COMPUTATION PROGRAMS OF THE ASTRONOMICAL VESSEL POSITION

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ABSTRACT

In open sea sailing, as opposed to sailing with satellite navigation systems, the astronomical vessel position (AVP) is not limited by military codes. If programs can be developed to solve the AVP directly, drawbacks of current methods for performing marine operations can be greatly improved. Complete AVP computations have two necessary points: compute the observed altitude and compute the AVP for two bodies. Thus, this paper utilizes Matlab® programming language to develop the observed altitude (ObsAltPro) and the AVP by using intercept method (IMPro-2) programs, respectively. Adopting the ObsAltPro can avoid the use of nautical almanacs and directly obtain the observed altitudes of various celestial bodies between 1986 and 2050. As for merits of using the IMPro-2, they can skip limits of tabular methods and directly use the dead reckoning (DR) as the input variable to replace the assumed position (AP). To address the deficiencies of the intercept method (IM), iteration computation is added into the latter program as a verification function. The above two programs, through demonstrated examples, have shown advantages of being simple, fast, and accurate.

I. INTRODUCTION

For ocean navigation, the main approaches of positioning are the classical method of astronomical vessel position (AVP) and advanced satellite navigation systems. Due to the complicated, tedious and time consuming nature of current AVP and the accuracy limitations due to tabular methods, the majority of navigators prefer using satellite navigation systems. However, the AVP has the advantage of not being limited by military codes. If assisted by computerized programs, the practical operation process can be greatly improved. Thus, in searching for computation software for the AVP, it was discovered that the software developed by the U.S. Navy is not purchasable [5].

Other softwares, NavPac developed by a British research organization [7], Navigator [12] and STARPILOT PC [13] developed by private companies, use the intercept method (IM). However, they do not have the verification function of iteration computation. This disparity between expectations and reality leads to the motivation of this article.

The task in developing the AVP computational programs is to understand the process. Due to the complicated astronomical algorithms that are involved with nautical almanacs, this paper will not discuss this topic in detail. Other than that, as the IM is the main way of the sight reduction methods, which requires more than two lines of position (LOPs) to obtain AVP this paper will develop the programs aiming at "the observed altitude of various celestial bodies," namely, ObsAltPro and "the intercept method for solving AVP for two bodies condition," namely, IMPro-2, respectively as shown in Fig. 1.

Apart from this introduction, the following sections are arranged as follows. Sections II organizes the computation procedures of ObsAltPro and IMPro-2. Based on this, sections III uses MATLAB® to develop the programs. Sections IV then verifies the self-developed programs through demonstrated examples. Finally, sections V provides some concrete conclusions.

II. ORGANIZING COMPUTATION PROCEDURES

1. Computation Procedures of Observed Altitude from Sextant Altitude

This paper uses the nautical almanac tabular formulae [6] to establish the computation procedures in conjunction with the semi-diameter (*SD*) of Sun and the horizontal parallaxes (*HP*) of the Moon, Venus, and Mars obtained from the works of Jean Meeus published in 1998 [2, 3, 10]. These formulae are only suitable for use between 1986 and 2050. The organized computation procedures are as follows:

Step 1: Calculate the dip of the horizon (*Dip*).

$$Dip = 0.0293° \times \sqrt{he}, \qquad (1)$$

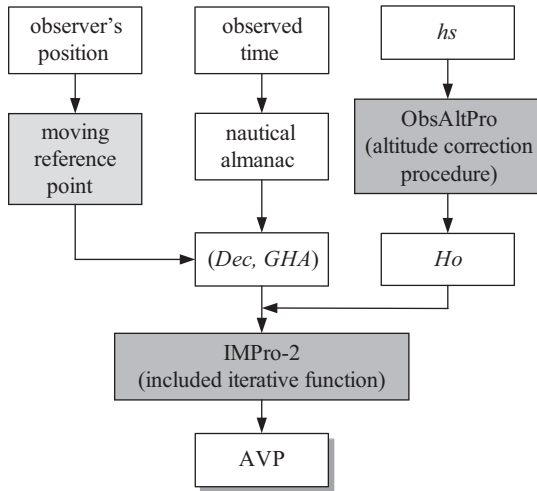where *he* is the height of eye above the horizon in meters.

Fig. 1. Computational programs of the AVP.

