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UNSTEADY MOTIONS OF SEPARATED SHEAR LAYER IN WAKE FLOW DUE TO UPSTREAM DISTURBANCE

Chin-Tsan Wang
Department of Mechanical and Electro-Mechanical Engineering, National I Lan University, I Lan, R.O.C., ctwang@niu.edu.tw

Jun-Ji Miau
Department of Aeronautics and Astronautics, National Cheng Kung University, Tainan, R.O.C.

Cheng Kuang Shaw
Department of Mechanical and Aerospace Engineering, University of California, Los Angeles, U.S.A.

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UNSTEADY MOTIONS OF SEPARATED SHEAR LAYER IN WAKE FLOW DUE TO UPSTREAM DISTURBANCE

Chin-Tsan Wang*, Jun-Ji Miau**, and Cheng Kuang Shaw***

Key words: span-wise motion of shear layer, low frequency variations, wake flow.

ABSTRACT

The phenomenon of physical wake flow is complex and remains relatively unknown, especially for the relationship between the span-wise motions of separated shear layers and low frequency variations embedded in the vortex shedding process of upstream disturbances. In this study experiments were executed by situating a mesh grid and controlling cylinder upstream. Results showed that the integral time scale of the span-wise motions for a separated shear layer was close to that of the low frequency variations for an upstream disturbance. This evidence supports the idea that span-wise motions of a separated shear layer and the low frequency variations should appear at same time in the vortex shedding process. In addition, the visibility of the low frequency variations obtained was fixed at about 10% of flow energy regardless of flow disturbance from upstream. Finally, a physical representation regarding the span-wise oscillation of the shear layer and the low frequency variations in the vortex shedding process was constructed and appeared in chaotic behavior. These findings will be useful to further the understanding of the wake flow model.

I. INTRODUCTION

The flow past a circular cylinder appears in a series of flow regimes ranging from low to high Reynolds numbers, and flow resulting from different sources has been shown to be sensitive to disturbances like flow conditions at the end of the cylinder and upstream turbulence [7, 11, 12, 24, 26]. With increasing Reynolds numbers the separated region elongates in the direction of the stream current, and wake instability also occur at Reynolds numbers of about 34-40 [6, 13, 17]. From Re = 50 to 200, a stable laminar vortex street is generated [17, 23] and eventually degenerates into an unstable state [25]. Here one should note that some of these apparent changes in flow behavior, whose flow modes were within the stable regime of the laminar motions, could be due to non-uniformity of the flow [10, 29]. Irregular disturbances in the vicinity of the vortex formation region are found at about Re = 200 [5, 13], and are regarded as the beginnings of transitional motion [23] and three-dimensional distortion of the initially straight vortex lines that will eventually degenerate into turbulence [5, 13].

Above the range of Re = 300 to 400, the phenomenon of the transition to turbulence first addressed by Bloor [5] begins with the formation of sinusoidal oscillations within the separated shear layers associated with transition waves. This formation is augmented in conditions with Reynolds numbers initially ranging from Re = 300 to 400, and then later decreased to $Re = 5 \times 10^4$. Wei and Smith [32] later indicated that the "transition wave" detected by Bloor was identical to the secondary vortices of three-dimensional flow structures and that its appearance could be the result of the transition to turbulent Strouhal vortices [3]. When the flow transition occurs immediately after the flow separation, it exhibits a high potential for flow reattachment at about $Re = 2 \times 10^5$. The exact conditions are dependent on surface roughness, free stream turbulence, etc, but generally speaking, the complexity of the three-dimensional flow is directly related to the magnitude of the Reynolds number, especially for Reynolds numbers above 3×10^6 [2, 8, 9, 13, 33].

Recent data showed that the phenomenon of three-dimensional wake flow led to the development of low frequency modulations during the vortex shedding process. Henderson [16] speculated that the modulation was not periodic, like the beating that would occur with a nonlinear interaction between the vortex shedding frequency and the oscillation frequency of the separated shear layer [22, 23]. Indeed, in the present wake flow model the three-dimensional flow characteristics and the presence of wide-band velocity fluctuations in the near wake region are expected.

