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Ming-Te Liang
*Department of Civil Engineering, China Institute of Technology, Taipei, Taiwan, R.O.C.*, mtliang@cc.chit.edu.tw

Ting- Bang Chu
*Department of Harbor and River Engineering, National Taiwan Ocean University, Keelung, Taiwan, R.O.C.*

Wen-Hu Tsao
*Department of Civil Engineering, China Institute of Technology, Taipei, Taiwan, R.O.C.*

Chi-Jang Yeh
*Join Engineering Consultants, Taipei, Taiwan, R.O.C.*



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Ming-Te Liang*, Ting- Bang Chu**, Wen-Hu Tsao*, and Chi-Jang Yeh***

Key words: multi-pole fuzzy pattern recognition evaluation method, D. E. R. evaluation method, Hamming weighted distance method, Euclidean weighted distance method.


## ABSTRACT

This paper proposes a technique for determining the repair ranking of existing reinforced concrete (RC) bridges using the multi-pole fuzzy pattern recognition evaluation method. The differences between bridges and grades are expressed by weighted generalized weight distance. To find the optimal relative membership degree of an evaluated bridge attributed to any grade, the operative principle is that the sum of the square of weighted generalized weight distance is minimum. Based on this principle, the problem of extreme value is established by the objective function in connection with constraint condition. Using the Lagrangean method, the multi-pole fuzzy pattern recognition evaluation method is thus established. In order to verify the applicability of this proposed method, five existing RC bridges in Taiwan are used as an illustrative example. The present study results indicate that the proposed method is reasonable, feasible and reliable. The studied results can be used as a crucial engineering decision-making tool for the repair, strengthening or demolition ranking of existing RC bridges.


## I. INTRODUCTION

During the thirty years or so after the Second World War, many bridges were designed and built with little or no thought given to the longer-term requirements of their future durability. That rather myopic point of view has changed, and bridge management is currently given the prommence it deserves, and it is now accepted that design is only a small part of the overall requirements. Bridge management is in effect the framework necessary to ensure a rational overall study of any given bridge, from conception to the end of its nominal life of 120 years, and beyond. Trustfully, the tragic instances of bridges lost to the destructive effects of corrosion, erosion, damage and overload will decrease, or even be eliminated.

The damage assessment of existing reinforced concrete (RC) bridges is very important to bridge management. Inspection [12] and condition assessment are now well-refined science, although assessment techniques applied in the field to determine actual damage state also need development. The D. (Degree) E. (Extent) R. (Relevancy) evaluation method [5, 20, 21, 22] is now widely used to evaluate the damage grade and repair ranking for existing RC bridges in Taiwan, and it consists of field measurement and visual inspection. Brown and Yao [2] used fuzzy set theory to assess the property of concrete strength, a concept which can be extended to evaluate the damage of existing structural engineering. Chou and Yuan [4] applied the fuzzy-Bayesian approach to compute the posterior probability based on visual inspection of existing structural components by incorporating fuzzy-set theory into Bayes' theorem. Zhao and Chen [38, 39] presented a fuzzy rule-based inference system for concrete bridge diagnosis, showing that the system has a high classification accuracy rate with a small number of rules. In the case of damage diagnosis, we may use the theory of analytical hierarchy process (AHP) [28, 29, 30, 31], deterministic method [9], probabilistic approaches [10, 27, 32, 34] vibration measurements and detection [7], neural network methods [33, 36], grey theory [18, 26, 23], fuzzy mathematics [23, 24, 25], fuzzy logic and probability-based estimation [8,35] as well as wavelet analysis [1, 6, 11] for bridge condition assessment.

Although these studies have provided much valuable information on the relationship between repair rankings and RC bridges, there are still many [12, 18, 26, 27, 38, 39] repair rankings that have not yet been determined. This paper describes a practical study investigating the repair ranking of existing RC bridges by using multi-pole fuzzy pattern recognition evaluation method. We investigated five existing RC bridges in Taiwan to verify the feasibility of the proposed method. The results of this study may be used to aid decision-making for repair, strengthening or demolition for these existing RC bridges.

Paper submitted 01/03/07; accepted 05/31/07. Author for correspondence: Ming-Te Liang (e-mail: mtliang@cc.chit.edu.tw).
*Department of Civil Engineering, China Institute of Technology, Taipei, Taiwan, R.O.C.
**Department of Harbor and River Engineering, National Taiwan Ocean University, Keelung, Taiwan, R.O.C.
***Join Engineering Consultants, Taipei, Taiwan, R.O.C.

## II. MULTI-POLE FUZZY PATTERN RECOGNITION EVALUATION METHOD

In the process of optimal selection, evaluating bridges simply as good or poor presents a crisp borderline, whereas it is actually transitional and fuzzy. Assume that n bridges in a bridge system are for optimal selection, forming a bridge alternative set. The assessment items for evaluating each bridge with good or poor establish an index set, $\overline{\mathrm{D}}$. According to the different assessment item attributes, $\overline{\mathrm{D}}$ is decomposed into B subsystems. Each subsystem has $c_1, c_2, c_3, \cdots\cdots c_m$ assessment items and satisfies

$$B = \bigcup_{i=1}^{m} c_i,\ c_i \cap c_k = \phi,\ \ i \neq k \tag{1}$$

where $\cup$, $\cap$ and $\phi$ stand for the union, intersection, and empty sets, respectively.

Propose a decision-making k subsystem which satisfies the constraint condition for supplying n bridge alternatives of optimal selection. The degree of optimism and pessimism of each alternative may be accorded to the m assessment item eigenvalues [21] for performing recognition. Now, there are n bridge alternatives and m assessment items for defining decision-making eigenvalue matrix X [21]

$$_kX = \begin{bmatrix} _kx_{11} & _kx_{12} & \cdots & _kx_{1n} \\ _kx_{21} & _kx_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ _kx_{m1} & _kx_{m2} & \cdots & _kx_{mn} \end{bmatrix} = (_kx_{ij}), \tag{2}$$

$$i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, n$$

where $_kx_{ij}$ is the eigenvalue of assessment item i of alternative j in the k subsystem.

The degree of optimism and pessimism of an alternative is based on the m assessment item eigenvalues, and it may be carried out the recognition for an alternative with z grade from optimistic to pessimistic grade. With respect to any assessment item, in the case of fuzzy concept, it may prescribe the optimistic (1 grade) and pessimistic (z grade) grades with values of 1 and 0 of relative membership grade, respectively. Because the optimum in the fuzzy concept appears gradually with variability in the transition stage, it is possible that relative membership grade from 1 to z grade will linearly decrease from 1 to 0. Thus, the decreasing difference of relative membership grade between front and rear grades is

$$\delta = \frac{1}{z-1},\ z > 2,\ z \in N \tag{3}$$

where $N$ is the natural number.

With regard to any assessment item, the relative membership grade standard vector of each grade from 1 to z grade is

$$s = (1,\ 1-\delta, 1-2\delta, \cdots, 0) = (S_h),\ h = 1, 2, \cdots, z. \tag{4}$$

Different assessment items are usually measured in different physical properties and units. Meanwhile, the optimal selection of alternative is a relative comparison to each selected alternative and is of relativity. To eliminate the concealed measurability due to different physical properties and unit and to increase convenience for calculation and optimal selection, let the absolute value of assessment item before decision-making transform to the relative value, i.e., relative membership grade [37]. Since there are different types of assessment item, the method of relative value transformed from absolute value is also different. Herein, we adopt the relative formulas suggested by Zadeh [40], as shown below:

A larger attribution of an assessment item indicates a better service state of the assessment item, and then we employ to increase semi-trapezoid type membership function to calculate the membership grade

$$r_{ij} = \frac{x_{ij} - \inf(x_i)}{Sup(x_i) - \inf(x_i)} \tag{5}$$

where $r_{ij}$ represents the relative membership grade of assessment items i under alternative j, $Sup(x_i) = \{Max(x_{ij}) \mid j = 1, 2, \cdots, n\}$ represents the maximum value of i assessment items during n membership grade, and $\inf(x_i) = \{Min(x_{ij}) \mid j = 1, 2, \cdots, n\}$ represents the minimum value of i assessment item during n membership grade.

A smaller attribution of an assessment item indicates a better service state of the assessment item, and then we use to decrease semi-trapezoid type membership function to calculate the membership grade

$$r_{ij} = \frac{Sup(x_i) - x_{ij}}{Sup(x_i) - \inf(x_i)} \tag{6}$$

Using Eqs. (5) and (6), the decision-making eigenvalue matrix of the k subsystem can be transformed as assessment item relative membership grade matrix

$$_kR = \begin{bmatrix} _kr_{11} & _kr_{12} & \cdots & _kr_{1n} \\ _kr_{21} & _kr_{22} & \cdots & _kr_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ _kr_{m1} & _kr_{m2} & \cdots & _kr_{mn} \end{bmatrix} = (_kr_{ij}) \tag{7}$$

where $_kr_{ij}$ stands for the relative membership grade of alternative j with respect to i assessment item in the k subsystem. Let the relative membership grades $r_{1j}, r_{2j}, \cdots, r_{mj}$ of m assessment items of alternative j compare with Eq. (4) item by item. Then we determine the relative membership grades of m assessment items of j alternative located in the closed grade internal $[a_{1j}, b_{1j}], [a_{2j}, b_{2j}], \cdots, [a_{mj}, b_{mj}]$. We also obtain the lower and upper bounds of grade of alternative j , i.e.,

$$\begin{cases} a_j = \{Min(a_{ij}) | \ i = 1,2,\cdots,m\} \\ b_j = \{Max(b_{ij}) | \ i = 1,2,\cdots,m\} \end{cases} \tag{8}$$

Assume that alternative ascribed to the relative membership grade matrix of each grade is

$${}_kU = \begin{bmatrix} {}_ku_{11} & {}_ku_{12} & \cdots & {}_ku_{1n} \\ {}_ku_{21} & {}_ku_{22} & \cdots & {}_ku_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ {}_ku_{z1} & {}_ku_{z2} & \cdots & {}_ku_{zn} \end{bmatrix} = ({}_ku_{hj}) \tag{9}$$

where ${}_ku_{hj}$ is the relative membership grade of alternative j with respect to grade h in the k subsystem.

