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MULTI-OBJECTIVE CONTROL DESIGN FOR STOCHASTIC LARGE-SCALE SYSTEMS BASED ON LMI APPROACH AND SLIDING MODE CONTROL CONCEPT

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MULTI-OBJECTIVE CONTROL DESIGN FOR STOCHASTIC LARGE-SCALE SYSTEMS BASED ON LMI APPROACH AND SLIDING MODE CONTROL CONCEPT

Koan-Yuh Chang* and Wen-Jer Chang**

Key words: stochastic large-scale systems, sliding mode control, upper bound covariance control, pole placement and linear matrix inequality approach.

ABSTRACT

In this paper, a controller $u_i(t)$ is designed for stochastic large-scale systems to achieve the following three objectives simultaneously: the pole placement constraint, H_∞ norm constraint and individual state variance constraint. In terms of the invariance property of sliding mode control, both the uncertain interconnection terms and an unknown nonlinear function will disappear on the sliding mode. Then, with the aid of upper bound covariance control theory, pole placement skill and H_∞ norm control theory, a controller, in which the control feedback gain matrix G_i is synthesized using linear matrix inequality approach, is derived to achieve the above multiple objectives. Finally, a simulation example is presented to illustrate the proposed method.

I. INTRODUCTION

It is known that some control objectives, such as the robust stability and noise attenuation, can be achieved if certain H_∞ bounds are maintained [17]. Hence, there have been lots of papers studying the feedback controller design with H_∞ norm constraints (see [1], [16] and [36]). In practice, we are always required to develop some ways for designing controllers to achieve multi-objective performance. [2] and [34] have discussed the H_∞ norm and variance constrained problem simultaneously. However, a Riccati equation approach applied to minimize a scalar cost index does not ensure satisfaction of individual variance constraints. A more straightforward method for designing controllers to achieve variance constraints of individual states is developed in [10], [13], [14], [19] and [32]. However, the approach described in [14] and [19] does not consider the presence of system perturbations; the closed-loop system may be unstable when it suffers from perturbations. An improved control method, called upper bound covariance control (UBCC), which satisfies variance constraints with perturbations is proposed in [10], [13], and [32]. Nevertheless, the drawback of the direct UBCC approach is that the state feedback gain designed in [10], [13], and [32] will become very large when the systems suffer from large perturbations. If a system is large-scale and the interconnections of the systems are uncertain then it is hard to design the state feedback control by using UBCC method directly. Based on the concept of sliding mode control (SMC) and with the aid of UBCC, a new control method is developed for the problems of local state upper bound covariance control (LSUBCC) in stochastic uncertain large-scale systems [6]. In [6], the aim of the so-called LSUBCC is to achieve the upper bound covariance control for each subsystem in the large-scale systems.

Pole location is directly associated with performance specifications, such as the settling time and rise time of a control system. The system poles are, in fact, not necessarily specified at exact locations, but assigned to a region (see [18], [21], [23] and [33]). For the regional pole constraint, a typical rule for evaluating the relative stability of closed-loop systems is to judge whether all of the poles are located within a prescribed circular region (Fig. 1). This specified circular region with center at $-q_i + j0$ $(q_i > 0)$ and radius ρ_i, $(\rho_i < q_i)$ is denoted by $\mathscr{D}(-q_i, \rho_i)$. This constraint is one of the most frequently employed performance requirements in system control design problems.

Owing to the advantages of simple design, easy implementation and insensitivity to system perturbations, the technique of SMC has become a successful synthesis method for a system control design and has been applied to many complex systems such as [15], [20], [22], [24], [27], [28] and [31]. The main characteristic of a SMC system is that the system dynamics in the sliding mode is made to be invariant if parameter uncertainties and/or perturbations satisfy a certain matching condition. However, the SMC for stochastic systems has been receiving relatively little attention until recently [5-9, 30]. Based on the concept of SMC design, [7] and [30] addressed the co-

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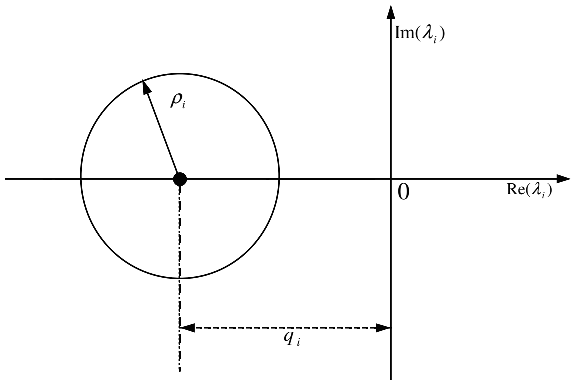

Fig. 1. Pole location region $\mathcal{D}(-q_i, \rho_i)$.

variance control problems in the case of stochastic model reference systems. In [8] and [9], the authors successfully extended the above approach to linear perturbed systems. Also, the authors applied this combined technique to deal with the covariance controller design problems for stochastic large-scale systems [5] and [6]. As an extension of the results in [5] and [6] and using the linear matrix inequality (LMI) method, this paper will deal with the same systems but involves a perturbations with nonlinear unknown function. Then, a useful controller is design which can force the systems to simultaneously achieve specified pole location constraints, H_∞ norm constraints, and individual variance constraints. Therefore, this proposed controller design will enable a quick and accurate response, noise attenuation, and robust stability. Here, we would like to point out that, according to recent research, some authors [25] are interested to follow our approach [7] to deal with the problems of uncertain stochastic systems with time-varying delay. Because LMI's intrinsically reflect constraints rather than optimality, many papers tend to offer more flexibility by combining several constraints on the system [4], [11] and [12]. Moreover, software like MatLab LMI Control Toolbox is now available to solve such LMI's in a fast and user-friendly manner.

This paper is organized as follows. Section 2 describes the system structure, formulates the design problems and also discusses on the issues related to the sliding phase and hitting phase of the system. In Section 3, the control feedback gain matrix G_i is constructed by using LMI method to satisfy multi-objective constraints. Also, a numerical example for the problem of stochastic large-scale systems is demonstrated in Section 4 to verify the proposed approach. Finally, a conclusion is made in Section 5. Here, we pre-define some notations which will be used in the consequent sections: $\|z(t)\|$ and $\|M\|$ are 2-norm and induced 2-norm of the vector $z(t)$ and the matrix M respectively. $\|z(t)\|_1$ is 1-norm of the vector $z(t)$, $(\cdot)^T$ is the transposition; $(\cdot)^*$ is the conjugate transpose and $\lambda(\cdot)$ is the eigenvalues. Moreover, a sign function $S_{iq}(t)$ is defined as

$$\mathrm{sgn}(S_{iq}(t)) = \begin{cases} 1 & S_{iq}(t) > 0 \\ 0 & S_{iq}(t) = 0 \\ -1 & S_{iq}(t) < 0 \end{cases} \text{ (see [35]).}$$

