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RISK EVALUATION OF SHIP DYNAMIC STABILITY IN REGULAR WAVES

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RISK EVALUATION OF SHIP DYNAMIC STABILITY IN REGULAR WAVES

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Key words: dynamic stability of marine vessels, risk evaluation, ship capsizing, random survival probability.

ABSTRACT

In this paper, a method combined with the existing approaches is proposed for evaluating and predicting the safety of ship rolling in regular waves. Based on the uncoupled equation of a single degree of freedom, the nonlinear ship rolling in waves is presented and the differential equation is integrated in the time domain by the fourth order Runge-Kutta algorithm. The erosion of the safe basins of a ship roll equation is simulated and the survival probability of a typical ship is studied according to the statistical and simulation results. The numerical results show that the survival probabilities of ship excited by the forces of the seas, especially in the regular wave status, can be predicted combining the statistical data and the basin erosion technique.

I. INTRODUCTION

The stability against capsizing in heavy seas is one of the most fundamental requirements considered by naval architects when designing a ship. Over the course of the past two and a half decades, the linear or small amplitude ship motion has completely understudied by the scholars [6]. But the development of the highly nonlinear or large amplitude ship motions has not been fully explored because of the complexity. The complicated phenomena of the nonlinear ship dynamics include the multiple solution, jumps between resonance, sensitivity to initial condition, sub harmonic and super harmonic response, chaotic motions etc. [20]. Recently, research using scale models in realistic wave conditions, combined with theoretical developments in non-linear systems dynamics has led to improved understanding and insight on the nature of the capsize process. Mathematical and numerical models followed of increasing sophistication are capable to predict the ability of the ship to resist capsize in a range of scenarios with sufficient engineering accuracy. Synchronously, the numerical simulation and probabilistic analysis is enabling develop instructions for ship handling based on risk. Generally, the mathematical models of the nonlinear ship dynamics relate to two different problem areas: dynamics of ship motion using a combination of frequency domain techniques, time domain simulation and properties of non-linear systems; and the stochastic nature of wave excitation, including the identification of sea wave spectra and of encountered wave groups consisting of high waves necessary to cause capsizing [2, 9, 11].

For the analysis of dynamic behavior of nonlinear systems i.e. nonlinear oscillations where ship rolling is included, several methods have developed. Cardo *et al*. studied the dynamic behavior by perturbation method [1]. Scanchez *et al*. research the nonlinear rolling of ships in regular beam seas by multiple time scales method [12]. Schmidt *et al*. studied the no-linear vibrations by time averaging method. Francescutto *et al*. used Krylov-Bogoliubov-Mitropolsky method to analyze the bifurcation in ship rolling comparing with experimental results [3]. Senjanovic advocated the harmonic balance method and harmonic acceleration method for the nonlinear resolution [14].

In reality, it is important to know the possibility modes of capsizing and the interrelationships of certain influence parameters based on the results of model testing and numerical simulations. If the rolling in beam seas is considered, it is possible to ignore the coupling of rolling and the other degrees of freedom of ship motion. Ship rolling in beam seas could be regarded as a simple pendulum. Thompson first introduced the concept of safe basin to the study of the nonlinear ship rolling motion and capsizing [17-19], plotting the evolving solution of a single-degree-of-freedom dynamic system as a trajectory in phase space rather than as a time history. Subsequently, Scanchez *et al*. studied the jumping and chaos of the ship roll for the nonlinear rolling model [12]. Tang *et al*. studied the subharmonic response of the coupled roll-pitch motion [16]. Ji *et al*. investigated the stability against capsizing in regular waves by nonlinear dynamical theories, and the stability of the stable solutions was analyzed by the bifurcation theory and Floquet theory [22]. Zhang *et al*. calculated the stability of ships with the theory of safe basins. The results show that the ships will capsize when the safe basins break in some conditions [5], whereas the capsizing probability of ships cannot be predicted quantitatively by all of the above mentioned methods.

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In this paper, the random differential equation of ship roll on random beam seas is established considering nonlinear damping, nonlinear restoring moment and random beam seas excitation. The capsizing process of the ship roll is simulated by applying the fourth-order Runge-Kutta algorithm to solve the nonlinear differential equation. The ferry is used for the study of capsizing process caused safe basin erosion according to the theory of safe basin. The nonlinear damping is obtained from previous experiment of the ship model rolling in a wave tank [11]. Other coefficients in the roll motion equation of the ship in waves are obtained through theoretical methods. And finally, the survival probability is obtained under different wave parameter excitation.

