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# EFFECTS OF NON-NEWTONIAN COUPLE STRESSES ON THE DYNAMIC CHARACTERISTICS OF WIDE RAYLEIGH STEP SLIDER BEARINGS

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# EFFECTS OF NON-NEWTONIAN COUPLE STRESSES ON THE DYNAMIC CHARACTERISTICS OF WIDE RAYLEIGH STEP SLIDER BEARINGS

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Key words: dynamic characteristics, Rayleigh step bearings, slider bearings, non-Newtonian effects, couple stresses.

## ABSTRACT

To provide more information in fluid-film lubricated bearings, the effects of non-Newtonian couple stresses on the dynamic characteristics of Rayleigh step slider bearings have been presented in this paper. According to the results, the effects of non-Newtonian couple stresses signify an improvement in the bearing characteristics. Comparing with the Newtonian-lubricant Rayleigh step bearing, the non-Newtonian couple stress effects provide an increase in the steady load-carrying capacity, the dynamic stiffness coefficient and the dynamic damping coefficient, as well as a reduction in the steady volume flow rate required. In order to guide the use of the present study, a numerical example and further evaluated results for the non-Newtonian-lubricant Rayleigh step bearing characteristics are also included for engineering applications.

## I. INTRODUCTION

Conventionally, investigations of lubrication performances of slider bearings concentrate upon the hydrodynamic thin-film mechanisms lubricated under the Newtonian-lubricant case (NLC). Typical studies can be observed in the steady-state analysis of various film shapes by Pinkus and Sternlicht [15] and Williams [20]. Further studies of the dynamic characteristics are considered in the exponential-film bearing by Lin and Hung [7], the Rayleigh step bearings by Lu *et al*. [13], and the tapered-land bearings by Lin *et al*. [11]. In order to improve the lubricating characteristics, a Newtonian lubricant blended with different kinds of additives has received a great concern. Since these types of complex fluids display non-Newtonian flow behaviors, the conventional continuum theory of a Newtonian fluid is not appropriable. In order to depict the flow behaviors of these types of non-Newtonian complex fluids, a micro-continuum theory of the couple stress fluids has been recommended by Stokes [19]. The Stokes micro-continuum theory has been originated from the conventional Newtonian continuum theory taking into account the intrinsic motion of material constituents, and allowing for the non-Newtonian effects such as the presence of non-symmetric stress tensors, body couples and couple stresses. This non-Newtonian theory of couple stress fluid model is purposeful to take account of the particle-size effects of various types of additives. It is important for engineering application of pumping fluids such as complex fluids, polymer-thickened oils, liquid crystals and animal bloods. Many investigators have applied this couple stress fluid model of Stokes micro-continuum theory to weight the lubrication performances of different kinds of hydrodynamic bearings, such as the squeeze-film mechanism with reference to synovial joints by Bujurke and Jayaraman [2] and Ahmad and Singh [1]; the squeeze-film convex-surface bearing by Lin [5]; the squeeze-film cylinder-plate bearing by Lin [8]; the rolling bearing by Bujurke and Naduvinami [3] and Das [4]; the journal bearing by Mokhiamer *et al*. [14]; the ball bearings by Sarangi *et al*. [17, 18]; the steady inclined plane slider bearing by Ramanaish [16]; and the dynamic inclined slider bearing by Lin *et al*. [10] and the dynamic exponential-film slider bearing by Lin *et al*. [12]. Although the dynamic characteristics of Rayleigh step slider bearings lubricated with a Newtonian-lubricant have been analyzed by Lu *et al*. [13], the study of dynamic characteristics for Rayleigh step slider bearings lubricated with non-Newtonian couple stress fluids is absent. In order to provide more information in bearing designing and selection, a further investigation is therefore motivated.

On the basis of the Stokes micro-continuum theory [19], the effects of non-Newtonian couple stresses upon the dynamic

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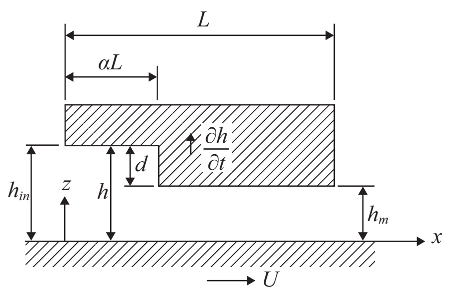


**Fig. 1. Physical geometry of a wide Rayleigh step slider bearing lubricated with a non-Newtonian couple stress fluid.**

characteristics of Rayleigh step slider bearings are of interest. A closed-form solution of the steady load capacity, the steady volume flow rate required, the dynamic stiffness coefficient and the dynamic damping coefficient. Comparing with the traditional Newtonian-lubricant case, the non-Newtonian fluid-lubricated Rayleigh step bearing characteristics are discussed and presented with different values of the non-Newtonian couple stress parameter, the riser location parameter and the shoulder parameter. Further numerical example and results are also presented for engineering applications.

