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# DISCRETE-TIME OUTPUT FEEDBACK SLIDING MODE CONTROL FOR UNCERTAIN SYSTEMS

Ming-Chang Pai\*

Key words: discrete-time output feedback, sliding mode control, uncertain systems, chattering phenomenon.

## ABSTRACT

This paper presents a discrete-time output feedback sliding mode control to stabilize a class of linear uncertain systems in which the state is unavailable and no estimated state is required. The proposed sliding mode controller is derived to guarantee the existence of quasi-sliding mode by using output feedback only and the stability of overall closed-loop system is assured. This scheme ensures the robustness against parametric uncertainties and disturbances. Neither chattering phenomenon will occur nor the knowledge of upper bound of uncertainties is required. Simulation results demonstrate the efficacy of the proposed control methodology.

## I. INTRODUCTION

The theory of sliding mode control (SMC) is often used in controlling of uncertain systems. The main merit of SMC is their fast response, good transient performance, insensitiveness to matching parameter uncertainties and external disturbances [2,3,9,17]. In practice, using computers or DSP chips to implement the controller becomes more and more important nowadays, and discrete-time SMC has gained more and more attractive attention recently. Different from continuous-time SMC, the motion of a discrete-time SMC system can approach the switching surface but cannot stay on it in practice. Therefore, only the quasi-sliding mode is ensured [6,12,15].

Several design methods for discrete-time SMC have been proposed in the literature [1,5-7,12,13,15,16,18,19]. However, all these papers are always limited to systems with full-state feedback. In practice, full measurement of the state vector is not feasible. Such situations would demand the use of observers or dynamic compensators, which would add to the system complexity. Because the direct output feedback in SMC has a desirable feature as it does not add any additional dynamics and is

simpler to implement, several research works are available to deal with SMC design using static output feedback [4,10,11,14,20]. Wang and Fan [20], Kwan [10,11] and Edwards *et al.* [4] have developed output feedback SMC to stabilize a class of uncertain systems. Pai and Sinha [14] developed an output feedback SMC method for time-varying mismatched uncertain systems. However, these control techniques are always limited to continuous-time systems.

In this paper, we extend the idea of Wang and Fan [20] and Kwan [10] from continuous-time output feedback SMC to discrete-time output feedback SMC. A discrete-time sliding mode controller using only output variable is proposed to stabilize uncertain systems robustly. The scheme is very simple in structure and easy to implement because no observer is needed. By using the proposed controller, the stability of system in the quasi-sliding mode is assured under certain conditions. The robustness against parametric uncertainties and disturbances is ensured. Neither chattering phenomenon will occur nor the knowledge of upper bound of uncertainties is required. Furthermore, it can be suitable for uncertain systems with both matched state delays and input delays.

This paper is organized as follows. Section 2 briefly states problem formulation and assumptions. Section 3 provides the proposed discrete-time output feedback SMC scheme. The selection of sliding surface, the design of sliding mode controller, and the stability of system in the quasi-sliding mode have been addressed. Section 4 presents results from numerical simulations. Finally, a conclusion is provided in section 5.

## II. PROBLEM FORMULATION AND ASSUMPTIONS

Consider a discrete-time uncertain system represented by:

$$\bar{x}_{k+1} = (\bar{A} + \Delta\bar{A})\bar{x}_k + (\bar{B} + \Delta\bar{B})u_k + f_k \quad (1.a)$$

$$y_k = \bar{C}\bar{x}_k \quad (1.b)$$

where  $\bar{x}_k \in R^n$  is the state vector,  $u_k \in R^m$  is the control input,  $y_k \in R^p$  is the output,  $\bar{A}$ ,  $\bar{B}$ , and  $\bar{C}$  are system matrices of appropriate dimensions,  $\Delta\bar{A}$ ,  $\Delta\bar{B}$  and  $f_k$  represent unknown parametric uncertainties and external disturbances respectively.