II. THEORETICAL EQUATION OF ROLLING MOTION

The first-order couplings from surge, heave and pitch to roll are zero and the couplings from sway and yaw to roll are not because of the port-starboard symmetry of ship. The typical single-degree-of-freedom roll differential equation of ships in beam seas is generally as follows [8, 21]:

$$(I_{44} + A_{44}(\omega))\ddot{\phi} + B_{44}(\omega)\dot{\phi} + \Delta GZ(\phi) = M_{sea}(t) \qquad (1)$$

where I_{44} is the inertia moments coefficient of the ship mass about the roll axis, A_{44} is the hydrodynamic inertia moment coefficient of the added mass, B_{44} is the hydrodynamic damping moment, Δ is the weight displacement of the ship, $GZ(\phi)$ is the righting arm of the ship, $M_{sea}(t)$ is the excitation moment (including wave and wind excitation), ϕ is the ship rolling angle.

Then (1) can be written as following:

$$(I_{44} + A_{44})\ddot{\phi} + B_{44}\dot{\phi} + C_{44}(\phi) = M_{sea}(t) \qquad (2)$$

where C_{44} is the hydrostatic restoring moment.

And according [7]

$$I_{44} + A_{44} = (T/2\pi)^2 \Delta g MG \qquad (3)$$

where T is the ship rolling period, MG is the metacentric height. The restoring moment is hydrostatic and given by a nonlinear odd function. It may be represented by a fifth-order polynomial as follows

$$C_{44}(\phi) = K_1\phi + K_3\phi^3 + K_5\phi^5 \qquad (4)$$

Substituting the above formulae (4) into (2), and dividing the result by the virtual moment of inertia, the final form of the differential equation of motion is obtained as follows

$$\ddot{\phi} + d\dot{\phi} + k_1\phi + k_3\phi^3 + k_5\phi^5 = m_{sea}(t) \qquad (5)$$

Table 1. Coefficients of the relative restoring moment.

number of receivers	$k_1(s^{-2})$	$k_3(s^{-2})$	$k_5(s^{-2})$	$k_7(s^{-2})$
3rd	0.671997033	-0.53920393	0	0
5th	0.630620936	-0.40482026	-0.08679720	0
7th	0.536133048	0.21992815	-1.128716074	0.4885682579

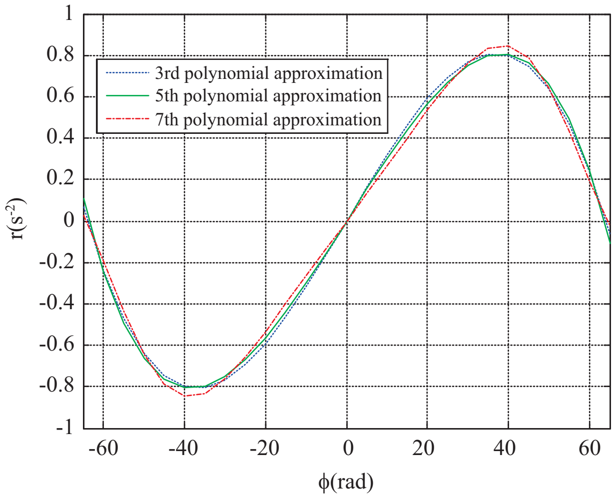


Fig. 1. Non-dimension restoring moments.

where

$$d = B_{44}/(I_{44} + A_{44})$$

$$k_i = K_i/(I_{44} + A_{44}) \qquad i = 1, 3, 5$$

$$m_{sea}(t) = M_{sea}(t)/(I_{44} + A_{44})$$

III. CHARACTERISTICS OF THE ANALYZED SHIP

Ship roll and survival analysis are illustrated in the case of ferry with the following particulars [11]. The Length between perpendiculars/breadth is 3.632; Length on waterline/breadth is 3.656; Breadth/draught is 4.182. Block coefficient is 0.603. Coefficients of damping moment were $d_1 = 0.01265913s^{-1}$ and $d_3 = 0.4954s^{-1}$. The virtual moment of inertia is defined according to the expression (1) is 29682 tm^2 without the bilge keels. The effective coefficients of restoring moment were shown in Table 1 and fitted by the curve in Fig. 1 in different polynomials.

