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ON STATE-FEEDBACK CONTROLLER SYNTHESIS FOR A CLASS OF SECOND-ORDER NONLINEAR SYSTEMS WITH UNCERTAINTIES

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ON STATE-FEEDBACK CONTROLLER SYNTHESIS FOR A CLASS OF SECOND-ORDER NONLINEAR SYSTEMS WITH UNCERTAINTIES

Jenq-Lang Wu* and Chee-Fei Yung*

Key words: nonlinear systems, robust control, robust control Lyapunov function, robust practical stability.

ABSTRACT

This paper presents a constructive method to design stabilizing controllers for a class of second-order uncertain nonlinear systems whose input parts may vanish at the origin. A sufficient condition for the existence of stabilizing state feedback controllers is provided. Under the *small control property*, a formula is proposed for constructing continuous stabilizing feedback laws. In the case that the *small control property* does not hold, another formula for constructing continuous state feedback controllers, with small control magnitude near the origin, to achieve robust practical stability is presented.

I. INTRODUCTION

For stability analysis and controller synthesis problems of nonlinear systems, Lyapunov based methods are the most important approaches. It is well known that for nonlinear systems in feedback linearizable form, strict feedback form, and feedforward form *et al.*, there are systematic ways to find stabilizing controllers and the corresponding Lyapunov functions, see [2, 4-6]. For example, consider a second-order nonlinear system in strict feedback form:

$$\dot{x}_1 = x_2 + \phi(x_1)$$

$$\dot{x}_2 = f(x_1, x_2) + g(x_1, x_2)u$$

If $g(x_1, x_2) \neq 0$ in a neighborhood of the origin, the backstepping approach can be used to derive stabilizing controller. Moreover, the corresponding Lyapunov function can be obtained. However, to applying the backstepping approach, it is necessary that the function $g(x_1, x_2)$ (called input part) is nonzero in a neighborhood of the origin. Moreover, the obtained controllers often achieve only local stability. If the input part $g(x_1, x_2)$ vanishes at the origin (i.e., $g(0, 0) = 0$), the backstepping approach cannot be used to solve its stabilization problem. In this case, how to design globally stabilizing controllers is interesting.

In this paper, we consider the stabilization problem for a class of uncertain nonlinear systems, whose input parts vanish at the origin. For simplification, we focus on the second-order case. Based on the control Lyapunov function approach (please see [1, 3, 5-10]), a simple sufficient condition for the existence of stabilizing controllers is derived. Then, based on the Sontag's formula (please see [10]), we propose a new formula for constructing globally and robustly stabilizing state feedback controllers. In [11], some results about the stabilization for polytopic nonlinear systems have been presented. It has been shown that for any polytopic nonlinear system in canonical form, robust stabilization is always possible. However, in [11], the case that the *small control property* does not hold has not been discussed. In addition, in [11], for polytopic nonlinear systems in canonical form, it is assumed that the input parts are always nonzero. In this paper, we consider the case that the input parts of the considered systems vanish at the origin. Moreover, we discuss both the cases that the *small control property* holds or not. If the *small control property* holds, the obtained feedback law is continuous in R^2. If the *small control property* does not hold, a new formula is provided for constructing continuous feedback laws, with small control magnitude near the origin, to achieve robust practical stability.

II. PROBLEM FORMULATION

Consider a second-order nonlinear system:

$$\dot{x}_1 = f_1(x_1, x_2)$$

$$\dot{x}_2 = f_2(x_1, x_2) + g(x_1, x_2)u \quad (1)$$

where $x = [x_1 \;\; x_2]^T \in R^2$ denotes the state, $u \in R$ is the control input. Suppose that smooth functions $f_1(.,.)$ and $g(.,.)$ are known, and the smooth function $f_2(.,.)$ is not exactly known.

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Consider the case that $f_2(x_1, x_2) = \sum_{i=1}^{m} \alpha_i \cdot f_{2i}(x_1, x_2)$ with unknown parameters $0 \le \alpha_i \le 1$, i = 1, 2, …, m, and $\sum_{i=1}^{m} \alpha_i = 1$. Let $g(x_1, x_2) = 0$ if and only if $x_2 = \eta(x_1)$ for some smooth function $\eta(.)$ with $\eta(0) = 0$.