## II. ANALYSIS OF NON-NEWTONIAN RAYLEIGH STEP BEARING CHARACTERISTICS

Fig. 1 shows the physical geometry of a wide Rayleigh step slider bearing of length $L$ lubricated with an incompressible Stokes non-Newtonian couple stress fluid [19]. The bearing has a sliding velocity $U$ and a squeezing velocity $\partial h/\partial u$. The film thickness can be described by the following.

$$h(x,t)=\begin{cases} h_{in}(t)=d+h_m(t), & 0\le x\le \alpha L \\ h_m(t), & \alpha L\le x\le L \end{cases} \tag{1}$$

where $h_{in}(t)$ is the inlet film thickness, $h_m(t)$ denotes the outlet film thickness, $d$ is the steady inlet-outlet film-thickness difference and $\alpha$ describes the riser location parameter. For the present analysis it is assume that the thin-film theory of lubrication [15, 20] is applicable, and the body forces and body couples are absent. According to the derivation by Lin *et al.* [10], the velocity component $u$ in the $x$-direction and the non-Newtonian couple-stress dynamic Reynolds-type equation can be written as:

$$u=\frac{1}{2\mu}\frac{\partial p}{\partial x}\left\{z^2-hz+2l^2\left[1-\frac{\cosh[(2z-h)/2l]}{\cosh(h/2l)}\right]\right\}+U\cdot\left(1-\frac{z}{h}\right) \tag{2}$$

$$\frac{\partial}{\partial x}\left[f(h,l)\frac{\partial p}{\partial x}\right]=6\mu U\frac{\partial h}{\partial x}+12\mu\frac{\partial h}{\partial t} \tag{3}$$

where $l$ can be identified as the molecular length of polar additives in a Newtonian fluid,

$$l=(\eta/\mu)^{1/2} \tag{4}$$

In this definition $\mu$ is the conventional shear viscosity, and $\eta$ is a new material constant possessing the dimension of momentum and is responsible for the non-Newtonian property of couple stresses. In addition, the function $f(h,l)$ is given by:

$$f(h,l)=h^3-12l^2\cdot h+24l^3\cdot\tanh(h/2l) \tag{5}$$

The $x$-direction volume flow rate can be evaluated by integrating the velocity component $u$ across the film thickness.

$$Q=\int_{z=0}^{h} u\cdot Ddz \tag{6}$$

Applying the expression of $u$ and performing the above integration, we can obtain the volume flow rate.

$$Q=\frac{1}{2}Uh\cdot D-\frac{1}{12\mu}\cdot\frac{\partial p}{\partial x}\cdot f(h,l)\cdot D \tag{7}$$

Expressed in a non-dimensional form, both the volume flow rate and the non-Newtonian couple-stress dynamic Reynolds-type equation can be presented as follows.

$$Q^*=\frac{1}{2}h^*-\frac{1}{12}f(h^*,C)\cdot\frac{\partial p^*}{\partial x^*} \tag{8}$$

$$\frac{\partial}{\partial x^*}\left[f^*(h^*,C)\frac{\partial p^*}{\partial x^*}\right]=6\frac{\partial h^*}{\partial x^*}+12\frac{\partial h^*}{\partial t^*} \tag{9}$$

In these equations, the non-dimensional variables and parameter are defined by:

$$f^*(h^*,C)=h^{*3}-12C^2\left[h^*-2C\tanh(h^*/2C)\right] \tag{10}$$

$$h^*(x^*,t^*)=\begin{cases} h_{in}^*(t^*)=\delta+h_m^*(t^*), & 0\le x^*\le\alpha \\ h_m^*(t^*), & \alpha\le x^*\le 1 \end{cases} \tag{11}$$

$$C=l/h_{ms},\ \ \delta=d/h_{ms} \tag{12}$$

where $C$ is the non-dimensional non-Newtonian couple stress parameter, $\delta$ is the non-dimensional shoulder parameter, and is $\alpha=\alpha L/L$ the riser location parameter. Observing the bearing geometry, we have two regions for the non-dimensional non-Newtonian couple-stress dynamic Reynolds-type equation.