For completing the description of discrete-time uncertain

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system (1), the following assumptions are assumed to be valid:

**Assumption 1.** The system  $(\bar{A}, \bar{B}, \bar{C})$  is controllable and observable.

**Assumption 2.**  $rank(\bar{C}\bar{B}) = m$ ; i.e.  $rank(\bar{B}) = m$ .

**Assumption 3.** There exist matrices of appropriate dimension  $\hat{D}_a$ ,  $\hat{D}_b$  and  $\hat{f}_k$  such that  $\Delta\bar{A} = \bar{B}\hat{D}_a$ ,  $\Delta\bar{B} = \bar{B}\hat{D}_b$  and  $\hat{f}_k = \bar{B}\hat{f}_k$ . These conditions are so-called matching conditions [3].

Using Assumption 3, discrete-time uncertain system (1) can be rewritten as

$$\bar{x}_{k+1} = \bar{A}\bar{x}_k + \bar{B}u_k + \bar{B}d_k \tag{2.a}$$

$$y_k = \bar{C}\bar{x}_k \tag{2.b}$$

where the generalized uncertainty  $d_k$  is constructed as  $d_k = \hat{D}_a\bar{x}_k + \hat{D}_b u_k + \hat{f}_k$  and is assumed to be bounded.

The objective of this paper is to develop a discrete-time output feedback sliding mode controller such that discrete-time uncertain system (2) is asymptotically stable even in the presence of parametric uncertainties and external disturbances.

In this paper, the switching function is defined as

$$S_k = Gy_k \tag{3}$$

where  $G \in R^{m \times p}$  is a constant matrix to be designed.

### III. MAIN RESULTS

The design of discrete-time SMC consists of two steps: The first step is to design a switching surface such that in the quasi-sliding mode system response acts like the desired dynamics. The second step is to design the control law in order that quasi-sliding mode is reached and sustained for all time.

#### 1. Design of switching surface

Define a state variable transformation

$$x_k = H\bar{x}_k \tag{4}$$

where  $H \in R^{n \times n}$  is an orthogonal transformation matrix such that

$$H\bar{B} = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} = B \tag{5}$$

where  $B_2 \in R^{m \times m}$  is a nonsingular matrix.

Using (4), equations (2.a) and (2.b) can be rewritten as

$$x_{k+1} = Ax_k + Bu_k + Bd_k \tag{6.a}$$

$$y_k = Cx_k \tag{6.b}$$

where  $A = H\bar{A}H^{-1}$  and  $C = \bar{C}H^{-1}$ .

From (5), the first  $(n-m)$  rows of  $B$  are zero. Therefore, the state vector  $x_k$  is partitioned as follows:

$$x_k = \begin{bmatrix} x_{1k} \\ x_{2k} \end{bmatrix} \tag{7}$$

where  $x_{1k} \in R^{n-m}$  and  $x_{2k} \in R^m$ . Then, the dynamics of  $x_{1k}$  can be written from (6.a) as

$$x_{1k+1} = A_{11}x_{1k} + A_{12}x_{2k} \tag{8}$$

where  $A_{11}$  and  $A_{12}$  are parts of the matrix  $A$  corresponding to the partition (7). Similarly, partitioning the matrix  $C$  according to (7), output equation (6.b) can then be rewritten as

$$y_k = C_1x_{1k} + C_2x_{2k} \tag{9}$$

where  $C_1 \in R^{p \times (n-m)}$  and  $C_2 \in R^{p \times m}$ .

Assuming that  $GC_2$  is nonsingular, equations (3) and (9) yield

$$x_{2k} = (GC_2)^{-1}(-GC_1x_{1k} + S_k) \tag{10}$$

Substituting (10) into (8)

$$x_{1k+1} = [A_{11} - A_{12}(GC_2)^{-1}GC_1]x_{1k} + A_{12}(GC_2)^{-1}S_k \tag{11}$$

Thus, discrete-time uncertain system (6) in the quasi-sliding mode can be reduced to the following  $n-m$  dimensional form

$$x_{1k+1} = (A_{11} - A_{12}KC_1)x_{1k} \tag{12.a}$$

where  $K = (GC_2)^{-1}G$ .

or

$$x_{1k+1} = A_m x_{1k} \tag{12.b}$$

where  $A_m = A_{11} - A_{12}KC_1$ .

