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# GENETIC ALGORITHMS FOR DESIGN OF PIPE NETWORK SYSTEMS

# Hong-Min Shau\*, Bi-Liang Lin\*\*, and Wen-Chih Huang\*\*\*

Key words: genetic algorithm, pipe network systems, optimal design, practical requirement.

#### ABSTRACT

In the last three decades, a significant number of methods for optimal design of pipe network systems have been developed using linear programming, non-linear programming, dynamic programming, enumeration techniques, and genetic algorithm (GA). This paper presents a genetic algorithm (GA) approach to the design of a pipe network systems. The objectives considered are minimization of the network cost in the practical requirement. Of all the preceding methods, GA based methods appear to be robust, as they can handle discrete pipes sizes with ease and produce a set of alternatives. By judgmentally selecting the Ruey-Fang district water supply system for the case study and by comparing the data gathered from the case study, this study also aims to verify the efficacy of the proposed method. The finding indicates that the proposed method is superior to enumeration techniques with respect to solution speed and cost.

## **INTRODUCTION**

Nowadays, designers often make use of commercial simulation packages such as KYPIPE, WATER, CYBERNET, EPANET2.0, WATERCAD and MIKENET [8]. The trial and error process is repeated until satisfactory alternatives are found. Users obtain at best near-optimal solutions. A number of researchers have used linear programming (Morgan and Goulter [10]; Fujiwara [3]; Kessler and Shamir [6]; Wu and Lin [14]) or nonlinear programming (Liebman *et al.* [8]; Brooke *et al.* [1]; Lansey and Mays [7]) to obtain an optimal design of a pipe network. In general, nonlinear programming packages use a constrained generalized reduced gradient (GRG) technique to identify a local optimal solution. The limitations of using linear and nonlinear programming packages to solve a real life complex pipe system include:

(1) The solution may specify sizes that are not commercial diameters, and therefore necessitates round-ing of final solution.

(2) The solution found by these optimal techniques is at most an optimal solution, not the global optimal solution.

(3) The size of network design depends on the number of constraints. A specific software may be used for the decision of the size.

Numerous researchers have proposed different optimal designs of pipe network systems as a combinatorial optimization problem and solved with enumeration and genetic algorithm (Gessler [4]; Lin *et al.* [9]; Simpson *et al.* [13]; Dandy *et al.* [2]; Savic and Walters [12]).

With respect to preliminary solutions to optimization problems of the network system of water supply, Goldberg [5], proposed well-founded frameworks. In principle, the GA is regarded a search procedure for the minimum or maximum of an unconstrained function by using random selection processes, which simulate reproduction, selection, crossover, and mutation of living creatures.

Pilar *et al.* [11] proposed a convergence optimization to modify the simple GA. They tried to explain how species are selected to change and how they are transformed into different species. The GA is regarded a search procedure applicable to different problems for optimal solutions. To meet the needs of nodes and the layout of hydraulic elements, the optimal design of a looped network for gravity systems is decided by the set of pipe size, which results in the minimum investment cost.

Simpson *et al.* [13] applied the GA to solve the pipe network systems, and compared it with the complete enumeration method and the nonlinear programming. Dandy *et al.* [2] improved Simpson's efficiency by applying the simple GA directly. They also

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compared the problems of New York tunnels with those of previous research documents. The result showed that the GA could generate a lower-cost design program than the methods previously applied.

Based on the document analysis, this research applied the GA to the optimal design of the pipe network systems. This study aims at exploring how to obtain optimal design of the pipe network systems with limited resources and insufficient time. The results of this study are also compared with those of previous studies.

# METHODOLOGY

Advanced by John Holland in the 1970s, the term "Genetic Algorithm" (GA) originated in the Darwinian notion of "Survival of the fittest in natural selection." Ever since its introduction, the GA has successfully solved the discontinuous, the non-differentiable, the non-convex, and the multiple peaks function optimization problems which traditional analytical and numerical optimization method failed to solve [5]. Although the pipe network systems is very complicated, its diameter variables can be easily coded. Over the past years scholars have proposed utilizing the GA to solve pipe network systems optimization.