Step 2: Calculate the apparent altitude (*ha*) of the observed body.

$$ha = hs \pm I \pm IC - Dip, \qquad (2)$$

where *hs* is the sextant altitude, *I* is the sextant instrument correction, and *IC* is the sextant index correction.

Step 3: Calculate refraction (*R*).
- The refraction formula at a standard temperature of 10° Celsius (C) and standard pressure of 1010 millibars (mb) is as follows:

$$R_O = \frac{0.0167°}{\tan\left[ha + \left(\dfrac{7.32}{ha + 4.32}\right)\right]}, \qquad (3)$$

in which R_O represents refraction for standard condition.
- If the temperature *T*°C and pressure *B* mb are known, then the refraction formula is as follows:

$$R = \left(\frac{0.28 \times B}{T + 273}\right) \times R_O, \qquad (4)$$

in which *R* represents refraction for non-standard atmospheric conditions.

Step 4: Calculate semi-diameter (*SD*).
- Sun: Computing the distance between the Sun and the Earth using the VSOP 87 method, namely obtaining the radius vector of the Sun (Δ*s*), and then further calculating the Sun semi-diameter in Eq. (5) [2].

$$SD = \frac{959.63''}{\Delta s}, \qquad (5)$$

where Δ*s* denotes the Sun radius vector in astronomical units (AU), and the Sun *SD* in arc minutes.
- Moon: Computing the distance between the Moon and the Earth using the ELP2000/82 method can obtain the radius vector of the Moon (Δ*m*). Then, the Moon *HP* is calculated in Eq. (6) [3], and substituting the *HP* into Eq. (7) yields the Moon semi-diameter (*SD*),

$$\sin HP = \frac{6378.14}{\Delta m}, \qquad (6)$$

$$SD = 0.2724° \times HP. \qquad (7)$$

Where Δ*m* denotes the Moon radius vector in kilometers, the Moon *HP* in degrees, and the Moon *SD* in arc minutes.
- Navigational stars and planets: the *SD* is ignored due to the center of their observed bodies is easier to determine.

Step 5: Calculate the parallax (*P*), from the *HP* and the *ha*, the formula is as follows:

$$P = HP \times \cos(ha). \qquad (8)$$

- Sun: Substituting the Sun *HP* 0.0024° in Eq. (8) to calculate the Sun *P*.
- Moon: After obtaining the Moon *HP* through Eq. (6), substituting the *HP* into Eq. (8) yields the Moon *P*.
- Venus and Mars: After computing the distances between Venus and the Earth, Mars and the Earth, with the VSOP 87 method, respectively, one can obtain the radius vector of Venus or Mars (Δ*st*). Then, the *HP* is calculated in Eq. (9) [3]. Substituting the *HP* into Eq. (8) yields the *P* of Venus or Mars.

$$\sin HP = \frac{\sin 8.794''}{\Delta st}, \qquad (9)$$

where Δ*st* denotes the radius vector of Venus or Mars in kilometers, and the Venus or Mars *HP* in degrees.
- Jupiter, Saturn, and navigational stars: the *P* is ignored due to their far distance from the Earth.

Step 6: Calculate the observed altitude (*Ho*).

$$Ho = ha - R \pm SD + P, \qquad (10)$$

where *Ho* represents observed altitude.

2. Computation Procedures of AVP by Using the Intercept Method for Two Bodies

The basic idea of the IM is to choose the assumed position (AP) at the nearby the dead reckoning (DR) position, and take it as the reference point to calculate the altitude and azimuth. By comparing the computed altitude with observed altitude,
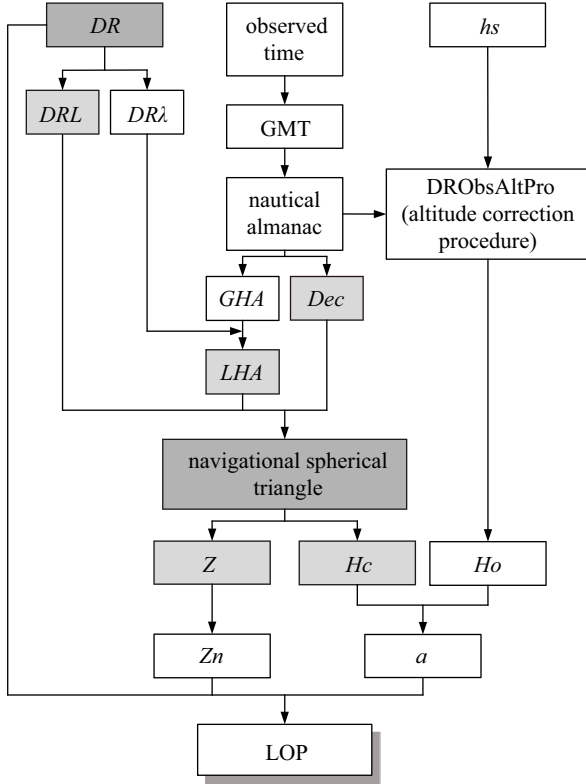
Fig. 2. The flowchart for solving the LOP by using the intercept method.