Because the alternative j is located in the grade internal $[a_j, b_j]$, the matrix ${}_kU$ must satisfy the normalized condition

$$\sum_{h=a_j}^{b_j} u_{hj} = 1, \forall j \tag{10}$$

Due to the different range of grade interval of each alternative j, overall considering z grades and n alternatives, the matrix ${}_kU$ must also satisfy the normalized constraint condition

$$\sum_{h=1}^{z} u_{hj} = 1, \forall j \tag{11}$$

Because $a_j \geq 1$ and $b_j \leq z$, they should also satisfy the normalized constraint conditions of Eqs. (10) and (11) and have

$$u_{hj} = 0, \ h < a_j \text{ or } h > b_j \tag{12}$$

In the k subsystem, the assessment item of each alternative has different weights. Thus, assume that the assessment item effective weight matrix of alternative is

$${}_kw = \begin{bmatrix} {}_kw_{11} & {}_kw_{12} & \cdots & {}_kw_{1n} \\ {}_kw_{21} & {}_kw_{22} & \cdots & {}_kw_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ {}_kw_{m1} & {}_kw_{m2} & \cdots & {}_kw_{mn} \end{bmatrix} = ({}_kw_{ij}) \tag{13}$$

$${}_kw_j = \left\{ \sum_{i=1}^{m} {}_kw_{ij} \middle| j = 1,2,\cdots,n \right\} = 1$$

In the k subsystem, the relative membership grade of m assessment items of alternative j can be expressed in vector terms as

$${}_kr_j = \left( {}_kr_{1j}, {}_kr_{2j}, \cdots, {}_kr_{mj} \right) \tag{14}$$

Based on Eq. (4), the standard relative membership grade of m assessment items of grade h is

$$s_h = \frac{z-h}{z-1}, \ h = 1,2,\cdots,z, \forall i \tag{15}$$

In the k subsystem, for the difference between alternative j and grade h, we can adopt the generalized weight distance [3] expressed by

$${}_kd_{h_j} = \left\{ \sum_{i=1}^{m} \left[ {}_kw_{ij} \left( \left| {}_kr_{ij} - s_h \right| \right) \right]^p \right\}^{1/p} \tag{16}$$

where P stands for distance parameter, P=1 is the Hamming distance, and P=2 is the Euclidean distance. The physical meaning of Eq. (16) is the difference between alternative j and grade h after considering the weight of the assessment item. To more completely describe the difference between alternative j and grade h, we define the weighted generalized weight distance between alternative j and grade h as

$$\begin{aligned} {}_kD_{hj} &= {}_ku_{hj}\ {}_kd_{h_j} \\ &= {}_ku_{hj} \left\{ \sum_{i=1}^{m} [\, {}_kw_{ij} \left( \left| {}_kr_{ij} - s_h \right| \right)^p \right\}^{1/p} \end{aligned} \tag{17}$$

It is noteworthy that ${}_kD_{hj}$ considers not only the weight of assessment item in the k subsystem but also the weight of the relative membership grade ${}_ku_{hj}$ of alternative j attributed to grade h.

In order to find the optimal relative membership grade of alternative j attributed to grade h, the optimal principle is that for n alternatives the square sum of the standard relative membership grade of estimation factor and the weighted generalized weight distance is of minimum value. According to this principle, we may establish the objective function:

$$\min\left\{ F\left({}_ku_{hj}\right) = \sum_{h=a_j}^{b_j} ({}_kD_{hj})^2 \right\} \tag{18}$$

Based on Eqs. (10) and (18), we may set up fuzzy linear programming to finding the problem of extreme value, and the Lagrangean method is adopted to solve this problem. Assume that ${}_k\lambda_j$ are the Lagrangean multipliers. Then based of Eqs. (10), (17), and (18) the corresponding Lagrangean function is

$$L\left({}_ku_{hj}, {}_k\lambda_j\right) = \sum_{h=a_j}^{b_j} {}_ku_{hj}^2\ {}_kd_{hj}^2 - {}_k\lambda_j \left( \sum_{h=a_j}^{b_j} u_{hj} - 1 \right) \tag{19}$$

After taking respective partial derivatives with respect to ${}_ku_{hj}$ and ${}_k\lambda_j$ for Eq. (19), we obtain

$${}_ku_{hj} = \frac{\lambda_j}{2\left\{ \sum_{i=1}^{m} \left[ {}_kw_{ij} \left( \left| {}_kr_{ij} - s_h \right| \right) \right]^p \right\}^{\frac{2}{p}}} \tag{20}$$

and

**Table 1. Weight for each item of a bridge in the D.E.R. evaluation method.**

| Item | Weight | Item | Weight | Item | Weight |
|---|---|---|---|---|---|
| Substructure protection | 6 | Minor element (diaphragm) | 6 | Abutment | 6 |
| Pier foundation | 8 | Deck or hinged plate | 7 | Wing masonry | 5 |
| Pier shaft | 7 | Guide passage | 3 | Retaining wall | |
| Supporting mat | 5 | Road embankment | | Friction layer | 3 |
| Seismic block | 5 | Guide passage | 2 | Drainage appliance | 4 |
| Restraining cable | | Protection fence | | Curb and pedestrian way | 2 |
| Expansion joint | 6 | River channel | 3 | Balustrade/ Protection fence | 3 |
| Major element (girder) | 8 | Guide passage | 3 | Other | 1 |
| | | Road embankment protection | 7 | | |
| | | Abutment foundation | | | |

$$\sum_{h=a_j}^{b_j} u_{hj} - 1 = 0 \tag{21}$$

From Eqs. (20) and (21), we have

$$\lambda_i = \frac{2}{\displaystyle\sum_{h=a_j}^{b_j} \frac{1}{\left\{\sum_{i=1}^{m}\left[{}_k w_{ij}\left(\left|{}_k r_{ij} - s_h\right|\right)\right]^p\right\}^{\frac{2}{p}}}} \tag{22}$$

According to Eqs. (20) and (22), we obtain

$${}_k u_{hj} = \frac{1}{\left\{\sum_{i=1}^{m}\left[{}_k w_{ij}\left(\left|{}_k r_{ij} - s_h\right|\right)\right]^p\right\}^{\frac{2}{p}} \displaystyle\sum_{h=a_j}^{b_j} \frac{1}{\left\{\sum_{i=1}^{m}\left[{}_k w_{ij}\left(\left|{}_k r_{ij} - s_h\right|\right)\right]^p\right\}^{\frac{2}{p}}}} \tag{23}$$

To carry out check to Eq. (23) and to avoid confusion, we change the subscript h to x for the second term to the denominator at the right side of Eq. (23). Thus, Eq. (23) can be rewritten as

$${}_k u_{hj} = \frac{1}{\displaystyle\sum_{x=a_j}^{b_j}\left\{\frac{\sum_{j=1}^{n}\left[{}_k w_{ij}\left(\left|{}_k r_{ij} - s_h\right|\right)\right]^p}{\sum_{i=1}^{m}\left[{}_k w_{ij}\left(\left|{}_k r_{ij} - s_x\right|\right)\right]^p}\right\}^{\frac{2}{p}}} \tag{24}$$

Eq. (24) is called the multi-pole fuzzy pattern recognition model. When $d_{hj} = 0$, i.e., $\left\{\sum_{i=1}^{m}\left[{}_k w_i\left(\left|{}_k r_{ij} - s_h\right|\right)\right]^p\right\}^{1/p} = 0$, then ${}_k u_{hj} = 1$ is a special case. This means that in the k subsystem the relative membership grade of assessment item, ${}_k r_{ij}$, completely belongs to the standard relative membership grade of assessment item, $s_h$, i.e., ${}_k u_{hj} = 1$. Thus, the complete form of the multi-pole fuzzy pattern recognition model may be expressed in terms of

$${}_k u_{hj} = \begin{cases} 0 & h<a_j \ or\, h>b_j \\ \dfrac{1}{\displaystyle\sum_{x=a_j}^{b_j}\left\{\frac{\sum_{j=1}^{m}\left[{}_k w_{ij}\left(\left|{}_k r_{ij} - s_h\right|\right)\right]^p}{\sum_{i=1}^{m}\left[{}_k w_{ij}\left(\left|{}_k r_{ij} - s_x\right|\right)\right]^p}\right\}^{\frac{2}{p}}} & a_j \le h \le b_j, d_{hj} \ne 0 \\ 1 & d_{hj} = 0 \end{cases} \tag{25}$$

If we use Eq. (25) then the optimal relative membership grade matrix of the alternative set ascribed to each grade in the k subsystem can be found and referred to as grade membership grade matrix

$${}_k\overline{U} = \begin{bmatrix} {}_k\overline{u}_{11} & {}_k\overline{u}_{12} & \cdots & {}_k\overline{u}_{1n} \\ {}_k\overline{u}_{21} & {}_k\overline{u}_{22} & \cdots & {}_k\overline{u}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ {}_k\overline{u}_{z1} & {}_k\overline{u}_{z2} & \cdots & {}_k\overline{u}_{zn} \end{bmatrix} = ({}_k\overline{u}_{hj}),\ h = 1, 2, \cdots, z;$$
$$j = 1, 2, \cdots, n \tag{26}$$

where ${}_k\overline{u}_{hn}$ stands for the relative membership grade of the alternative n with respect to grade h in the k subsystem.

