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# EXPERIMENTAL STUDY ON THE MOTIONS OF A MOORING SHIP IN MIXED WAVES


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# EXPERIMENTAL STUDY ON THE MOTIONS OF A MOORING SHIP IN MIXED WAVES

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Key words: physical model experiment, mixed waves, bimodal spectrum, mooring (LNG) ship, movements, energy distribution, changes of wave conditions.

## ABSTRACT

By employing the method of bimodal spectral simulations, a series of experiments were performed on a 266,000 $m^3$ LNG ship moored to an island berth in mixed waves. In the physical model experiments, a special effort was made to investigate how changing wave conditions affect the motion responses of the mooring ship in mixed waves. Therefore, this paper mainly engages in the analysis of such effects resulted from wave condition changes of mixed waves from such perspectives as the distribution of high- and low-frequency energy, the periodic changes of low-frequency swells, the wave-height changes of low-frequency swells, and the wave-height changes of high-frequency wind waves.

## I. INTRODUCTION

It is a very important issue for port design and application to study the motions of a mooring ship. If a mooring ship has a relatively large motion, the loading and unloading operations of the ship will be affected, the ship will have to be berthed for a longer time, and possible accidents, such as the breaking of mooring lines, may happen, which lead to greater losses [5, 19, 24]. Many scholars [25, 26, 31] have pointed that swell wave is one of the major factors that cause substantial motions of mooring ships.

Substantial observational data analyses of sea waves show that there are few pure wind waves or pure swells on sea. In most cases, swells together with wind waves appear in the form of mixed waves. The spectral structure of mixed waves is very complex and diverse. In most cases, the spectra of mixed waves are bimodal or multimodal, except for a few cases unimodal. Waves can be divided into two parts by bimodal spectra: the low frequency waves (swells) and the high frequency waves (wind waves). Each part has their own characteristic parameters, which characterizes a spectral shape of mixed waves in the form of bimodal peaks. Since the 1970s, scholars both at home and abroad have begun to do substantial researches on the bimodal spectral mixed waves. Huang *et al*. [10], Chen *et al*. [6], Pan *et al*. [14], Toffoli *et al*. [22], and Pascoal *et al*. [15, 16] have studied the statistical properties of the measured bimodal spectra of waves. Soares [9] has discussed the probability of occurrence of double-peaked wave spectra. Arena *et al*. [2, 3] has done research into nonlinear peak to trough distributions in sea states with double-peaked spectra. Petrova *et al*. [17, 18] has studied wave height distributions in bimodal sea states from offshore basins. Ochi and Hubble [29] proposed a theoretical expression of the bimodal spectra of waves. Soares [8], Huang *et al*. [11], Guan *et al*. [7] proposed different expressions of bimodal spectral waves according to the measured values. Petrova *et al*. [17, 18] achieved the generation of bimodal spectral waves in an offshore test basin. No application of bimodal spectral mixed waves was found in the ship mooring field [4, 20, 21, 27] as well as the marine area [1, 13, 23, 28]. In order to better reflect the effects upon a mooring ship in an environment with natural waves, it is necessary to carry out the research to the effect that the bimodal spectra of mixed waves have on mooring ships.

In this paper, a physical model experiment of a mooring ship was performed on a 266,000 $m^3$ LNG ship by employing bimodal spectra to simulate the waves mixed by swells and wind waves that often occur at the same time. A special effort was made to investigate and study the energy distribution of the mixed waves and the effects on the mooring ship's motion responses with the change of wave conditions in the physical model experiment.

## II. DESIGN OF THE EXPERIMENT

### 1. Simulations of the Mooring Ship

The model scale was set 1:60 in accordance with the requirements of *Wave Model Test Regulation* [12]. The experiment was performed on a 266,000 $m^3$ LNG ship moored

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**Table 1. Dimensions of the 266,000 $m^3$ LNG ship.**

| Parameters | Unit | Laden | Ballast |
|---|---|---|---|
| Length Over all | m | 345 | |
| Length between Perpendiculars | m | 320 | |
| Breadth | m | 55 | |
| Depth | m | 27.2 | |
| Draft | m | 12 | 9.6 |
| Displacement Volume | t | 184008 | 147206 |
| Height of gravitational center | m | 24 | 19.2 |
| Natural Period of Roll | s | 16.24 | 10.83 |
| Natural Period of Pitch | s | 9.48 | 8.53 |