In a fixed pipe network systems, each pipe and its auxiliary facilities may be regarded as a stage. In the process of design, there are various commercial pipe sizes available for selection. Therefore, the design of the water distribution system is used to precede network simulation under different loading demands as well as to select varied combinations of substitute elements in order to obtain the pressure distribution of the system and gain the water head of each node. To comply with both theoretical and practical needs, the researchers established the optimal mode of the water distribution system planning and design, coupled with each water distribution element. The optimal model is shown below:

Maximized Fitness = 
$$\frac{1}{Cost}$$
  
 $Cost = \sum_{i=1}^{n} \sum_{j=1}^{m_i} F_i(D_{ij}) L_i X_{ij} + penalty_i$  (1)

Subject to 
$$\sum_{k} Q_{in} - \sum_{k} Q_{out} = Q_{k}, k \in \text{node}$$
 (2)

$$\sum_{l} h_{l} - DH_{l} = 0, \ l \in \text{loop}$$
(3)

$$HMIN_k \le H_k \le HMAX_k, \ k \in \text{node}$$
(4)

$$VMIN_i \le V_i \le VMAX_i, \ i = 1, 2, \dots, n$$
(5)

$$X_{ij} = 0, 1, i = 1, 2, ..., n \text{ and } j = 1, 2, ..., m_i$$
 (6)

$$\sum_{j=1}^{m_i} X_{ij} = 1, \ i = 1, 2, ..., n \text{ and } j = 1, 2, ..., m_i$$
(7)

and 
$$L_i, D_i \ge 0$$
 (8)

Where  $D_i$  = the diameter in stage *i*;  $L_i$  = the pipe length in stage *i*; *n* = the number of potential pipes in the network;  $m_i$  = the number of options in the ith stage;  $F_i(D_{ij})$  = the pipes (link) cost function as a function of the diameter,  $D_{ij}$ ;  $D_{ij}$  = the diameter of option *j* at stage *i*; penalty<sub>i</sub> = the penalty in stage *i*;  $Q_{in}$  = the inflow rate at node *k*;  $Q_{out}$  = the outflow rate at node *k*;  $Q_k$  = the demand at node *k*;  $h_l$  = the head loss at loop *l*;  $DH_l$  = the difference in water level from the initial node to the last node at loop *l*;  $HMIN_k$  = the minimum head level at node *k*;  $VMIN_i$  = the minimum velocity limitation at node *i*;  $VMAX_i$  = the maximum velocity limitation at node *i*; and  $X_{ij}$  = the 0-1 variables for option *j* in stage *i*.

# APPLING THE GA TO THE PIPE NETWORK SYSTEMS DESIGN

The Improved GA adopted in this study combines the gray coding with the elitist strategy. Compared with the basic GA, the improved GA can increase the probability of finding the global maximum. Figure 1 shows the application of the GA on the flow path of the design of the pipe network systems. The relative parameter used in GA is tested and evaluated for the case the Ruey-Fang water supply system in Taipei County.

# 1. The coding of the diameter variables

If a chromosome is to store correct messages, the



Fig. 1. The flow chart of GA on the design of the pipe network systems.

gene inside it must have a fixed coding method. In principle, either binary basic coding pattern or gray coding can be applied to diameter coding. By gray code is meant that the set (of numbers) differs from the previous set (of numbers) only in one number. For example, the 011 is differs from the previous 001 only in one number. The corresponding relationship between the diameter variable and the code is shown in Table 1.

# 2. Producing the initial population

Using random sampling produces the initial population. The size of the population depends on the dimension of the pipe network systems. Choosing a suitable population size may enable the researchers to use the GA efficiently and effectively. To comply with the needs of the selected case, the researchers test the numbers from 10 through 1000.

# 3. The objective function value evaluation

Once the initial population is decoded, researchers need to go on with counting the cost of the responding diameter one by one. The sum is the value of objective function as shown in Eq. (1).