the difference of two altitudes (called the intercept, *a*) can be obtained. Therefore, once the AP, the computed azimuth angle (*Z*) of the body, and the intercept are all determined, the LOP can be plotted according to the three elements. Hence, the IM is essentially a kind of trial-and-error method. If the computation method is adopted, the choice of the initial position, such as the DR position in place of the AP, can be unconstrained. The flowchart for solving the LOP by using the IM is shown in Fig. 2. It should be note that an observed body's position variables in celestial equator coordinate, such as the declination (*Dec*) and the Greenwich hour angle (*GHA*), can be obtained at observed time by using the nautical almanac or related software. Besides, a celestial body's local hour angle (*LHA*) is the algebraic sum of its *GHA* and the observer's longitude (*λ*).

Computing an observed body's position variables in celestial horizon coordinate, such as the computed altitude (*Hc*) and azimuth angle (*Z*), the IM is a navigational spherical triangle problem. That is, when the two sides (co-latitude and polar distance) and the included angle (*LHA*) of an oblique spherical triangle are given, the problem is to solve the third side (co-altitude) and the outer angle (*Z*). When the simplicity and minimum error propagation are taken into account in the computation, the optimal formulae are the side cosine and the four-parts formula [8, 9]. The formulae are as follows:

$$\sin Hc = \sin(DRL)\cdot\sin(Dec) + \cos(DRL)\cdot\cos(Dec)\cdot\cos LHA,$$
(11)

$$\tan Z = \frac{\sin LHA}{[\cos(DRL)\cdot\tan(Dec)] - [\sin(DRL)\cdot\cos LHA]},$$
(12)

in which *DRL* is the DR latitude, *Dec* is declination, *LHA* is local hour angle, *Hc* is computed altitude, and *Z* is azimuth angle.

Some important issues should be noted here. The first is that in the above equations if sign of *Dec* is the same as *DRL*, it is treated as a positive quantity. On the contrary, it is treated as a negative quantity. The second is that the side cosine formula can be reduced to Eq. (11) because sine functions are odd and cosine functions are even. In addition, if the computation method is used, the accuracy of the obtained AVP can be increased and verified by the iteration computation [4, 8].

Basically, the classical problem of celestial navigation, in its simplest form, is the determination of an AVP; one intersection is resulted from the different LOPs of two celestial bodies, observed at known GMT on a known date. However, the most case is observing altitudes of the same or different celestial bodies at different time [14]. Thus, the running fix concept is usually adopted and after the course, speed, and period of two observing times have been identified. To this point, for complete computation procedure, the "moving reference point," should be taken into consideration. This is explicated below.

In order to move the circle of position (COP) or LOP to the same time, the running fix concept to do so is to use the rhumb line sailing in conjunction with the moving reference point, the geographical position (GP) of the COP or the DR of the LOP, is adopted [1, 5, 8]. The reason for obtaining a celestial fix is that each would have to be advanced or retired to the desired time for celestial fix, and making proper allowance for the travel of the vessel during the intervening time. The computation procedures are as follows:

Step 1: Calculate the moving distance (*d*) and difference of latitude (*l*) by using

$$d = s \times t,$$
(13)

and

$$l = d \times \cos C,$$
(14)

where *s* is speed, *t* is time interval, and *C* is course angle.

Step 2: Calculate the new reference point latitude (*L₂*) by using

$$L_2 = L_1 \pm l,$$
(15)

where L_1 is the reference point latitude.