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

1. System Description

Consider a linear time-invariant large-scale system consisting of n uncertain interconnected stochastic subsystems established on a filtered probability space $\left(\Omega, \boldsymbol{F}, \left(\boldsymbol{F}_t\right)_{t\in R^+}, \boldsymbol{P}\right)$ and each subsystem is described as

$$\dot{x}_i(t) = A_i x_i + B_i(u_i(t) + f_i(x_i(t))) + \sum_{\substack{j=1 \\ j\neq i}}^{n} A_{ij} x_j(t) + D_i w_i(t),\ i = 1,2,\cdots,n \quad \text{(1a)}$$

$$y_i(t) = F_i x_i(t) \quad \text{(1b)}$$

where $x_i(t) \in R^{n_i\times 1}$, $u_i(t) \in R^{m_i\times 1}$, and $w_i(t) \in R^{m_i\times 1}$ are the state variable, control and white noise of the $i-\text{th}$ subsystem, respectively. A_i and $A_{ij} \in R^{n_i\times n_i}$; B_i and $D_i \in R^{n_i\times m_i}$; $F_i \in R^{m_i\times n_i}$, where A_{ij} is a bounded uncertainty satisfying $\|A_{ij}\| \le \eta_{ij}$. Here, $f_i(x_i(t)) \in R^{m_i\times 1}$ is unknown nonlinear function satisfying

$$\|f_i(x_i(t))\| \le \beta_i \|x_i(t)\| \quad (2)$$

with $\beta_i > 0$ a known constant. Moreover, we suppose that $n_i > m_i$ is satisfied and the white noise $w_i(t)$ of (1) satisfies (3)

$$E(w_i(t)) = 0,\ E(x_i(0)w_i^T(t)) = 0,\ E(w_i(t)w_i^T(t)) \triangleq W_i = I_i, \quad (3)$$

where $x_i(0)$ denotes the initial state and I_i denotes the identity matrix. Assume that (A_i, B_i) is a stabilizable pair and A_{ij} satisfies the matching condition (4)

$$rank\,[B_i : A_{ij}] = rank\,[B_i]. \quad (4)$$

2. Problem Formulation of the Controller Design

Now, the goal of this paper is to design the control $u_i(t)$ for each subsystem to satisfy the following objectives.

Objective (i): Constraints on pole placement region

The issue of transient response of the designed closed-loop system is addressed by properly specifying the locations of its poles. In this paper, the pole placement region in the complex z_i-plane is described by the LMI condition [12]

$$\mathcal{D} = \left\{ z_i \in \mathcal{C} : f_{\mathcal{D}}(z_i) = U_i + z_i N_i + \bar{z}_i N_i^T < 0 \right\} \quad (5)$$

where $\mathcal{C}$ denotes the set of complex number; $U_i = U_i^T$ and N_i are real matrix parameters for choosing a suitable convex region by defining the characteristic function $f_{\mathcal{D}}(z_i)$. Specifically, we consider the region of the disk $\mathscr{D}(-q_i, \rho_i)$ with center at $(-q_i, 0)$ and radius $0 < \rho_i < q_i$ for the closed-loop pole of the system. The disk region $\mathscr{D}(-q_i, \rho_i)$ in the complex plane $z_i = \hat{x}_i + j\hat{y}_i$ can be described as

$$\left(q_i + \hat{x}_i\right)^2 + \hat{y}_i^2 = \left(q_i + z_i\right)\left(q_i + \overline{z}_i\right) < \rho_i^2 . \tag{6}$$

By the property of Schur's complement [4], we have the characteristic function $f_{\mathscr{D}}(z_i)$ of (6) within the disk region $\mathscr{D}(-q_i, \rho_i)$ as follows:

$$f_{\mathscr{D}}(z_i) = \begin{bmatrix} -\rho_i & q_i + z_i \\ q_i + \overline{z}_i & -\rho_i \end{bmatrix} < 0 . \tag{7}$$

In comparison with the definition of LMI condition in (5), the matrix parameters for the disk region $\mathscr{D}(-q_i, \rho_i)$ are

$$U_i = \begin{bmatrix} -\rho_i & q_i \\ q_i & -\rho_i \end{bmatrix}, \ N_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}. \tag{8}$$

The considered system (1) is called $\mathscr{D}$-stable if the eigenvalues of the system are located in the disk region as shown in Fig. 1, i.e.,

$$\lambda(A_i + B_i G_i) \in \mathscr{D}(-q_i, \rho_i) . \tag{9}$$

That is, the closed-loop poles of the system are specified in terms of the system matrix $A_i + B_i G_i$ and required to lie within the disk region $\mathscr{D}(-q_i, \rho_i)$ with suitable chosen parameters $q_i > \rho_i > 0$.

Objective (ii): Constraints on H_∞ norm

In the system (1), the effect of the noise input $w_i(t)$ on the output $y_i(t)$ should be kept small for the system. Under the assumption, the system (1) is controlled to be stable, let $H_i(s)$ denote the closed-loop transfer function from $w_i(t)$ to $y_i(t)$. The desired H_∞ performance level is described as (10)

$$\left\|H_i(s)\right\|_\infty = \sup \frac{\left(E\int_0^\infty y_i^T(t) y_i(t) dt\right)^{1/2}}{\left(E\int_0^\infty w_i^T(t) w_i(t) dt\right)^{1/2}} < \gamma_i \tag{10}$$

where the performance level upper bound γ_i can be implemented as a constraint to be met or a parameter to be minimized during the controller construction.

Objective (iii): Constraints on upper bound of local state covariance

Besides the signal amplitude considered in the output channel, we are also interested in the state covariance of the system (1) induced by the external disturbance input $w_i(t)$. The individual steady state variance of each subsystem satisfies the following constraint:

$$[X_{kk}]_i \triangleq Var(x_{ik}(t)) \le [\tilde{X}_{kk}]_i \le (\sigma_k^2)_i \quad k = 1, 2, \cdots, n_i \tag{11}$$

where $Var(x_{ik}(t))$ and $(\sigma_k)_i$, respectively, denote the $k-$th variance value and root mean square (RMS) constraints for variance of the $i-$th subsystem, $[\tilde{X}_{kk}]_i$ denotes the $k-$th diagonal element of local state upper bound covariance matrix $\tilde{X}_i$, $[X_{kk}]_i$ denotes the $k-$th diagonal element of the matrix X_i. Here, X_i denotes the local state covariance matrix of the $i-$th subsystem within the definition of following

$$X_i = \lim_{t\to\infty} E(x_i(t) x_i^T(t)) . \tag{12}$$

Remark 1: The objective (iii) is called the LSUBCC problem [6].

It is seen from (1a), the presence of the uncertain interconnected term $\sum_{\substack{j=1 \\ j\neq i}}^{n} A_{ij} x_j(t)$ and unknown nonlinear function $f_i(x_i(t))$ not only will make the LSUBCC problem with state feedback control be much more difficult but also let the multiple objectives for system requirement design become harder. In order to overcome these difficulties, the invariance property of SMC may be a good way to handle the problems. From our previous work [6], the following subsections 2.3 and 2.4 are reviewed briefly. The details, contained lemmas and proofs of lemma, can be found in [6].

3. Sliding Phase of the System

First, we define the switching function $S_i(t)$ corresponding to the $i-$th subsystem as follows

$$S_i(t) = C_i x_i(t) - \int_0^t (C_i A_i + C_i B_i G_i) x_i(\tau) d\tau \tag{13}$$

where $S_i(t) = \begin{bmatrix} S_{i1}(t) & \cdots & S_{iq}(t) & \cdots & S_{im_i}(t) \end{bmatrix}^T \in R^{m_i \times 1}$; C_i and $G_i \in R^{m_i \times n_i}$ are constant matrices to be designed. C_i is chosen such that $C_i B_i \neq 0$ and $C_i D_i = 0$, and G_i is the control feedback gain matrix to be determined so that the local state covariance can fit the requirement in the sliding mode. The switching function $S_i(t)$ in (13) is well defined for the solution $x_i(t)$ of the system (1).