#### 4. Counting the fitness

The higher the fitness degree is, the better qualities the chromosome possesses, and the more likely they are to be reproduced in the future. With respect to the problem of searching for the maximized objective value, the fitness degree can be directly represented by the objective function. As to the design of the pipe network systems, seeking for the lowest cost is regarded as a problem of the minimized objective function. The function of fitness degree needs to be further treated by using the objective function. This research adopted the reciprocal of the objective function as the fittest degree. The smaller the value of the objective function is, the greater the fittest degree is. If the hydraulic analysis of the system design program does not conform to the design criteria, a penalty factor will be given. An amount of 1,000,000 NT\$ was proposed by this study as the penalty. Eq. (1) is the fitness used in this research.

#### 5. Reproduction and selection

By using a predetermined method researchers may select from the present a parent population for matching and reproduction of their offspring. Generally, the roulette wheel method is implemented for the selection of the parent population. This research, however, utilized the tournament selection method. After evenly mixing the parent population, the researchers selected two from it to compare their fitness degree and pick up one that is more likely to evolve. Again, the researchers selected two more from it separately to compare their fitness degree and pick up one that is more likely to evolve. This is to prove that the parent population with better adaptability will further evolve. It is assumed

	Diameter	Eight pipes		Sixteen	pipes
		Binary coding	Gray coding	Binary coding	Gray coding
1	0.10	000	000	0000	0000
2	0.15	001	001	0001	0001
3	0.20	010	011	0010	0011
4	0.25	011	010	0011	0010
5	0.30	100	110	0100	0110
6	0.35	101	111	0101	0111
7	0.40	110	101	0110	0101
8	0.45	111	100	0111	0100
9	0.50			1000	1100
10	0.60			1001	1101
11	0.70			1010	1111
12	0.80			1011	1110
13	0.90			1100	1010
14	1.00			1101	1011
15	1.10			1110	1001
16	1.20			1111	1000

Table 1. The corresponding relationship chart between the diameter variable and the code

that better genes can therefore be passed on to and inherited by the offspring.

### 6. The elitist strategy

The GA determines the reproduction probability of the offspring's word strings on the basis of fitness degree. However, this does not mean that the word strings of the offspring, which have the optimal fitness, are sure to guarantee reproduction. Therefore, the best individuals may be preset for up-to-now reservation. However, the number of individuals to be reserved depends on practical problems. The researchers use the numbers 100.

#### 7. The crossover and the crossover rate

The selected paired chromosomes produce the offspring by exchanging genes with each other. By means of this, the offspring can possess some of the excellent genes from their parents, and chromosomes with better adaptability are recombined accordingly. Three methods are generally adopted for gene crossover, including single point crossover, dual point crossover, and uniform crossover. This research adopted single point crossover and uniform crossover. The purpose of crossover is to let chromosomes exchange useful information with each other, and get higher fitness in order to have better chromosomes in the next generation and have preferable performance. In case we would like to keep certain specific chromosomes in the next generation, we may adopt crossover rate. The rates of crossover depend on individual problems. Appropriate crossover rate is of significance to quality training. This research set the crossover rate at 0.5, 0.6, or 0.7.

# 8. The mutation

Mutation aims to obtain more information. However, mutation rates tend to be very small. This is because that if mutation rate is too high, chromosomes might lose some information in the process of mutation. Mutation rate, however, is flexible. If chromosomes are found to cease to progress after a specific period of time, we may increase the mutation rate to enable more changes to take place. In general, two handling methods are adopted:

#### (1) The gene mutation

When the occurring random numbers are smaller than the preset mutation rate, the system will randomly choose genes, and change the gene values. (2) The term formation mutation

# (2) The transformation mutation:

This is used to change the gene's location in a certain section of the chromosome without changing its

value. The setting of the mutation rate directly influences the result of the search for excellent individuals. If the value is too small, mutation will become dysfunctional, thus leading to premature convergence. Large mutation rate, on the contrary, may keep the offspring from inheriting good genes from their parent population. The rates of mutation, therefore, depend on the nature of individual problem. Its suitable value is usually obtained by the experiment. In this study, the researchers set the mutation rate at 0.02. By now, the parent population has evolved into the second generation. By repeating the above-mentioned steps 3 through 8, we may hope to obtain better variable solutions through generations of mathematical calculations. And if possible, we may even get a solution that is converged on and approximates the optimal solution of the whole.