Generally speaking, in fuzzy synthesis evaluation and decision-making, the principle of maximum membership grade is usually used as the recognition model because it is concise and direct. However, the principle of maximum membership grade is its non-suitability. If a mistake is made in this principle then an unreasonable judgment will be occurred, yielding a wrong result that affects decision making. In order to illustrate this phenomenon, we use a simple example described as follows:

$${}_1\overline{u}_{11} = \max\left({}_1\overline{u}_{11}, {}_1\overline{u}_{21}, \cdots, {}_1\overline{u}_{z1}\right) \tag{27}$$

$${}_1\overline{u}_{11} \le \sum_{h=2}^{z} {}_1\overline{u}_{h1} \tag{28}$$

Eq. (27) denotes that in the first subsystem the membership grade of the first alternative with respect to the first grade is maximum. Eq. (28) indicates that the sum of membership grade of the first alternative with respect to from the second grade to the z grade is larger than the membership grade of the first grade. Using the principle of maximum membership grade, we judge that the first alternative belongs to the first grade. However, the first alternative is in fact not ascribed to the membership grade of the first grade over attributing to the membership grade of the first grade. Therefore, we adopt the vector form of grade eigenvalue ${}_k H$ [3]

**Table 2. D.E.R. values for each composite member of the Dah-jin bridge in Taiwan.**

Bridge name : Dah-jin bridge    Bridge No. :    Assessment date : 1997 / 09 / 17-26

General assessment items

| Assessment items | D E R | Assessment items | D E R | Assessment items | D E R |
|---|---|---|---|---|---|
| 1.Guide passage road embankment | | 5.Abutment foundation | | 9.Drainage appliance | |
| 2.Guide passage protection fence | | 6.Abutment | 2 1 1 | 10.Stone curb and Pedestrian way | |
| 3.River channel | 2 1 1 | 7.Wing masonry | | 11.Balustrade protection fence | 4 1 4 |
| 4.Guide passage road embankment protection | | 8.Friction layer | 2 2 2 | 21.Other | |

Detail assessment items

| Assessment items | 12 Substructure protection | 13 Pier foundation | 14 Pier shaft | 15 Supporting mat | 16 Seismic block | 17 Expansion joint | 18 Major member | 19 Minor member | 20 Deck |
|---|---|---|---|---|---|---|---|---|---|
| | D E R | D E R | D E R | D E R | D E R | D E R | D E R | D E R | D E R |
| | | | 2 1 1 | 3 1 2 | 2 1 2 | 3 4 3 | 2 3 2 | 2 1 1 | 2 1 2 |
| | | | 2 1 1 | 3 1 2 | | 3 4 3 | 2 3 2 | | 2 1 2 |
| | | | 2 1 1 | | | 3 4 3 | 2 3 2 | | 2 1 2 |
| | | | | | | 3 4 3 | 2 3 2 | | |
| | | | | | | 3 4 3 | 2 3 2 | | |
| | | | | | | 3 4 3 | 2 3 2 | | |
| | | | | | | | 2 3 2 | | |

N/A Without this item    U/I unable to assess    R/U unable to judge relative importance

| Assessment grade D | Range E | Importance R with respect to bridge | Emergency |
|---|---|---|---|
| N/A Good Mediate Poor Severe | U/I Local Global | R/U small Large | Route 5years 1year urgenc |
| 0 1 2 3 4 | 0 1 2 3 4 | 0 1 2 3 4 | 0 1 2 3 4 |

$$
\begin{aligned}
{}_kH &= (1,\ 2, \cdots, z)\left({}_k\bar{u}_{hn}\right) \\
&= \left({}_kH_1,\ {}_kH_2, \cdots, {}_kH_n\right)
\end{aligned}
\tag{29}
$$

to carry out optimal alternative selection. The alternative corresponding to the minimum grade eigenvalue ${}_kH = \left\{Min\left(H_j\right) \middle| j = 1, 2, \cdots, n\right\}$ is the optimal alternative of decision-making optimal selection.

The grade eigenvalue of n alternative of each subsystem is found from Eq. (29). The alternative grade eigenvalue of all subsystem consists of the decision-making eigenvalue matrix in more advanced layers. The subsystem weight establishes the effective weight of the more advanced layer subsystem. These data are used as input data. Using both the multi-pole fuzzy optimal selection model and grade eigenvalue, we apply the overall system layer for repeated calculations. Finally, the grade eigenvalue of individual alternative is obtained and this is the ranking index.

When we perform the optimal selection for decision-making, the grade z should be greater than or equal to z. The larger the value of z is, the higher the accuracy of optimal selection for decision-making has. However, the volume of calculation is also larger. In practical situations, taking z=2 should satisfy the requirement of accuracy. When the difference between grade eigenvalues is very small, i.e., it is very difficult to determine the decision-making of optimal selection. Thus, in order to promote the accuracy of decision-making of optimal selection we may take z=5. This means that the grades are divided into excellent, good, medium, below medium and poor.

## III. ILLUSTRATIVE EXAMPLE

The D.E.R. evaluation method [5, 20, 21, 22] divides RC bridges into 21 assessment items. Table 1 indicates the weight for each assessment item of an RC bridge in the D.E.R. evaluation method. Fig. 1 shows the analytic hierarchy model for an existing RC bridge in the D.E.R. evaluation method. We use the Dah-jin [13], Shuang-yuan [14], Jong-jang [15], Tou-chyan-shi [16] and Lin-bian [17] bridges in Taiwan as illustrative examples. Table 2 denotes each composite member D.E.R. value for the Dah-jin bridge. The composite member D.E.R. values of the other bridges are not indicated due to space limitations. Based on each composite member D.E.R. value of these five RC bridges, we establish assessment index. This assessment index is obtained by the product value of D, E, and R values, superimposing them and finally taking their average. Taking the fourteen the assessment items, i.e., pier shaft, of the Dah-jin bridge (see Table 2) as an illustrative example, we calculate the decision-making eigenvalue of assessment item $\left[(2\times1\times1)+(2\times1\times1)+(2\times1\times1)\right]/3=2$. Table 3 displays the decision-making eigenvalue of each bridge for different assessment items. Due to the characteristics of the D.E.R. evaluation method, the assessment index of an RC bridge is in the range from D×E×R×=1×1×1=1 to D×E×R=4×4×4=64. The larger the value of D×E×R has, the more the severe the damage is (see Table2). The value of D×E×R=0 indicates that this assessment item did not exist or it was impossible to assess or judge its relative importance, and so it does not need to be considered in the damage grade of the whole bridge. To avoid calculation, we replace 0 by ×.

**Table 3. Decision-making eigenvalue of each bridge subject to different assessment items.**

| Subsystem | Bridge decision-making eigenvalue (D*E*R)/n | | | | | |
|---|---|---|---|---|---|---|
| | Assessment items | Dah-jin | Shuang-yuan | Jong-jang | Tou-chyan-shi | Lin-bian |
| General assessment items | Guide Passage road embankment | x | x | x | x | x |
| | Guide Passage protection fence | x | x | x | x | x |
| | River channel | 2 | x | x | 32 | x |
| | Guide passage road embankment protection | x | x | x | x | x |
| | Abutment foundation | x | x | x | x | x |
| | Abutment | 2 | 2 | 2 | 3 | x |
| | Wing masonry | x | x | x | 6 | x |
| | Friction layer | 8 | x | x | x | 2 |
| | Drainage appliance | x | x | 2 | x | x |
| | Stone curb and Pedestrian way | x | x | x | x | x |
| | Balustrade Protection fence | 16 | 6 | 16 | x | 2 |
| | Other | x | x | x | x | x |
| Detail assessment items | Substructure protection | x | x | x | 64 | x |
| | Pier foundation | x | x | 9.6 | 23.07692 | x |
| | Pier shaft | 2 | 2.117647 | 2 | 2.666667 | 2 |
| | Supporting mat | 6 | x | x | x | x |
| | Seismic block | 4 | x | x | 5 | x |
| | Expansion joint | 36 | 8 | 12.375 | 18 | 10 |
| | Major member | 12 | 4.4 | 2.142857 | 3 | 9.333333 |
| | Minor member | 2 | x | 2 | 2.210526 | 4.25 |
| | Deck | 4 | 3.925926 | 2 | 2.5 | 2 |

According to this method and using Table 3, we set up the decision-making eigenvalue matrix for this subsystem as Table 3:

$$_{general}X = \begin{bmatrix} \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ 2 & \times & \times & 32 & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ 2 & 2 & 2 & 3 & \times \\ \times & \times & \times & 6 & \times \\ 8 & \times & \times & \times & 2 \\ \times & \times & 2 & \times & \times \\ \times & \times & \times & \times & \times \\ 16 & 6 & 16 & \times & 2 \\ \times & \times & \times & \times & \times \end{bmatrix}$$

and

$$_{detail}X = \begin{bmatrix} \times & \times & \times & 64 & \times \\ \times & \times & 9.600 & 23.077 & \times \\ 2 & 2.118 & 2 & 2.667 & 2 \\ 6 & \times & \times & \times & \times \\ 4 & \times & \times & 5 & \times \\ 36 & 8 & 12.375 & 18 & 10 \\ 12 & 4.4 & 2.143 & 3 & 9.333 \\ 2 & \times & 2 & 2.211 & 4.25 \\ 4 & 3.926 & 2 & 2.5 & 2 \end{bmatrix}$$

According to the concept of fuzzy membership grade and the essence of decision-making eigenvalue, Eq. (6) of the decreasing semi-trapezoid type membership function can be modified as

$$r_{ij} = \frac{Sup(x_{ij}) - x_{ij}}{Sup(x_{ij}) - \inf(x_{ij})} \tag{30}$$

where $Sup\left(x_{ij}\right) = 64$ and $\inf\left(x_{ij}\right) = 1$.