Differentiating the equation (13) with respect to time, choosing $C_i D_i = 0$ and using equation (1), we obtain $\dot{S}_i(t) = C_i \sum_{\substack{j=1 \\ j\neq i}}^{n} A_{ij} x_j(t) + C_i B_i u_i(t) + C_i B_i f_i(x_i(t)) - C_i B_i G_i x_i(t)$. In the sliding mode, $\dot{S}_i(t) = 0$ holds, then we get the equivalent control as follows

$$u_{ieq}(t) = G_i x_i(t) - (C_i B_i)^{-1} C_i \sum_{\substack{j=1 \\ j\neq i}}^{n} A_{ij} x_j(t) - f_i(x_i(t)) . \tag{14}$$

Substituting $u_{ieq}(t)$ into (1), the sliding mode dynamic equation is

$$\dot{x}_i(t)=(A_i+B_iG_i)x_i(t)+(I_i-B_i(C_iB_i)^{-1}C_i)\sum_{\substack{j=1\\j\neq i}}^{n}A_{ij}x_j(t)+D_iw_i(t) \quad (15a)$$

$$y_i(t)=F_ix_i(t). \quad (15b)$$

If $\sum_{\substack{j=1\\j\neq i}}^{n}A_{ij}x_j(t)$ is regarded as a disturbance and (4) holds, by the invariance property of SMC, the dynamics (15) is insensitive to the disturbance. Thus (15) is reduced to

$$\dot{x}_i(t)=(A_i+B_iG_i)x_i(t)+D_iw_i(t) \quad (16a)$$

$$y_i(t)=F_ix_i(t). \quad (16b)$$

4. Hitting Phase of the System

This subsection tries to find the controller $u_i(t)$ on the $i-$th subsystem such that the states of the system (1) can be forced to the sliding surface. Let us define a Lyapunov function for each subsystem as

$$V_i(S_i(t))=S_i^T(t)S_i(t)=S_{i1}^2(t)+\cdots+S_{iq}^2(t)+\cdots+S_{im_i}^2(t). \quad (17)$$

From our previous work [6], one has the following results.

Lemma 2.1 [6]

Consider the system (1) with the solution $x_i(t)$. If a Lyapunov function $V_i(S_i(t))$ is given as (17), (3) holds and $C_iD_i=0$ is chosen, then we have

$$\frac{d}{dt}V_i(S_i(t))=2S_i^T(t)\dot{S}_i(t). \quad (18)$$

Lemma 2.2 [6]

Consider the system (1), if $C_iD_i=0$ and let the controller $u_i(t)$ be

$$u_i(t)=G_ix_i(t)-(C_iB_i)^{-1}[k_i\|C_i\|\cdot\sum_{\substack{j=1\\j\neq i}}^{n}\|x_j(t)\|+\beta_i\|C_i\|\cdot\|B_i\|\cdot\|x_i(t)\|+\alpha_i]\mathrm{sgn}(S_i(t)),$$
$$i=1,2,\cdots,n \quad (19)$$

where $k_i>\max_{1\le j\le n}\eta_{ij}$, α_i is an arbitrary positive number and $\mathrm{sgn}(S_i(t))=\left[\mathrm{sgn}(S_{i1}(t))\ \cdots\ \mathrm{sgn}(S_{iq}(t))\ \cdots\ \mathrm{sgn}(S_{im_i}(t))\right]^T$. Then the state $x_i(t)$ will be forced to the sliding surface.

Proof:

Differentiating (13) and choosing $C_iD_i=0$ and multiplying it by $S_i(t)$, we get

$$S_i^T(t)\dot{S}_i(t)=$$
$$S_i^T(t)\left[C_iB_iu_i(t)+C_i\sum_{\substack{j=1\\j\neq i}}^{n}A_{ij}x_j(t)+C_iB_if_i(x_i(t))-C_iB_iG_ix_i(t)\right]. \quad (20)$$

Substituting (19) into (20), then (20) becomes

$$S_i^T(t)\dot{S}_i(t)=S_i^T(t)[C_i\sum_{\substack{j=1\\j\neq i}}^{n}A_{ij}x_j(t)-$$
$$(k_i\|C_i\|\cdot\sum_{\substack{j=1\\j\neq i}}^{n}\|x_j(t)\|+\beta_i\|C_i\|\cdot\|B_i\|\cdot\|x_i(t)\|+\alpha_i)\cdot\mathrm{sgn}(S_i(t))+C_iB_if_i(x_i(t))]. \quad (21)$$

Therefore, (18) becomes

$$\frac{d}{dt}V_i(S_i(t))=2S_i^T(t)C_i\sum_{\substack{j=1\\j\neq i}}^{n}A_{ij}x_j(t)-2k_i\|C_i\|\cdot\sum_{\substack{j=1\\j\neq i}}^{n}\|x_j(t)\|\cdot\|S_i(t)\|_1-2\beta_i\|C_i\|\cdot\|B_i\|\cdot\|x_i(t)\|\cdot\|S_i(t)\|_1$$
$$-2\alpha_i\|S_i(t)\|_1+2S_i^T(t)C_iB_if_i(x_i(t)) \quad (22)$$

$$\le 2\left|S_i^T(t)C_i\sum_{\substack{j=1\\j\neq i}}^{n}A_{ij}x_j(t)\right|-2k_i\|C_i\|\cdot\sum_{\substack{j=1\\j\neq i}}^{n}\|x_j(t)\|\cdot\|S_i(t)\|_1-2\beta_i\|C_i\|\|B_i\|\cdot\|x_i(t)\|\cdot\|S_i(t)\|_1$$
$$+2\left|S_i^T(t)C_iB_if_i(x_i(t))\right|-2\alpha_i\|S_i(t)\|_1$$

$$\le 2\|C_i\|\sum_{\substack{j=1\\j\neq i}}^{n}\|A_{ij}\|\|x_j(t)\|\|S_i(t)\|-2k_i\|C_i\|\cdot\sum_{\substack{j=1\\j\neq i}}^{n}\|x_j(t)\|\cdot\|S_i(t)\|_1-2\beta_i\|C_i\|\cdot\|B_i\|\cdot\|x_i(t)\|\cdot\|S_i(t)\|_1$$
$$+2\|C_i\|\cdot\|B_i\|\cdot\|f_i(x_i(t))\|\cdot\|S_i(t)\|-2\alpha_i\|S_i(t)\|_1$$

$$\le 2\|C_i\|\left(\max_{1\le j\le n}\eta_{ij}\right)\sum_{\substack{j=1\\j\neq i}}^{n}\|x_j(t)\|\cdot\|S_i(t)\|-2k_i\|C_i\|\cdot\sum_{\substack{j=1\\j\neq i}}^{n}\|x_j(t)\|\cdot\|S_i(t)\|_1-2\beta_i\|C_i\|\|B_i\|\cdot\|x_i(t)\|\cdot\|S_i(t)\|_1$$
$$+2\|C_i\|\cdot\|B_i\|\cdot\|f_i(x_i(t))\|\cdot\|S_i(t)\|-2\alpha_i\|S_i(t)\|_1$$

$$\le -2\alpha_i\|S_i(t)\|<0 \quad (23)$$

where $\|S_i(t)\|\le\|S_i(t)\|_1$, $k_i>\max_{1\le j\le n}\|\eta_{ij}\|$ and equation (2) are used. That means the state $x_i(t)$ will be forced to reach the sliding surface. The proof is completed.