Paper submitted 04/15/08; accepted 08/01/08. Author for correspondence: Chin-Tsang Wang (e-mail: ctwang@niu.edu.tw).
**Department of Mechanical and Electro-Mechanical Engineering, National I Lan University, I Lan, R.O.C.*
***Department of Aeronautics and Astronautics, National Cheng Kung University, Tainan, R.O.C.*
****Department of Mechanical and Aerospace Engineering, University of California, Los Angeles, U.S.A.*

The present work was motivated by existing concerns on the quality of the vortex-shedding signals obtained from a trapezoidal cylinder installed in a cylindrical pipe [21]. As pointed out in previous work [21], pressure signals measured on a trapezoidal cylinder contain a substantial component of low frequency variations as the Reynolds numbers were larger than 10^3. It was also noted from the energy spectra obtained that the energy associated with the low frequency variations increases with the Reynolds number in cases where the low frequency variations are composed of frequencies lower than the vortex shedding frequency. Miau *et al.* [19] performed further experiments with a trapezoidal cylinder of the same cross-sectional shape and a circular cylinder to examine the relation between the low frequency variations embedded in the base pressure signals and the velocity signals measured in the nearby wake region. The conclusion obtained by Miau *et al.* [19] in this study was that the low frequency variations embedded in the vortex shedding process are closely linked with the unsteady variations of the vortex formation's length [19]. Additionally, in work done by Unal and Rockell [28], Bloor and Gerrard [6, 14], and Belevin [4] concerning the flow patterns over a circular cylinder, the variations of the time-mean base pressure and the vortex formation length against Reynolds numbers in the range of 10^2-10^5 are well documented. Bearman [1] showed that the time-mean base pressure varied with the vortex formation length behind a blunt trailing edge, where the base pressure was modified due to a splitter plate of variable length attached further downstream. However, the instantaneous relationship between base pressure and vortex formation length subjected to a steady incoming flow was rarely reported. Basically, the relation found by Miau *et al.* [19] was that the variations of base pressure and vortex formation length with respect to time tend to be out of phase.

According to visualizations of flow structure, Najjar & Balachandar [22] conjectured that the formation of stream-wise and span-wise vortices are not in perfect synchronization, and hence the existence of the low frequency unsteadiness that would result from this imbalance or phase mismatch. However, the phenomenon of span-wise separated shear layer structures originating from the bluff body would be very complex at higher Reynolds number above the order of 10^4 because of its tendency to develop into turbulent motion.

In spite of the aforementioned extensive investigations of wake flow, many aspects of the span-wise motion of separated shear layers in wake flow remains completely unknown. This work is basically a follow-up of a series of research of Miau and members of his group [19, 20, 21]. The purposes of this work were to experiment and deduce the relationship between span-wise separated shear layers and low frequency variations due to flow disturbance from upstream, and eventually formulate a physical representation of the wake flow behind a bluff body.

II. EXPERIMENTAL APPARATUS

A closed and low-speed wind tunnel applied to measure base pressure signals with a square test section measuring 150 mm by 150 mm was employed mainly for qualitative measurements at Reynolds number on the order of 10^4. The turbulence intensity of the wind tunnel was approximately 0.6% of the mean velocity measured at the centerline of the test section. The bluff body was a trapezoidal cylinder installed with the wider side facing the incoming flow. The maximum width was 32 mm, denoted as *D*. Therefore, the blockage ratio based on the frontal area of the bluff body was approximately 21%.

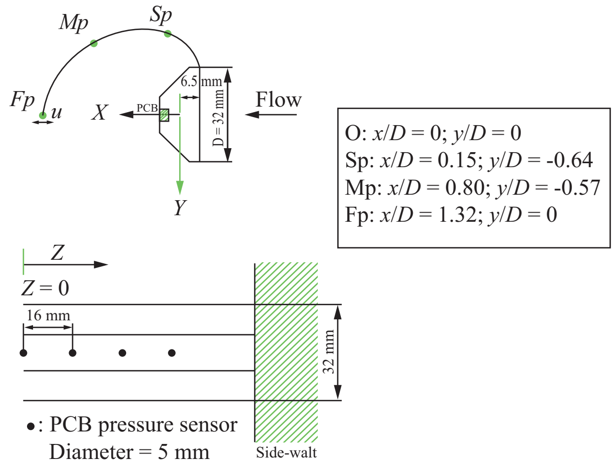

Fig. 1. Experimental arrangement of the trapezoidal cylinder (a) three measured points *Sp*, *Mp* and *Fp* for span-wise velocity (b) situations of four piezoelectric pressure sensors.