Equation (12) can be considered as a linear output feedback problem. The matrix  $G$  can be selected to satisfy Lemma 1 of [20], i.e.

$$rank(C_2K - I) \leq p - m \tag{13}$$

#### 2. Design of Discrete-Time Sliding Mode Controller

After designing the switching surface, the next phase is to design the control law such that quasi-sliding mode is reached and stayed thereafter. Before design the controllers, we first give a lemma proposed by Sarpturk [15].

##### Lemma 1

A necessary and sufficient condition for a discrete-time SMC to assure both sliding motion and convergence onto the hyper-plane is

$$\|S_{k+1}\| < \|S_k\| \tag{14}$$

Condition (14) can be further decomposed into the following two inequalities:

$$S_k^T [S_{k+1} - S_k] < 0 \tag{15}$$

$$S_k^T [S_{k+1} + S_k] > 0 \tag{16}$$

where (15) and (16) are called sliding condition and convergence condition, respectively.

**Lemma 2**

Consider the equivalent reduced-order system (12.b). Let  $\|\lambda_m\| < 1$  be the maximum real part of the eigenvalues of  $A_m$ . Then,  $\|x_{1k}\|$  is bounded by  $w_k$  for all  $k$ , where  $w_k$  is the solution of

$$w_{k+1} = \lambda_w w_0 + \rho \left( \|A_{12} (GC_2)^{-1} S_k\| \right) \tag{17}$$

,  $0 < \lambda_m < \lambda_w < 1$ ,  $\rho = \|N\| \|N^{-1}\|$ ,  $w_0 > \rho \|x_{10}\| > 0$  and  $N$  is a transformation matrix.

**Proof:**

Since matrices  $K$  and  $G$  can be designed such that  $A_m$  is stable, all the eigenvalues of  $A_m$  lie inside the unit circle of the z-plane, and can be expressed as  $\lambda_1, \lambda_2, \dots, \lambda_{n-m}$ . The solution of (11) can be obtained as

$$x_{1k} = NJ^k N^{-1} x_{10} + N \left( \sum_{i=0}^{k-1} J^i N^{-1} [A_{12} (GC_2)^{-1} S_{k-i-1}] \right) \tag{18}$$

where  $J = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_{n-m})$  and  $N$  is a transformation matrix. Then, from (18)

$$\begin{aligned} \|x_{1k}\| &\leq \rho \|J^k\| \|x_{10}\| + \rho \left( \sum_{i=0}^{k-1} \|J^i\| \|A_{12} (GC_2)^{-1} S_{k-i-1}\| \right) \\ &\leq \rho \|\lambda_m^k\| \|x_{10}\| + \rho \left( \sum_{i=0}^{k-1} \|\lambda_m^i\| \|A_{12} (GC_2)^{-1} S_{k-i-1}\| \right) \end{aligned} \tag{19}$$

where  $\rho = \|N\| \|N^{-1}\|$ .

The solution of (17) is given by

$$w_k \leq \|\lambda_w^k\| w_0 + \rho \left( \sum_{i=0}^{k-1} \|\lambda_w^i\| \|A_{12} (GC_2)^{-1} S_{k-i-1}\| \right) \tag{20}$$

Comparing the right-hand sides of (19) and (20), and noting the fact that  $0 < \lambda_m < \lambda_w < 1$ , it yields

$$w_k \geq \|x_{1k}\| \tag{21}$$

for all  $k$ . The proof is completed.