For $0 \le x^* \le \alpha$: Region 1

$$\frac{\partial}{\partial x^*}\left\{ f^*(h^*,C)\cdot\frac{\partial p_1^*}{\partial x^*}\right\} = 12V^* \quad (13)$$

$$h^*(x^*,t^*) = \delta + h_m^*(t^*) \quad (14)$$

For $\alpha \le x^* \le 1$: Region 2

$$\frac{\partial}{\partial x^*}\left\{ f^*(h^*,C)\cdot\frac{\partial p_2^*}{\partial x^*}\right\} = 12V^* \quad (15)$$

$$h^*(x^*,t^*) = h_m^*(t^*) \quad (16)$$

where $V^* = dh_m^*/dt^*$ represents the non-dimensional squeezing velocity. The corresponding boundary conditions within Region 1 are:

$$\text{at } x^* = \alpha : Q_1^* = Q_{1\alpha}^* \quad (17)$$

$$\text{at } x^* = 0 : p_1^* = 0 \quad (18)$$

$$\text{at } x^* = \alpha : p_1^* = p_{1\alpha}^* \quad (19)$$

The corresponding boundary conditions for Region 2 are:

$$\text{at } x^* = \alpha : Q_2^* = Q_{2\alpha}^* \quad (20)$$

$$\text{at } x^* = \alpha : p_2^* = p_{2\alpha}^* \quad (21)$$

$$\text{at } x^* = 1 : p_2^* = 0 \quad (22)$$

Evaluating the volume flow rate (8) with the flow boundary condition (17) and solving the non-Newtonian dynamic Reynolds-type Eq. (13) with the pressure boundary conditions (18) and (19), we can obtain:

$$Q_{1\alpha}^* = (\delta + h_m^*)/2 - V^*\cdot\alpha - I_1(h_m^*,V^*)/12 \quad (23)$$

$$p_{1\alpha}^* = 12V^*\cdot f_A(\alpha,h_m^*) + I_1(h_m^*,V^*)\cdot f_B(\alpha,h_m^*) \quad (24)$$

where $I_1$ denotes the integration functions, and

$$f_A(x^*,h_m^*) = \int_{x^*=0}^{x^*} x^*/f^*(h^*,C)dx^* \quad (25)$$

$$f_B(x^*,h_m^*) = \int_{x^*=0}^{x^*} 1/f^*(h^*,C)dx^* \quad (26)$$

Evaluating the volume flow rate (8) with the flow boundary condition (20) and solving the non-Newtonian dynamic Reynolds-type Eq. (15) with the pressure boundary conditions (21) and (22), we can obtain:

$$Q_{2\alpha}^* = h_m^*/2 - V^*\cdot\alpha - I_2(h_m^*,V^*)/12 \quad (27)$$

$$p_{2\alpha}^* = 12V^* f_C(\alpha,h_m^*) + I_2(h_m^*,V^*) f_D(\alpha,h_m^*) \quad (28)$$

where $I_2$ denotes the integration functions, and

$$f_C(x^*,h_m^*) = \int_{x^*=1}^{x^*} x^*/f^*(h_m^*,C)dx^* \quad (29)$$

$$f_D(x^*,h_m^*) = \int_{x^*=1}^{x^*} 1/f^*(h_m^*,C)dx^* \quad (30)$$

Now equating the values of the volume flow rate at the position $x^* = \alpha$ requires:

$$Q_{1\alpha}^* = Q_{2\alpha}^* \text{ at } x^* = \alpha \quad (31)$$

Similarly, equating the values of the film pressure at the position $x^* = \alpha$ requires:

$$p_{1\alpha}^* = p_{2\alpha}^* \text{ at } x^* = \alpha \quad (32)$$

Applying the conditions (31) and (32), we obtain the expressions of the integration functions.

$$I_1 = \frac{12V^* f_C(\alpha,h_m^*) - 12V^* f_A(\alpha,h_m^*) - 6\delta f_D(\alpha,h_m^*)}{f_B(\alpha,h_m^*) - f_D(\alpha,h_m^*)} \quad (33)$$

$$I_2 = I_1 - 6\delta \quad (34)$$

The non-Newtonian dynamic film force can be calculated by integrating the film pressure over the fluid-film region. Expressed in a non-dimensional form, we have:

$$F^* = \int_{x^*=0}^{1} p^* dx^* = \int_{x^*=0}^{\alpha} p_1^* dx^* + \int_{x^*=\alpha}^{1} p_2^* dx^* \quad (35)$$