1. Encounter Frequency

Waves incident on the structure or ship can be described as head seas, following seas, beam seas, or quartering seas depending on the incident direction. The ship heading angle is defined as the angle between the vessel direction and the wave propagation at a celerity velocity c. The wave direction are changed around the exact beam wave, $\chi = 90°$. We consider

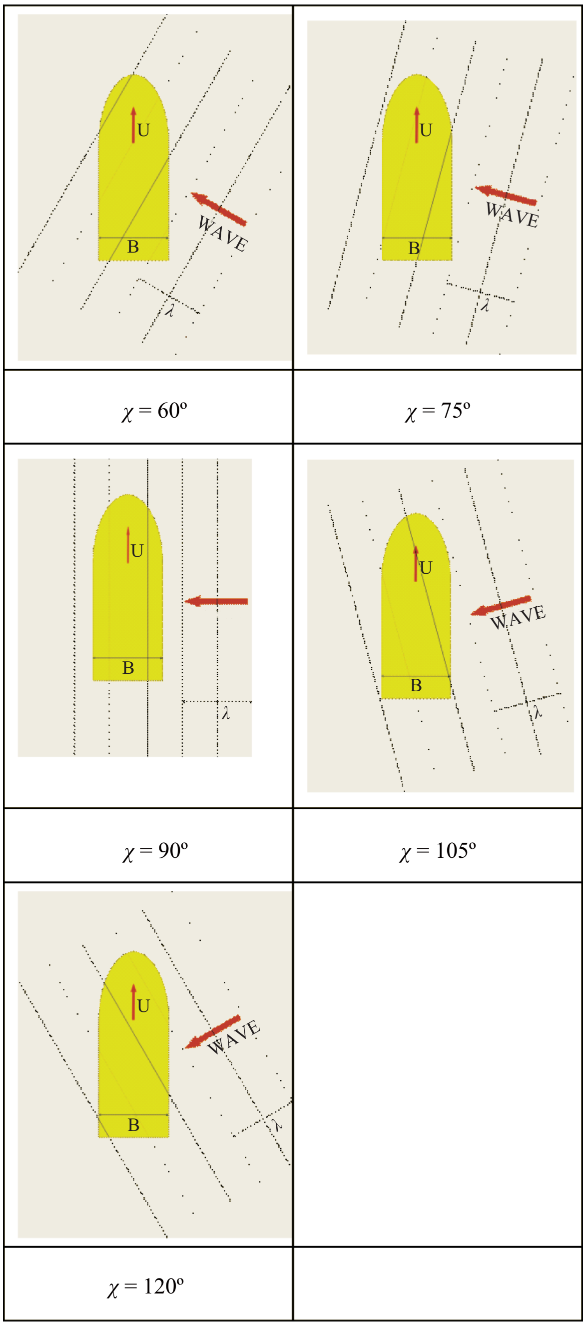


Fig. 2. Definition of heading angle [24].

the heading angle changes from χ = 60° to 120° as shown in Fig. 2 [24]. The ship speed is chosen as 0 m/s, 1 m/s, 2 m/s, 3 m/s, and 4 m/s.

The motion of a ship, forward or otherwise, affects the way incident waves are viewed by someone aboard the vessel. If the waves are incident on the ship at some angle, χ, then the component of the speed of the ship in the direction of wave propagation is $U_a = -U \cos \chi$. The wave crests move at the phase speed, $C_P = \omega / k$ and the relative speed between the ship and the wave is

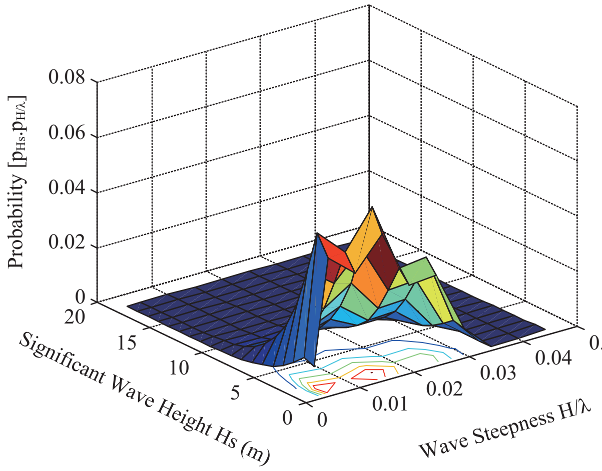


Fig. 3. Probability of wave height and steepness.

