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# CPT-BASED SIMPLIFIED LIQUEFACTION ASSESSMENT BY USING FUZZY-NEURAL NETWORK

Shuh-Gi Chern\* and Ching-Yinn Lee\*

Key words: CPT, fuzzy-neural, network, liquefaction.

#### ABSTRACT

Due to the difficulty and the cost of obtaining high quality undisturbed samples, simplified methods based on in-situ tests such as the standard penetration test (SPT) and the cone penetration test (CPT) are preferred by geotechnical engineers for evaluation of earthquake induced liquefaction potential of soils. Because of the increasing popularity worldwide of the CPT for site characterization, significant progress on the CPTbased methods has been made.

In most existing CPT-based methods, empirically determined curves are used to predict liquefaction and non-liquefaction. These empirical curves are generally relied on engineering judgment and are essentially performance functions that were established based on field observations of soil performance during earthquakes at sites where in-situ CPT data are available. The performance functions can be referred to as the limit state functions and the empirical curves are generally limit state functions such that the curve are generally limit state curve. The limit state for liquefaction evaluation is defined by CRR being equal to CSR, in which CRR is liquefaction resistance of a soil that is generally expressed as cyclic resistance ratio, and CSR is the cyclic stress ratio, i.e., the seismic load that causes liquefaction. In this study, a fuzzy-neural network with 466 CPT field observations is developed first to evaluate liquefaction potential of soils. Then a search procedure is presented to locate data points on the limit state function. Finally, regression is used to determine a simple formula of limit state curve that can easily evaluate cyclic liquefaction potential of soils.

#### I. INTRODUCTION

Liquefaction is known as one of the most destructive phenomena caused by earthquake and has been widely seen in loose saturated soil deposit (Niigata, 1964; Alaska, 1964; Tangshan, 1979; Loma Prieta, 1989; Kobe, 1995; Turkey, 1998; Chi-Chi, Taiwan, 1999). In view of serious damages caused by earthquake induced liquefaction, geotechnical engineers are actively engaged in the study of soil liquefaction induced by earthquakes. As of now, they have developed many assessment methods for soil liquefaction. However, it is hard to choose a suitable empirical equation for regression analysis due to the high uncertainty of earthquake environments and soil characteristics. Thus, many scholars and experts attempt to seek analytical models that are simpler, easier, and more reasonable and accurate than traditional empirical equations for soil liquefaction analysis.

Many of the existing assessment methods were developed from observations of the performance of sites during earthquakes. Previously, geotechnical engineers generally accepted the simple liquefaction analytical model developed by STP-N due to computer speed and analytical ability. In recent years, data processing and analytical ability have greatly increased and CPT has the advantages in being a fast, continuous and accurate measurement of soil parameters. At the same time, the related testing data continued to accumulate. So the potential of applying CPT to liquefaction research has grown significantly. For example, Shibata and Teparaksa [12], Stark and Olsen [13], Olson [8], Robertson and Companella [9], Robertson and Wride [10], and Juang and Chen [4, 6] all adopted CPT based liquefaction to establish soil liquefaction models and acquired great achievement.

To assess soil liquefaction induced by earthquakes, it is necessary to find the correlation between soil parameters and earthquake factors. However, the relationship between them is highly non-linear. Therefore, an induction cannot be made by pure linear regression or empirical rules. Artificial neural network simulates human thinking and learning and finds corresponding rules with mapping relationship between inputs and outputs for complicated non-linear problems. Many Scholars (Goh [3], Juang *et al.* [7]) approved that neural network method is a powerful and effective tool and is more accurate and reliable than conventional method to deal with liquefaction problem. This study attempts to combine fuzzy theory to establish a neuro-fuzzy system. Subtractive clustering algo-

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Seismic Liquefied Zone

Normalized In-Situ Soil Strength

Fig. 2. The methodology for establishing the limit state.

$$LI = f_{II}(q_{c1N}, I_c, \sigma_v, CSR_{7.5})$$
(1)

Fig. 1. Flow chart for neuro-fuzzy system analysis.

rithm is used to extract hidden classification rules from data and analyze the system in the study with the divided-andconquer methodology. Through neural network's learning and reminding ability, parameters are used as inputs and outputs, then the complicated relationship among parameters can be found. Consequently, the successfully trained and tested neurofuzzy neural networks is combined with regression analysis to establish a limit state, a multiple dimension boundary that separate zone of liquefaction from zone of non- liquefaction. The flow chart of neuro-fuzzy system is shown in Fig. 1. Details of the neuro-fuzzy network program and modeling details are described in Chern *et al.* [2].