# CASE STUDY AND DISCUSSION

#### 1. Practical design cases and computer program

The Ruey-Fang water supply system in Taipei County used in other studies or reports are selected to illustrate the practical application of the GA. There are 26 pipelines, 20 nodes, and 2 water intake points. One of intake is water treatment plant; the other is the Kung-Liao system support. Table 2 is the cost per unit length of DIP used in the optimal design of Ruey-Fang system. The Pipe and Node Characteristics for Ruey-Fang system is in Table 3. From operation data, the maximum day of water-consuming amount of the entire district is 15,480 CMD, and the minimum day is 5,710 CMD. The Ruey-Fang system hydraulic analysis figure shows in Figure 3. The results and the discussions are provided below. A computer program was developed based on the GA, and written in FORTRAN. The modified Hardy-Cross method is adopted in this study for hydraulic analysis simulation. 0.0001 CMS is used for the water convergence precision at node. The remaining adopted data related are the same as those in the report.

#### 2. The finding and results

#### (1) The population size:

The larger the system is, the larger the population size should be accordingly. As a result, there will be sufficient points scattered in the solution area. For this reason, 100 populations will suffice for this case study. (2) The result of the solution:

When the selected parameter is suitable, the GA can identify an economical program, which costs much, lower than the case using the enumeration method previously used. This means that the range of the practicable diameter, which is formerly fixed, can be subject

No.	Diameter (M)	Material cost (NT\$/M)	Construction cost (NT\$/M)	Total cost (NT\$/M)	
1	0.10	516	348	864	
2	0.15	732	427	1159	
3	0.20	958	514	1472	
4	0.25	1088	612	1700	
5	0.30	1367	707	2074	
6	0.35	1619	1019	2638	
7	0.40	2061	1051	3112	
8	0.45	2468	1338	3806	
9	0.50	2908	1483	4391	
10	0.60	3889	1688	5577	
11	0.70	6059	2065	8124	
12	0.80	7549	2570	10119	
13	0.90	9290	3120	12410	
14	1.00	11185	3739	14924	
15	1.10	14193	4419	18612	
16	1.20	16522	5248	21770	

Table 2. The cost per unit length of DIP used in the optimal design of Ruey-Fang system



Fig. 2. The best value and average value of each generation.

to further investigation and readjustment. (3) The number of generation:

The GA does not guarantee finding the global optimal solution. However, the GA can always find the near optimal solution with convergent generation. For limited time, 1,000 generation is preceded. Figure 2 is the best value and average value of each generation. (4) The time of the solution:

The case at issue clearly indicates that, in principle, satisfactory solution may be attained. The solution time is mainly related to the number of evolving generations and the size of the population. The experiencebased practical design determines on the basis of flow volume the pipe diameter, water pressure, application environment and economic cost. While it takes three days to analyze by using the enumeration method, the GA method need only to take a couple of hours to analyze. The GA method undoubtedly saves a lot of time. Also, the GA analysis may handle a larger volume than other methods. The GA method is found to be more reliable.

#### (5) The value of the penalty function:

The purpose of the penalty function is to decrease the fitness degree of the solution that does not conform to the head limitation equation, and terminates its process of evolution. If the optimal solution is located at