$$U_r = U_a + C_P = -U \cos \chi + \omega / k \tag{6}$$

Then the waves have a phase speed such as

$$U_r = \omega_e / k = -U \cos \chi + \omega / k \tag{7}$$

Using the dispersion relationship for waves in deep water we can rewrite the equation for encounter frequency as

$$\omega_e = \omega - \omega^2 (U / g) \cdot \cos \chi \tag{8}$$

2. Excitation Moment from Wave of the Sea

The outer excitation moment from the wave excitation of regular wave can be approximated by a harmonic sinusoidal function $M(t)$, which is presented in the following form [13].

$$M_{wave}(t) = (I_{44} + A_{44}) a_0 \omega_n^2 \pi (H / \lambda) \sin \chi \cos \omega_e t \tag{9}$$

$$\omega_n = [g \Delta GM / (I_{44} + A_{44})]^{\frac{1}{2}} \tag{10}$$

where a_0 is the effective wave slope coefficient, ω_n is the initial rolling natural frequency, H is the wave height, λ is the wave length, ω_e is the encounter frequency

Divided by the $I_{44} + A_{44}$:

$$m_{wave}(t) = a_0 \omega_n^2 \pi (H / \lambda) \sin \chi \cos \omega_e t \tag{11}$$

where a_0, the effective wave slope coefficient, is taken to a constant 0.729 [4]. ω_n and ω equal 1.32 rad/s and 3 rad/s respectively, and the terms of wave significant height $H_{1/3}$ = 5 m and wave length λ are chosen according to the probability distribution as Fig. 3 [10, 15]. The wave climate statistics of the North Atlantic are taken from Bales [1]. Table 2 displays the distribution of significant wave heights and nominal wave steepness.

Table 2. Probability of wave height (m) and steepness.

Height/ Steep	0.5≤	0.5~1.5	1.5~2.5	≤3.5
0.005	0.049181	0.056463	0.040649	0.027162
0.01	0.040666	0.046686	0.033611	0.022459
0.015	0.052417	0.060178	0.043324	0.028949
0.02	0.030787	0.035346	0.025446	0.017004
0.025	0.031639	0.036323	0.02615	0.017474

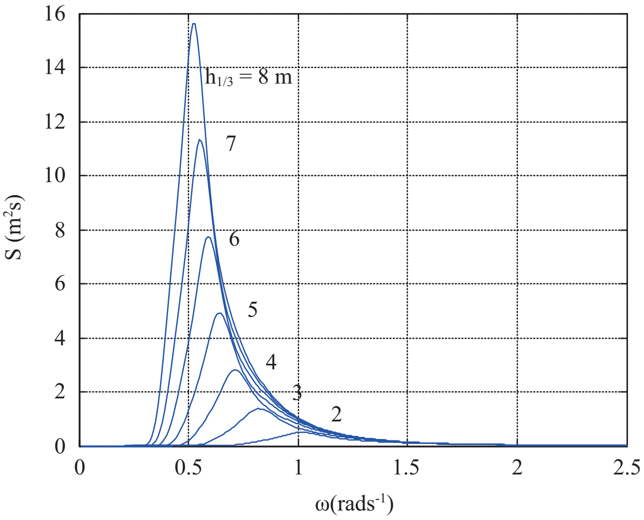


Fig. 4. Wave energy spectrum.

The wave energy spectrum depends on ocean statistics and assumed that the vessel is exposed to the Adriatic Sea environment in the analysis. For the closed sea, a specific wave energy spectrum has been defined on the basis of the observations and measurements [13]. The equation of narrow band wave spectrum as

$$S(\omega) = 0.862 \times (0.0135 g^2 / \omega^5) \times \exp[-5.183/(\omega^4 h_{1/3}^2)]1.63^p \qquad (12)$$

where

$$p = \exp[-(\omega - \omega_m)^2 / (2\sigma^2 \omega_m^2)]$$

$$\omega_m = 0.32 + 1.80/(h_{1/3} + 0.6)$$

$$\sigma = \begin{cases} 0.08 & \omega < \omega_m \\ 0.10 & \omega > \omega_m \end{cases}$$

Significant wave height $H_{1/3}$ = 2~8 m, the wave energy spectrum curve and relative excitation moment are shown in Figs. 4 and 5 separately.