The coordinate system displayed in Fig. 1 and applied in this experiment is described as follows: *X* represents the stream-wise axis, whose positive direction points downstream, *Y* is the vertical axis, whose positive direction points downward, and *Z* represents the span-wise axis. The origin, or where *X* = 0, *Y* = 0 and *Z* = 0, is located at the geometrical center of the trapezoidal cylinder.

Here, a split fiber probe, DANTEC 55R55, was utilized to detect the span-wise motions of the separated shear layer because of its outstanding ability in simultaneously sensing the flow direction and fluctuation. In measuring the stream-wise velocity in the nearby wake region, the major axis of the split fiber was aligned parallel to the trapezoidal cylinder such that one of the films was situated on the forward side and the other was situated on the rear side. Hence, by comparing the outputs of the two sensors, one could further identify the flow direction. In this case, the split fiber was oriented perpendicular to the cylinder, thereby allowing the films on both sides to measure the span-wise velocity component. Three measured points, indicated in Fig. 1(a) as *Sp*, *Mp* and *Fp*, were selected along the separated shear layers. During the experiment, the split fiber probe was oriented such that its major axis was perpendicular to span-wise axis of the cylinder, thus allowing the thermal films on the probe to sense and record span-wise velocity fluctuations. In addition, the span-wise base pressure signals were measured by four piezoelectric pressure sensors, PCB 103A, applied and situated in the base of trapezoidal

Table 1. The values of cross-correlation coefficient at time lag (τ = 0) for different spanwise base pressures based on filtered data and raw data at upstream undisturbed for *Re* = 16300, respectively.

$\Delta Z/D$	C.C (τ = 0) for f_c = 5.5 Hz	C.C (τ = 0) for Raw data
0.5	0.38	0.32
1	0.35	0.30
1.5	0.34	0.18

Table 2. Integral time scales of spanwise velocity fluctuations for different downstream measured points measured along the shear layer.

R	16570	25540	31150
I_t (Ts)	2	2	2
f_c (Hz)	5.6	8.6	10.5
I_{Sp} (Ts)	1.8	1.8	1.8
I_{Mp} (Ts)	1.6	1.7	1.7
I_{Fp} (Ts)	1.6	1.7	1.6

cylinder along the span-wise direction from $\Delta z/D$ = 0.5 to 1.5 as shown in Fig. 1(b). Finally, an disturbance upstream at Lx/D = 4, caused by a grid whose mesh size M = 15.6 mm/mesh and mesh diameter is 3.2 mm at $Re = 2.56 \times 10^4$ and situated by a controlling cylinder whose diameter d = 1 mm and 3 mm at a distance x/D = -0.625 to bluff body and $Re = 2.35 \times 10^4$, were used for studying the effect of upstream flow disturbed.

III. EXPERIMENTAL RESULTS AND DISCUSSION

As for the occurrence of low frequency variations in wake flow and its physical behavior, it is still completely unknown in spite of continuous numerical analysis. A portrayal of physical model describing the relation between the low frequency variations and vortex formation length was first addressed by Miau *et al*. [19]. This explanation that the low frequency variations were associated with a separated shear layer seems logical and natural because of the fact its existence is embedded in the vortex shedding process. Through flow visualization, Najjar & Balachandar [22] theorized that the imbalance/phase mismatch between the formation of stream-wise and span-wise vortices would induce the appearance of the low frequency variations. However, this explanation lacks direct quantitative measurements for support. Moreover, current findings of the relationship between the two flow characteristics need further clarification.