Using Eq.(30), the decision-making eigenvalue can be transferred as a relative membership grade, as shown in Table 4. The relative membership grade matrix may be expressed by the matrix form, as Table 4:

$$_{general}R = \begin{bmatrix} \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ 0.9841 & \times & \times & 0.5079 & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ 0.9841 & 0.9841 & 0.9841 & 0.9683 & \times \\ \times & \times & \times & 0.9206 & \times \\ 0.8889 & \times & \times & \times & 0.9841 \\ \times & \times & 0.9841 & \times & \times \\ \times & \times & \times & \times & \times \\ 0.7619 & 0.9206 & 0.7619 & \times & 0.9841 \\ \times & \times & \times & \times & \times \end{bmatrix}$$

and

**Table 4. Relative membership grade of each bridge.**

| Subsystem | Assessment items | Bridge decision-making eigenvalue (D*E*R)/n | | | | |
|---|---|---|---|---|---|---|
| | | Dah-jin | Shuang-yuan | Jong-jang | Tou-chyan-shi | Lin-bian |
| General assessment items | Guide Passage road embankment | x | x | x | x | x |
| | Guide Passage protection fence | x | x | x | x | x |
| | River channel | 0.9841 | x | x | 0.5079 | x |
| | Guide passage road embankment protection | x | x | x | x | x |
| | Abutment foundation | x | x | x | x | x |
| | Abutment | 0.9841 | 0.9841 | 0.9841 | 0.9683 | x |
| | Wing masonry | x | x | x | 0.9206 | x |
| | Friction layer | 0.8889 | x | x | x | 0.9841 |
| | Drainage appliance | x | x | 0.9841 | x | x |
| | Stone curb and Pedestrian way | x | x | x | x | x |
| | Balustrade Protection fence | 0.7619 | 0.9206 | 0.7619 | x | 0.9841 |
| | Other | x | x | x | x | x |
| Detail assessment items | Substructure protection | x | x | x | 0.0000 | x |
| | Pier foundation | x | x | 0.8635 | 0.6496 | x |
| | Pier shaft | 0.9841 | 0.9823 | 0.9841 | 0.9735 | 0.9841 |
| | Supporting mat | 0.9206 | x | x | x | x |
| | Seismic block | 0.9524 | x | x | 0.9365 | x |
| | Expansion joint | 0.4444 | 0.8889 | 0.8194 | 0.7302 | 0.8571 |
| | Major member | 0.8254 | 0.9460 | 0.9819 | 0.9683 | 0.8677 |
| | Minor member | 0.9841 | x | 0.9841 | 0.9808 | 0.9484 |
| | Deck | 0.9524 | 0.9536 | 0.9841 | 0.9762 | 0.9841 |

**Table 5. Damage range of each bridge.**

| | | *Dah-jin* | *Shuang-yuan* | *Jong-jang* | *Tou-chyan-shi* | *Lin-bian* |
|---|---|---|---|---|---|---|
| *General assessment items* | *Grade* | *1-2* | *1-2* | *1-2* | *1-3* | *1-2* |
| *Detail assessment items* | *Grade* | *1-4* | *1-2* | *1-2* | *1-5* | *1-2* |

$$
{}_{\text{detail}}R = \begin{bmatrix}
\times & \times & \times & 0.0000 & \times \\
\times & \times & 0.8635 & 0.6496 & \times \\
0.9841 & 0.9823 & 0.9841 & 0.9735 & 0.9841 \\
0.9206 & \times & \times & \times & \times \\
0.9524 & \times & \times & 0.9365 & \times \\
0.4444 & 0.8889 & 0.8194 & 0.7302 & 0.8571 \\
0.8254 & 0.9460 & 0.9819 & 0.9683 & 0.8677 \\
0.9841 & x & 0.9841 & 0.9808 & 0.9484 \\
0.9524 & 0.9536 & 0.9841 & 0.9762 & 0.9841
\end{bmatrix}
$$

Taking z=5, we obtain the standard vector of relative membership grade $s = (1\ ,\ 0.75\ ,\ 0.5\ ,\ 0.25\ ,\ 0)$ and $h = 1, 2, 3, 4, 5$. Employing Eq. (8), and comparing in turn the relative membership grade matrix with the standard vector of relative membership grade, we obtain the range of damage grade of each bridge, as shown in Table 5. In the general assessment items, the damage of Tou-chyan-chi bridge is clearly severe. The difference of damage grades for the other bridges is very small and fuzzy. In the detail assessment items, the damage to the Ton-chyan-shi and Dah-jin bridges are very severe. There are no differences for the other bridges.

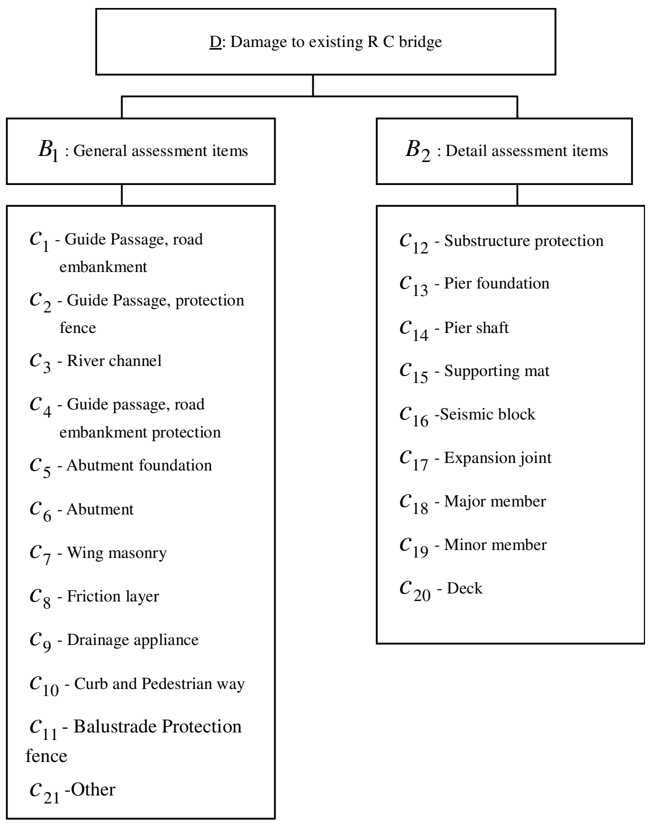


**Fig. 1. Analytic hierarchy model for an existing RC bridge.**

The effective weight of each assessment item of five existing RC bridges is indicated in Table 6. Based on Table 6, we may respectively establish the effective weight matrices of general

**Table 6. Effective weight of each assessment item of the bridges.**

| Subsystem | Assessment items | Effective Weight | | | | |
|---|---|---|---|---|---|---|
| | | Dah-jin | Shuang-yuan | Jong-jang | Tou-chyan-shi | Lin-bian |
| General assessment items | Guide Passage road embankment | 0 | 0 | 0 | 0 | 0 |
| | Guide Passage protection fence | 0 | 0 | 0 | 0 | 0 |
| | River chan-nel | 0.2 | 0 | 0 | 0.214286 | 0 |
| | Guide passage road embankment protection | 0 | 0 | 0 | 0 | 0 |
| | Abutment foundation | 0 | 0 | 0 | 0 | 0 |
| | Abutment | 0.4 | 0.666667 | 0.461538 | 0.428571 | 0 |
| | Wing ma-sonry | 0 | 0 | 0 | 0.357143 | 0 |
| | Friction layer | 0.2 | 0 | 0 | 0 | 0.5 |
| | Drainage appliance | 0 | 0 | 0.307692 | 0 | 0 |
| | Stone curb and Pedes-trian way | 0 | 0 | 0 | 0 | 0 |
| | Balustrade Protection fence | 0.2 | 0.333333 | 0.230769 | 0 | 0.5 |
| | Other | 0 | 0 | 0 | 0 | 0 |
| Detail assessment items | Substructure protection | 0 | 0 | 0 | 0.113208 | 0 |
| | Pier founda-tion | 0 | 0 | 0.190476 | 0.150943 | 0 |
| | Pier shaft | 0.159091 | 0.25 | 0.166667 | 0.132075 | 0.205882 |
| | Supporting mat | 0.113636 | 0 | 0 | 0 | 0 |
| | Seismic block | 0.113636 | 0 | 0 | 0.09434 | 0 |
| | Expansion joint | 0.136364 | 0.214286 | 0.142857 | 0.113208 | 0.176471 |
| | Major member | 0.181818 | 0.285714 | 0.190476 | 0.150943 | 0.235294 |
| | Minor member | 0.136364 | 0 | 0.142857 | 0.113208 | 0.176471 |
| | Deck | 0.159091 | 0.25 | 0.166667 | 0.132075 | 0.205882 |