The main objective of this paper is to find a function $p(.)$ such that the state feedback controller $u = p(x)$ globally and asymptotically stabilizes the system (1) for all possible uncertainty. If continuous *stabilizing* controllers cannot be found, then a continuous controller $u = h(x)$ is developed to achieve robust practical stability.

Remark 1: It should be noted that the function $g(x_1, x_2)$ has not been assumed to be nonzero in a neighborhood of the origin. If $f_1(x_1, x_2) = x_2 + \phi(x_2)$ for some smooth function $\phi(\cdot)$, $f_2(x_1, x_2)$ is precisely known, and $g(x_1, x_2) \neq 0$ for all $x \in R^2$, the system (1) is in strict-feedback form that is well studied in the literature. In this paper, we relax these assumptions, and therefore, the structure of system (1) can be seen as an extension of strict-feedback form. ■

Remark 2: It is a little restriction that assuming $g(x_1, x_2) = 0$ if and only if $x_2 = \eta(x_1)$. In fact, we can easily extend our results to the more general case that $g(x_1, x_2) = 0$ if and only if x satisfying $\varphi(x_1, x_2) = 0$ for some smooth function $\varphi(\cdot,\cdot)$. We make the assumption only for simplification. ■

III. MAIN RESULTS

For system (1), since $g(x_1, x_2)$ vanishes at the origin, the feedback linearization and the backstepping approaches cannot be used to derive stabilizing controllers.

Based on the robust control Lyapunov function approach, in the following theorem a sufficient condition for the existence of robust stabilizing feedback laws for the system (1) will be proposed. Moreover, a universal formula for constructing stabilizing controllers will be presented.

Theorem 1: Consider the system (1). Suppose $g(x_1, x_2) = 0$ if and only if $x_2 = \eta(x_1)$. Then, there exists a feedback law which can globally asymptotically stabilize the system if $x_1 f_1(x_1, \eta(x_1)) < 0$ for all $x_1 \neq 0$. Moreover, in this case, the feedback law

$$u = p(x) = \begin{cases} \min\limits_{i\in\{1,2,\ldots,m\}} \left\{ -\dfrac{a_i(x) + \sqrt{a_i^2(x) + b^4(x)}}{b(x)} \right\}, & \text{if } b(x) > 0 \\ 0, & \text{if } b(x) = 0 \\ \max\limits_{i\in\{1,2,\ldots,m\}} \left\{ -\dfrac{a_i(x) + \sqrt{a_i^2(x) + b^4(x)}}{b(x)} \right\}, & \text{if } b(x) < 0 \end{cases} \tag{2}$$

is one such controller, where

$$a_i(x) = x_1 \cdot f_1(x_1, x_2) - (x_2 - \eta(x_1)) \cdot \frac{d\eta(x_1)}{dx_1} \cdot f_1(x_1, x_2)$$
$$+ (x_2 - \eta(x_1)) \cdot f_{2i}(x_1, x_2), \quad i = 1, 2, \ldots, m$$

$$b(x) = (x_2 - \eta(x_1)) \cdot g(x_1, x_2).$$

Proof: Choose $V(x) = 0.5x_1^2 + 0.5(x_2 - \eta(x_1))^2$ as a candidate robust control Lyapunov function. Note that $V(x)$ is positive definite by definition. It is clear that

$$\begin{aligned} \dot{V}(x) &= x_1 \cdot f_1(x_1, x_2) - (x_2 - \eta(x_1)) \cdot \frac{d\eta(x_1)}{dx_1} \cdot f_1(x_1, x_2) \\ &\quad + (x_2 - \eta(x_1)) \cdot \sum_{i=1}^{m} \alpha_i \cdot f_{2i}(x_1, x_2) \\ &\quad + (x_2 - \eta(x_1)) \cdot g(x_1, x_2) u \\ &= \sum_{i=1}^{m} \alpha_i \cdot \Big(x_1 \cdot f_1(x_1, x_2) - (x_2 - \eta(x_1)) \\ &\quad \times \left(\frac{d\eta(x_1)}{dx_1} \cdot f_1(x_1, x_2) + f_{2i}(x_1, x_2) \right) \Big) + b(x) u \\ &= \sum_{i=1}^{m} \alpha_i \cdot a_i(x) + b(x) u \\ &= \sum_{i=1}^{m} \alpha_i \cdot (a_i(x) + b(x) u) \end{aligned} \tag{3}$$