Pipe	J1	J2	Length	Diameter	С	Flow
No.			(M)	(M)		(CMD)
1	1	2	320	0.5	100	-9304
2	2	3	520	0.5	100	-8760
3	2	4	1100	0.1	100	-543
4	4	7	380	0.1	100	657
5	3	5	930	0.5	100	-7420
6	5	6	180	0.2	100	-5485
7	6	9	60	0.2	100	-3417
8	6	7	80	0.2	100	-2067
9	7	10	400	0.1	100	-869
10	10	17	420	0.1	100	-329
11	9	11	110	0.1	100	-1500
12	11	12	220	0.1	100	-362
13	12	13	120	0.2	100	1189
14	9	14	130	0.1	100	-1377
15	14	15	210	0.1	100	-514
16	15	16	114	0.1	100	-64
17	11	14	90	0.1	100	-136
18	12	15	90	0.1	100	-551
19	13	16	90	0.3	100	-10998
20	16	17	150	0.3	100	-10522
21	5	8	190	0.1	100	-1394
22	8	18	390	0.1	100	-394
23	18	20	200	0.1	100	617
24	18	19	280	0.1	100	189
25	19	20	100	0.3	100	14717
26	13	19	320	0.5	100	13987
Node	Joint	Source	Demand	Total head	Elevation	Operation
			(CMD)	(M)	(M)	head (M)
1	1	1	9304	100.51	80.0	20.51
2	3	0	0	100.19	78.0	22.19
3	2	0	-1340	99.73	75.0	24.73
4	2	0	-1200	85.73	70.0	15.73
5	3	0	-540	99.12	70.0	29.12
6	3	0	0	93.26	68.0	25.26
7	3	0	-540	92.83	67.0	25.83
8	2	0	-1000	84.81	68.5	16.31
9	3	0	-540	92.45	67.0	25.45
10	2	0	-540	80.28	64.0	16.28
11	3	0	-1000	82.97	66.0	16.97
12	3	0	-1000	81.60	64.0	17.60
13	3	0	-1800	81.83	62.0	19.83
14	3	0	-1000	82.88	65.0	17.88
15	3	0	-1000	80.39	63.0	17.39
16	3	0	-540	80.36	61.0	19.36
17	2	0	-10850	78.10	60.0	18.10
18	3	0	-1200	81.99	66.5	15.49
19	3	0	-540	82.51	64.0	18.51
20	2	1	15335	85.32	65.0	20.32

 Table 3. Pipe and node characteristics for optimal design of Ruey-Fang system



Fig. 3. System hydraulic analysis chart of Ruey-Fang system (Maximum hour).

the head limitation of 15 m or 40 m, it may be the solution that does not correspond to the head limitation equation. The value of the penalty function has a great influence on the optimal solution.

# (6) The bit number of the diameter:

It represents the size of the scale the system combination establishes. While 3 bits means there are eight optional commercial diameters for each pipe, 4 bits means there are sixteen optional commercial diameters for each pipe. The program will find the bit number automatically in each stage.

# (7) The coding method:

The method of coding will affect the result of implementation. Both coding method of binary or grey will converge to the optimal solution, but the grey will find the solution efficiently.

### 3. Evaluation of benefits

 The method of Minimum Cost with water pipes is calculated to be 11,592,856 NT\$ with remaining head 15.49 M-29.12 M as shown in Table 3. The system hydraulic analysis figure shows in Figure 4.

- (2) By using the GA method, the simplified systematic solution, and the heuristic algorithm, this research designs a model that provides the greatest benefits to people. The model then draws up specifications to search for a feasible and economic project.
- (3) It utilizes documents and the traditional drinking water system to design an optimal model and find a solution. By applying the GA and the concept of evolutionism the researchers set up an optimal model for the future.

# CONCLUSION

1. When applying the GA to the pipe network systems of different sizes, we may get various solutions by proceeding evolution with diverse parameter values (e.g. the generation number of evolution, the population number of each generation, the penalty function factor, and the coding method, *etc.*) This research adopted a specific area to test and verify their respective proper parameter values. The finding shows the GA has the property of continuous improvement. Although it cannot guarantee a global optimal solution, it can



Fig. 4. Optimal solution with system hydraulic analysis chart of Ruey-Fang system.

always help us find the known optimal solutions for the case, and come up with the satisfactory solutions in a short time.

- 2. The result of the GA excellence selection shows that there is no need to seek a large number of combinations to obtain the excellence solutions. A few combinations are sufficient enough to enable us to obtain satisfactory results. Thus, we can prove that the GA is indeed an excellence searching method with high efficiency.
- 3. No matter we apply the GA or the boundary concealed enumeration method to solve their problems of the pipe network systems, we can always acquire the alternatives of the appointed number. These alternatives, which can be further analyzed and compared to enable us to have a better understanding of the whole system, provide us with meaningful information and also help to improve the quality of decision-making.
- 4. In addition to pipes, the optimal design with practical requirement of the pipe network systems should also take reservoirs, pumping stations, and relative auxiliary facilities into consideration. Meanwhile, system allocation should be closely connected to the geographic information system. Hence, the integration of an optimal method and geographical information system is the solution to the current planning and designing problems. Such an approach is worth further exploration.

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