#### **II. LIMIT STATE**

Juang et al. [7] presented a CPT-based limit state function for assessing the cyclic liquefaction resistance of sandy soils, the liquefaction resistance of a soil is generally expressed as cyclic resistance ratio CRR, while the seismic load that causes liquefaction is expressed as cyclic stress ratio CSR. The limit state is defined by CRR = CSR, i.e., the CRR is equal to the critical or maximum CSR that a soil can resist without the occurrence of liquefaction. The method developed by Juang et al. [7] to establish a limit state function is based on an artificial neural network modeling and analysis of 225 field liquefaction performance records. First, a neural network model is developed to predict the occurrence of liquefaction based on historic field performance records. Second, a search procedure is developed to locate data points on the limit state surface. Third, another neural network model is created to approximate the multi-variable limit state function.

To develop the intended limit state function for liquefaction evaluation, a function called the liquefaction indicator function LI (Eq. (1)) that maps the input variables into the output variable is established first by Juang *et al.* [7].

where  $q_{c1N}$  is normalized cone tip resistance;  $I_c$  is soil type index;  $\sigma_v$ ' is effective overburden stress; and  $CSR_{7.5}$  is cyclic stress ratio *CSR* adjusted to a reference earthquake magnitude of 7.5.

The methodology for establishing the limit state is illustrated with Fig. 2 (Juang *et al.* [5]). For each case in the collected database, if liquefaction was observed, the limit state boundary may be reached by decreasing seismic load (path A in Fig. 2) or increasing normalized soil strength (path B in Fig. 2). Initially, with a small decrease in seismic load, the output is likely to remain the same. However, by continuing this process of decreasing seismic load, the output will eventually indicate non-liquefaction. The updated seismic load that produces a change of liquefaction indication is the critical load that defines the limit state for the given soil conditions (Juang *et al.* [5]). Similarly, if noliquefaction was observed, the search for the limit state boundary could involve an increase in the seismic load (path C in Fig. 2) or a decrease in the normalized soil strength (path D in Fig. 2).

Though a CPT-based limit state function for assessing the cyclic liquefaction, resistance of sandy soils was presented by Huang *et al.* [5], a simple limit state curve or equation was not presented by Juang *et al.* [5]. In this study, a fuzzy-neural network with 466 CPT field observations is developed first to evaluate liquefaction potential. Then a search procedure is presented to locate data points on the limit state function. Finally, regression analysis is used to determine the limit state curve that an empirical equation correlating cyclic resistance ratio *CRR* and normalized cone tip resistance  $q_{c1N}$  is obtained. By comparing cyclic resistance ratio *CRR* and earthquake induced cyclic stress ratio *CSR*, liquefaction potential may easily be assessed.

## III. MODELING OF FIELD LIQUEFACTION RECORDS

The collected case records are evaluated using the neuro-

 Table 1. The maximum and minimum value of the reference data set.

	М	$\sigma_0(kPa)$	$\sigma'_0(kPa)$	$q_c(Mpa)$	$a_{\max}(g)$	$q_{c1N}$
Max.	7.8	364.5	227.5	25.00	0.80	300.06
Min.	5.9	16.7	16.7	0.18	0.08	2.69

fuzzy networks developed by Chern *et al.* [2]. The data base includes 466 CPT-base field liquefaction records from more than 11 major earthquakes between 1964 and 1999. The data consists of 21 case records from Japan, 85 from China, 7 from Canada, 219 from the USA, and 134 from Taiwan. This represents 250 sites that liquefied and 216 sites that did not liquefy. 5 parameters had been recorded in all 466 sites are: (1) earthquake magnitude, M; (2) total overburden pressure,  $\sigma_0$ ; (3) effective overburden pressure,  $\sigma'_0$ ; (4)  $q_c$  value from CPT; and (5) maximum ground acceleration,  $a_{\text{max}}$ . The range between maximum and minimum values of each parameter is summarized in Table 1. Parameter values of all 466 case records are presented in paper written by Chern et al. [2]. The input representing liquefaction potential is given a binary value of 1 for liquefied sites and a value of -1 for non-liquefied sites. Training was done iteratively until the Root-Mean Square Error (RMSE) is smaller than threshold set point 0.1. Even though the system fails to attain the expected training goal, the calculation will be terminated after 10,000 times of iteration and output results.