Step 3: Calculate the meridional difference (*m*) by using

$$ML_1 = a \ln \left[\tan \left(45° + \frac{L_1}{2} \right) \times \left(\frac{1 - e \sin L_1}{1 + e \sin L_1} \right)^{\frac{e}{2}} \right], \quad \text{(16-a)}$$

$$ML_2 = a \ln \left[\tan \left(45° + \frac{L_2}{2} \right) \times \left(\frac{1 - e \sin L_2}{1 + e \sin L_2} \right)^{\frac{e}{2}} \right], \quad \text{(16-b)}$$

and

$$m = ML_1 \sim ML_2, \quad \text{(16-c)}$$

where a is the equator's arc minute length, e is earth flattening, ML_1 is the meridional parts at L_1, ML_2 is the meridional parts at L_2.

Step 4: Calculate the new reference point longitude (λ_2) by using

$$DLo = m \times \tan C, \quad \text{(17)}$$

$$\lambda_2 = \lambda_1 \pm DLo, \quad \text{(18)}$$

where DLo is difference of longitude, and λ_1 is the reference point longitude.

However, as Eq. (17) can be seen, when the course angle is 90°, $\tan C = \infty$, and the DLo can not be calculated. In this case, the parallel sailing should be used for calculating the DLo. The formula is as follows:

$$DLo = d \times \sec L, \quad \text{(19)}$$

where L is the reference point latitude.

It is worth mentioning, that because of an irregular oblate spheroid of the earth, parameters of meridional parts are taken from the 1984 world geodetic system (WGS-84) [11], in which

$$a = \frac{21600'}{2\pi}, \ e = \sqrt{2f - f^2}, \text{ and } f = \frac{1}{298.257223563},$$

The parameters of the Global Positioning System (GPS) and Electronic Chart Display and Information System (ECDIS) both adopt WGS-84; therefore the applications of developed program in this paper are quite broad.

III. DEVELOPING COMPUTATION PROGRAMS

As explained in Sections II, this paper uses computation procedures of observed altitude from sextant altitude as the basis in developing the observed altitude program of various celestial bodies between 1986 and 2050, named ObsAltPro and its flowchart is shown in Fig. 3. As for computation procedures of the AVP, the IM method is used as the basis to
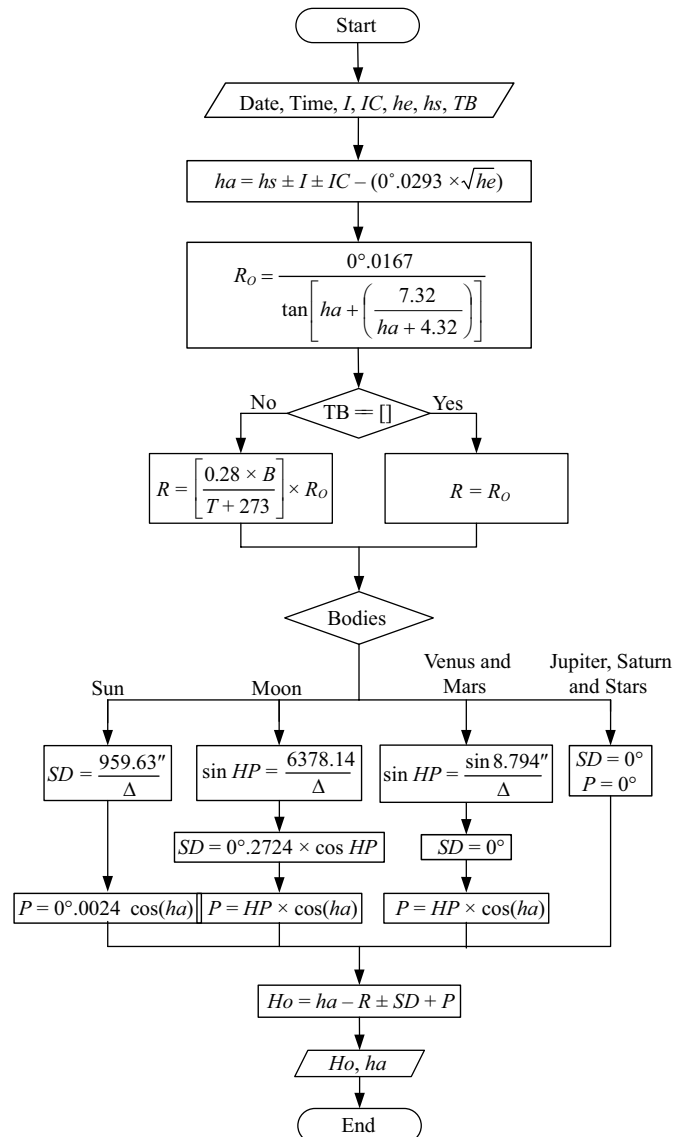
Fig. 3. The flowchart of the ObsAltPro program.

develop the IM program for two bodies, named IMPro-2, and the iteration computation function is provided for the accuracy of the AVP shown in Fig. 4. Moreover, to ensure the completion of the IMPro-2, moving reference point computation is included as shown in Fig. 5. For friendly use, the above two computation programs were both developed using the graphical user interface (GUI) of the MATLAB® programming language.