In the sequel, we consider the control law for discrete-time uncertain system (6) as

$$u_k = -\alpha (GCB)^{-1} S_k - d_{k-1} \tag{22}$$

where the generalized disturbance  $d_k$  defined in (2) can be estimated through the following relation

$$d_{k-1} = (GCB)^{-1} (GCx_k - GCAx_{k-1} - GCBu_{k-1}) \tag{23}$$

and  $\alpha > 0$  will be designed later. Since  $\text{rank}(CB) = m$ ,  $(GCB)^{-1}$  exists.

**Theorem 1**

Consider the discrete-time uncertain system (6) with the proposed control law (22) and switching function (3). If there exist matrices  $K$  and  $G$  such that eigenvalues of the matrix  $A_{11} - A_{12}KC_1$  are within the unit circle in the Z-plane, then

(a) the quasi-sliding mode condition  $\|S_{k+1}\| < \|S_k\|$  will be satisfied outside the region A, where the region A is defined

$$\text{as } \Omega_A = \left\{ S_k : \|S_k\| \leq \max_{r_1 < \alpha < (2-r_1)} \left[ \frac{\eta_k}{\alpha - r_1}, \frac{\eta_k}{2 - \alpha - r_1} \right] \right\} \text{ and}$$

$$r_1 = \|GCA - GC\| \|(GC_2)^{-1}\| \text{ and}$$

$$\eta_k = \|GCA - GC\| \|H_1\| \|w_k\| + \varepsilon_k \text{ with}$$

$$H_1 = \begin{bmatrix} I_{n-m} \\ -(GC_2)^{-1} GC_1 \end{bmatrix}, \quad w_k \text{ satisfying (17) and}$$

$$\|GCB(d_k - d_{k-1})\| \leq \varepsilon_k.$$

(b) the quasi-sliding mode of discrete-time uncertain system (6) is stable.

**Proof :**

First, we prove that  $\|S_{k+1}\| < \|S_k\|$  is satisfied. For this, the proof includes two parts.

Part I. (Sliding condition). From (3), (6) and (22), the difference between  $S_{k+1}$  and  $S_k$  can be expressed as

$$\begin{aligned} S_{k+1} - S_k &= GCx_{k+1} - GCx_k \\ &= GCAx_k + GCBu_k + GCBd_k - GCx_k \\ &= (GCA - GC)x_k - \alpha S_k + GCB(d_k - d_{k-1}) \end{aligned} \tag{24}$$

Pre-multiplying (24) by  $S_k^T$

$$\begin{aligned} S_k^T [S_{k+1} - S_k] &= -\alpha \|S_k\|^2 + S_k^T [(GCA - GC)x_k \\ &\quad + GCB(d_k - d_{k-1})] \end{aligned}$$

$$< -\alpha \|S_k\|^2 + \|S_k\| [\|GCA - GC\| \|x_k\| + \|GCB(d_k - d_{k-1})\|]$$

$$< -\alpha \|S_k\|^2 + \|S_k\| [\|GCA - GC\| \|x_k\| + \varepsilon_k] \tag{25}$$

However, the state vector  $x_k$  in (25) is not available. But, the upper bound on its norm can be estimated as follows. From (10), the state vector  $x_k$  can be rewritten as

$$x_k = \begin{bmatrix} x_{1k} \\ x_{2k} \end{bmatrix} = H_1 x_{1k} + \begin{bmatrix} 0 \\ (GC_2)^{-1} S_k \end{bmatrix} \quad (26)$$

where  $H_1 = \begin{bmatrix} I_{n-m} \\ -(GC_2)^{-1} GC_1 \end{bmatrix}$ .