and detail assessment items as follows:

$$_{\text{general}}w = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0.214 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0.667 & 0.461 & 0.429 & 0 \\ 0 & 0 & 0 & 0.357 & 0 \\ 0.2 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0.308 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.333 & 0.231 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$_{\text{detail}}w = \begin{bmatrix} 0 & 0 & 0 & 0.113 & 0 \\ 0 & 0 & 0.190 & 0.151 & 0 \\ 0.159 & 0.250 & 0.167 & 0.132 & 0.206 \\ 0.114 & 0 & 0 & 0 & 0 \\ 0.114 & 0 & 0 & 0.094 & 0 \\ 0.136 & 0.214 & 0.143 & 0.113 & 0.176 \\ 0.182 & 0.286 & 0.190 & 0.151 & 0.235 \\ 0.136 & 0 & 0.143 & 0.113 & 0.176 \\ 0.159 & 0.250 & 0.167 & 0.132 & 0.206 \end{bmatrix}$$

Substituting the relative membership grade matrix, standard vector of relative membership grade and effective weight matrix into Eq. (25) and taking P=1, the grade membership grade matrix of general assessment items is determined as follows:

$$_{general}\overline{U} = \begin{bmatrix} 0.8221 & 0.9706 & 0.8811 & 0.5935 & 0.9954 \\ 0.1779 & 0.0294 & 0.1189 & 0.3028 & 0.0046 \\ 0 & 0 & 0 & 0.1037 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Taking the Dah-jin bridge as an illustrative example, the damage grade of all general assessment items is located in the range of first and second grades. Thus, the grade membership grade with respect to the third, fourth and fifth grades of general assessment items for the Dah-jin bridge are all zero. However, the grade membership grade with respect to the first grade, $_{general}\overline{U}_{11}$, is

$$_{general}\overline{U}_{11} = \frac{1}{\sum_{x=1}^{2}\left\{\frac{\sum_{i=1}^{m}\left[_{general}w_{i1}\left(_{general}r_{i1} - s_1\right)\right]}{\sum_{i}^{m}\left[_{general}w_{i1}\left(_{general}r_{i1} - s_x\right)\right]}\right\}^2}$$

$$= \frac{1}{\left(\frac{_{general}d_{11}}{_{general}d_{11}}\right)^2 + \left(\frac{_{general}d_{11}}{_{general}d_{21}}\right)^2}$$

where

$$\sum_{i=1}^{m}\left[_{general}w_{i1}\left(_{general}r_{i1} - s_1\right)\right] = \left[0.2\left(|0.9841-1|\right)\right] + \left[0.4\left(|0.9841-1|\right)\right]$$
$$+\left[0.2\left(|0.8889-1|\right)\right] + \left[0.2\left(|0.7619-1|\right)\right]$$
$$\cong 0.0794 = {}_{general}d_{11}$$

and

$$\sum_{i=1}^{m}\left[{}_{general}w_{i1}\left({}_{general}r_{i1}-s_2\right)\right]=\left[0.2\left(\left|0.9841-0.75\right|\right)\right]+\left[0.4\left(\left|0.9841-0.75\right|\right)\right]+\left[0.2\left(\left|0.8889-0.75\right|\right)\right]+\left[0.2\left(\left|0.7619-0.75\right|\right)\right]\cong 0.1706={}_{\text{general}}d_{21}$$

Accordingly, we have

$$_{\text{general}}\overline{\mathrm{U}}_{11}=\frac{1}{\left(\frac{0.0794}{0.0794}\right)^2+\left(\frac{0.0794}{0.1706}\right)^2}\cong 0.8221$$

Putting the relative membership grade matrix, the standard vector of relative membership grade and the effective weight matrix into Eq. (25), and taking P=1, then the grade membership grade matrix of detail assessment items is calculated as follows:

$$_{\text{detail}}\overline{U}=\begin{bmatrix}0.6151 & 0.9254 & 0.8988 & 0.4292 & 0.8595\\ 0.2794 & 0.0746 & 0.1012 & 0.3532 & 0.1405\\ 0.0763 & 0 & 0 & 0.1274 & 0\\ 0.0292 & 0 & 0 & 0.0576 & 0\\ 0 & 0 & 0 & 0.0327 & 0\end{bmatrix}$$

Employing the Dah-jin bridge as an illustrative example, the damage grade of all detail assessment items is stated in the range of the first and fourth grades. Therefore, the grade membership grade with respect to the fifth grade of the detail assessment items for the Dah-jin bridge is zero. Nevertheless, the grade membership grade with respect to the first grade, $_{\text{detail}}\overline{U}_{11}$, is

$$_{\text{detail}}\overline{U}_{11}=\frac{1}{\sum_{x=2}^{5}\left\{\frac{\sum_{i=1}^{m}\left[{}_{\text{detail}}w_{i1}\left({}_{\text{detail}}r_{i1}-s_1\right)\right]}{\sum_{i=1}^{m}\left[{}_{\text{detail}}w_{i1}\left({}_{\text{detail}}r_{i1}-s_x\right)\right]}\right\}^2}=\frac{1}{\left(\frac{{}_{\text{detail}}d_{11}}{{}_{\text{detail}}d_{11}}\right)^2+\left(\frac{{}_{\text{detail}}d_{11}}{{}_{\text{detail}}d_{21}}\right)^2+\left(\frac{{}_{\text{detail}}d_{11}}{{}_{\text{detail}}d_{31}}\right)^2+\left(\frac{{}_{\text{detail}}d_{11}}{{}_{\text{detail}}d_{41}}\right)^2}$$

where

$$\sum_{i=1}^{m}\left[{}_{\text{detail}}w_{i1}\left({}_{\text{detail}}r_{i1}-s_1\right)\right]=\left[0.159\left(\left|0.9841-1\right|\right)\right]+\left[0.114\left(\left|0.9206-1\right|\right)\right]+\left[0.114\left(\left|0.9524-1\right|\right)\right]+\left[0.136\left(\left|0.4444-1\right|\right)\right]+\left[0.182\left(\left|0.8254-1\right|\right)\right]+\left[0.136\left(\left|0.9841-1\right|\right)\right]+\left[0.159\left(\left|0.9524-1\right|\right)\right]\cong 0.1342={}_{\text{detail}}d_{11}$$

$$\sum_{i=1}^{m}\left[{}_{\text{detail}}w_{i1}\left({}_{\text{detail}}r_{i1}-s_2\right)\right]=\left[0.159\left(\left|0.9841-0.75\right|\right)\right]+\left[0.114\left(\left|0.9206-0.75\right|\right)\right]+\left[0.114\left(\left|0.9524-0.75\right|\right)\right]+\left[0.136\left(\left|0.4444-0.75\right|\right)\right]+\left[0.182\left(\left|0.8254-0.75\right|\right)\right]+\left[0.136\left(\left|0.9841-0.75\right|\right)\right]+\left[0.159\left(\left|0.9524-0.75\right|\right)\right]\cong 0.1991={}_{\text{detail}}d_{21}$$

$$\sum_{i=1}^{m}\left[{}_{\text{detail}}w_{i1}\left({}_{\text{detail}}r_{i1}-s_3\right)\right]=\left[0.159\left(\left|0.9841-0.5\right|\right)\right]+\left[0.114\left(\left|0.9206-0.5\right|\right)\right]+\left[0.114\left(\left|0.9524-0.5\right|\right)\right]+\left[0.136\left(\left|0.4444-0.5\right|\right)\right]+\left[0.182\left(\left|0.8254-0.5\right|\right)\right]+\left[0.136\left(\left|0.9841-0.5\right|\right)\right]+\left[0.159\left(\left|0.9524-0.5\right|\right)\right]\cong 0.3809={}_{\text{detail}}d_{31}$$

and

$$\sum_{i=1}^{m}\left[{}_{\text{detail}}w_{i1}\left({}_{\text{detail}}r_{i1}-s_4\right)\right]=\left[0.159\left(\left|0.9841-0.25\right|\right)\right]+\left[0.114\left(\left|0.9206-0.25\right|\right)\right]+\left[0.114\left(\left|0.9524-0.25\right|\right)\right]+\left[0.136\left(\left|0.4444-0.25\right|\right)\right]+\left[0.182\left(\left|0.8254-0.25\right|\right)\right]+\left[0.136\left(\left|0.9841-0.25\right|\right)\right]+\left[0.159\left(\left|0.9524-0.25\right|\right)\right]\cong 0.6158={}_{\text{detail}}d_{41}$$

Therefore, we obtain

$$_{\text{detail}}\overline{U}_{11}=\frac{1}{\left(\frac{0.1342}{0.1342}\right)^2+\left(\frac{0.1342}{0.1991}\right)^2+\left(\frac{0.1342}{0.3809}\right)^2+\left(\frac{0.1342}{0.6158}\right)^2}\cong 0.6151$$