IV. DEMONSTRATED EXAMPLES

Example 1: At 10-00-00 GMT (07-00-00 local time), June 16, 1994, the navigator obtains a sight of the Moon's upper limb. The sextant altitude (hs) is 20°6.7'. Height of eye is 18 feet; there is no index error. Determine the observed altitude (Ho) of the Moon [1].
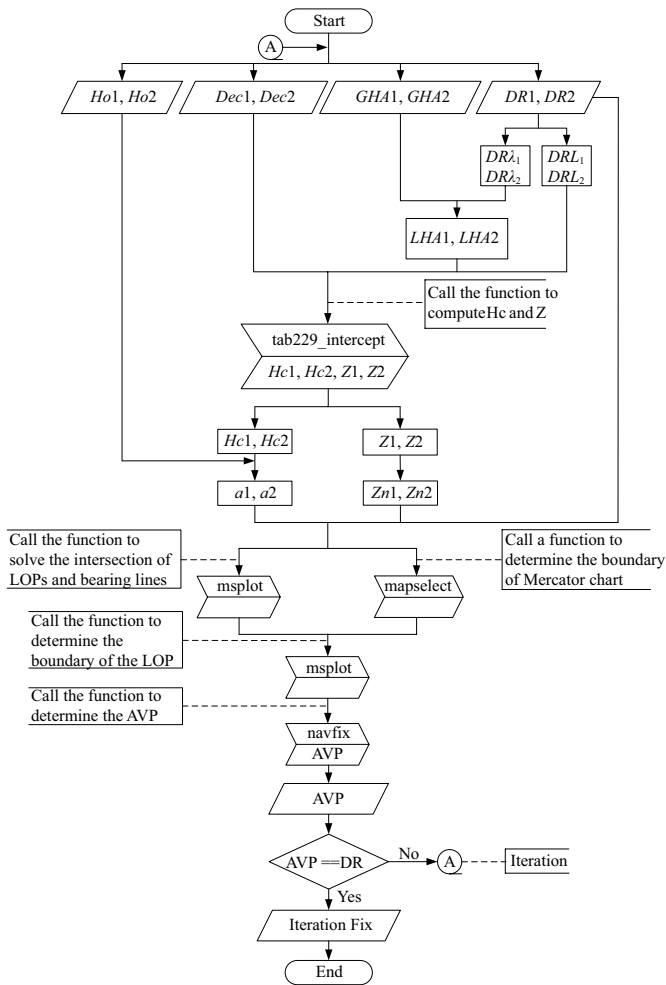
Fig. 4. The flowchart of the IMPro-2 program.

Fig. 5. The flowchart of moving reference point function in IMPro-2.

Required: The observed altitude can be determined in the following two approaches for sight reduction.

- Using the nautical almanac to solve the observed altitude.
- Using developed program ObsAltPro to solve the observed altitude directly.

Solution:

- Extracted from American Practical Navigator (No. 9), result of the Moon observed altitude is 26°37.1' [1].
- Executing ObsAltPro shows the Moon observed altitude to be 26°37.1', as shown in Fig. 6.