Using the expressions of the film pressure and performing the integrations, we can obtain the non-dimensional non-Newtonian dynamic film force.

$$\begin{aligned} F^*(h_m^*,V^*) &= 12V^* F_A(h_m^*) + 12V^* F_C(h_m^*) \\ &+ I_1(h_m^*,V^*) F_B(h_m^*) + I_1(h_m^*,V^*) F_D(h_m^*) - 6\delta F_D(h_m^*) \end{aligned} \quad (36)$$

where the related functions are defined in **Appendix A**. Following the similar procedures of the study of magneto-hydrodynamic characteristics by Lin *et al*. [9], we can obtain the non-Newtonian steady volume flow rate $Q_s^* = Q_{1\alpha}^*(h_{ms}^*, 0)$, the non-Newtonian steady load-carrying capacity $W_s^* = F^*(h_{ms}^*, 0)$, the non-Newtonian dynamic stiffness coefficient $S_d^* = -(\partial F^*/\partial h_m^*)_s$ and the non-Newtonian dynamic damping coefficient $D_d^* = -(\partial F^*/\partial V^*)_s$.

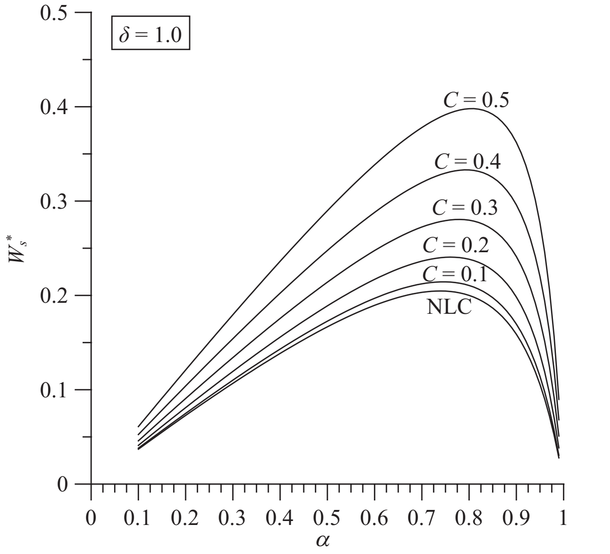


**Fig. 2. Variation of the steady load-carrying capacity $W_s^*$ as a function of $\alpha$ for different $C$ under $\delta$ = 1.**

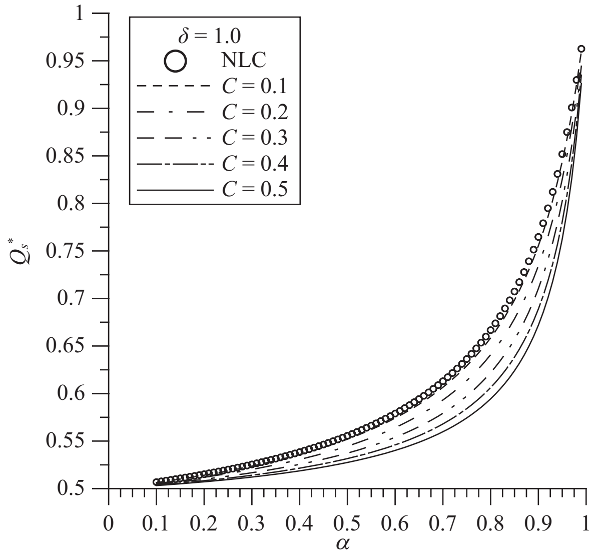


**Fig. 3. Variation of the steady volume flow rate $Q_s^*$ with $\alpha$ for different $C$ under $\delta$ = 1.**

$$Q_s^* = Q_{1\alpha}^*(h_{ms}^*, 0) = \frac{1}{2}\cdot(\delta + h_{ms}^*) - \frac{1}{12}\cdot I_{1s} \tag{37}$$

$$\begin{aligned} W_s^* &= F^*(h_{ms}^*, 0) \\ &= I_{1s}F_B(h_{ms}^*) + I_{1s}F_D(h_{ms}^*) - 6\delta F_D(h_{ms}^*) \end{aligned} \tag{38}$$

$$\begin{aligned} S_d^* = {} & 6\delta\cdot\left(\frac{\partial F_D}{\partial h_m^*}\right)_s - I_{10}\cdot\left[\left(\frac{\partial F_B}{\partial h_m^*}\right)_s + \left(\frac{\partial F_D}{\partial h_m^*}\right)_s\right] \\ & -\left(\frac{\partial I_1}{\partial h_m^*}\right)_s\cdot\left[(F_B)_s + (F_D)_s\right] \end{aligned} \tag{39}$$

$$D_d^* = -12\cdot[(F_A)_s + (F_C)_s] - \left(\frac{\partial I_1}{\partial V^*}\right)_s\cdot\left[(F_B)_s + (F_D)_s\right] \tag{40}$$