Since $b(x) = 0$ if and only if $x_2 = \eta(x_1)$, it is obvious that $a_i(x)\big|_{b(x)=0} = x_1 \cdot f_1(x_1, \eta(x_1)) < 0$ $\forall x \neq 0$ and $\forall i \in$ {1, 2, …, m}. This implies that $V(x)$ is a robust control Lyapunov function for system (1). Therefore, stabilizing feedback laws for system (1) exist [10].

Now, we prove that the feedback law (2) asymptotically stabilizes (1). If $b(x) > 0$, then

$$\begin{aligned} & a_i(x) + b(x) p(x) \\ &= a_i(x) + b(x) \cdot \min_{i\in\{1,2,\ldots,m\}} \left\{ -\frac{a_i(x) + \sqrt{a_i^2(x) + b^4(x)}}{b(x)} \right\} \\ &\le a_i(x) + b(x) \cdot \left(-\frac{a_i(x) + \sqrt{a_i^2(x) + b^4(x)}}{b(x)} \right) \\ &= -\sqrt{a_i^2(x) + b^4(x)} \\ &< 0. \end{aligned}$$

Therefore,

$$\dot{V}(x) = \sum_{i=1}^{m} \alpha_i \cdot \left(a_i(x) + b(x)p(x) \right) \le \sum_{i=1}^{m} \alpha_i \cdot \left(-\sqrt{a_i^2(x) + b^4(x)} \right) < 0.$$

Similarly, if $b(x) < 0$, we can show that $\dot{V}(x) < 0$ by using the feedback law (2).

Finally, note that $b(x) = 0$ if and only if $x_2 = \eta(x_1)$. If $x \neq 0$ is such that $b(x) = 0$, then

$$\dot{V}(x)\Big|_{b(x)=0} = x_1 \cdot f_1(x_1, \eta(x_1)) < 0 .$$

Therefore, the feedback law (2) globally asymptotically stabilizes system (1). ■

As in [10], we say that the robust control Lyapunov function $V(x)$ of system (1) satisfies the *small control property* if for each $\varepsilon > 0$ there is a $\delta > 0$ such that, for all $x \neq 0$ satisfying $\|x\| < \delta$, there is some u with $\|u\| < \varepsilon$ such that $\sum_{i=1}^{m} \alpha_i \cdot \left(a_i(x) + b(x)u \right) < 0$ for all possible uncertainty (or equivalently, $a_i(x) + b(x)u < 0$ $\forall i \in \{1, 2, \ldots, m\}$).

Lemma 1: If $V(x)$ satisfies the *small control property*, then the controller (2) is continuous in R^2.

Proof: Since $V(x)$ satisfies the *small control property*, the function

$$p_i(x) \equiv \begin{cases} -\dfrac{a_i(x) + \sqrt{a_i^2(x) + b^4(x)}}{b(x)}, & \text{if } b(x) \neq 0 \\ 0, & \text{if } b(x) = 0 \end{cases}$$

is continuous in R^2 [10]. Then, by definition, it is clear that the feedback law (2) is continuous in R^2. ■

However, if $V(x)$ does not satisfy the *small control property*, the feedback law (2) will be discontinuous at the origin. In this case, $b(x^j)$ may converge to zero faster than $a(x^j)$ as x^j approaching the origin. Although this causes no problems regarding uniqueness of solutions [10], it can cause an undesired property that the feedback law (2) may have extremely large control magnitude near the origin. In the following, for the case that the *small control property* does not hold, we provide a new formula for constructing continuous feedback laws, with small control magnitude near the origin, to achieve robust practical stability. For some practical applications, robust practical stability is enough.