Similarly, we may obtain the grade membership grade of the other four bridges. Moreover, the grade eigenvalue of the general and detail assessment items of each bridge can be calculated by Eq. (29) and are respectively

$$_{\text{general}}H=(\ 1.1779\ ,\ 1.0294\ ,\ 1.1189\ ,\ 1.5102\ ,\ 1.0046\ )$$

and

$$_{\text{detail}}H=(\ 1.5197\ ,\ 1.0746\ ,\ 1.1012\ ,\ 1.9114\ ,\ 1.1405\ )$$

Using the Dah-jin bridge as an illustrative example, its grade membership grade vectors of general and detail assessment items are respectively $[0.8221 , 0.1779 , 0 , 0 , 0]^T$ and $[0.6151 , 0.2794 , 0.0763, 0.0292 , 0]^T$. Thus, its grade eigenvalue of general and detail assessment items are respectively

$$_{\text{general}}H_1=\left[1,2,3,4,5\right]\begin{bmatrix}0.8221\\0.1779\\0\\0\\0\end{bmatrix}=1.1779$$

and

$$_{\text{detail}}H_1 = [1, 2, 3, 4, 5] \begin{bmatrix} 0.6151 \\ 0.2794 \\ 0.0763 \\ 0.0292 \\ 0 \end{bmatrix} = 1.5197$$

In the same manner, we may find the grade eigenvalue vector of general and detail assessment items. Both the grade eigenvalue vector of general and detail assessment items can determine the decision-making eigenvalue matrix of subsystem as

$$X = \begin{bmatrix} 1.1779 & 1.0294 & 1.1189 & 1.5102 & 1.0046 \\ 1.5197 & 1.0746 & 1.1012 & 1.9114 & 1.1405 \end{bmatrix}$$

The larger the value of decision-making eigenvalue has, the more severe the damage grade of an existing RC bridge is.

In accordance with the concept of fuzzy membership grade and the essence of decision-making eigenvalue, Eq. (6) of the decreasing semi-trapezoid type membership function can be modified as

$$r_{ij} = \frac{Sup(x_{ij}) - x_{ij}}{Sup(x_{ij}) - \inf(x_{ij})} \tag{31}$$

where $Sup\left(x_{ij}\right) = 5$ and $\inf\left(x_{ij}\right) = 1$.

Using Eq. (31), the decision-making eigenvalue of subsystem can be transferred as the relative membership grade. The relative membership grade matrix is

$$R = \begin{bmatrix} 0.9555 & 0.9927 & 0.9703 & 0.8725 & 0.9989 \\ 0.8701 & 0.9814 & 0.9747 & 0.7722 & 0.9649 \end{bmatrix}$$

Table 7 shows the subsystem weight [24]. Based on Table 7, we may construct the subsystem weight matrix as follows:

$$w = \begin{bmatrix} 0.2542 & 0.2432 & 0.2364 & 0.2090 & 0.1500 \\ 0.7458 & 0.7568 & 0.7636 & 0.7910 & 0.8500 \end{bmatrix}$$

Substituting the relative membership grade matrix, the standard vector of relative membership grade and the subsystem weight matrix into Eq. (25) yields the grade membership grade matrix

$$U = \begin{bmatrix} 0.6321 & 0.9954 & 0.9863 & 0.0416 & 0.9817 \\ 0.3679 & 0.0046 & 0.0137 & 0.9584 & 0.0183 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Using Eq. (29), we obtain the grade eigenvalue of each bridge

$$H = (\text{Dah-jin, Shuang-yuan, Jong-jang, Tou-chyan-shi, Lin-bian})$$
$$= (1.3679,\ 1.0046,\ 1.0137,\ 1.9584,\ 1.0183)$$

The larger the grade eigenvalue has, the greater the damage grade for an existing RC bridge is and the sooner the repair must be carried out. Thus, the repair ranking is the Tou-chyan-shi, Dah-jin, Lin-bian, Jong-jang and Shuang-yuan bridges. This means that the Tou-chyan-shi bridge is the first one to be repaired among the selected five existing RC bridges. This also implies that the assessment items with higher weight value such as pier foundation, major element (girder), pier shaft and deck (see Table 1) are taken priority of repair.

As a result, the flow chart of computation procedure of multi-pole fuzzy pattern recognition evaluation method is summarized as shown in Fig. 2.

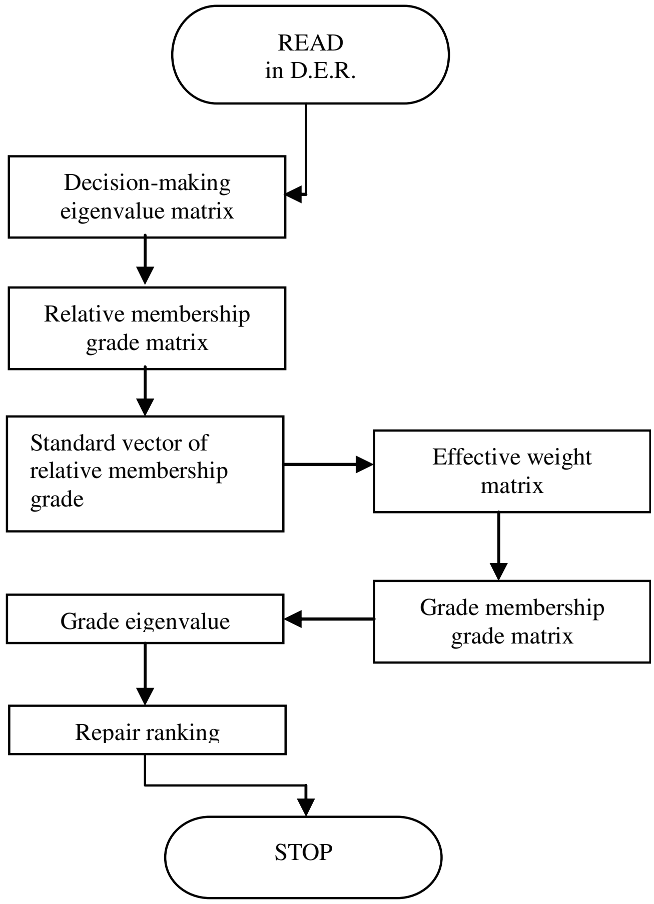


**Fig. 2. Flow chart for multi-pole fuzzy pattern recognition evaluation method.**

**Table 7. Effective subsystem weight of each bridge.**

| | Dah-jin | Shuang-yuan | Jong-jang | Tou-chyan-shi | Lin-bian |
|---|---|---|---|---|---|
| General assessment items | 0.25424 | 0.24324 | 0.23636 | 0.20896 | 0.15000 |
| Detail assessment items | 0.74576 | 0.75676 | 0.76364 | 0.79104 | 0.85000 |

## IV. DISCUSSION

The weighted distance, multi-pole fuzzy pattern recognition evaluation and D.E.R. evaluation methods discussed in sequence.

### 1. Weighted Distance Method

The Hamming and Euclidean weighted distance methods are now employed to find the distance of each bridge with respect to "good" (membership grade=1) and "poor" (membership grade=0).

*1) The Hamming Weighted Distance Method*

The Hamming weighted distance method can be expressed by

$$d_{sj}^{H} = \sum_{i=1}^{m} w_{ij}\left(\left|r_{is} - r_{ij}\right|\right) \tag{32}$$

where $d_{sj}^{H}$ stands for the Hamming weighted distance of alternative j with respect to alternative s; $w_{ij}$ stands for the weight of alternative j with respect to assessment item i; and $r_{is}$ and $r_{ij}$ stand for the membership grades of alternatives s and j with respect to assessment item i, respectively. The conditions of $r_{is} = 0$ and $r_{is} = 1$ stand for the distance of alternative j with respect to "poor" and "good", respectively. The calculated results are described as follows:

A. $r_{is} = 0$

Considering only the detail assessment items, we obtain: Dah-jin bridge ( 0.8658), Shuang-yuan bridge (0.9447), Jong-jang bridge (0.9372), Tou-chyan-shi bridge (0.7838) and Lin-bian bridge (0.9280). Thus, the repair ranking is: Tou-chyan-shi bridge → Dah-jin bridge → Lin-bian bridge → Jong-jang bridge → Shuang-yuan bridge.

Synthesizing both the general and detail assessment items, we then have: Dah-jin bridge (0.8797), Shuang-yuan bridge (0.9492), Jong-jang bridge (0.9362), Tou-chyan-shi bridge (0.7981) and Lin-bian bridge (0.9364). Therefore, the repair ranking is: Tou-chyan-shi bridge → Dah-jin bridge → Jong-jang bridge → Lin-bian bridge → Shuang-yuan bridge.

B. $r_{is} = 1$

Considering only the detail assessment items, we then obtain: Dah-jin bridge (0.1342), Shuang-yuan bridge (0.0553), Jong-jang bridge (0.0628), Tou-chyan-shi bridge (0.2162) and Lin-bian bridge (0.0720). Accordingly, the repair ranking is: Tou-chyan-shi bridge → Dah-jin bridge → Lin-bian bridge → Jong-jang bridge → Shuang-yuan bridge.

Combiningly both the general and detail assessment items, we then determine: Dah-jin bridge (0.1203), shuang-yuan bridge (0.0508), Jong-jang bridge (0.0638), Tou-chyan-shi bridge (0.2019), Lin-bian bridge (0.0636). Therefore, the repair ranking is: Tou-chyan-shi bridge → Dah-jin bridge → Jong-jang bridge → Lin-bian bridge → Shuang-yuan bridge.