***Remark 2*:** Lemma 2.2 can be derived from the Theorem 4.1 of [6] with suitable modification.

III. DESIGN OF CONTROL FEEDBACK GAIN MATRIX G_i TO SATISFY THE MULTIPLE OBJECTIVES

In this section, the design of control feedback gain matrix G_i is constructed for the system (16) to achieve the multi-objective performance constraints in terms of suitable LMI conditions. It was known that local state covariance X_i defined in (12) satisfies the following Lyapunov equation

$$(A_i+B_iG_i)X_i+X_i(A_i+B_iG_i)^T+D_iD_i^T=0. \quad (24)$$

Equation (24) was proposed for a system's stability design [1] and served as the starting point of the derivation in this paper.

1. Constraints on Pole Placement Region

This subsection will derive a constraint, which can be found in Lemma 3.2, for pole placement on system (16). To attain this goal, the following Lemma 3.1 is helpful.

Lemma 3.1:

Consider the system (16). Let G_i be given and $\gamma_i > 0$ be a fixed scalar. If there exists a positive definite matrix $\tilde{X}_i$ satisfying

$$(A_i + B_i G_i)\tilde{X}_i + \tilde{X}_i(A_i + B_i G_i)^T + \gamma_i^{-2}\tilde{X}_i\tilde{R}_i\tilde{X}_i + D_i D_i^T + q_i^{-1}(A_i + B_i G_i)\tilde{X}_i(A_i + B_i G_i)^T + q_i^{-1}(q_i^2 - \rho_i^2)\tilde{X}_i = 0 \quad (25)$$

where $\tilde{R}_i = F_i^T F_i$. Then all the closed-loop poles of $(A_i + B_i G_i)$ are located within $\mathcal{D}(-q_i, \rho_i)$ and $\|H_i(s)\|_\infty \le \gamma_i$. Furthermore, in this case, we have $X_i \le \tilde{X}_i$.

Proof:

Let λ_i and $v_i \in \mathcal{C}$ (complex), respectively, be the eigenvalue and the right eigenvector of $(A_i + B_i G_i)$, then $(A_i + B_i G_i)v_i = \lambda_i v_i$ and $(A_i + B_i G_i)^T v_i = \overline{\lambda}_i v_i$ in which $\lambda_i = \hat{x}_i + j\hat{y}_i$ and $\overline{\lambda}_i = \hat{x}_i - j\hat{y}_i$. Substituting this expression into (25), we have

$$v_i^*\left[\lambda_i\tilde{X}_i + \overline{\lambda}_i\tilde{X}_i + q_i^{-1}\lambda_i\overline{\lambda}_i\tilde{X}_i + q_i^{-1}(q_i^2 - \rho_i^2)\tilde{X}_i\right]v_i = -v_i^*(\gamma_i^{-2}\tilde{X}_i\tilde{R}_i\tilde{X}_i + D_i D_i^T)v_i$$

$$\Rightarrow \left[2\hat{x}_i + q_i^{-1}(\hat{x}_i^2 + \hat{y}_i^2) + q_i^{-1}(q_i^2 - \rho_i^2)\right]v_i^*\tilde{X}_i v_i = -v_i^*(\gamma_i^{-2}\tilde{X}_i\tilde{R}_i\tilde{X}_i + D_i D_i^T)v_i$$

$$\Rightarrow \left[q_i^{-1}(\hat{x}_i^2 + 2\hat{x}_i q_i + q_i^2 + \hat{y}_i^2 - \rho_i^2)\right]v_i^*\tilde{X}_i v_i = -v_i^*(\gamma_i^{-2}\tilde{X}_i\tilde{R}_i\tilde{X}_i + D_i D_i^T)v_i . \quad (26)$$

Since $q_i > 0$, $\tilde{X}_i > 0$ and $\gamma_i^{-2}\tilde{X}_i\tilde{R}_i\tilde{X}_i + D_i D_i^T \ge 0$, from (26) we obtain

$$\left[q_i^{-1}(\hat{x}_i^2 + 2\hat{x}_i q_i + q_i^2 + \hat{y}_i^2 - \rho_i^2)\right]v_i^*\tilde{X}_i v_i < 0 \quad (27)$$

$$\Rightarrow (\hat{x}_i + q_i)^2 + \hat{y}_i^2 - \rho_i^2 < 0 \quad (28)$$

which means that all eigenvalues of $(A_i + B_i G_i)$ should locate in a specified disk $\mathcal{D}(-q_i, \rho_i)$. Consider (25) and by the inducement of the Fact 1.2 of [29] we obtain $\|H_i(s)\|_\infty \le \gamma_i$, since $q_i^{-1}(A_i + B_i G_i)\tilde{X}_i(A_i + B_i G_i)^T + q_i^{-1}(q_i^2 - \rho_i^2)\tilde{X}_i \ge 0$. Subtracting (24) from (25) and using $\gamma_i^{-2}\tilde{X}_i\tilde{R}_i\tilde{X}_i + q_i^{-1}(A_i + B_i G_i)\tilde{X}_i(A_i + B_i G_i)^T + q_i^{-1}(q_i^2 - \rho_i^2)\tilde{X}_i \ge 0$, the inequality $X_i \le \tilde{X}_i$ will be gotten from Theorem 4.2 of [21] due to the fact that $(A_i + B_i G_i)$ is stable.
The proof is completed.

Lemma 3.2

Consider the system (16). If there exist positive definite matrix $\tilde{X}_i$ and matrix L_i satisfying

$$\begin{bmatrix} -\rho_i\tilde{X}_i & A_i\tilde{X}_i + B_i L_i + q_i\tilde{X}_i \\ q_i\tilde{X}_i + \tilde{X}_i A_i^T + L_i^T B_i^T & -\rho_i\tilde{X}_i \end{bmatrix} < 0 \quad (29)$$

where $L_i = G_i\tilde{X}_i$. Then the closed-loop poles of $A_i + B_i G_i$ are located within disk LMI region $\mathcal{D}(-q_i, \rho_i)$.

Proof:

Given a prescribed disk LMI region $\mathcal{D}(-q_i, \rho_i)$ in the left-hand side of complex z_i − plane , and the matrix $(A_i + B_i G_i)$ of the system (16) is $\mathcal{D}$-stable (i.e., $\lambda_i(A_i + B_i G_i) \in \mathcal{D}(-q_i, \rho_i)$). Then, from Lemma 3.1, there exists a positive definite matrix $\tilde{X}_i$ satisfying

$$(A_i + B_i G_i)\tilde{X}_i + \tilde{X}_i(A_i + B_i G_i)^T + q_i^{-1}\left((A_i + B_i G_i)\tilde{X}_i(A_i + B_i G_i)^T + (q_i^2 - \rho_i^2)\tilde{X}_i\right) < 0. \quad (30)$$

Equation (30) has the form of (27) with $\lambda_i(A_i + B_i G_i) \in \mathcal{D}(-q_i, \rho_i)$ and can deduce to (28) which is in fact the same as (6). Therefore, the matrix inequality for $\mathcal{D}$-stability (30) is equivalent to the pole placement condition (7).