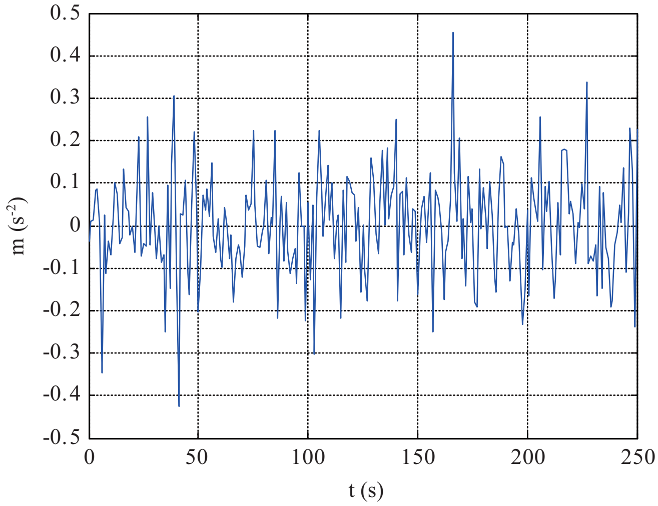


Fig. 5. Time history of relative excitation moment ($H_{1/3}$ = 2.5 m, x = 90°).

IV. SIMULATION

For the calculation of rolling and capsizing of the intact ship in harmonic excitations, the simulation of rolling without considering the excitation from the wind and substitute the moment of wave for the out excitation moment. The differential equation is a typical nonlinear dynamical system. The safe basins of system can be defined using a surrounded area A in the space of phase trajectories. The trajectory start from the safe basins will remain in the area A when the time t tends to infinity. Otherwise, the trajectory start beyond the safe basins will escape the area A; such a trajectory is unstable and may destroy or collapse the system. The acreage and shape of the safe basins will change when the parameter of the system changes.

Firstly, the evolution of the safe basins is studied numerically when the excitation parameter changes its value and others are chosen from above discussed. The surrounded area A is defined as follows:

$$A = \{(x, y) : -2 \le x \le 2, -2 \le y \le 2, y = \dot{x}\}$$

Then A is divided into 80×80 lattices, and the lattice points are taken as the initial values for the solutions of system. If the solution of system remains in the area A for a sufficiently long time up to t = 2000s, such a solution can be approximately taken as a safe solution, and the corresponding lattice may be taken as part of the safe basins; if the solution of system escapes the area A, such a solution is taken to be an unsafe solution, and the corresponding lattice is beyond the safe basins. The whole governing equation is numerically integrated by the fourth-order Runge–Kutta algorithm, and the numerical results are shown in following. The green region denotes the safe basins while the blank region represents the unsafe area.