Define

$$\hat{p}(x) = \begin{cases} \min\limits_{i \in \{1,2,\ldots,m\}} \left\{ -\dfrac{a_i(x) + \sqrt{a_i^2(x) + b^4(x)}}{b(x) + \mu \cdot \left| \rho - \|x\| \right| \cdot} \right\}, & \text{if } b(x) > 0 \\ 0, & \text{if } b(x) = 0 \\ \max\limits_{i \in \{1,2,\ldots,m\}} \left\{ -\dfrac{a_i(x) + \sqrt{a_i^2(x) + b^4(x)}}{b(x) - \mu \cdot \left| \rho - \|x\| \right| \cdot} \right\}, & \text{if } b(x) < 0 \end{cases}$$

for some $\mu > 0$ and $\rho > 0$. Then, we have the following result.

Theorem 2: Consider the system (1). Suppose that $g(x_1, x_2) = 0$ if and only if $x_2 = \eta(x_1)$, and that $x_1 f_1(x_1, \eta(x_1)) < 0$ for all $x_1 \neq 0$. Moreover, suppose the robust control Lyapunov function $V(x) = 0.5x_1^2 + 0.5\left(x_2 - \eta(x_1)\right)^2$ of system (1) does not satisfy the *small control property*. Then, the feedback law

$$u = h(x) = \begin{cases} p(x), & \text{if } \|x\| \ge \rho \\ \hat{p}(x), & \text{if } \|x\| < \rho \end{cases}$$

is continuous in R^2 and is able to achieve robust practical stability (that is, $\lim\limits_{t \to \infty} \|x(t)\| < \rho$).

Proof: Substituting $u = h(x)$ into (3) yields

$$\dot{V}(x) = \sum_{i=1}^{m} \alpha_i \cdot \left(a_i(x) + b(x)h(x) \right).$$

If $\|x\| \ge \rho$, $h(x) = p(x)$. Therefore, $\lim\limits_{t \to \infty} \|x(t)\| < \rho$ since $\dot{V}(x) = \sum_{i=1}^{m} \alpha_i \cdot \left(a_i(x) + b(x)p(x) \right) < 0$ for $\|x\| \ge \rho$.

Now we prove the continuity of $h(x)$. We first show that $\hat{p}(x)$ is continuous in the region $\left\{ x \in R^2 \,\middle|\, \|x\| < \rho \right\}$. By the definition of $\hat{p}(x)$, in this region the only possible discontinuous points are the origin and those points that satisfying $b(x) = 0$. It is clear that

$$\lim_{j \to \infty} \hat{p}(x^j) = 0 \tag{4}$$

if x^j is a sequence of states that converging to the origin. Moreover, note that $b(x) + \mu \cdot \left| \rho - \|x\| \right| \neq 0$ if $b(x) > 0$ and $b(x) - \mu \cdot \left| \rho - \|x\| \right| \neq 0$ if $b(x) < 0$. Let x^j be a sequence of states that satisfying $b(x^j) > 0$ and converging to a point $\bar{x} \neq 0$ with $\|\bar{x}\| < \rho$ and $b(\bar{x}) = 0$. Then, $a_i(\bar{x}) < 0$ and, therefore,

$$\lim_{j \to \infty} \min_{i \in \{1,2,\ldots,m\}} \left\{ -\frac{a_i(x^j) + \sqrt{a_i^2(x^j) + b^4(x^j)}}{b(x^j) + \mu \cdot \left| \rho - \|x^j\| \right| \cdot} \right\}$$

$$= \min_{i\in\{1,2,\ldots,m\}} \left\{ -\frac{a_i(\bar{x}) + \sqrt{a_i^2(\bar{x})}}{\mu \cdot \left|\rho - \|\bar{x}\|\right| \cdot} \right\}$$

$$= 0.$$

Similarly, if x^j is a sequence of state that satisfying $b(x^j) < 0$ and converging to a point $\bar{x} \neq 0$ with $\|\bar{x}\| < \rho$ and $b(\bar{x}) = 0$, we can show that

$$\lim_{j\to\infty} \max_{i\in\{1,2,\ldots,m\}} \left\{ -\frac{a_i(x^j) + \sqrt{a_i^2(x^j) + b^4(x^j)}}{b(x^j) - \mu \cdot \left|\rho - \|x^j\|\right| \cdot} \right\} = 0 .$$