1. Span-wise Coherence of Separated Shear Layer without Upstream Disturbance

The three-dimensional flow characteristics in the nearby wake region were examined through the correlation of base pressure signals obtained by four identical piezoelectric pressure sensors situated at the rear surface of the cylinder at a distance of 16 mm from one another (as shown in Fig. 1(b)). The results of the cross-correlation coefficients at conditions of τ = 0, Re = 16300, and $\Delta Z/D$ = 0.5, 1 and 1.5 with the reference sensor fixed at the mid-span of the cylinder are listed in Table 1 for upstream undisturbed. Evaluation of the data in Table 1 reveals that the values of the correlation coefficients at τ = 0 regarding the raw signals or low pass filter traces based on the cut-off frequency chosen (f_c = 5.5 Hz) and determined from (6) of Miau *et al.* [20] were less than 0.4, thereby showing that the three dimensionality of the wake flow is more organized and the span-wise characteristic length of low frequency variations is larger than the characteristic length scale of the bluff body. Though not shown here in this paper, by comparing the values of the low-pass traces with raw data, the span-wise coherence of low frequency variations with a span-wise length scale of 1.5 D or more at Re = 25600 were measured with similar results [16, 18, 19, 20, 27]. This outcome is reasonable given the three-dimensional nature of flow.

2. Unsteady Spanwise Motions of the Separated Shear Layer without Upstream Disturbance

Concerning the low frequency variations embedded in the vortex shedding process, Henderson indicated that the modulation of force was found to originate in the intrinsic structure of the wake [16], and Najjar & Balachandar [22] found that the low frequency variations are a result of the imperfect synchronization between the stream-wise and span-wise vortices. These statements all suggest that some relationship between span-wise motion of a separated shear layer and the low frequency variations exists in the vortex shedding process. To test this hypothesis, the split fiber probe was applied at three measured points *Sp*, *Mp* and *Fp* (shown in Fig. 1(a)) and corresponding to (x/D, y/D, z/D) = (0.15, -0.64, 0), (0.80, -0.57, 0) and (1.32, 0, 0), respectively. Spatially-speaking, *Sp* was located near the harp edge of the cylinder, *Fp* was located close to the rear end of the vortex formation region, and *Mp* was located between *Sp* and *Fp*.

Table 2 shows that the integral time scales of the span-wise motion of the separated shear layer measured at the three points. The integral time scale was reduced from the low-passed signals. The cut-off frequencies of low frequency variations chosen by the results of Miau *et al.* [19] were 5.6 Hz, 8.6 Hz and 10.5 Hz corresponding to Re = 16570, 25540 and 31150, respectively. Analysis of the data in Table 2 indicate that the integral time scales characterizing span-wise motion of separated shear layers for different downstream points along the shear layers were on the same order as the integral time scales of the low frequency variations [20]. Hence, the relative lack of difference between the I_{Sp}, I_{Mp}, and I_{Fp} values seems to suggest that the unsteady, span-wise motions of the separated shear layer is associated with the low frequency variations seen in the vortex shedding process. The low frequency varia-

Table 3. The values of cross-correlation coefficients at time lag (τ = 0) for different two spanwise base pressures based on filtered data and raw data at *Re* = 25600 and upstream distance *Lx*/*D* = 4 disturbed by a grid.

$\Delta Z/D$	C.C (τ = 0) for f_c = 8.9 Hz	C.C (τ = 0) for Raw data
0.5	0.38	0.38
1	0.36	0.23
1.5	0.35	0.23

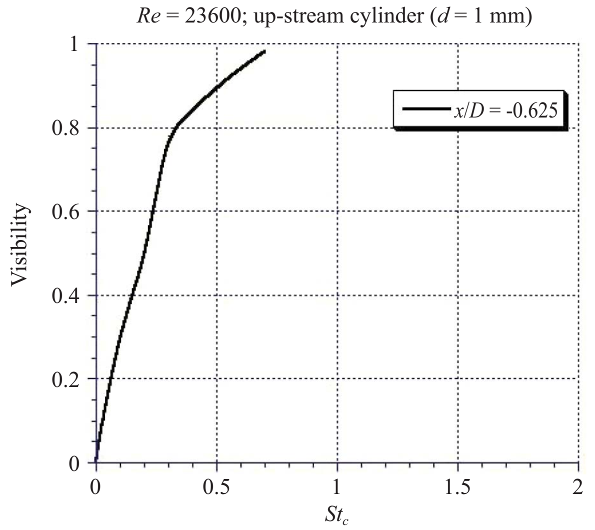

Fig. 2. The distribution of visibility of base pressure fluctuations versus the non-dimensional cut-off frequency at upstream controlling cylinder and *Re* = 23600.