1. Parts of Simulation Result

According to the distribution of nominal wave steepness,

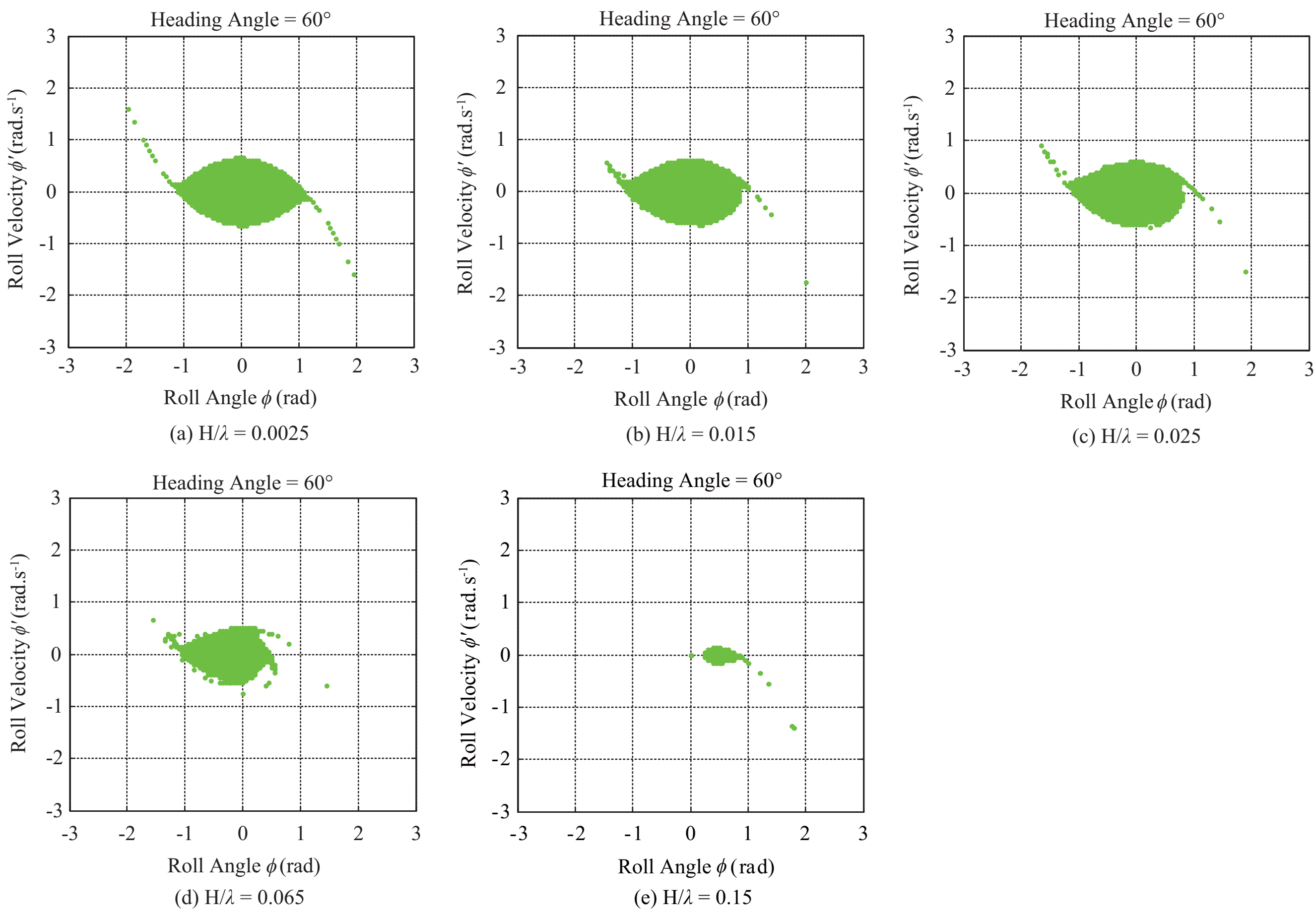


Fig. 6. Erosion of safe basins V = 4 m/s, heading angle = 60° [24].

Table 3. Coefficients variables of the following simulation.

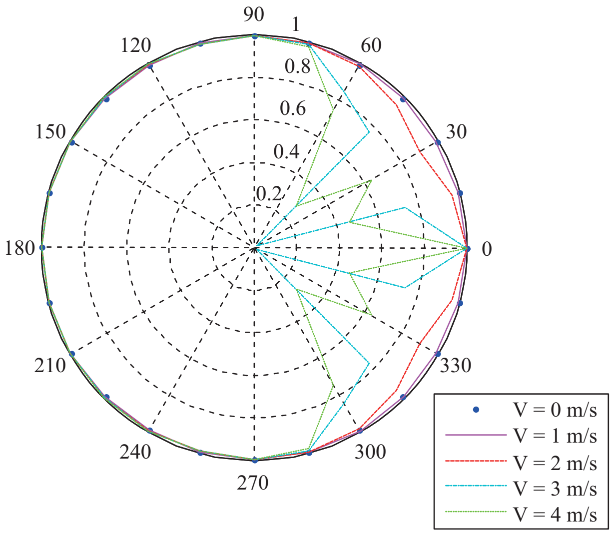
ω_n (rad/s)	ω (rad/s)	$H_{1/3}$ (m)	U (m/s)	a_0	χ (°)
1.32	3	5	4	0.729	60

the steepness of wave is chosen as 0.0025, 0.015, 0.025, 0.065, 0.15 and the heading angle is chosen as 60°. Table 3 lists the details of the simulation parameters. And the erosions of the safe basin in different steepness are shown in Fig. 6 [24].