Explanation: For the same results, this example is verified that self-developed program is correct. Moreover, the ObsAltPro program, directly performs calculations using formulae, adopts GUI expression to demonstrate the advantages of being simple, fast and accurate. Besides, computerized program is more versatile than tabular method due to its design. That is, the ObsAltPro program does not require the use of nautical almanacs, and using this program can directly obtain the observed altitude of various celestial bodies between 1986 and 2050.
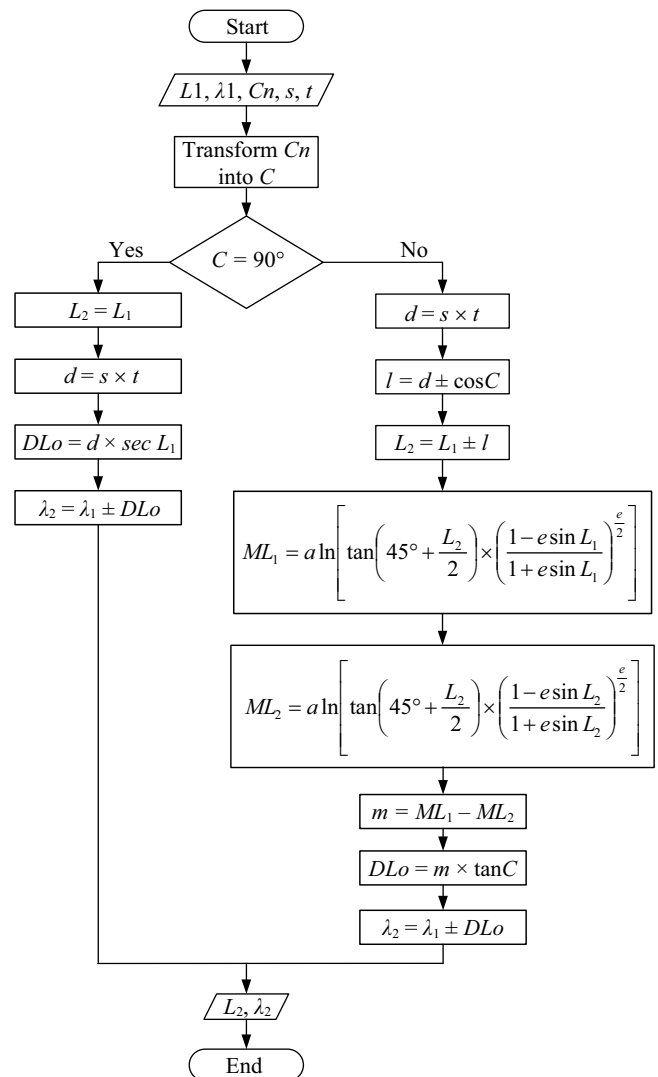
Example 2: On June 16, 1994, at 05-15-23 local time, at DR position L 30° N, λ 045° W, the navigator already took a sight of the Sun's upper limb. The ship is keeping on course 030° and speed 10 knots. Again, at 07-00-00 local time, another sight of the Moon was obtained, as explicated in Example 1. Thus, the navigator wants to adopt the running fix concept to determine 0700 celestial fix, namely AVP. Therefore, the navigator records the needed information and further reduces it from the nautical almanac for sight reduction as shown in Table 1 [1].

Required: The AVP can be determined by using self-developed program, IMPro-2, to solve the AVP.

Solution: First, use the moving reference point function of IMPro-2 to obtain the post-moving reference position L 30°15.1' N, λ 044°50.0' W, as shown in the upper-left of Fig. 7. Then, treat the obtained position as the DR position of the panel, and enter the needed information, such as the *Dec*,

Table 1. Extract of relevant information for sight reduction in example 2.

Body	ZT	*Ho*	*Dec*	*GHA*
Sun	05-15-23	2°48.1'	23°20.5' *N*	303°42.1'
Moon	07-00-00	26°37.1'	0°13.8' *N*	346°15.0'

Source: [1] *The American Practical Navigator*, pp. 303-307 (2002).

Fig. 6. Using the ObsAltPro to solve the observed altitude in example 1.

Table 2. Extract of relevant information for sight reduction in example 3.

Body	ZT	*Ho*	*GP*
Alkaid	20-03-06	77°34.9′	49°25.7′ *N* 003°14.2′
Capella	20-04-08	15°19.3′	45°58.4′ *N* 131°24.8′