This shows that the function $\hat{p}(x)$ is continuous in $\left\{x \in R^2 \,\middle|\, \|x\| < \rho\right\}$. Then, by the definition of $h(x)$, its continuity is clear since $\hat{p}(x) = p(x)$ on the boundary $\|x\| = \rho$. ■

IV. EXAMPLE

Consider the following system ($0 \le \alpha_1 \le 1$, $0 \le \alpha_2 \le 1$, and $\alpha_1 + \alpha_2 = 1$):

$$\dot{x}_1 = f_1(x_1, x_2)$$

$$\dot{x}_2 = \alpha_1 \cdot f_{21}(x_1, x_2) + \alpha_2 \cdot f_{22}(x_1, x_2) + g(x_1, x_2)u \qquad (5)$$

where

$$f_1(x_1, x_2) = x_1 x_2 + 0.1 \cdot x_1^3 x_2^2,$$

$$f_{21}(x_1, x_2) = \cos(x_2) \cdot x_1^2 x_2 + x_1,$$

$$f_{22}(x_1, x_2) = \sin(x_1) \cdot x_1 x_2^2 - x_2,$$

and

$$g(x_1, x_2) = (x_1^2 + 2x_2) \cdot (1 + x_2^2).$$

It should be noted that $g(0, 0) = 0$. We can see that $g(x_1, x_2) = 0$ if and only if $x_2 = \eta(x_1) \equiv -0.5x_1^2$, and that $x_1 f_1(x_1, \eta(x_1)) < 0$ for all $x_1 \neq 0$. Therefore,

$$V(x) = 0.5x_1^2 + 0.5\left(x_2 - \eta(x_1)\right)^2 = 0.5x_1^2 + 0.5\left(x_2 + 0.5x_1^2\right)^2$$

is a robust control Lyapunov function for the system (4). Let

$$a_1(x) = x_1 \cdot f_1(x_1, x_2) - (x_2 - \eta(x_1)) \cdot \frac{d\eta(x_1)}{dx_1} \cdot f_1(x_1, x_2) + (x_2 - \eta(x_1)) \cdot f_{21}(x_1, x_2)$$
$$= (x_1 + x_1 x_2 + 0.5x_1^3) \cdot (x_1 x_2 + 0.1x_1^3 x_2^2) + (x_2 + 0.5x_1^2) \cdot (\cos(x_2) x_1^2 x_2 + x_1)$$

$$a_2(x) = x_1 \cdot f_1(x_1, x_2) - (x_2 - \eta(x_1)) \cdot \frac{d\eta(x_1)}{dx_1} \cdot f_1(x_1, x_2) + (x_2 - \eta(x_1)) \cdot f_{22}(x_1, x_2)$$
$$= (x_1 + x_1 x_2 + 0.5x_1^3) \cdot (x_1 x_2 + 0.1x_1^3 x_2^2) + (x_2 + 0.5x_1^2) \cdot (\sin(x_1) \cdot x_1 x_2^2 - x_2)$$

$$b(x) = (x_2 - \eta(x_1)) \cdot g(x_1, x_2) = 2(x_2 + 0.5x_1^2)^2 \cdot (1 + x_2^2).$$

Since $b(x) \ge 0 \ \forall x$, let

$$p(x) = \begin{cases} \min_{i\in\{1,2\}} \left(-\frac{a_i(x) + \sqrt{a_i^2(x) + b^4(x)}}{b(x)} \right), & \text{if } b(x) > 0 \\ 0, & \text{if } b(x) = 0 \end{cases}$$

It can be verified that the robust control Lyapunov function $V(x)$ does not satisfy the *small control property*. Therefore, the controller $u = p(x)$ may have extremely large control magnitude near the origin. By Theorem 2, define

$$\hat{p}(x) = \begin{cases} \min_{i\in\{1,2\}} \left(-\frac{a_i(x) + \sqrt{a_i^2(x) + b^4(x)}}{b(x) + \mu \cdot \left|\rho - \|x\|\right| \cdot} \right), & \text{if } b(x) > 0 \\ 0, & \text{if } b(x) = 0 \end{cases}$$

Then, the feedback law

$$u = h(x) = \begin{cases} p(x), & \text{if } \|x\| \ge \rho \\ \hat{p}(x), & \text{if } \|x\| < \rho \end{cases}$$

is continuous in R^2 and is able to achieve robust practical stability with $\lim_{t\to\infty} \|x(t)\| < \rho$.