The feasibility of pole region (7) is equal to the matrix inequality condition

$$M_D(\hat{A}_i, \tilde{X}_i) = U \otimes \tilde{X}_i + N \otimes (\hat{A}_i\tilde{X}_i) + N^T \otimes (\hat{A}_i\tilde{X}_i)^T < 0 \quad (31)$$

where ⊗ denotes the Kronecker product of matrices. Equation (31) was proven in [11] and as a counterpart of Gutman's theorem for LMI regions. Since the expressions of $M_D(\hat{A}_i, \tilde{X}_i)$ in (31) and $f_{\mathcal{D}}(z_i)$ in (7) are related by the substitution $(\tilde{X}_i,\ \hat{A}_i\tilde{X}_i,\ \tilde{X}_i\hat{A}_i^T) \leftrightarrow (1,\ z_i,\ \overline{z}_i)$, the matrix inequality condition for the disk region $\mathcal{D}(-q_i, \rho_i)$ as shown in (7) can be written as

$$\begin{bmatrix} -\rho_i\tilde{X}_i & q_i\tilde{X}_i + \hat{A}_i\tilde{X}_i \\ q_i\tilde{X}_i + \tilde{X}_i\hat{A}_i^T & -\rho_i\tilde{X}_i \end{bmatrix} < 0. \quad (32)$$

By the substitution of $\hat{A}_i = A_i + B_i G_i$ and $L_i = G_i\tilde{X}_i$, we have the following LMI condition in terms of the matrix variables $\tilde{X}_i$ and L_i for addressing the disk LMI region $\mathcal{D}(-q_i, \rho_i)$ constraints of the closed-loop poles:

$$\begin{bmatrix} -\rho_i\tilde{X}_i & A_i\tilde{X}_i + B_i L_i + q_i\tilde{X}_i \\ q_i\tilde{X}_i + \tilde{X}_i A_i^T + L_i^T B_i^T & -\rho_i\tilde{X}_i \end{bmatrix} < 0. \quad (33)$$

The proof is completed.

2. Constraints on H_∞ Norm

In considering the performance related to both the amplitude attenuation level in the output channel and the presence of stochastic external input are based on the result of the Lyapunov equation in (24).

Lemma 3.3

In the system (16), let $\gamma_i > 0$ be a fixed scalar. If there exist positive definite matrix $\tilde{X}_i$ and matrix L_i such that the following LMI condition holds

$$\begin{bmatrix} A_i\tilde{X}_i + B_iL_i + \tilde{X}_iA_i^T + L_i^TB_i^T + D_iD_i^T & \tilde{X}_iF_i^T \\ F_i\tilde{X}_i & -\gamma_i^2 I_i \end{bmatrix} < 0 \qquad (34)$$

where $L_i = G_i\tilde{X}_i$. Then, the H_∞ norm constraint (10) is satisfied. Furthermore, in this case, we have

$$X_i \le \tilde{X}_i \qquad (35)$$

Proof:

First, we define the Lyapunov function for the system dynamics (16) as following:

$$V(x_i(t)) = x_i^T(t)\tilde{X}_i^{-1}x_i(t) \qquad (36)$$

where $\tilde{X}_i = \tilde{X}_i^T > 0$ is the upper bound of X_i. According to Itô's differential rule [26], the time derivative of the quadratic Lyapunov function is

$$\begin{aligned} &\frac{d}{dt}V(x_i(t)) = x_i^T(t)\tilde{X}_i^{-1}(A_i + B_iG_i)x_i(t) \\ &+ x_i^T(t)(A_i + B_iG_i)^T\tilde{X}_i^{-1}x_i(t) + trace\left(D_i^T\tilde{X}_i^{-1}D_i\right). \end{aligned} \qquad (37)$$

The asymptotic stability of system (16) can be established if $\frac{d}{dt}V(x_i(t)) < 0$ can be satisfied. Now, we can rewrite (10) to obtain

$$\lim_{t\to\infty}\gamma_i^{-2}E[\int_0^t y_i^T(\tau)y_i(\tau)d\tau] - \lim_{t\to\infty}E[\int_0^t w_i^T(\tau)w_i(\tau)d\tau] < 0. \qquad (38)$$

Then, we define

$$\begin{aligned} J_i(t) &= E\left(\int_0^t\left[\gamma_i^{-2}y_i^T(\tau)y_i(\tau) - w_i^T(\tau)w_i(\tau)\right]d\tau\right) \\ &= E\left\{\int_0^t\left(\left[\gamma_i^{-2}y_i^T(\tau)y_i(\tau) - w_i^T(\tau)w_i(\tau)\right] + \frac{dV(x_i(\tau))}{d\tau}\right)d\tau\right\} - E[V(x_i(\tau))] + E[V(x_i(0))]. \end{aligned}$$

Also we have

$$\begin{aligned} J_i(t) &\le E\left\{\int_0^t\left(\left[\gamma_i^{-2}y_i^T(\tau)y_i(\tau) - w_i^T(\tau)w_i(\tau)\right] + \frac{dV(x_i(\tau))}{d\tau}\right)d\tau\right\} \\ &= E\left\{\int_0^t\left(\left[\gamma_i^{-2}\left(x_i^T(\tau)F_i^TF_ix_i(\tau)\right) - w_i^T(\tau)w_i(\tau)\right] + \frac{dV(x_i(\tau))}{d\tau}\right)d\tau\right\}. \end{aligned} \qquad (39)$$

Substituting (37) into (39), then we can obtain

$$J_i(t) \le E\left(\int_0^t \begin{bmatrix} x_i(\tau) \\ w_i(\tau) \end{bmatrix}^T \begin{bmatrix} (A_i+B_iG_i)^T\tilde{X}_i^{-1} + \tilde{X}_i^{-1}(A_i+B_iG_i) + \gamma_i^{-2}F_i^TF_i & \tilde{X}_i^{-1}D_i \\ D_i^T\tilde{X}_i^{-1} & -I_i \end{bmatrix}\begin{bmatrix} x_i(\tau) \\ w_i(\tau) \end{bmatrix}d\tau\right). \qquad (40)$$

By letting $t \to \infty$ and combining the condition in (38), the following inequality can be derived.

$$E\left(\int_0^\infty \begin{bmatrix} x_i(\tau) \\ w_i(\tau) \end{bmatrix}^T \begin{bmatrix} (A_i+B_iG_i)^T\tilde{X}_i^{-1} + \tilde{X}_i^{-1}(A_i+B_iG_i) + \gamma_i^{-2}F_i^TF_i & \tilde{X}_i^{-1}D_i \\ D_i^T\tilde{X}_i^{-1} & -I_i \end{bmatrix}\begin{bmatrix} x_i(\tau) \\ w_i(\tau) \end{bmatrix}d\tau\right) < 0 \qquad (41)$$

By Schur's complement, the inequality condition (41) is equivalent to

$$(A_i + B_iG_i)^T\tilde{X}_i^{-1} + \tilde{X}_i^{-1}(A_i + B_iG_i) + \gamma_i^{-2}F_i^TF_i + \tilde{X}_i^{-1}D_iD_i^T\tilde{X}_i^{-1} < 0. \qquad (42)$$

Pre- and post-multiplying both sides of (42) by $\tilde{X}_i$, we then have

$$(A_i + B_iG_i)\tilde{X}_i + \tilde{X}_i(A_i + B_iG_i)^T + \gamma_i^{-2}\tilde{X}_iF_i^TF_i\tilde{X}_i + D_iD_i^T < 0. \qquad (43)$$

Substituting $L_i = G_i\tilde{X}_i$ into (43) and using Schur's complement again, the inequality of (34) can be achieved. Moreover, subtracting (24) from (43), the inequality $X_i \le \tilde{X}_i$ can also be obtained due to the positive item $\gamma_i^{-2}\tilde{X}_iF_i^TF_i\tilde{X}_i > 0$. The proof is completed.