Using (26) and (21), the upper bound of the state vector  $\|x_k\|$  can be expressed as

$$\begin{aligned} \|x_k\| &\leq \|H_1\| \|x_{1k}\| + \|(GC_2)^{-1}\| \|S_k\| \\ &\leq \|H_1\| \|w_k\| + \|(GC_2)^{-1}\| \|S_k\| \end{aligned} \quad (27)$$

Substituting (27) into (25)

$$\begin{aligned} S_k^T [S_{k+1} - S_k] &< -\alpha \|S_k\|^2 + \|S_k\| [\|GCA - GC\| \|H_1\| \|w_k\| \\ &+ \|GCA - GC\| \|(GC_2)^{-1}\| \|S_k\| + \varepsilon_k] \\ &< -(\alpha - r_1) \|S_k\| \left[ \|S_k\| - \frac{\eta_k}{\alpha - r_1} \right] \end{aligned}$$

where  $r_1 = \|GCA - GC\| \|(GC_2)^{-1}\|$  and  $\eta_k = \|GCA - GC\| \|H_1\| \|w_k\| + \varepsilon_k$ .

The sliding condition (15) will be satisfied if  $\|S_k\| \geq \frac{\eta_k}{\alpha - r_1}$  and  $\alpha - r_1 > 0$ . Therefore, with a known bound of initial state  $w_0$ , a stable  $A_m$ ,  $\|S_k\| \geq \frac{\eta_k}{\alpha - r_1}$  and  $\alpha - r_1 > 0$ , the sliding condition  $S_k^T [S_{k+1} - S_k] < 0$  is achieved.

Part II. (Convergence condition). From (3), (6) and (22), the sum between  $S_{k+1}$  and  $S_k$  can be expressed as

$$\begin{aligned} S_{k+1} + S_k &= GCAx_k + GCBu_k + GCBd_k + S_k \\ &= (GCA - GC)x_k - \alpha S_k + GCB(d_k - d_{k-1}) + 2S_k \end{aligned} \quad (28)$$

Pre-multiplying (28) by  $S_k^T$  and using (27)

$$\begin{aligned} S_k^T [S_{k+1} + S_k] &= (2 - \alpha) \|S_k\|^2 + S_k^T [(GCA - GC)x_k \\ &+ GCB(d_k - d_{k-1})] \\ &> (2 - \alpha) \|S_k\|^2 - \|S_k\| [\|GCA - GC\| \|x_k\| + \varepsilon_k] \\ &= (2 - \alpha) \|S_k\|^2 - \|S_k\| [r_1 \|S_k\| + \eta_k] \\ &= (2 - \alpha - r_1) \|S_k\| \left[ \|S_k\| - \frac{\eta_k}{(2 - \alpha - r_1)} \right] \end{aligned}$$

If  $\|S_k\| \geq \frac{\eta_k}{2 - \alpha - r_1}$  and  $(2 - \alpha - r_1) > 0$ , then

$S_k^T [S_{k+1} + S_k] > 0$ , which implies that the convergence condition is achieved. From Part I, Part II and Lemma 1, if  $S_k \geq \max_{r_1 < \alpha < (2 - r_1)} \left\{ \frac{\eta_k}{\alpha - r_1}, \frac{\eta_k}{2 - \alpha - r_1} \right\}$ , it concludes  $\|S_{k+1}\| < \|S_k\|$ , which indicates that switching function  $S_k$  is decreasing outside  $\Omega_A$ .

Once the quasi-sliding mode condition  $\|S_{k+1}\| < \|S_k\|$  is satisfied, the system state trajectories will approach the switching surface in finite time. From (12), it shows that the dynamical equation of the quasi-sliding mode. Since matrices  $K$  and  $G$  can be designed such that eigenvalues of the matrix  $A_{11} - A_{12}KC_1$  are within the unit circle in the Z-plane, it is obvious that the quasi-sliding mode of discrete-time uncertain system (6) is stable. The proof is completed.

It is noted from (22) that there is no switching action in the proposed controllers, which means that chattering phenomenon will never happen. Also, the other advantage of (22) is that upper bound of the uncertainty  $d_k$  needs not to be known beforehand when the controller is implemented. Hence, it will increase the applicability of the proposed control scheme.