tions and span-wise motions of separated shear layer should theoretically coexist during the vortex shedding process.

3. Effect of Upstream Flow Disturbed by Mesh Grid

Investigating further into the disturbance of the span-wise motion of a separated shear layer due to flow originating upstream was more of a challenge. Here, grids of various sizes situated upstream were used to generate different flow intensities. Roughly speaking, turbulence generated by grid should be seen as homogeneous at the distance $Lx/M > 100$ [3]. Table 3 shows the span-wise correlation coefficients of base pressure at $\tau = 0$ for raw signals and low-passed traces with a cut-off frequency of f_c = 8.9 Hz taken from an upstream flow disturbed at Lx/D = 4 by a grid with mesh size M = 15.6 mm/mesh and mesh diameter is 3.2 mm at $Re = 2.56 \times 10^4$. When comparing the results shown in Tables 1 and 3, it is revealed that the values in Table 3 show very similar features with those of Table 1. This is reasonable because the low frequency variations are intrinsic characteristics [30]. This evidence indicates that the three dimensional structure of wake flow is organized in relation to its low frequency variations.

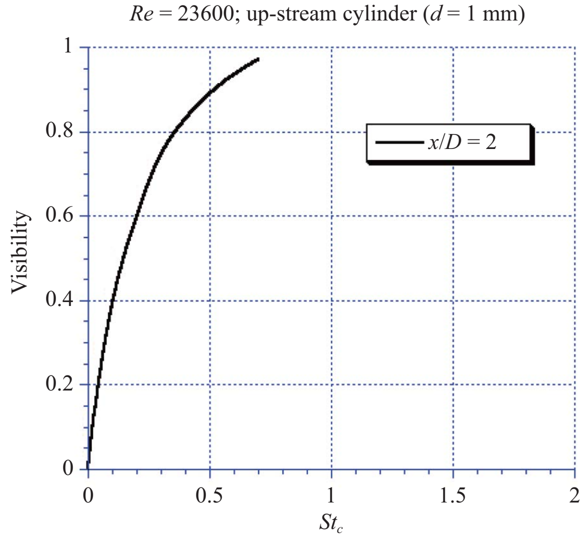

Fig. 3. The distribution of visibility of for velocity fluctuations measured at *X*/*D* = 2 versus the non-dimensional cut-off frequency for upstream controlling cylinder and *Re* = 23600.

4. Effect of Visibility on Upstream Controlling Cylinder

The need for measuring the vortex signals for the shedder in order to determine the visibility of the low frequency variations is important because the energy of low-frequency variations increases significantly in accordance with the Reynolds number, thus degrading the vortex shedding signals [21]. Therefore, a quantity, called visibility V_L, was proposed to describe the significance of the low frequency variations in flow signals disturbed by a controlling cylinder situated upstream.

$$V_L = \frac{(Q_L)_{rms}}{(Q)_{rms}} \tag{1}$$

In (1), $(Q_L)_{rms}$ denotes the root-mean-square value of the fluctuations in the low-passed signal trace, and $(Q)_{rms}$ denotes the root-mean-square value of fluctuations in the raw signal trace measured. Hence, V_L signifies the relative importance of the filtered low-frequency flow fluctuations to the total fluctuations.