In these equations, the subscript "*s*" denotes the bearing operating under the steady state, and the associated functions and quantities are defined in **Appendix A**.

## III. RESULTS AND DISCUSSION

Based upon the above analysis, the non-Newtonian-lubricant Rayleigh step bearing characteristics are dominated by three parameters: the non-Newtonian couple stress parameter, $C = l/h_{ms} = (\eta/\mu)^{1/2}/h_{ms}$, the riser location parameter $\alpha = \alpha L/L$, and the shoulder parameter, $\delta = d/h_{ms}$, Without loss of generality, the non-dimensional steady minimum film thickness is taken to be $h_{ms}^* = 1$ to present the bearing characteristics.

Fig. 2 shows the variation of the non-dimensional steady load-carrying capacity with the riser location parameter $\alpha$ for different values of $C$ under the shoulder parameter $\delta$ = 1. Observing the NLC, the steady load-carrying capacity increases with the value of $\alpha$ until a critical riser location is achieved, and thereafter falls as the value of $\alpha$ continues to increase. In the presence of non-Newtonian couple stresses ($C$ = 0.1), a higher load-carrying capacity is obtained. In addition, increasing values of the couple stress parameter ($C$ = 0.2, 0.3, 0.4 and 0.5) increases the increment of the load-carrying capacity.

Fig. 3 shows the variation of the non-dimensional steady volume flow rate with the riser location parameter $\alpha$ for different values of $C$ under the shoulder parameter $\delta$ = 1. It is observed that the required volume flow rate increase with increasing values of the riser location parameter. Comparing with the NLC, the effects of non-Newtonian couple stresses ($C$ = 0.2, 0.3, 0.4 and 0.5) provide further reductions of the flow rates required for the bearing.

Fig. 4 depicts the variation of the non-dimensional dynamic stiffness coefficient with the riser location parameter $\alpha$ for different values of $C$ under the shoulder parameter $\delta$ = 1. It is found that the maximum stiffness coefficient occurs within the range of large riser location parameters. Comparing with the NLC, the effects of non-Newtonian couple stresses ($C$ = 0.1) provide a higher bearing stiffness. Increasing values of the couple stress parameter ($C$ = 0.2, 0.3, 0.4 and 0.5) provide further increments of the bearing stiffness.

Fig. 5 describes the variation of the non-dimensional dynamic damping coefficient with the riser location parameter $\alpha$ for different values of $C$ under the shoulder parameter $\delta$ = 1. The damping coefficients are found to decrease with increasing values of the riser location parameter. Comparing with the NLC, the Rayleigh step bearing lubricated with non-Newtonian couple stress fluids are observed to provide