**Remark 1**

In general, it is usually desired to have a minimum bound of  $\Omega_A$  in order to increase the accuracy of control if the uncertainty  $d_k$  exists. Hence, the designed parameter  $\alpha$  in the control law (22) will be selected such that that the least upper bound of  $\min_{r_1 < \alpha < (2 - r_1)} \|S_k\| = \min_{r_1 < \alpha < (2 - r_1)} \left\{ \frac{\eta_k}{\alpha - r_1}, \frac{\eta_k}{2 - \alpha - r_1} \right\}$  is achieved.

**Remark 2**

From remark 1, it shows that the least upper bound of  $\min_{r_1 < \alpha < (2 - r_1)} \|S_k\| = \min_{r_1 < \alpha < (2 - r_1)} \left\{ \frac{\eta_k}{\alpha - r_1}, \frac{\eta_k}{2 - \alpha - r_1} \right\}$  will increase the accuracy of control, which implies the  $\eta_k$  will affect the accuracy of control. From Theorem 1, it shows the  $\eta_k$  is related to the generalized uncertainty difference. It is obvious that the smaller generalized uncertainty difference will increase the more accuracy of control.

**IV. ILLUSTRATIVE EXAMPLES**

To illustrate the utilization of the proposed approach, we consider an aircraft model which is used in [10,11,20]. The state-space model of the aircraft is given by

$$\begin{bmatrix} \dot{\varphi} \\ \dot{q} \\ \dot{\delta}_e \end{bmatrix} = \begin{bmatrix} -0.277 & 1 & -0.0002 \\ -17.1 & -0.178 & -12.2 \\ 0 & 0 & -6.67 \end{bmatrix} \begin{bmatrix} \varphi \\ q \\ \delta_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 6.67 \end{bmatrix} u$$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi \\ q \\ \delta_e \end{bmatrix}$$

where  $\varphi$  is the attack angle,  $q$  is the pitch rate,  $\delta_e$  is the elevator angle,  $u$  is the command to the elevator, and  $y$  is the measurement vector. The parametric uncertainty and disturbance are respectively assumed to be

$$\hat{D}_a = 0.1[\sin(t) + \cos(2t) \quad \sin(3t)\cos(t) \quad -1 + \sin(2t)\cos(3t)]$$

$$\hat{f} = [0 \quad 0 \quad 0.5]$$

The discrete-time transformed system (2) for each sampling time can be easily obtained using the Matlab program function `c2d` [8] with  $T_s = 0.001$  second and given as

$$\bar{x}_{k+1} = \begin{bmatrix} 0.9997 & 0.001 & 0 \\ -0.0171 & 0.9998 & -0.0122 \\ 0 & 0 & 0.9934 \end{bmatrix} \bar{x}_k + \begin{bmatrix} 0 \\ 0 \\ 0.0066 \end{bmatrix} u_k$$

$$+ \begin{bmatrix} 0 \\ 0 \\ 0.0066 \end{bmatrix} d_k$$

Since the matrix  $\bar{B}$  is of the form in (5), we need not transform the system. Define  $x_{1k} = [\varphi_k \quad q_k]^T$  and  $x_{2k} = \delta_{ek}$ , then

$$A_{11} = \begin{bmatrix} 0.9997 & 0.001 \\ -0.0171 & 0.9998 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 \\ -0.0122 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

According to (13), we choose  $G = [-0.4635 \quad 1]$  to place the eigenvalues of the reduced-order system (12) at  $0.9969 \pm 0.0031i$ . The initial state vector is chosen as  $x_0 = [2 \quad 0 \quad 0]^T$ . Also,  $\lambda_w$ ,  $\rho$  and  $w_0$  in (17) are taken to be 0.998, 5.7333 and 11.5, respectively. Then, the controller (22) with  $\alpha = 1$  can be designed as

$$u_k = -150 S_k - d_{k-1}$$

where  $S_k$  and  $d_{k-1}$  are given in (3) and (23), respectively.