Figures 2 and 3 show the distribution of V_L to St_c for the base pressure signals and velocity traces at Re = 23600 with respect to upstream flow disturbed by a cylinder whose diameter was d = 1 mm and situated at $(x/D, y/D, z/D)$ = (-0.625, 0, 0). It can be seen by the results of Figs. 2 and 3 that the values of V_L to the base pressure signals and velocity traces measured at x/D = 2 and based on the optimal cut-off frequency St_c = 0.0237 [21] for the low frequency variations are fixed and about 10% of flow energy regardless of upstream disturbances because of its intrinsic nature [30]. Similar results (not shown in this paper) were also obtained when using a cylinder of diameter d = 3 mm ranging from x/D = -0.625 to -8.75 at upstream Re = 23570.

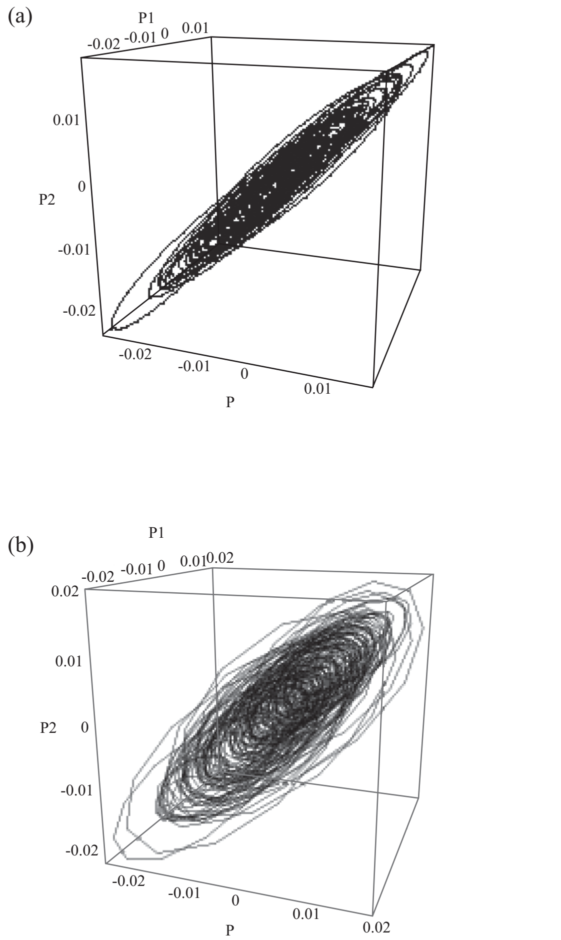

Fig. 4. Attractor of self-similar laceration for low frequency variations embedded in base pressure signals measured at (a) *Re* = 10500 (b) *Re* = 26800, respectively.

Based on the features of the Figs. 2 and 3 mentioned above, the conclusion obtained indicates that the effect of upstream flow disturbed by a controlling cylinder on the visibility of low frequency variations is relatively insignificant.

5. Unsteady Motions of Separated Shear Layer in Wake Flow

The phenomenon of physical wake flow is complex and remains relatively unknown, especially for the relationship between the span-wise motions of separated shear layers and low frequency variations embedded in the vortex shedding process.

Additional evidence proposed by new findings is addressed as follows: the attractor of self-similar laceration for low frequency variations exiting in pressure fields at *Re* = 10500 and 26800 has been found and its chaotic characteristic is shown in Fig. 4. This result also suggests that the characteristic of self-similar trajectories of attractor for self-similar laceration manifests during the vortex shedding processes [31]. As for its intrinsic low frequency variations [30], the visibility of low frequency variations is about 10% of flow energy regardless of the whether there are upstream disturbances or not.

As shown through the use of three dimensional flow patterns, the separated shear layer features low frequency variations in space and time during the vortex shedding process. The shedding vortices do not necessarily align perfectly along the span-wise axis in wake flow because of its chaotic behavior.

IV. CONCLUSION

The experimental results presented are summarized below:

1. The integral time scales of span-wise motions for the separated shear layers are on the same order as the integral time scale of low frequency variations regardless of the existence of upstream disturbances.
2. The span-wise motions of separated shear layers related to the low frequency variations in the vortex shedding process yielded extremely similar integral time scales, both on the same order of magnitude.
3. The visibility of low frequency variations is fixed at about 10% of flow energy regardless of the disturbances upstream.

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