#### **IV. ANALYSIS RESULTS**

466 collected sets of data are randomly divided into 350 sets as training group and 116 sets as testing group. Depending on different parameter combinations and number of hidden neurons, 4 different neural network models are established. Table 2 shows analysis results of 4 different models. In table 2, "no. of error" means the number of cases with wrong predictions; "error rate" means the ratio of "no. of error" with total field observations; "no. of hidden neuros means member of neuros used in the hidden layer. Subtractive clustering algorithm is used to divide a large data sets into several clusters that all data points at least belong to one cluster and no empty cluster exists. Through optimization analysis, all training data sets are separated into 4 clusters [2]. "No. of elements training cluster" means number of training data sets in the 4 clusters that belong to.

Also as shown in Table 2, the input parameters in models C4 and C4H6 are earthquake magnitude M, effective overburden pressure  $\sigma'_0$ , cone resistance  $q_c$ , and maximum ground acceleration  $a_{max}$ , However, model C4H6 has one more hidden neuron than C4 model. Both two models developed by this study have 4% error rate in training phase. Model C4H6 has one less error in testing phase than model C4. However, both models have nearly overall 96% success rate for judging liquefaction. It shows that very good results can be achieved in

 
 Table 2. Results and details of designed fuzzy-neural networks.

	Input variables	No. of elements	No. of	No. of Error		Error rate (%)		Total
Model		in every training	hidden	Training	Testing	Training	Testing	error
		cluster	neurons					rate (%)
C4	$M, \sigma_0, q_c, a_{\max}$	217, 116, 82, 79	5	14	6	4.0	5.17	4.29
C4H6	$M, \sigma'_0, q_c, a_{\max}$	217, 116, 82, 79	6	14	5	4.0	4.31	4.08
C5	$M, \sigma_0, \sigma'_0, q_c, a_{\max}$	190, 93, 114, 89	5	7	5	2.0	4.31	2.58
C5N	$M, \sigma_0, \sigma'_0, q_{clN}, a_{max}$	190, 91, 113, 91	5	11	5	3.14	4.31	3.43

Table 3. Relative importance (%) of input parameters.

Cluster	Relative importance (%)						
	М	$a_{\rm max}$	$\sigma_0'$	$\sigma_{0}$	$q_c$		
Ι	23.8	18.7	16.1	28.2	13.3		
II	17.1	26.5	22.6	7.3	26.5		
III	10.0	28.8	18.8	21.5	20.9		
IV	16.5	31.4	10.7	17.5	23.9		

this study system with only 5 hidden neurons. Compared with model C4, models C5 and C5N additionally consider the effect of total overburden pressure  $\sigma_0$  on liquefaction occurrence. In model C5N, normalized cone resistance  $q_{c1N}$  is considered as input parameter instead of cone resistance  $q_c$ . From analysis results, model C5 with only overall error rate 2.58% has better accuracy than model C5N both in training and testing phase. Apparently, model C5 is the best model in this study for liquefaction assessment. Models C5 and C5N with additional consideration of  $\sigma_0$  have better success rate than models C4 and C4H6 without consideration of  $\sigma_0$ , shows that  $\sigma_0$  is an important factor for the assessment of liquefaction. In this study, system output value is between 1 and -1. When it is larger than 0, it is within the liquefaction zone. When it is smaller than 0, it is in non-liquefaction zone.