With the designed parameter setting and initial condition  $y_0 = [2 \quad 0]^T$ , the results of simulation are shown in Figs. 1-4. Fig. 1 shows that output trajectories of the system, which con-

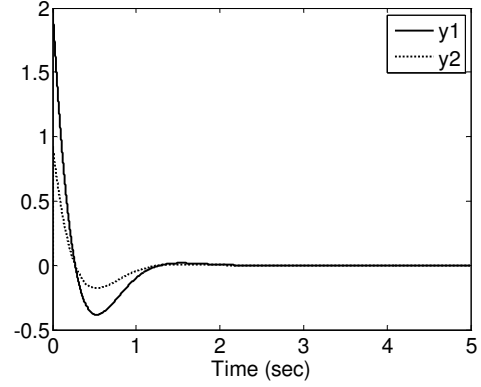


Fig. 1. Output trajectories  $q_k$  and  $\delta_{ek}$ .

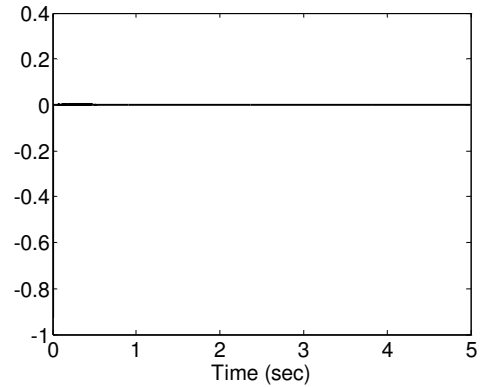


Fig. 2. Switching surface variable  $S_k$ .

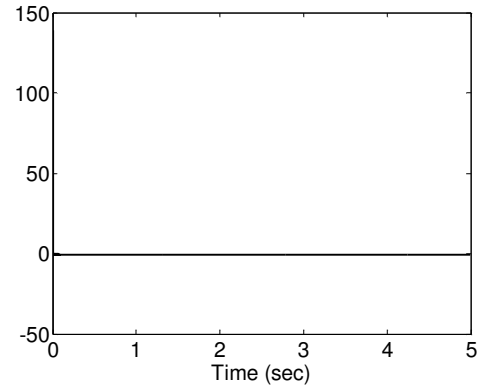


Fig. 3. Control input  $u_k$ .

firm the stability and robustness of the closed-loop system to parametric uncertainties and disturbances. Fig. 2 shows the switching surface variable. Fig. 3 shows that the control input with reasonable magnitudes. It can be seen that the chattering phenomenon is eliminated. Fig. 4 shows the performance of  $w_k$  and  $\|x_{1k}\|$ . It can be seen that the  $\|x_{1k}\|$  is smaller than  $w_k$  for all  $k$ .

## V. CONCLUSION

In this paper, a discrete-time output feedback SMC scheme

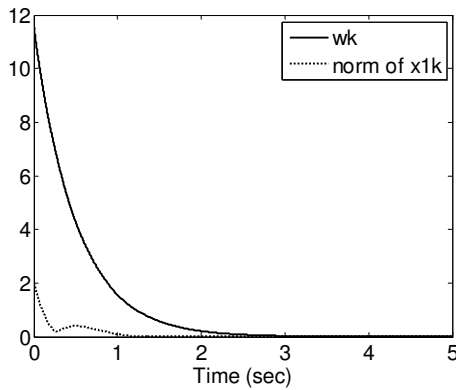


Fig. 4. Performance of  $w_k$  and  $\|x_k\|$ .

has been proposed to stabilize a class of uncertain systems. The design technique is simple and efficient. The proposed controller use only output variable and no observer is required. The selection of switching surface and the existence of the quasi-sliding mode are two important issues, which have been addressed. It has been shown that the stability of the closed-loop system is guaranteed. Neither chattering phenomenon will occur nor the knowledge of upper bound of uncertainties is required beforehand. Furthermore, it can be suitable for uncertain systems with both matched state delays and input delays. Simulation studies have confirmed the validity of this proposed control system.

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