**Table 1. Non-Newtonian bearing characteristics and comparison with the NLC by Lu *et al.* [13].**

| $\alpha = 0.72$ | $\delta$ | Lu *et al.* [13] | Present study: Non-Newtonian Rayleigh step bearing | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | $C = 0$ | $C = 0.1$ | $C = 0.2$ | $C = 0.3$ | $C = 0.4$ | $C = 0.5$ |
| $W_s^*$ | 0.5 | .182 | .18162 | .19482 | .23121 | .28876 | .36670 | .46497 |
| | 1.0 | .204 | .20432 | .21368 | .23851 | .27579 | .32431 | .38404 |
| | 1.5 | .178 | .17806 | .18332 | .19707 | .21718 | .24288 | .27418 |
| | 2.0 | .146 | .14609 | .14902 | .15666 | .16774 | .18183 | .19892 |
| | 2.5 | .119 | .11882 | .12053 | .12497 | .13142 | .13962 | .14958 |
| $Q_s^*$ | 0.5 | .608 | .60811 | .60483 | .59764 | .58996 | .58359 | .57882 |
| | 1.0 | .622 | .62162 | .61498 | .60072 | .58591 | .57393 | .56510 |
| | 1.5 | .606 | .60599 | .59865 | .58322 | .56766 | .55537 | .54648 |
| | 2.0 | .587 | .58696 | .58019 | .56615 | .55226 | .54145 | .53372 |
| | 2.5 | .571 | .57073 | .56485 | .55277 | .54094 | .53183 | .52536 |
| $S_d^*$ | 0.5 | .442 | .44178 | .49327 | .63279 | .84969 | 1.14100 | 1.50774 |
| | 1.0 | .381 | .38104 | .40767 | .47492 | .57157 | .69597 | .85023 |
| | 1.5 | .259 | .25896 | .27006 | .29705 | .33450 | .38247 | .44245 |
| | 2.0 | .171 | .17149 | .17626 | .18765 | .20339 | .22382 | .24977 |
| | 2.5 | .116 | .11625 | .11846 | .12373 | .13106 | .14077 | .15330 |
| $D_d^*$ | 0.5 | .496 | .49579 | .52988 | .62448 | .77536 | .98112 | 1.24170 |
| | 1.0 | .273 | .27293 | .28591 | .32106 | .37528 | .44740 | .53742 |
| | 1.5 | .165 | .16454 | .17082 | .18786 | .21423 | .24945 | .29354 |
| | 2.0 | .109 | .10882 | .11279 | .12377 | .14111 | .16462 | .19433 |
| | 2.5 | .078 | .07819 | .08128 | .08995 | .10391 | .12307 | .14747 |

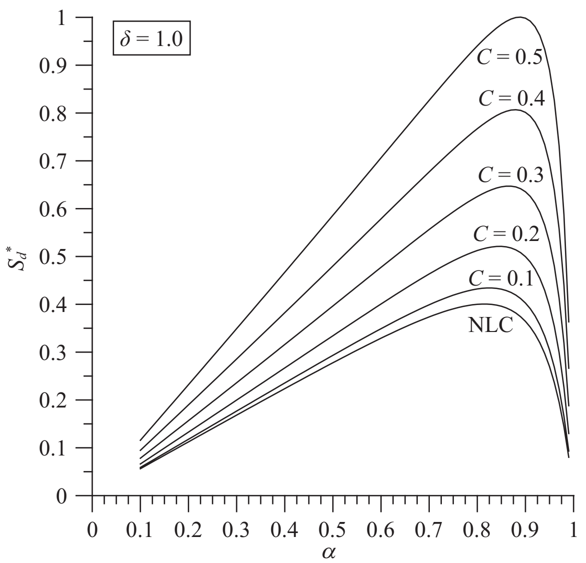


**Fig. 4. Variation of the dynamic stiffness coefficient $S_d^*$ with $\alpha$ for different $C$ under $\delta = 1$.**

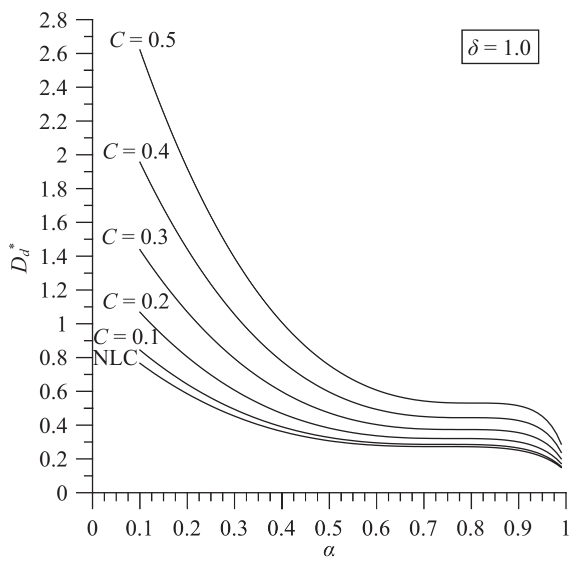


**Fig. 5. Variation of the dynamic damping coefficient $D_d^*$ with $\alpha$ for different $C$ under $\delta = 1$.**

higher damping coefficients. On the whole, better dynamic damping characteristics are provided for the Rayleigh step bearing with a larger couple stress parameter and a smaller riser location parameter.

