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SCALE EFFECTS IN SCOUR PHYSICAL-MODEL TESTS: CAUSE AND ALLEVIATION

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SCALE EFFECTS IN SCOUR PHYSICAL-MODEL TESTS: CAUSE AND ALLEVIATION

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Key words: scale effect, similarity law, scour, physical model.

ABSTRACT

Scale effect occurs when a prototype hydraulic process is simulated at a laboratory scale due to dissatisfaction of similarity laws. It might lead to considerable deviation when the model scour depth is extrapolated to prototype value. Three popular experimental approaches including prediction-equation-targeted flume test, series-model test and similitude-model test are reviewed with emphasis upon their merits and limitations in reducing or alleviating scale effects. Scale laws guiding scour physical-model design are discussed for performing cost-effective model tests. An empirical equation is further derived from data of clear-water pier scour experiments to examine the test results in up-scale extrapolating. It is suggested that scale effect in a scour physical model test could be efficiently reduced if both the mobility similarity of bed particles and Froude number similarity are satisfied simultaneously.

I. INTRODUCTION

Local scour at bridge piers, groins, jetties, breakwaters, offshore pipelines and drilling rig legs etc. is an important engineering issue facing both engineers and researchers. Due to the complexity of the scour processes physical modeling has been still a reliable means to predict the maximum scour depth for design purpose. Nonetheless, scale effects arise when similarity laws are not satisfied at the laboratory scale. They may cause considerable deviations when the maximum scour depth is extrapolated from model to prototype value. If such values are over-estimated investment in protection engineering might be wasted; otherwise, engineering safety is at risk. Therefore, it is critically important for experimenters to reduce or alleviate scale effects when performing scour physical-model tests.

Though scale effects in physical hydraulic model tests have long been recognized [e.g. 3, 4, 6, 8, 10, 14, 15, 18] and been investigated through physical modeling [e.g. 7, 17] or numerical modeling [e.g. 20], there is still lack of a generally-accepted remedy for alleviating them. A common sense exists that the mobility similarity of bed particles should be fulfilled to the maximum possible degree in a scour experiment, but experimenters did argued how to implement this requirement [see 1 and 3]. Above all, this is because of the complicate structure-flow (wave)-sediment interactions in local scour processes, but also because of different experimental exercises and experiences.

This paper reviews three popular experimental approaches including prediction-equation-targeted flume test, series-model test and similitude-model test with emphasis upon their merits and limitations in reducing or alleviating scale effects. Moreover, scale laws guiding scour physical-model design and an empirical equation derived from clear-water pier scour experiment data reported in the literature [12] are further presented to help to perform cost-effective model tests and examine better the test results for up-scale extrapolation.

II. PREDICTION-EQUATION-TARGETED FLUME TEST

Many experimenters chose alluvial sediments to conduct scour flume tests and then obtained pier-scour prediction equations by dimensional analysis and parameter regression, such as FDOT (Florida Department of Transportation) equation [5]. Generally speaking, predicted prototype values using these equations are larger than filed observations and such deviations are attributed to scale effects in flume tests [3, 4, 10]. They are considered mainly brought in by the hydraulic-modeling constraint imposed by the lower size limit to which non-cohesive alluvial sediment can be geometrically scaled down [3, 4]. Experimenters argued that such limitations would cause model B/D_{50} (B is the pier width, D_{50} is the median grain size of sediment) is larger than prototype value; larger B/D_{50} consequently causes the distortion of flow pattern in a model test, that is, the exaggeration of velocity

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gradient and flow vortices in the three dimensional flow field around a pier; as a result, a larger value of maximum scour depth in a model test is produced [3].

Ettema *et al*. [2] re-explained that the geometric distortion of B/D_{50} causes larger frequency of wake vortex shedding from the bed behind the pier, producing larger scour holes. On the contrary, Lee and Sturm [10] suggested that it is the distortion of the large-scale unsteadiness of the horseshoe vortex in front of a model pier that produces larger scour depths.