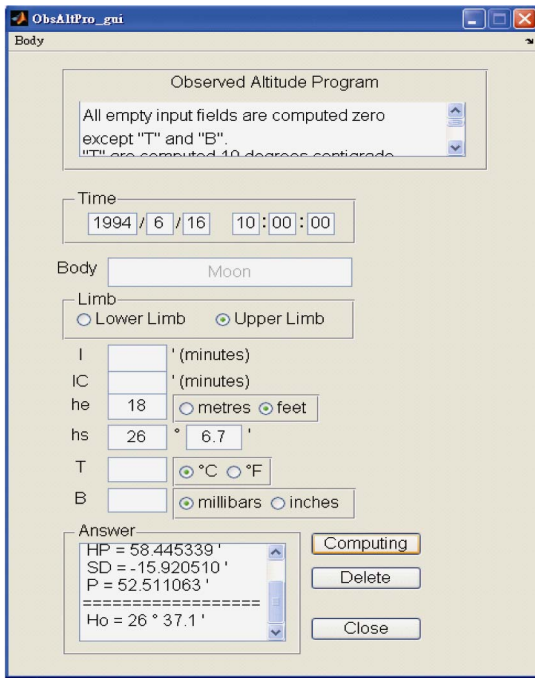Source: [5] *Dutton's Nautical Navigation*, pp. 332-335 (2004).
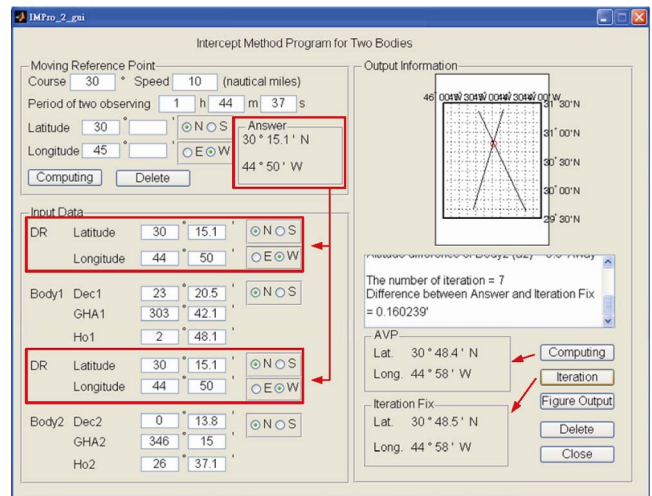
Fig. 7. Using the IMPro-2 to solve the AVP in example 2.

GHA, and *Ho*, of the two celestial bodies (The Sun and the Moon), as shown in the lower-left of Fig. 7. Clicking the computing function can obtain an AVP of *L* 30°48.4' *N*, *λ* 044°58.0' *W*. Further, clicking the iteration function can yield the accurate AVP of *L* 30°48.5' *N*, *λ* 044°58.0' *W*, as shown in the lower-right of Fig. 7.

Explanation: This designed example is only demonstrated how to use IMPro-2 to solve the AVP.

Example 3: The 2004 DR position of a vessel is *L* 41°34.8' *N*, *λ* 017°00.5' *W*. At 20-03-58, the star **Capella** is observed with a sextant. At 20-02-56, shortly before the above observation, another star, the **Alkaid** is spotted. The navigator records the needed information and further reduces it from the nautical almanac for sight reduction as shown in Table 2 [5].

Required: The AVP can be determined in the following three approaches for sight reduction.

- Using the IM in conjunction with the inspection tables (No. 229) to solve the *AP*, *Zn* and *a*, and plot the LOP. (Tabular method)

- Treating AP as the initial reference point, using developed program IMPro-2 to solve the possible AVP and verify it. (AP to run IMPro-2)
- Treating DR as the initial reference point, using developed program IMPro-2 to solve the possible AVP and verify it. (DR to run IMPro-2)

Solution:
- Extracted from Dutton's Nautical Navigation, results of *AP*, *Zn* and *a* are listed in Table 3 and the graphical AVP is *L* 41°38.6' *N*, *λ* 017°08.1' *W* [5].
- AP was treated to run IMPro-2, AVP is *L* 41°39.0' *N*, *λ* 017°07.6' *W*. Then, executing the iteration function yields fixed AVP is *L* 41°39.1' *N*, *λ* 017°07.3' *W*, as shown in Fig. 8.
- DR was treated to run IMPro-2, AVP is *L* 41°39.1' *N*, *λ* 017°07.3' *W*. Then, executing the iteration function yields fixed AVP is *L* 41°39.1' *N*, *λ* 017°07.3' *W*, as shown in Fig. 9.

Discussion: In actuality, this example is validated and the accurate AVP is pointed at *L* 41°39.1' *N*, *λ* 017°07.3' *W* from the paper of Hsu, T.P. *et al.* [8]. Consequently, it is demonstrated that DR running IMPro-2 can obtain the true AVP, more accurate than tabular method or AP running IMPro-2. However, executing the iteration function showed that, regardless of whether AP running IMPro-2 or DR running

Table 3. Three elements for plotting the LOPs by using the intercept method (the tabular method) in example 3.

