002).

19. Yeh, M. Y., *Modification of Fick's Diffusion Model for Chloride Diffusing in Concrete,* Master Dissertation, Department of Construction Engineering, National Yunlin University of Science & Technology, Yun-Lin, Taiwan (2007).

20. Zhang, M. H. and Gjorv, O. E., "Permeability of high strength lightweight concrete," *ACI Materials Journal*, Vol. 88, No. 5, pp. 463-469 (1991).