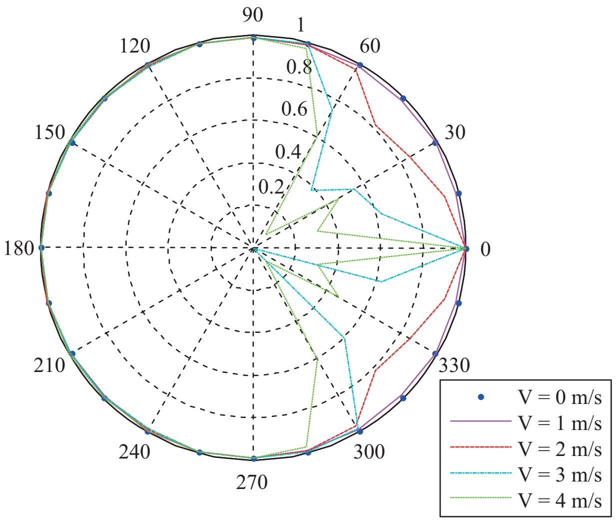
From the calculation results, we can see that the boundaries of the safe basins of system are smooth without any erosion in the case when H/λ = 0.0025 (Fig. 6(a)). Then the ship is absolutely safety even though in the high navigation speed. When the steepness H/λ_1 > 0.0025, the safe basins are eroded increasingly with an increase value of steepness as shown in Figs. 6(b), 6(c) and 6(d); and in the case when H/λ_2 > 0.15 (Fig. 6(e)), the safe basins disappear nearly. Hence, wave steepness H/λ_1 and H/λ_2 > are two significant critical points for the evolution of erosion.

2. Ship Survival Probability

This section describes the probability of the ship survival, the probability of survival of ferry is discussed approximately based on the simulation results. The principle of survival probability is that the ratio between the acreage of the safe basin with and without considering wave excitation because the differences of safe basin acreage exist in the two simulation conditions. According to the areas ratio of the erosion basins with and without wave excitation, the estimation of ship survival probability can be measured [13]. The diagram, as showing in Fig. 7, gives the capsizing probability as the function of the ship heading angle, ship speed and wave steepness. The diagram of Fig. 7(a) constructed for the intact ship considering the sea condition with wave steepness H/λ = 0.045 and the diagram of Fig. 7(b) for wave steepness H/λ = 0.065. The ship speed is 0, 1, 2, 3 and 4 m/s respectively, the heading angle is a variable change from 0 to 180°. As well, the survival probability in the condition of wave steepness H/λ = 0.045 and H/λ = 0.065 are quite different. As the wave steeper, the probability of capsizing is more dangerous. Table 4 lists the minimum values of survival probability in this extreme sea conditions. The simulation result shows that quartering seas are very dangerous for the stability of voyaging ships, and the vessels have to reduce speed or change heading direction in order to avoid capsizing.



(a) $H/\lambda = 0.045$



(b) $H/\lambda = 0.065$

Fig. 7. Polar diagram of predicted ship survival probability, $H_{1/3}$ = 5 m.

V. CONCLUDING REMARKS

This paper concludes with a discussion of the probability of ship capsizing in regular waves combined with statistical and nonlinear safe basin method. One advantage of using this method is the prediction of probability for safety assessment quantitatively. Risk assessment is very important in operational procedures and management inasmuch it would reveal weak points of ships. Risk analysis and development of risk based criteria requires estimation of probability of capsizing in various dangerous situations simulating respective capsizing scenarios, especially the most important hazards created by forces of the seas. This paper focus on the hazards from the beam seas and a preliminary approach is proposed. The excitations are expressed based on the hazard scenarios of the sea status and the following conclusions can be drawn from the present study.

Table 4. Minimum survival probability of the ship, $H_{1/3}$ = 5 m.

Steepness	Velocity (m/s)	Head-Angle (degree)	Probability
0.065	0	90	0.9927
0.065	1	45	0.9832
0.065	2	45	0.8156
0.065	3	45	0.3844
0.065	4	45	0.0826

1. The instantaneous states of ship roll and the forces of the seas have intensive effect to the probability of ships capsizing. Both of them should be considered simultaneously when estimating the safety of ships rather than one of them. The simulation results have shown that there is no safe basin erosion when the ship is in the calm sea.
2. The method of evaluating the survival probability of ship capsizing in time domain base on the combination of statistical and nonlinear safe basin method has pointed out. The safe basin erosion is very sensitive to the alteration of the excitation forces. It is an effective technique for analysis the stability assessment when ship navigates in random waves and excited by the forces of the sea.
3. The problem is the limitation of this method to a limited domain of parts. The random characters, such as the random properties of irregular wave, chaos of nonlinear dynamic system etc cannot be expressed in this method.

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