***Remark 3*:** The equation (43) also can be derived from (25).

3. Constraints on Upper Bound of Local State Covariance

In this subsection, the upper bound of local state covariance constraint is deduced to the following Lemma 3.4.

Lemma 3.4

Consider the desired upper bound of local state covariance constraint on the system as described in (11). Let $(\sigma_k)_i > 0$ is given. If there exists positive definite matrix $\tilde{X}_i$ such that the following LMI condition holds

$$\begin{bmatrix} (\sigma_k^2)_i & I_{ki}\tilde{X}_i \\ \tilde{X}_iI_{ki}^T & \tilde{X}_i \end{bmatrix} \ge 0, \quad k = 1, 2, \cdots, n_i \qquad (44)$$

where $I_{ki} = [0 \ \cdots \ 1 \ \cdots \ 0] \in R^{1\times n_i}$ denotes a row vector with the $k-$th element is 1 and others are 0. Then, the upper bound of local state covariance constraint can be achieved.

Proof:

Rewriting (11), one has

$$I_{ki}\tilde{X}_iI_{ki}^T \le (\sigma_k^2)_i, \quad k = 1, 2, \cdots, n_i \qquad (45)$$

or equivalently,

$$(\sigma_k^2)_i - I_{ki}\tilde{X}_i\tilde{X}_i^{-1}\tilde{X}_iI_{ki}^T \ge 0, \quad k = 1, 2, \cdots, n_i. \qquad (46)$$

Using the property of Schur's complement [4], (45) can be reformulated as following inequality

$$\begin{bmatrix} (\sigma_k^2)_i & I_{ki}\tilde{X}_i \\ \tilde{X}_iI_{ki}^T & \tilde{X}_i \end{bmatrix} \ge 0, \quad k = 1, 2, \cdots, n_i, \qquad (47)$$

which is the same as (46). Therefore the proof is completed.

Now, the individual LMI conditions as described in Lemma 3.2~3.4 for addressing various interesting performance constraints are summarized in the following main theorem.

Main theorem

In the system (16), given $\gamma_i > 0$, $(\sigma_k)_i > 0$, ρ_i and $q_i > 0$. The multi-objective performance constraints (i)~(iii) as described in (5), (10) and (11) are satisfied if there exist positive definite matrix $\tilde{X}_i$ and matrix L_i such that the following LMIs holds,

$$\begin{bmatrix} A_i\tilde{X}_i + B_iL_i + \tilde{X}_iA_i^T + L_i^TB_i^T + D_iD_i^T & \tilde{X}_iF_i^T \\ F_i\tilde{X}_i & -\gamma_i^{-2}I_i \end{bmatrix} < 0 \quad (48)$$

$$\begin{bmatrix} -\rho_i\tilde{X}_i & A_i\tilde{X}_i + B_iL_i + q_i\tilde{X}_i \\ q_i\tilde{X}_i + \tilde{X}_iA_i^T + L_i^TB_i^T & -\rho_i\tilde{X}_i \end{bmatrix} < 0 \quad (49)$$

$$\begin{bmatrix} (\sigma_k^2)_i & I_{ki}\tilde{X}_i \\ \tilde{X}_iI_{ki}^T & \tilde{X}_i \end{bmatrix} \geq 0, \ k = 1,\ 2,\ \cdots,\ n_i\ . \quad (50)$$

Proof:

Following the proofs of Lemma 3.2~3.4, one knows that the multi-objective performance (i)~(iii) can be achieved by the suitable convex optimization problem as shown in LMIs (48), (49) and (50). In other word, if matrices $\tilde{X}_i$ and L_i exist and satisfy these LMIs, then the control feedback gain G_i achieving the multi-objective performance constraints (i)~(iii) can be synthesized by

$$G_i = L_i\tilde{X}_i^{-1}. \quad (51)$$

The proof is completed.

Remark 4: By using the main theorem, one can minimize the H_∞ performance level γ_i for noise attenuation. Then, the resulting solution of the control feedback gain G_i can also achieve the pole placement constraint as well as upper bound on local state covariance constraint.

Main theorem shows that the multi-objective (i)~(iii) can be achieved by a convex optimization problem with LMI constraints. If the above LMIs are feasible, then we can obtain the control feedback gain G_i. In next section, a numerical example is provided to verify the proposed method. Now, one should check whether $\|H_i(s)\|_\infty \leq \gamma_i$ holds or not, the following lemma will be helpful.

Lemma 3.5 [3]

Consider the system (16). There exists a positive scalar γ_i to satisfy $\|H_i(s)\|_\infty \leq \gamma_i$ if and only if M_{γ_i} has no eigenvalues on the imaginary axis,

where $M_{\gamma_i} \triangleq \begin{bmatrix} A_i + B_iG_i & \gamma_i^{-1}D_iD_i^T \\ -\gamma_i^{-1}F_i^TF_i & -(A_i + B_iG_i)^T \end{bmatrix}$.

The design procedures presented in this paper is outlined as following

Initial status: The system (1) is given with certain assumptions in section 2.

Objective: Find the controller $u_i(t)$ such that the goals (i), (ii) and (iii) are achieved.

Step 1: Choose C_i to satisfy $C_iD_i = 0$ and $C_iB_i \neq 0$.

Step 2: From the main theorem, get the feasible solution of control feedback gain G_i as (51).

Step 3: Set the switching function $S_i(t)$ as (13).

Step 4: The controller $u_i(t)$ is obtained from (19).