## 1. Numerical Example

In order to guide the use of the Rayleigh step slider bearing lubricated with non-Newtonian couple stress fluids, a numerical example modified from Lin *et al*. [6] and Lin *et al*. [9] is illustrated: inlet-outlet film thickness differences, $\delta = (0.50, 1.00, 1.50, 2.00, 2.50) \times 10^{-4}\ m$; steady outlet film thickness, $h_{ms} = 0.0001\ m$; length of the first part of the bearing, $\alpha L = 0.072\ m$; length of the bearing, $L = 0.1\ m$; lubricant viscosity, $\mu = 2.45 \times 10^{-6}\ Pa \cdot s$; couple-stress material constants, $\eta = (0, 2.45, 9.80, 22.05, 39.20, 61.25) \times 10^{-16}\ N \cdot s$. With the aid of the definitions of the parameters, we can calculate the values of the three parameters: $\alpha$ = 0.72; $\delta$ = 0.5, 1.0, 1.5, 2.0, 2.5; $C$ = 0, 0.1, 0.2, 0.3, 0.4, 0.5. In accordance with the equations derived, the steady and dynamic characteristics of the Rayleigh step slider bearing lubricated with non-Newtonian couple stress fluids are presented in Table 1. Furthermore, the results under the same parameters of the NLC by Lu *et al*. [13] are also included for comparison. It is observed that for the non-Newtonian couple stress parameter $C$ = 0, the present calculated results are close to the NLC by Lu *et al*. [13]. This close agreement signifies a support for the present study of the non-Newtonian couple-stress Rayleigh step slider bearing.

**2. Finite-Width Rayleigh Slider Bearings**

In the present study, we have investigated the steady and dynamic characteristics of wide Rayleigh step bearings. Extending the present study, the non-Newtonian couple-stress dynamic Reynolds-type equation for Rayleigh step slider bearings with finite width can be observed from the previous derivation by Lin *et al.* [10]. In the future, it is possible for us to derive a finite-difference close-form solution for finite-width Rayleigh bearings extended from the idea of this paper.

## IV. CONCLUSIONS

On the basis of the Stokes micro-continuum theory [19], the effects of non-Newtonian couple stresses upon the steady and dynamic characteristics of Rayleigh step slider bearings have been investigated in the present paper. According to the results obtained and discussed, conclusions can be drawn as follows.

A closed-form solution is obtained for the non-Newtonian steady performances and the non-Newtonian dynamic characteristics for the Rayleigh step slider bearings. Comparing with the Newtonian-lubricant case, the effects of non-Newtonian couple stresses for the Rayleigh step slider bearing provides of a significant improvement in the steady load-carrying capacity, the dynamic stiffness and damping coefficients, as well as a reduction in the required volume flow rate. To assist the industrial applications of engineers, a numerical example and the calculated results for the Rayleigh step slider bearing lubricated with non-Newtonian couple stress fluids are included. Comparing with the Newtonian-lubricant case by Lu *et al*. [13], a close agreement signifies a support for the present study of the non-Newtonian couple-stress Rayleigh step slider bearing.

## APPENDIX A: THE ASSOCIATED FUNCTIONS AND QUANTITIES

$$f_{A\alpha} = f_A(\alpha, h_m^*) \quad \text{(A1)}$$

$$f_{C\alpha} = f_C(\alpha, h_m^*) \quad \text{(A2)}$$

$$(f_{B\alpha})_s = f_B(\alpha, h_{ms}^*) \quad \text{(A3)}$$

$$(f_{D\alpha})_s = f_D(\alpha, h_{ms}^*) \quad \text{(A4)}$$

$$F_A(h_m^*) = \int_{x^*=0}^{\alpha} f_A(x^*, h_m^*)dx^* \quad \text{(A5)}$$

$$F_B(h_m^*) = \int_{x^*=0}^{\alpha} f_B(x^*, h_m^*)dx^* \quad \text{(A6)}$$

$$F_C(h_m^*) = \int_{x^*=\alpha}^{1} f_C(x^*, h_m^*)dx^* \quad \text{(A7)}$$

$$F_D(h_m^*) = \int_{x^*=\alpha}^{1} f_D(x^*, h_m^*)dx^* \quad \text{(A8)}$$

$$\frac{\partial f^*}{\partial h_m^*} = 3h^{*2} - 12C^2 \cdot \tanh^2\left(\frac{h^*}{2C}\right) \quad \text{(A9)}$$

$$\frac{\partial f_{B\alpha}}{\partial h_m^*} = -\int_{x^*=0}^{\alpha} \frac{1}{f^{*2}} \cdot \frac{\partial f^*}{\partial h_m^*} dx^* \quad \text{(A10)}$$