While keeping the geometric similarity, i.e. H/B value (H is water depth) constant from prototype to model these flume tests normally fulfilled dynamic similitude through flow intensity [10]. Technically, the V/V_c (V is the flow velocity while V_c is the critical flow velocity for the incipient motion of bed sediment) were kept the same from prototype to model if underscaled alluvial sediment was used as model bed material. But this demanding for constant V/V_c does not guarantee simultaneously Froude number similarity, more often than not, the flow velocity V in the flume test is larger than what is required by the Froude number criterion and larger scour hole is then produced [4].

Such a type of flow velocity deviation is not due to aforementioned 'geometric distortion' of B/D_{50} from prototype to model but to the dissatisfaction of movable-bed resistance similarity in flume tests when underscaled alluvial sediment is used as model material. The movable-bed resistance determines flow energy dissipation and its similarity can be represented by

$$\lambda_n = \frac{\lambda_R^{2/3}\lambda_S^{1/2}}{\lambda_V} \tag{1}$$

where λ_n is the Manning roughness scale ratio from the prototype value to model value, λ_R is the hydraulic radius scale ratio, λ_S is the surface gradient scale ratio and λ_V is the flow velocity scale ratio. Only when Froude number similarity (i.e. $\lambda_V = \lambda_h^{1/2}$) is fulfilled and the model geometry is not distorted, Eq. (1) could be reduced as:

$$\lambda_n = \lambda_h^{1/6} \tag{2}$$

in which λ_h is the geometric vertical scale ratio and assumed to equal to λ_R. Eq. (1) is generally applicable to rough turbulent flow regime [18]. For the smooth and transitional turbulent regime an appropriate equation to calculate the movable-bed drag coefficient should be invoked.

As the movable-bed resistance in a model that uses underscaled alluvial sediment as model bed material is larger than what is required by Eq. (1), the model water depth becomes consequently larger while the velocity becomes simultaneously smaller. In order to keep the water depth to the required value by geometric similarity, experimenters had to simultaneously increase flow discharge and distort the surface gradient. In so doing the flow velocity became resultantly larger than the value required by Froude number similarity and larger values of the maximum scour depth were thus produced.

Nonetheless, if carefully tuned, the scour depth caused by deviated flow velocity might be increased just enough to compensate the effect of larger movable-bed resistance. Thus the scale effects caused by the dissatisfaction of movable-bed resistance similarity and Froude number similarity could be reduced to a minimum.

III. SERIES-MODEL TESTS

1. Method

The method of series-model test is to conduct a series of undistorted model tests at different geometric scales according to Froude number similitude and then extrapolate the test results to prototype scour depth. According to Sha [14] the method is explained as follows:

Assuming the prototype scour depth Y_p has a power law relationship with the prototype depth variable h_p, one has

$$Y_p = K_p h_p^{n_p} \tag{3}$$

where K_p is a constant, n_p is a power.

Similarly, assuming the model scour depth Y_m maintains a power-law relationship with the model depth variable h_m, one further has

$$Y_m = K_m h_m^{n_m} \tag{4}$$

where K_m is a constant, n_m is the power.

Let Eq. (3) is divided by Eq. (4) and assume n_p equals n_m, one obtains

$$\lambda_Y = \lambda_K \lambda_h^{n_h} \tag{5}$$

where the constant scale λ_K is an unknown variable, it is mainly caused by geometric-scale reduction. It is also assumed a power-law relationship with the depth variable, that is

$$\lambda_K = C\lambda_h^{n_K} \tag{6}$$

At the prototype, $\lambda_h = 1$, $\lambda_K = 1$, C thus equals 1. Putting Eq. (6) into Eq. (5) yields:

$$\lambda_Y = \lambda_h^{n_h + n_K} \tag{7}$$

Let $n = n_h + n_K$, the scale equation of a series model becomes:

$$\lambda_Y = \frac{Y_p}{Y_m} = \lambda_h^n \tag{8}$$

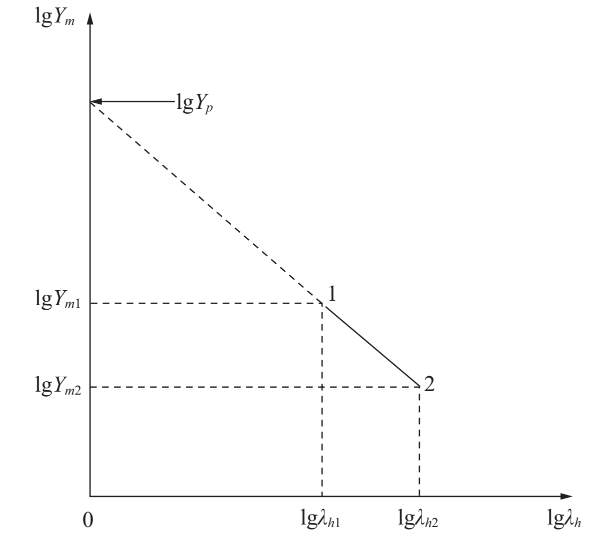

Fig. 1. Y_m and λ_h logarithm-logarithm relationship (based on [14]).

where n is a comprehensive power, it means that the scour depth scale λ_Y maintains a power-law relationship with the model geometric scale λ_h if all other variables keep a power-law relationship with λ_h in a series model test [14].

In order to extrapolate the scour depth to the prototype value experimenters can draw a double-logarithm coordinates: the ordinate is the geometric scale λ_h while the abscissa is the model scour depth Y_m (Fig. 1). Generally speaking, the series-model tests need at least two different geometric scales (i.e. λ_{h1}, λ_{h2} in Fig. 1), and the prototype value Y_p could be obtained by fitting a straight line on the double-logarithm figure.

Let λ_{h_0} is the geometric scale that satisfies both the mobility similarity of bed particle and Froude number similarity. If the prototype sediment is used as model material λ_{h_0} will equal 1; Y_m is right the prototype scour depth Y_p when the connection line of $\lg\lambda_{h1}$ and $\lg\lambda_{h2}$ is straightly extended to ordinate = 0 on the figure. When the model bed material is different from the prototype sediment λ_{h_0} equals a particular value that satisfies both the mobility similarity and Froude number similarity; the results obtained in a series model test are firstly extended to $\lambda_h = \lambda_{h_0}$ to obtain a scour depth, which is then extrapolated to the prototype value by multiplying λ_{h_0}. How to determine λ_{h_0} when model bed material is different from the prototype sediment is explained later in the section "similitude-model test".

2. Merits and Limitations

It is called scale-independent characteristics if a system demonstrates the same physical behavior at different geometric scales [9]. There exists a non-dimensional number that is fractal invariant. For example, the slope of the linear line in a logarithm-logarithm coordinates consisting of grain cumulative percentage of concentration (abscissa) and grain size (ordinate) is a fractal-dimension number, which indicates the sorting degree of sediment [11]. The slope of the linear line on the logarithm-logarithm coordinates for the series-model tests also indicates the 'fractal' characteristics. It represents the scale-independent degree of the model scour depth on the model geometric scale.

However, it might be over simple as the series-model tests only assume power-law relationships for arguments with dependent variables; meanwhile, scale effects due to dissatisfaction of both movable-bed resistance similarity and Froude number similitude do exist at each geometric-scale test, as what happens in aforementioned 'prediction-equation-targeted flume test', as a result, there is possibly a deviation accumulation when the test results are extrapolated to prototype value. The extrapolated prototype value is generally prone to be over-estimated. Nevertheless, the advantage of this method is that the prototype scour depth could be directly extrapolated from model results without resorting to an empirical equation.

Another advantage of a series-model test is able to use prototype-sized sediment as model bed material. This can avoid the vexing problem in determining the critical velocity for the incipient motion of prototype sediment. But the prototype sediment is not feasible to act as model bed material if the grain size is too fine with cohesive force possibly emerging. In such a case, lightweight material with coarser diameter than the prototype size should be adopted. This is explained in the following section.

IV. SIMILITUDE-MODEL TEST

1. Scale Laws

There is no doubt that scale effects could be reduced to a minimum if a scour test could simultaneously satisfy both the bed-particle mobility similarity (i.e. movable-bed resistance similarity) and Froude number similarity. Such a type of model test might be called 'similitude-model test'. In comparison with the series-model test a similitude-model test could save considerable test costs and time as it needs only one geometric-scale test. The prototype scour depth can be directly obtained by multiplying the test result by the corresponding geometric scale λ_h.