IV. A NUMERICAL EXAMPLE

A linear time-invariant stochastic large-scale system is written in the form of two subsystems

$$\dot{x}_1(t) = A_1x_1(t) + B_1(u_1(t) + f_1(x_1(t))) + A_{12}x_2(t) + D_1w_1(t) \quad (52a)$$

$$y_1(t) = F_1x_1(t) \quad (52b)$$

$$\dot{x}_2(t) = A_2x_2(t) + B_2(u_2(t) + f_2(x_2(t))) + A_{21}x_1(t) + D_2w_2(t) \quad (53a)$$

$$y_2(t) = F_2x_2(t) \quad (53b)$$

where $x_1(t) = \begin{bmatrix} x_{11}(t) & x_{12}(t) \end{bmatrix}^T$, $A_1 = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $A_{12} = \delta\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$, $D_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $f_1(x_1(t)) \leq 0.6\|x_1(t)\|$, $F_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}$; $x_2(t) = \begin{bmatrix} x_{21}(t) & x_{22}(t) \end{bmatrix}^T$, $A_2 = \begin{bmatrix} 0 & 1 \\ -4 & 5 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $A_{21} = \delta\begin{bmatrix} 0 & 0 \\ 3 & 3 \end{bmatrix}$, $D_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $f_2(x_2(t)) \leq 0.4\|x_2(t)\|$ and $F_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, in which $\delta \in [-1, 1]$ is an uncertainty. The goal is to seek the control such that the steady state of the system satisfies the following requirements.

$$q_1 = 16,\ \rho_1 = 15 \quad (54)$$

$$q_2 = 18,\ \rho_2 = 17 \quad (55)$$

$$\|H_1(s)\|_\infty \leq 1 \quad (56)$$

$$\|H_2(s)\|_\infty \leq 0.8 \quad (57)$$

$$Var(x_{11}(t)) < 2.5,\ Var(x_{12}(t)) < 3 \quad (58)$$

$$Var(x_{21}(t)) < 1,\ Var(x_{22}(t)) < 2. \quad (59)$$

Suppose $x_1(0) = \begin{bmatrix} x_{11}(0) & x_{12}(0) \end{bmatrix}^T = \begin{bmatrix} 4 & 7 \end{bmatrix}^T$, $x_2(0) = \begin{bmatrix} x_{21}(0) & x_{22}(0) \end{bmatrix}^T = \begin{bmatrix} 5 & 8 \end{bmatrix}^T$ and the white noises $w_1(t)$ and $w_2(t)$ satisfy (3). Then, the proposed design procedure can be carried out as follows.

Step 1: Choosing $C_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}$ and $C_2 = \begin{bmatrix} -2 & 1 \end{bmatrix}$ such that $C_1D_1 = 0$, $C_2D_2 = 0$, and $C_1B_1 \neq 0$, $C_2B_2 \neq 0$. Then the corresponding two sliding modes are

$$\dot{x}_1(t) = (A_1 + B_1G_1)x_1(t) + D_1w_1(t) \quad (60a)$$

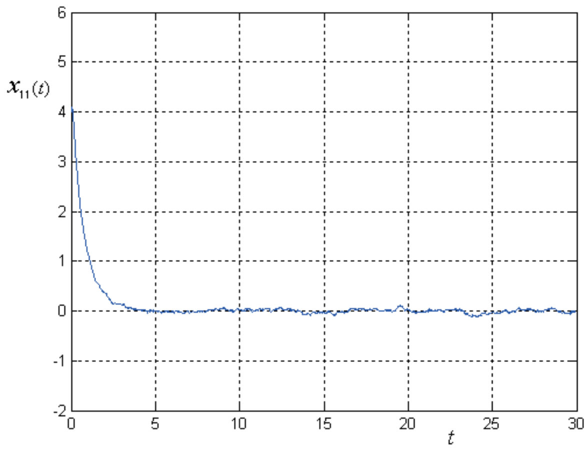

Fig. 2. Time response of state $x_{11}(t)$.

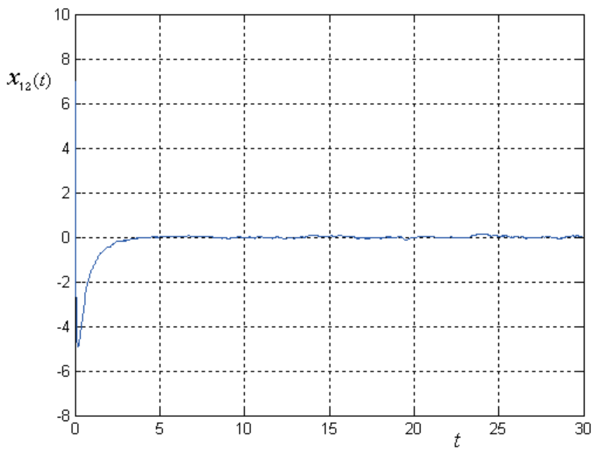

Fig. 3. Time response of state $x_{12}(t)$.

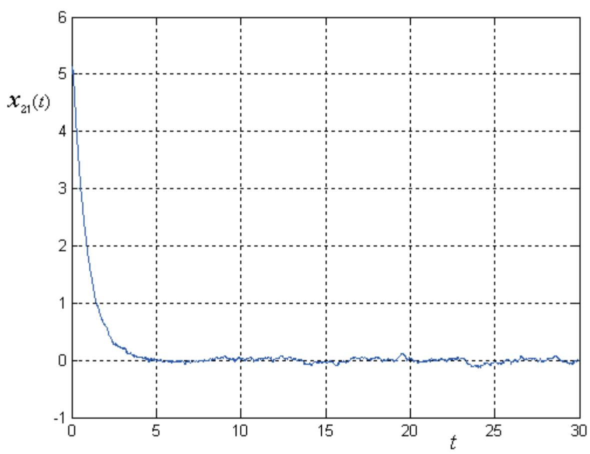

Fig. 4. Time response of state $x_{21}(t)$.

$$y_1(t) = F_1 x_1(t) \tag{60b}$$

and

$$\dot{x}_2(t) = (A_2 + B_2 G_2) x_2(t) + D_2 w_2(t) \tag{61a}$$

$$y_2(t) = F_2 x_2(t) \tag{61b}$$

respectively.

Step 2: Following the LMI algorithms in main theorem, we can construct a feasible solution of the local state upper

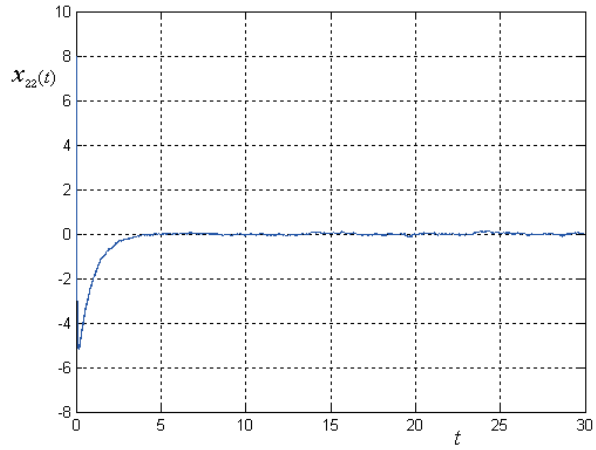

Fig. 5. Time response of state $x_{22}(t)$.

bound covariance matrices $\tilde{X}_1 = \begin{bmatrix} 1.1836 & -1.5122 \\ -1.5122 & 2.7282 \end{bmatrix}$ and $\tilde{X}_2 = \begin{bmatrix} 0.6671 & -0.7542 \\ -0.7542 & 1.7888 \end{bmatrix}$ in which their diagonal elements satisfy the performance constraints (58) and (59). And the related matrices are $L_1 = [9.0473 \quad -31.3765]$ and $L_2 = [6.8812 \quad -38.1604]$, respectively. Therefore, the control feedback gain matrices for each subsystem are as follows

$$G_1 = [-24.1562 \quad -24.8901] \tag{62}$$

and

$$G_2 = [-26.3698 \quad -32.4508]. \tag{63}$$

Step 3: From (13), the switching functions of two subsystems are

$$S_1(t) = [0 \quad 1] x_1(t) - \int_0^t [-26.1562 \quad -21.8901] x_1(\tau) d\tau \tag{64}$$

and

$$S_2(t) = [-2 \quad 1] x_2(t) - \int_0^t [-30.3698 \quad -29.4508] x_2(\tau) d\tau. \tag{65}$$