Table 3 shows the relative importance for the different models and the different parameter combinations in this study. "Relative importance" shows the significance of a parameter compared with the others in the model. A parameter with very high relative importance means that the parameter is very significant in the model. Model C4 and C4H6 have similar relative importance for parameters M and  $a_{max}$ , while different relative importance for parameters  $\sigma'_0$  and  $q_c$ . The trend of parameters' relative importance for model C5 is generally similar to model C5N. For importance in models C5 and C5N, the earthquake parameter factor ( M and  $a_{max}$ ) is 43%, the in-site stress factor ( $\sigma'_0$  and  $\sigma_0$ ) is 38%, and cone resistance factor ( $q_c$  and  $q_{c1N}$ ) is 19%. The neuro-fuzzy system established in this study follows divide-and-conquer methodology for assessment of liquefaction potential. Table 4 is relative importance for the four clusters of the C5 model. It can be seen that the main factor for liquefaction assessment in different cluster is different. For example, the liquefaction potential for the 1st cluster is mainly affected by M and  $\sigma_0$ .

cluster of model C5. Relative importance (%) Cluster  $a_{\rm max}$ Μ  $\sigma'_0$  $\sigma_0$  $q_c$ I 23.8 18.7 16.1 28.2 13.3 Π 17.1 26.5 22.6 7.3 26.5 Ш 10.0 28.8 18.8 21.5 20.9 23.9 IV 16.5 31.4 10.7 17.5

Table 4. Relative importance (%) of parameters in each

The 2nd cluster is mainly controlled by  $a_{\text{max}}$ ,  $\sigma'_0$  and  $q_c$ . The 3rd cluster is mainly controlled by  $a_{\text{max}}$ . The 4th cluster is mainly controlled by  $a_{\text{max}}$  and  $q_c$ . Since soil liquefaction induced by earthquake involves great uncertainty and is deeply affected by local geological condition, local stress condition and local earthquake parameters, the model is not limited to use the same neural network for analysis but follows data characteristics for classification and finds hidden rules. It provides a sub-network of learning for similar data clusters and meets the efficiency principle.

## V. DETERMINATION OF LIMITS STATE FUNCTION

The optimum neural network model C5 developed in this study for the assessment of liquefaction potential is used to establish a search procedure to locate data points on the limit state surface. Then regression analysis is used to approach the multi-variable limit state function. The five input variables in neural network model C5 are earthquake magnitude M, effective overburden pressure  $\sigma'_0$ , total effective overburden pressure  $\sigma_0$ , cone resistance  $q_c$  and maximum ground acceleration  $a_{\text{max}}$ . From the definition of limit state as shown in Fig. 2, limit state liquefaction parameters can be obtained by two ways. For each case in the collected database, if liquefaction was observed (target value is 1), the limit state boundary may be reached by decreasing seismic load (path A in Fig. 2) or increasing normalized soil strength (path B in Fig. 2). Considering path A as an example, a new data pattern is formed by decreasing seismic load (M or  $a_{max}$ ) while maintaining a constant soil resistance. Take M as an example, Table 1 shows that the difference between maximum and minimum M values is 1.9, with a small decrease (about 1% of difference between maximum and minimum values) in M, i.e., decrease M value  $1.9 \times 1\%$  each time while maintain the remaining four variables constant. With a new input M, neural network C5 would produce a new output. Initially, with a small decrease in M, the output is likely to remain the same. Continuing this process of decreasing M, the output will eventually indicate non-liquefaction (output value in model C5 is eventually smaller than 0). The updated earthquake magnitude M that produce a change of liquefaction indication is the critical M value that define the limit state for the given  $a_{\text{max}}$ ,  $\sigma'_0$ ,  $\sigma_0$  and  $q_c$  parameters. Similarly, if no liquefaction was observed, the



Fig. 3. The correlation between  $CSR_{7.5}$  and  $q_{c1N}$ .

search for the limit state boundary could involve an increase in the seismic load (path C in Fig. 2) or a decrease in the normalized soil strength (path D in Fig. 2). Because limit state liquefaction parameters can be obtained by two ways, hence 2 or 1 or even none limit state data pattern may be obtained rather than just one from a data set.