*2) The Euclidean Weighted Distance Method*

The Euclidean weighted distance method can be determined indicated by

$$d_{sj}^{E} = \left\{\sum_{i=1}^{m}\left[w_{ij}\left(\left|r_{is} - r_{ij}\right|\right)\right]^{2}\right\}^{1/2} \tag{33}$$

where $d_{sj}^{E}$ is the Euclidean weighted distance of alternative j with respect to alternative s, $w_{ij}$ is the weight of alternative j with respect to assessment item i, and $r_{is}$ and $r_{ij}$ are the membership grades of alternatives s and j with respect to assessment item i, respectively. The conditions of $r_{is} = 0$ and $r_{is} = 1$ are the distances of alternative j with respect to "poor" and "good", respectively. The calculated results are depicted as follows:

**Table 8. Membership grade of of each bridge grade. (The Hamming weighted distance method for detail assessment items.)**

| | Dah-jin | Shuang-yuan | Jong-jang | Tou-chyan-shi | Lin-bian |
|---|---|---|---|---|---|
| z=2 | 1.02346 | 1.00341 | 1.00447 | 1.07074 | 1.00598 |
| z=3 | 1.14994 | 1.01521 | 1.02022 | 1.32706 | 1.02750 |
| z=5 | 1.51965 | 1.07457 | 1.10119 | 1.91137 | 1.14048 |
| z=9 | 2.18475 | 1.38592 | 1.47581 | 3.02197 | 1.66253 |
| z=17 | 3.36167 | 1.93256 | 2.03056 | 5.19398 | 2.23529 |
| z=21 | 4.04130 | 2.11225 | 2.33912 | 6.28896 | 2.40888 |

A. $r_{is} = 0$

Considering only the detail assessment items, we then obtain: Dah-jin bridge (0.3381), Shuang-yuan bridge (0.4759), Jong-jang bridge (0.3863), Tou-chyan-shi bridge (0.3018) and Lin-bian bridge (0.4180). Therefore, the repair ranking is: Tou-chyan-Shi bridge → Dah-jin bridge → Jong-jang bridge → Lin-bian bridge → Shuang-yuan bridge.

Integratingly both the general and detail assessment items, we then obtain: Dah-jin bridge ( 0.2822), Shuang-yuan bridge (0.4009), Jong-jang bridge (0.3247), Tou-chyan-shi bridge (0.2641) and Lin-bian bridge (0.3703). As a result, the repair ranking is: Tou-chyan-shi bridge → Dah-jin bridge → Jong-jang bridge → Lin-bian bridge → Shuang-yuan bridge.

B. $r_{is} = 1$

Considering only the detail assessment items, we then obtain: Dah-jin bridge: (0.0832), Shuang-yuan bridge (0.0310), Jong-jang bridge (0.0370), Tou-chyan-shi bridge (0.1290) and Lin-bian bridge (0.0413). Thus, the repair ranking is: Tou-chyan-shi bridge → Dah-jin bridge → Lin-bian bridge → Jong-jang bridge → Shuang-yuan bridge

Combining both the general and detail assessment items, we then obtain: Dah-jin bridge: (0.0635), Shuang-yuan bridge (0.0244), Jong-jang bridge (0.0312), Tou-chyan-shi bridge (0.1046) and Lin-bian bridge (0.0352). Therefore, the repair ranking is: Tou-chyan-shi bridge → Dah-jin bridge → Lin-bian bridge → Jong-jang bridge → Shuang-yuan bridge.

## 2. Multi-pole Fuzzy Pattern Recognition Evaluation Method

The values P=1, P=2, and z=2, z=3, z=5, z=9, z=17, and z=21 are used to explore the bridge grade eigenvalue of considering only the detail assessment items and synthetically considering both the general and detail assessment items, respectively.

*1) P=1 (the Hamming Weighted Distance Method)*

A. Considering only the detail assessment items

The calculated results are shown in Table 8. When z=2, z=3, z=5, z=9, z=17 and z=21, the repair ranking is: Tou-chyan-shi bridge → Dah-jin bridge → Lin-bian bridge → Jong-jang bridge → Shuang-yuan bridge.

**Table 9. Membership grade of each bridge grade. (The Hamming weighted distance method for both general and detail assessment items.)**

| | Dah-jin | Shuang-yuan | Jong-jang | Tou-chyan-shi | Lin-bian |
|---|---|---|---|---|---|
| z=2 | 1.00039 | 1.00001 | 1.00002 | 1.00435 | 1.00003 |
| z=3 | 1.01845 | 1.00018 | 1.00046 | 1.14274 | 1.00058 |
| z=5 | 1.36790 | 1.00459 | 1.01368 | 1.95838 | 1.01829 |
| z=9 | 2.00760 | 1.19336 | 1.42847 | 2.97930 | 1.63023 |
| z=17 | 3.05627 | 1.97326 | 2.00207 | 4.87372 | 2.02412 |
| z=21 | 3.84996 | 2.00200 | 2.22960 | 5.86110 | 2.14539 |

**Table 10. Membership grade of grade of each bridge. (The Euclidean weighted distance method for detail assessment items.)**

| | Dah-jin | Shuang-yuan | Jong-jang | Tou-chyan-shi | Lin-bian |
|---|---|---|---|---|---|
| z=2 | 1.05712 | 1.00422 | 1.00911 | 1.15439 | 1.00969 |
| z=3 | 1.30398 | 1.01861 | 1.03981 | 1.57900 | 1.04347 |
| z=5 | 1.80872 | 1.08765 | 1.17044 | 2.26406 | 1.19623 |
| z=9 | 2.58316 | 1.38603 | 1.63805 | 3.63436 | 1.71791 |
| z=17 | 4.14200 | 1.93098 | 2.18668 | 6.39131 | 2.26499 |
| z=21 | 5.05862 | 2.14658 | 2.53454 | 7.77131 | 2.47211 |

**Table 11. Membership grade of each bridge grade (The Euclidean weighted distance method for both general and detail assessment items.)**

| | Dah-jin | Shuang-yuan | Jong-jang | Tou-chyan-shi | Lin-bian |
|---|---|---|---|---|---|
| z=2 | 1.00326 | 1.00002 | 1.00008 | 1.02981 | 1.00009 |
| z=3 | 1.13580 | 1.00033 | 1.00171 | 1.59830 | 1.00199 |
| z=5 | 1.85435 | 1.00828 | 1.03979 | 2.05939 | 1.05383 |
| z=9 | 2.38031 | 1.24862 | 1.74653 | 3.45970 | 1.82486 |
| z=17 | 3.88740 | 1.97368 | 2.05487 | 5.93740 | 2.07177 |
| z=21 | 4.66675 | 2.01083 | 2.58064 | 7.10535 | 2.30249 |

B. Synthetically considering both the general and detail assessment items

The calculated results are displayed in Table 9. When z=2, z=3, z=5, z=9 and z=17, the repair ranking is: Tou-chyan-shi bridge → Dah-jin bridge → Lin-bian bridge → Jong-jang bridge → Shuang-yuan bridge. However, when z=21, the repair ranking is: Tou-chyan-shi bridge→Dah-jin bridge→Jong-jang bridge→Lin-bian bridge→Shuang-yuan bridge.

*2) P=2 (the Euclidean Weighted Distance Method)*

A. Considering only the detail assessment items

The reckoned results are indicated in Table 10. When z=2, z=3, z=5, z=9 and z=17, the repair rankings are: Tou-chyan-shi bridge → Dah-jin bridge → Lin-bian bridge → Jong-jang bridge → Shuang-yuan bridge. Nevertheless, when z=21, the repair ranking is: Tou-chyan-shi bridge → Dah-jin bridge → Jong-jang bridge → Lin-bian bridge → Shuang-yuan bridge.

B. Combiningly both the general and detail assessment items

The calculated results are denoted in Table 11. When z=2, z=3, z=5, z=9 and z=17, the repair rankings are: Tou-chyan-shi bridge → Dah-jin bridge → Lin-bian bridge → Jong-jang bridge → Shuang-yuan bridge. However, when z=21, the repair ranking is: Tou-chyan-shi bridge → Dah-jin bridge → Jong-jang bridge → Lin-bian bridge → Shuang-yuan bridge.

## 3. D.E.R. Evaluation Method

Now we use both the condition index (CI) and priority index (PI) of the D.E.R. evaluation method [24] to find the repair ranking, as shown below:

A. CI

Tou-chyan-shi (78.5670) → Dah-jin (86.5996) → Jong-jang (92.1536) → Lin-bian (92.1810) → Shuang-yuan bridge (93.4331).

B. PI

Tou-chyan-shi (77.1508) → Dah-jin (85.2273) → Lin-bian (91.3526) → Jong-jang (92.2547) → Shuang-yuan bridge (92.9964).

We can observe that the rankings of only the Jong-jang and Lin-bian bridges have differences in the three repair ranking methods mentioned above. The principal reason for this is unification, done average distribution afresh. The unification causes the bridge damage grade to approach the damage grade of multiple items with weight. In other words, if most of the bridge members are in good condition and are performing fully, while with only a few bridge members subject to severe damage and failure, then the use of an evaluation method including unification property will neglect the failing bridge members and result in a wrong judgment.