Body	The three plotting elements of LOP	
Alkaid	$AP \begin{pmatrix} 42°N \\ 017°14.2'W \end{pmatrix}$	$Zn = 047.9°$ $a = 10.4'\,Away$
Capella	$AP \begin{pmatrix} 42°N \\ 017°24.8'W \end{pmatrix}$	$Zn = 318.8°$ $a = 24.2'\,Away$

Source: [5] *Dutton's Nautical Navigation*, pp. 332-335 (2004).

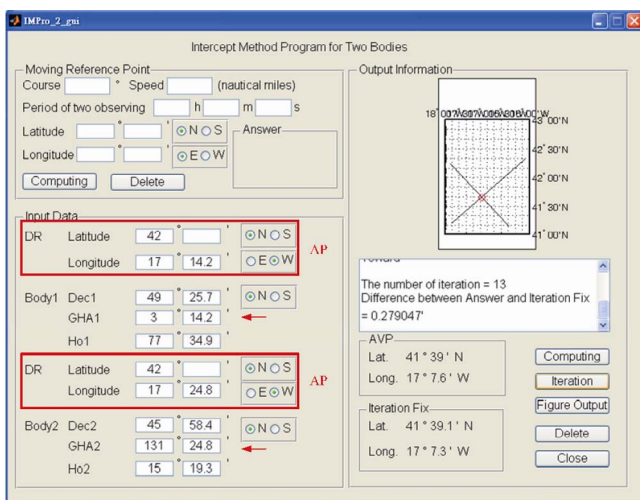**Fig. 8. Treating AP to run IMPro-2 for solving the AVP in example 3.**
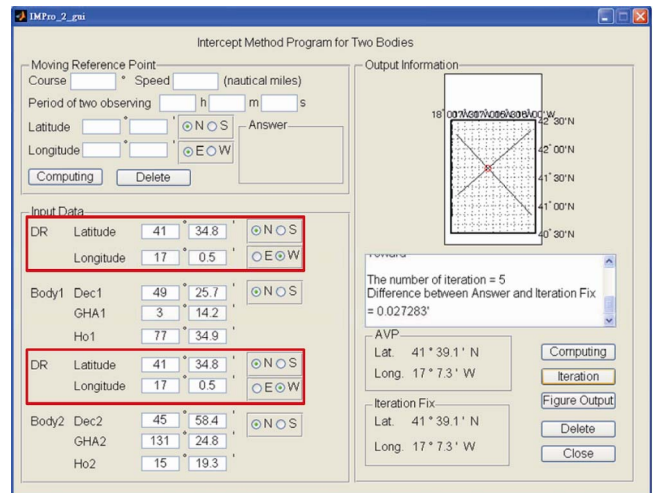
IMPro-2, the post-iteration AVP always is *L* 41°39.1' *N*, λ 017°07.3' *W*, as shown in Figs. 8 and 9, further demonstrating the accuracy of AVP. Besides, IMPro-2 can also demonstrate the manual plotting process; the figure in the upper-right part of this GUI, as shown in Figs. 8 and 9, can be expanded, outputted, and saved.

V. CONCLUSIONS

This paper proposes two computerized programs, ObsAlt-Pro and IMPro-2, allowing users to easily and quickly complete computations of the AVP. Actually, the computerized solution is always more it avoids than tabular methods because it is free of rounding error. The above two programs, through demonstrated examples, have shown advantages of being simple, fast, and accurate. In summary, the characteristics of these two computation programs as are follows:

- Running ObsAltPro does not require information of nautical almanac at all, thereby it avoids its limitations and errors. The observed altitudes of various celestial bodies from 1986 to 2050 can be directly obtained.
- Due to the inherent nature of the IM, a trial-and-error method, and improving drawbacks of the IM. The developed

Fig. 9. Treating DR to run IMPro-2 for solving the AVP in example 3.

IMPro-2 not only directly uses the DR as the input variable to replace the AP but also adds an iteration function to verify the determined AVP.

- For complete solving the classical problem of celestial navigation, the moving reference point function should be taken into account. The developed IMPro-2 adopted rhumb line sailing in conjunction with parallel sailing to deal with this problem. Again, for conventional user, the developed IMPro-2 can also demonstrate the plotting figure to help illustrate the result.

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