Shields parameter could represent bed particle mobility. The bed-particle mobility similarity requires the same Shields parameter in prototype and model. According to the method to calculate Shields parameter [15], i.e.

$$\theta = \frac{U_{fm}^2}{g(s-1)D} \tag{9}$$

with $U_{fm} = \sqrt{\dfrac{f}{2}} U_m$, here f is the friction coefficient of movable bed, U_m is the maximum current velocity or wave orbital velocity or combination of both above sea bed in the absence of a structure, s is the relative density of the sediment (ρ_s/ρ), g is the gravity acceleration and D is the particle diameter.

From Eq. (9) one can derive the scale ratio of the particle diameter as:

$$\lambda_D = \lambda_f \lambda_{\frac{1}{s-1}} \lambda_V^2 \tag{10}$$

According to Chezy formula (i.e. $V = C\sqrt{RJ}$, R is hydraulic radius, $R \approx h$, J is flow surface gradient) and the relationship of Chezy coefficient C with the friction coefficient f:

$$C = \sqrt{\frac{8g}{\lambda}} \tag{11}$$

One can obtain the scale ratio of the friction coefficient:

$$\lambda_f = \frac{\lambda_h^2}{\lambda_l \lambda_V^2} \tag{12}$$

in which λ_l is the geometric horizontal scale ratio.

Eq. (12) is applicable to a rough turbulent flow regime. If the flow satisfies the Froude number similarity, i.e. $\lambda_V = \sqrt{\lambda_h}$, Eq. (12) becomes:

$$\lambda_f = \lambda_h / \lambda_l \tag{13}$$

Substituting Eq. (13) into Eq. (10) yields:

$$\lambda_D = \lambda_{\frac{1}{s-1}} \lambda_l / \eta^2 \tag{14}$$

where η is the geometric distortion ratio (i.e. λ_l/λ_h).

Eq. (14) satisfies both movable-bed resistance similarity and Froude number similarity. If a scour test is designed following Eq. (14) it could be indeed a similitude model with negligible scale effect.

In case that the prototype sediment is fine and a model should be geometrically undistorted, coarser lightweight material could be chosen to eliminate the cohesive force. But Eq. (14) shows its limitation as it requires large model geometry so much so that it is not feasible to conduct such a test due to facility constraints. Therefore, if the model geometry has to be reduced to an appropriate level one might use the following scale law derived according to the incipient velocity similarity $\lambda_{V_c} = \lambda_V$ [19]:

$$\lambda_D = \lambda_{\frac{1}{s-1}}^{3/2} \lambda_l / \eta \tag{15}$$

Eq. (15) fulfills Froude number similarity but neglects the influence of movable-bed resistance, but it takes much into account the influence of model material density and the model geometry could be thus reduced to a smaller scale than what is required by Eq. (14). For example, if the prototype bed sediment has a D_{50} (the median grain diameter) of 0.12 mm, one wishes to choose coarser lightweight material as model bed particle to avoid cohesive force. Now crashed wood (powder) with a density 1050 kg/m^3 and D_{50} of 0.3 mm is available. The undistorted model geometric scale could be determined as 75 by Eq. (15) rather than 13 by Eq. (14). It should be pointed out that Eq. (15) could be transformed identical to Eq. (14) if the resistance influence of movable bed is taken into account [19].

The practice keeping the flow intensity V/V_c [10] the same in prototype and model test, as discussed in section II, is essentially satisfying the incipient velocity similarity $\lambda_{V_c} = \lambda_V$ (not necessarily the Froude number similarity).

Meanwhile, Eq. (15) does not fulfill the requirement of movable-bed resistance similarity. In order to examine the consequent scale effects at model tests designed according to Eq. (15) or 'the same' V/V_c practice the present study suggests a simple empirical equation to do so. It is explained in the next section.

2. Deviation Analysis

Melville and Chiew [12] reported test data from clear-water pier scour experiments. Clearwater scour occurs for mean flow velocities lower than the threshold velocity for bed sediment entrainment, i.e. $V \leq V_c$. The experiments were conducted in four different flumes, three at the University of Auckland and one at the Nanyang Technological University. Uniform sands were used at each venue, with sediment diameters ranging from 0.8 to 0.96 mm and flow intensity V/V_c ranging from 0.46 to 0.957. The present study only chooses data that demonstrate varying flow velocities while flow depth, pile diameter and the diameter and density of model sand are kept constant (Table 1). So pier equilibrium scour depths only vary with V/V_c.