For several different choices of uncertain parameters ($(\alpha_1, \alpha_2) = (1, 0)$, $(\alpha_1, \alpha_2) = (0.8, 0.2)$, $(\alpha_1, \alpha_2) = (0.6, 0.4)$, and $(\alpha_1, \alpha_2) = (0.4, 0.6)$), Fig. 1(a) shows the trajectories and control inputs of the system (5) with $u = h(x)$, $\mu = 0.002$, and $\rho = 0.2$, and Fig. 1(b) shows the trajectories and control inputs of the system (5) with $u = p(x)$. It can be seen that in the cases of $u = h(x)$ with $\mu = 0.002$ and $\rho = 0.2$, the state trajectories almost coincide with those of the cases $u = p(x)$ in the first 200 seconds. However, the control magnitudes near the origin for $u = h(x)$ are much smaller comparing with those of $u = p(x)$. The control magnitudes of $u = p(x)$ tend to grow up without bound as state converging to the origin.

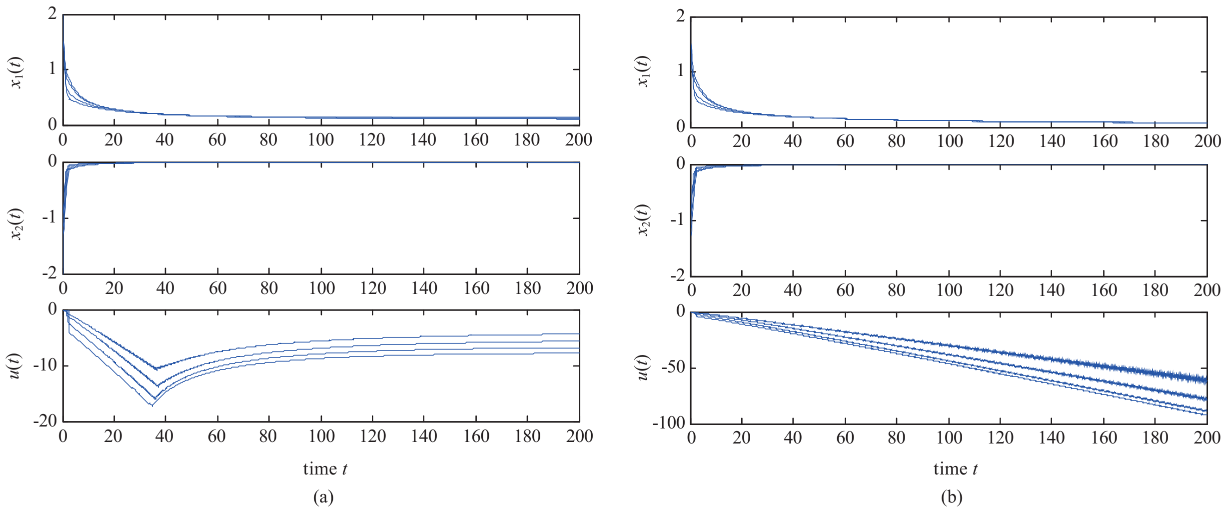

Fig. 1. (a) The state trajectories and control inputs of system (5) with controllers $u = h(x)$ ($\mu = 0.002$ and $\rho = 0.2$), (b) the state trajectories and control inputs of system (5) with controllers $u = p(x)$.

V. CONCLUSIONS

In this note, an approach for finding state feedback controllers to globally asymptotically stabilize a class of second-order polytopic nonlinear systems, whose input parts vanish at the origin, has been presented. A sufficient condition for the existence of stabilizing feedback laws has been derived. Based on the Sontag's formula, a new formula has been provided for constructing continuous stabilizing controllers if the *small control property* holds. For the case that the *small control property* does not hold, another formula has been proposed for constructing continuous feedback laws, which have small control magnitude near the origin, to achieve robust practical stability.

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