$$\frac{\partial f_{D\alpha}}{\partial h_m^*} = -\int_{x^*=1}^{\alpha} \frac{1}{f^{*2}} \cdot \frac{\partial f^*}{\partial h_m^*} dx^* \quad \text{(A11)}$$

$$\frac{\partial F_B}{\partial h_m^*} = -\int_{x^*=0}^{\alpha} \cdot \int_{x^*=0}^{x^*} \frac{1}{f^{*2}} \cdot \frac{\partial f^*}{\partial h_m^*} dx^* \cdot dx^* \quad \text{(A12)}$$

$$\frac{\partial F_D}{\partial h_m^*} = -\int_{x^*=\alpha}^{1} \cdot \int_{x^*=1}^{x^*} \frac{1}{f^{*2}} \cdot \frac{\partial f^*}{\partial h_m^*} dx^* \cdot dx^* \quad \text{(A13)}$$

$$I_{1s} = I_1(h_{m0}^*, 0) = \frac{-6\delta \cdot (f_{D\alpha})_s}{(f_{B\alpha})_s - (f_{D\alpha})_s} \quad \text{(A14)}$$

$$\frac{\partial I_1}{\partial h_m^*} = (-6\delta) \cdot \frac{f_{B\alpha} \cdot (\partial f_{D\alpha}/\partial h_m^*) - f_{D\alpha} \cdot (\partial f_{B\alpha}/\partial h_m^*)}{(f_{B\alpha} - f_{D\alpha})^2} \quad \text{(A15)}$$

$$\frac{\partial I_1}{\partial V^*} = \frac{12 \cdot (f_{C\alpha} - f_{A\alpha})}{f_{B\alpha} - f_{D\alpha}} \quad \text{(A16)}$$

## APPENDIX B: NOTATIONS

| | |
|---|---|
| $C$ | non-dimensional couple stress parameter, $C = l/h_{ms}$ |
| $d$ | steady thickness difference between the inlet and the outlet, $d = h_{ins} - h_{ms}$ |
| $D$, $L$ | width and length of the bearing |
| $D_d$, $D_d^*$ | dynamic damping coefficient, $D_d^* = D_d h_{ms}^3/\mu L^3 D$ |
| $f$, $f^*$ | function defined in the Reynolds-type equation, $f^*(h^*, l^*) = f(h, l)/h_{ms}^3$ |
| $F$, $F^*$ | dynamic film force, $F^*(h^*_m, V^*) = F h_{ms}^2/\mu U L^2 D$ |
| $h$, $h^*$ | film thickness, $h^*(x^*, t^*) = h(x, t)/h_{ms}$ |
| $h_{in}$ | inlet film thickness, $h_{in}(t) = d + h_m(t)$ |

$h_{ins}$ steady inlet film thickness, $h_{ins} = d + h_{ms}$
$h_m$, $h_m^*$ outlet film thickness, $h_m^*(t^*) = h_m(t)/h_{ms}$
$h_{ms}$ steady outlet film thickness
$I_1$, $I_2$ integration functions, $I_1 = I_1(h_m^*, V^*)$, $I_2 = I_2(h_m^*, V^*)$
$I_{1s}$ steady integration function, $I_{1s} = I_1(h_{ms}^*, 0)$
$l$ molecular length of polar suspensions in a Newtonian fluid, $l = (\eta/\mu)^{1/2}$
$p$, $p^*$ dynamic pressure, $p^* = ph_{ms}^2/\mu UL$
$Q$, $Q^*$ volume flow rate, $Q^* = Q/Uh_{ms}D$
$Q_s$, $Q_s^*$ volume flow rate, $Q_s^* = Q_s/Uh_{ms}D$
$S_d$, $S_d^*$ dynamic stiffness coefficient, $S_d^* = S_d h_{ms}^3/\mu UL^2D$
$t$, $t^*$ time, $t^* = Ut/L$
$u$, $w$ velocity components in the $x$- and $z$-directions
$U$ sliding velocity of the lower part
$V^*$ non-dimensional squeezing velocity, $V^* = dh_m^*/dt^*$
$W_s$, $W_s^*$ steady load-carrying capacity, $W_s^* = W_s h_{ms}^2/\mu UL^2D$
$x$, $z$ Cartesian coordinates
$x^*$ non-dimensional coordinate, $x^* = x/L$
$\alpha$ riser location parameter
$\delta$ shoulder parameter, $\delta = d/h_{ms}$
$\eta$ material constant responsible for the couple stress fluid property
$\mu$ lubricant viscosity

Subscript
$s$ steady state

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