Based upon Table 1 the present study regresses a simple equation:

$$\frac{E}{B} = 4.438 \frac{V}{V_c} - 1.94 \tag{16}$$

where E is the equilibrium scour depth and B is the pier diameter. The correlation coefficient of the regression equation R^2 is 0.899 with 95% confidence bounds.

As explained in section II, through increasing flow discharge and the flow surface gradient the water depth in a model test could satisfy geometric similarity, but the model V/V_c becomes normally larger than what is supposed to be. Using Eq. (16) one could examine the deviation of the scour depths between the model V/V_c and prototype V/V_c. Of course, the precision of Eq. (16) might be further improved with more observation

Table 1. Equilibrium scour depths of a cylindrical pier by varying flow intensities (data from [12]).

No.	Flow depth (m)	D_{50} (mm)	Pier diameter (m)	Velocity (m/s)	V/V_c	Equilibrium scour depth (m)
N2	0.07	0.96	0.07	0.245	0.654	0.077
N3	0.07	0.96	0.07	0.218	0.623	0.069
N4	0.07	0.96	0.07	0.231	0.671	0.079
N5	0.07	0.96	0.07	0.198	0.623	0.039
N13	0.07	0.96	0.07	0.304	0.85	0.121
N20	0.07	0.96	0.07	0.251	0.7	0.091
N22	0.07	0.96	0.07	0.179	0.519	0.025
N25	0.07	0.96	0.07	0.185	0.556	0.037
N26	0.07	0.96	0.07	0.185	0.556	0.035

data with regard to varying V/V_c, in particular, those produced by constant flow intensity and pier geometry but different diameters and/or densities of model sands.

Alternatively, depending on the prototype sceneries the scale effects of employing a smaller scale model might be corrected by introducing an appropriate correction parameter [see 13]; or they might be reduced by the so-called "composite modeling" that involves mutually-calibrated physical modeling and numerical modeling [see 16].

V. CONCLUDING REMARKS

Local scour has been one of the major concerns for the safety of marine and hydraulic structures. Owing to the complex *in situ* structure-flow (wave)-sediment interactions movable-bed physical modeling has been and will continue to be an efficient means to predict the maximum local scour depth. Nonetheless, scale effects are produced when similarity laws are not fulfilled from prototype to model. They might be reduced to a minimum by running tests in large enough facilities [8]. When this is not feasible one needs to have a good understanding of the important processes acting in the prototype situation and of the merits and limitations of an experiment technique before designing a geometrically-smaller model test.

The present study has reviewed three existing experimental approaches including prediction-equation-targeted flume test, series-model test and similitude-model test. Each approach has its own advantage and limitations in reducing or alleviating scale effects and in extrapolating model scour depths to prototype values.

Whatever an approach might be, the key principle in designing a scour physical-model test is to satisfy both the movable-bed resistance similarity (particle mobility similarity) and Froude number similarity simultaneously to the maximum degree. A similitude-model test could meet this strict similitude requirement but it often demands for very large model geometry, undistorted or distorted. A series-model method by conducting at least two geometrically-smaller undistorted model tests could use the prototype sediment as model bed material and extrapolate the test result directly to prototype value on a log-log diagram; meanwhile, it could also use smaller-than-prototype natural sand as model bed material; in so doing, the geometric vertical scale ratio λ_{h_0} satisfying both movable-bed resistance similarity and Froude number similarity on the log-log diagram could be obtained from the 'similitude model' analysis as represented by Eq. (14).

More often than not the popular experimental approach keeping the flow intensity V/V_c the same from prototype to model does not satisfy movable-bed resistance similarity, and deviates Froude number similarity too. In such a case, Eq. (15) that might be exercised to choose the appropriate geometric scale or the density and grain size of model sand if the prototype sediment can not be geometrically scaled down and lightweight material is desired as model sand. When natural sediment is used as model sand, the deviation degrees of scour depths at such model tests might be assessed using Eq. (16).

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