Using the algorithm described above, a total 529 limit state data patterns (rather than originally input 466 collected sets of data) are obtained. Each data pattern consists of M,  $\sigma_0'$ ,  $\sigma_0$ ,  $q_c$  and  $a_{\text{max}}$  factors. These data patterns are then used to compute normalized cyclic stress ratio  $CSR_{7.5}$  and normalized cone resistance  $q_{c1N}$  by using the following equations suggested by Juang *et al.* [5].

$$CSR_{7.5} = 0.65 \left(\frac{\sigma_0}{\sigma'_0}\right) \left(\frac{a_{\max}}{g}\right) \times \frac{r_d}{MSF}$$
(2)

in which  $r_d$  is shear stress reduction factor; *MSF* is magnitude scaling factor.

$$r_d = 1.0 - 0.00765z; z \le 9.15 m \tag{3a}$$

$$r_d = 1.174 - 0.0267z; 9.15 < z < 23 m \tag{3b}$$

$$MSF = \left(\frac{10^{2.24}}{M^{2.56}}\right) = \left(\frac{M}{7.5}\right)^{-2.56}$$
(4)

$$q_{c1N} = \frac{q_c/100}{\left(\frac{\sigma_0}{100}\right)^{0.5}}$$
(5)

in which dimension in  $q_c$  and  $\sigma'_0$  is kPa. Figure 3 shows the

		Case No.	No. of Error	Error rate %
	Total sets of case	466	111	24%
Collected sets of data in this study	Case of Liquefaction	250	56	22%
	Case of non-Liquefaction	216	55	25%
	Total sets of case	170	36	21%
Case records from Baziar (2003)	Case of Liquefaction	104	16	15%
	Case of non-Liquefaction	66	20	30%
	Total sets of case	226	61	27%
Case records from Juang (2003)	Case of Liquefaction	133	40	30%
	Case of non-Liquefaction	93	21	23%

Table 5. Comparison result by using the limit state developed in this study.

correlation between  $CSR_{7.5}$  and  $q_{c1N}$ . Through regression analysis, a limit state curve as shown by the following equations can be obtained.

$$CSR_{7.5} = 0.10071 \cdot e^{0.00857 \times q_{c1N}} \tag{6}$$

or

$$\ln(CSR_{7.5}) = 0.00857 \times q_{c1N} - 2.29554 \tag{7}$$

The limit state expressed in (6) or (7) is formulated in the general format of the simplified procedure pioneered by Seed and Idriss [11].  $CSR_{7.5}$  is the maximum cyclic shear stress ratio that the soil can sustain without the occurrence of liquefaction. That means  $CSR_{7.5}$  obtained from equation (6) or (7) is the cyclic liquefaction resistance *CRR* of a soil (i.e., *CRR* = CSR<sub>7.5</sub>). The case records from Baziar et al. [1] and Juang et al. [7] are evaluated using the limit state equation (6) or (7) developed in this study. Based on (6) or (7), the cyclic liquefaction resistance of soil is determined once  $q_{c1N}$  is known. By comparing cyclic liquefaction resistance with earthquake induced cyclic shear stress (usually expressed as cyclic shear stress ratio CSR), occurrence of liquefaction can be decided. Table 5 shows the comparison results. As shown in Table 5, the success rates are all higher than 70% based on the simplified equations developed in this study. This rate of successful prediction is about the same or a little higher than those normally achieved by the traditional methods used in practice. However, from Fig. 3, the limit state function obtained from this study does not seem to be better than the one obtained by simply drawing a boundary curve between the liquefied and non-liquefied data points. The prediction could be better if the four very biased nonliquefaction data points far above the limit state curve are removed.

#### VI. CONCLUSION

Fuzzy-Neural network is used in the study to evaluate liquefaction potential subjected to earthquake loadings. To achieve this object, a total 529 limit state patterns consisting M,  $\sigma'_0$ ,  $\sigma_0$ ,  $q_c$  and  $a_{\text{max}}$  factors are obtained first. These data patterns are then used to compute normalized critical cyclic stress ratio  $CSR_{7.5}$  and normalized cone resistance  $q_{c1N}$ . Subsequently, regression is used to determine a simple correlation equation between  $CSR_{7.5}$  and  $q_{c1N}$ . Finally, based on this empirical equation, cyclic liquefaction resistance  $CSR_{7.5}$  of sandy soils can be computed. The computed liquefaction resistance  $CSR_{7.5}$  is compared with earthquake induced cyclic stress CSR to decide whether liquefaction be occurred or not. Liquefaction will occur if the CSR of soils exceed its liquefaction resistance  $CSR_{7.5}$ .

The proposed simplified procedure is illustrated with the help of case studies. From the comparison results, it is found that the developed empirical equation may provide a very simple and accurate method with success rate as high as nearly 80% for assessing liquefaction potential.

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