Step 4: From (19), the desired controllers of two subsystems are

$$\begin{aligned} u_1(t) &= [-24.1562 \quad -24.8901] x_1(t) \\ &\quad - \left[1.42 \|x_2(t)\| + 0.6 \|x_1(t)\| + 2\right] \operatorname{sgn}(S_1(t)) \end{aligned} \tag{66}$$

$$\begin{aligned} u_2(t) &= [-26.3698 \quad -32.4508] x_2(t) \\ &\quad - \left[9.5034 \|x_1(t)\| + 0.8944 \|x_2(t)\| + 2\right] \operatorname{sgn}(S_2(t)) \end{aligned} \tag{67}$$

where $k_1 = 1.42$, $k_2 = 4.25$, $\beta_1 = 0.6$, $\beta_2 = 0.4$ and $\alpha_1 = \alpha_2 = 2$ are chosen.

From the above design procedure, we can conclude that the local state upper bound covariance $\tilde{X}_i$ will be achieved if the system is driven by the controller (66)~(67).

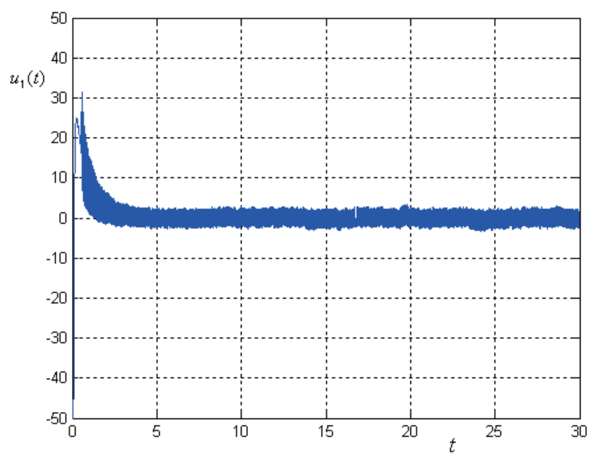

Fig. 6. Time response of state $u_1(t)$.

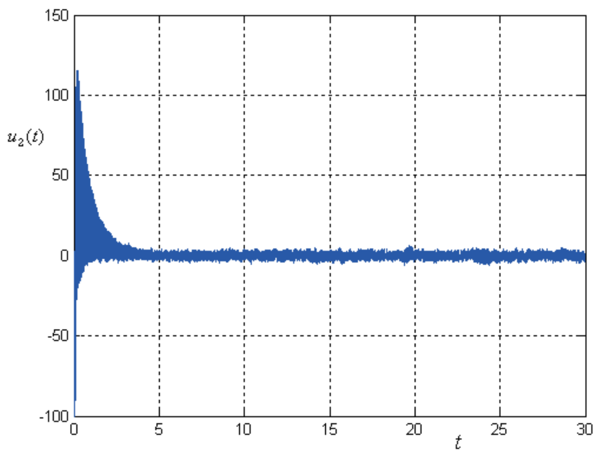

Fig. 7. Time response of state $u_2(t)$.

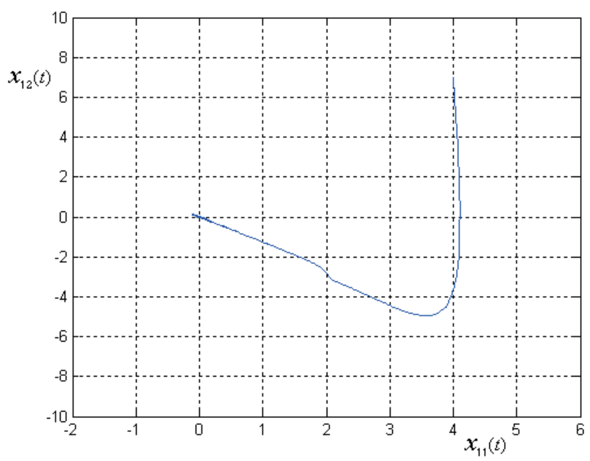

Fig. 8. The phase plane of $(x_{11}(t),\ \ x_{12}(t))$.

The simulation result for the controlled state responses $x_{11}(t)$, $x_{12}(t)$, $x_{21}(t)$, and $x_{22}(t)$ are shown in Figs. 2~5, respectively. The time response of $u_1(t)$ and $u_2(t)$ are shown in Figs. 6~7, respectively. The phase plane of $\left(x_{11}(t)\ ,\ x_{12}(t)\right)$ and $\left(x_{21}(t)\ ,\ x_{22}(t)\right)$ are shown in Figs. 8 and 9, respectively. It is

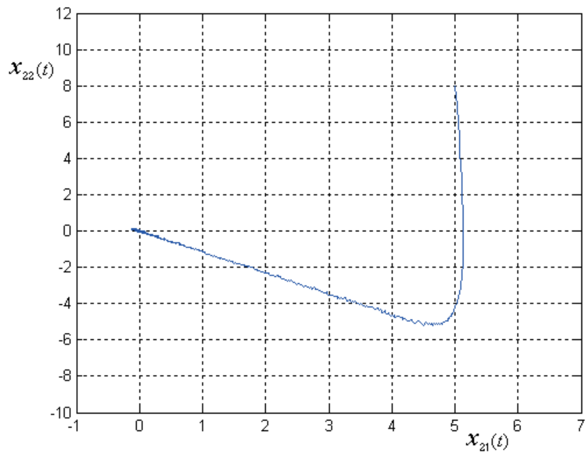

Fig. 9. The phase plane of $(x_{21}(t),\ \ x_{22}(t))$.

easy to check that the matrices M_{γ_1} and M_{γ_2}, which were defined in Lemma 3.5, have no eigenvalues on the imaginary axis; hence the H_∞ norm constraints (56) and (57) are satisfied. Moreover, the variances of $x_{11}(t)$, $x_{12}(t)$, $x_{21}(t)$, and $x_{22}(t)$ are 0.2347, 0.3796, 0.4254 and 0.4886, respectively. Therefore, the individual variance constraints (58) and (59) are also satisfied. We also check the poles of subsystem (52) and subsystem (53) locating in -1.2684, -20.6217, -1.1549 and -26.2959, respectively those satisfy the pole location constraint of (54) and (55), respectively.

V. CONCLUSIONS

This paper has applied the invariance property of SMC to the LSUBCC such that the both uncertain interconnection terms and an unknown nonlinear function can be ignored for the large-scale systems. Since the utilization of SMC and UBCC, the control feedback gain matrix G_i constructed by LMI approach not only achieves the multi-objective performance constraints for the closed-loop system but also determines the sliding surface of the system. Finally, a numerical example is adopted to illustrate the proposed method. The results of simulation show that the presented approach is effective in designing a multi-objective controller for the large-scale systems. In this paper, a new scheme of combination of SMC and LMI methods has been successfully demonstrated and the application of this scheme to some high performance complex systems will be developed in the future.

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