In the D.E.R. evaluation method, $Ic_{ii}$ is the submember condition index of each bridge member, and is expressed by

$$Ic_{ii} = 100 - \frac{100 \times D \times E \times R}{b} = 100 \times \left( \frac{b - D \times E \times R}{b - 0} \right) \quad (34)$$

where b=4×4×4.

We discover that the value of $Ic_{ii}$ is equivalent to an increase by 100 times of the cost type membership function. The maximum and minimum upper limits of $Ic_{ii}$ are b and 0, respectively.

$Ic_i$ is the condition index of each bridge member and is represented by

$$Ic_i = \frac{\sum Ic_{ii}}{n} = \frac{1}{n} \sum \left( Ic_{ii} - 0 \right) \quad (35)$$

The significance of $Ic_i$ is the Hamming weighted distance method with a grade of "poor."

$CI$ is the condition index (CI) of each bridge and is denoted by

$$CI = \frac{\sum_{i=1}^{21} (Ic_i \times w_i)}{\sum_{i=1}^{21} w_i} = \sum_{i=1}^{21} \left[ \frac{w_i}{\sum_{i=1}^{21} w_i} (Ic_i - 0) \right] \quad (36)$$

where $w_i$ stands for the weight of each bridge member.

$PI$ is the priority index (PI) of each bridge and is expressed by

$$PI = \frac{\sum_{i=12}^{20} (Ic_i \times w_i)}{\sum_{i=12}^{20} w_i} = \sum_{i=12}^{20} \left[ \frac{w_i}{\sum_{i=12}^{20} w_i} \left( Ic_i - 0 \right) \right] \quad (37)$$

The essence of $CI$ and $PI$ is the Hamming weighted distance

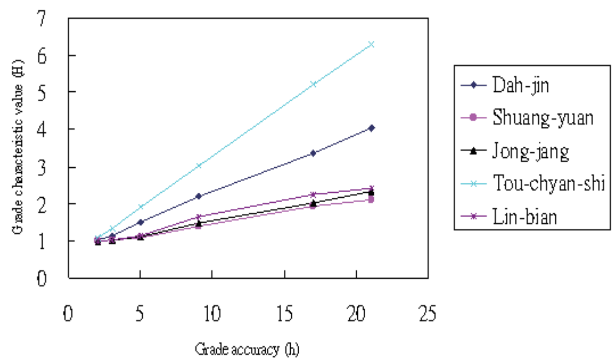


**Fig. 3. Relationship between grade characteristic value and grade accuracy.**
**(The Hamming weighted distance method for detail assessment items.)**

method with a grade of "poor."

According to the Euclidean weighted distance method, both the condition index (CI) and priority index (PI) of the D.E.R. evaluation method can be modified as the condition index of submember :

$$Ic_{ii} = 100\times\left(1-\frac{64-D\times E\times R}{b}\right) \tag{38}$$

where b=4×4×4. The condition index of a member is:

$$Ic_i = \frac{1}{\sqrt{n}}\sqrt{\sum Ic_{ii}^2} \tag{39}$$

The condition index is:

$$CI = \sqrt{\sum_{i=1}^{21}\left[\frac{w_i}{\sum_{i=1}^{21} w_i}(Ic_i)\right]^2} \tag{40}$$

The priority index is:

$$PI = \sqrt{\sum_{i=12}^{20}\left[\frac{w_i}{\sum_{i=12}^{20} w_i}(Ic_i)\right]^2} \tag{41}$$

The larger the value of CI or PI has, the more severe the damage to the bridge is, and the sooner the bridge must be repaired. Based on the modified formulas of Eqs. (38)-(41), we determine that the repair ranking is:

Tou-chyan-shi → Dah-jin → Lin-bian → Jong-jang → Shuang-yuan bridge.

CI (10.775)→(6.560)→(4.725)→(3.593)→(3.461)

PI (13.284)→(8.574)→(5.545)→(4.412)→(4.320)

This result is good in agreement with the result predicted by the proposed method.

In the case of the multi-pole fuzzy pattern recognition evaluation method, we discover from Figs. 3-6 that when the grade accuracy is promoted, the repair ranking approaches that of D.E.R. evaluation method. In the illustrative example of this study, no matter what the distance parameter, the repair ranking obtained from the proposed method with grade accuracy under z=17 is completely the same as that of the Euclidean weighted distance method. In addition, the repair ranking obtained from the proposed method with grade accuracy above z=17 is exactly the same as that of the CI of the D.E.R. evaluation method. Accordingly, the influence of the distance parameter with respect to unification degree is of only little value. The major influence is in the promotion of grade accuracy.

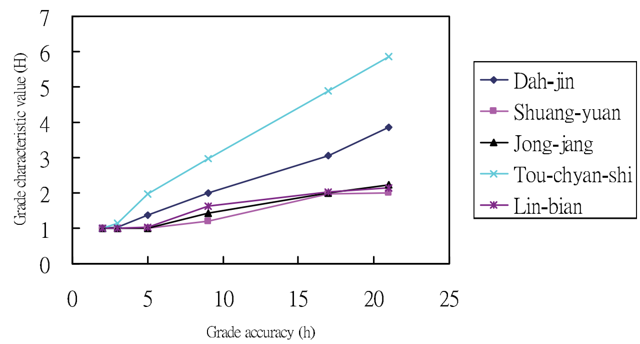


**Fig. 4. Relationship between grade characteristic value and grade accuracy.**
**(The Hamming weighted distance method for both general and detail assessment items.)**

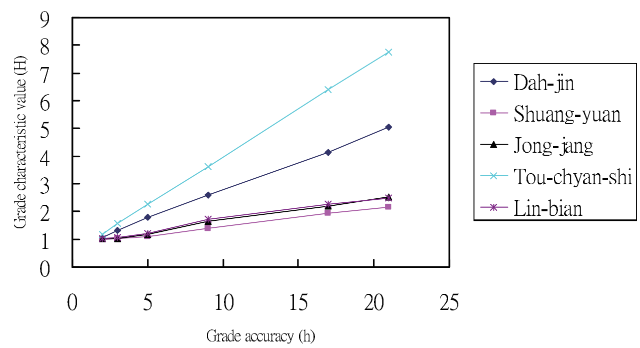


**Fig. 5. Relationship between grade characteristic value and grade accuracy.**
**(The Euclidean weighted distance method for detail assessment items.)**

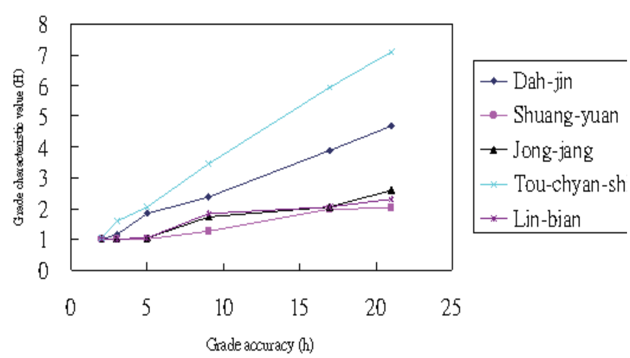


**Fig. 6. Relationship between grade characteristic value and grade accuracy.**
**(The Euclidean weighted distance method for both general and detail assessment items.)**

**Table 12. Comparisons on the general and detail assessment items of existing RC bridge for the repairing ranks obtained from the proposed, Hamming and Euclidean weighted distance, and D. E. R. evaluation methods.**

| Methods | Repairing ranks 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Proposed<br>Hamming($r_{is}$ = 0, $r_{is}$ = 1, detail)<br>Euclidean($r_{is}$ = 1, detail, general and detail)<br>Proposed associated with Hamming(detail, general and detail, z = 2, 3, 5, 9, 17, 21)<br>Proposed associated with Euclidean(detail, z = 2, 3, 5, 9, 17)<br>D. E. R.(PI)<br>Modified D. E. R.(CI, PI) | Tou-chyan-shi | Dah-jin | Lin-bian | Jong-jang | Shuang-yuan |
| Hamming($r_{is}$ = 0, $r_{is}$ = 1, general and detail)<br>Euclidean($r_{is}$ = 0, detail, general and detail)<br>Proposed associated with Hamming(general and detail, z = 21)<br>Proposed associated with Euclidean(detail, z = 21)<br>D. E. R.(CI) | Tou-chyan-shi | Dah-jin | Jong-jang | Lin-bian | Shuang-yuan |

Based on the Hamming or Euclidean weighted distance, we list up all the comparisons on the general and detail assessment items of existing RC bridges for the repairing ranks obtained from the proposed, weighted distance, and D. E. R. evaluation methods as shown in Table 12. It is worthy to point out that the result obtained by the proposed method may be acceptable.

Hereafter, in order to show the generic availability and applicability of the proposed method, we suggest that much more some existing RC bridges should be examined to convince the reliability of this method.

## V. CONCLUSIONS

This paper describes the theory of multi-pole fuzzy pattern recognition evaluation method for determining the repair ranking of existing RC bridges, using five existing RC bridges as an illustrative example. In the case of unification problem, the proposed, D.E.R. evaluation, Hamming and Euclidean weighted distance methods have also been discussed. Clearly, the optimistic and pessimistic selections are attributed to different categories. Nevertheless, the application range of proposed method is wide through the adjustment grade accuracy for employing to the optimistic and pessimistic selections. It is significant that the proposed method may be feasible, reasonable and reliable. The study results may be used as a decision-making tool for repair, strengthening or demolition of existing RC bridges.

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