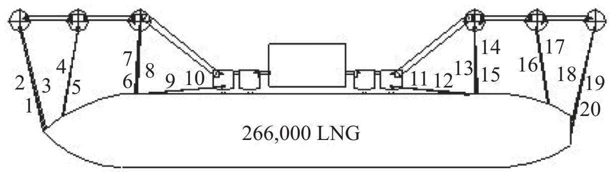


**Fig. 1. Berth layout and diagrammatic illustration of mooring patterns of the LNG ship.**

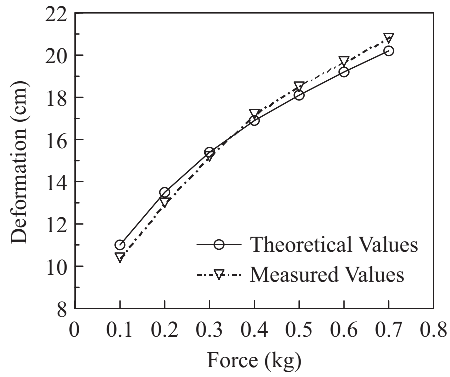


**Fig. 2. The modeling results of force-deformation curves of breast lines.**

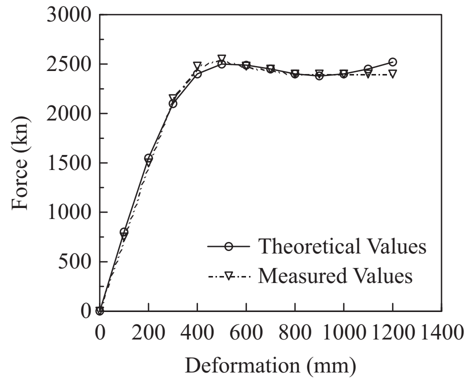


**Fig. 3. Force-deformation curves to the modeling results of fenders.**

to an island berth. The dimensions of the ship are given in Table 1. The model ship was built based on the 3D hull shape definition of a prototype LNG ship at a geometric scale of 1:60; the weight balance method was used to meet different requirements of load and weight distribution; the LNG ship's main particulars such as its center of gravity, the periods of roll and pitch, etc. were consistent with similar dynamic conditions.

## 2. Simulations of the Structure of Island Berth

In the same way, the simulations of the structure of island berth were fulfilled by reducing the prototype on the geometric scale 1:60. The simulation of island berth structure can ensure both the geometric similarity and the similarity of the location of caisson piers, as well as the stability of caisson pier. In addition, the outer shells of the caisson piers are made of wood, filled with gravels and small lead weights inside. The top of each caisson pier is connected with the upper wooden part of the berth, which has formed a unity. A number of weights can be evenly added to the surface of the upper structure to make the overall structure of the berth rigid and stable enough. The layout of the berth is shown in Fig. 1.

## 3. Simulations of Mooring Lines

The mooring lines used for mooring the LNG ship were arranged in a symmetrical way by the number of 3:2:3:2. That is to say, there were 3 ropes for both head line and stern line, 2 for both additional head line and stern line, 3 for both forward breast line and after breast line, and 2 for head spring line and stern spring line (Line Numbers refer to Fig. 1). When the locations of mooring dolphins and positions of the ship's mooring pipes are fixed, the length of lines will automatically satisfy geometric similarity. In the experiment, the nylon ropes with a diameter of $\Phi$ = 75 mm were chosen for use. Two strands of the ropes were twisted into one mooring line, and two of three strands of the ropes were twisted into one mooring line and the other strand was directly used as another line. The lines used for simulations are made of cotton ropes, the mass of whose per unit length can satisfy gravity similar rule, as well as which are allowed sufficient spare length for use. The lines were hung heavy weights in advance to make them lose elasticity completely. When simulating the lines, the elastic similar rules of lines should be taken into consideration. Wilson Equations can be used for the calculation of the force-deformation of simulating lines. The elastic pieces of steel were adopted to simulate the elasticity of the lines. In the experiment, an initial tension of 100 KN was loaded on each line in accordance with the prototype lines. The Fig. 2 shows the curve graphs of force-deformation of forward breast and after breast lines, which manifests good simulation results.

## 4. Simulations of Fenders

The main similarity conditions of fenders refer to the similarity of the curves of force-deformation and energy-deformation of fenders between the prototype and the model. The standard SUC-2250H cell rubber fenders were taken into

**Table 2. Characteristic Parameters of the Mixed Waves with the Same Energy.**

| Wave Types | frequency of wave | $H_{1/3}$/m | $T$/s | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Major Contribution of Wind Waves | high frequency | 1.0 | 6 | | | | | | | |
| | low frequency | 0.5 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| Same Contribution of Wind Waves and Swells | high frequency | 0.8 | 6 | | | | | | | |
| | low frequency | 0.8 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| Major Contribution of Swells | high frequency | 0.5 | 6 | | | | | | | |
| | low frequency | 1.0 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |

**Table 3. Characteristic Parameters of the Mixed Waves under Different Wave Conditions.**

| Working Conditions | frequency of wave | $T$/s | $H_{1/3}$/m | | | | |
|---|---|---|---|---|---|---|---|
| 1 | high frequency | 6 | 0.8 | | | | |
| | low frequency | 12 | 0.5 | 0.8 | 1.0 | 1.2 | 1.5 |
| 2 | high frequency | 6 | 0.5 | 0.8 | 1.0 | 1.2 | 1.5 |
| | low frequency | 12 | 0.8 | | | | |

use with a layout of two in one row. When simulating the layout of two in one row was simulated into the layout of one cell rubber fender in one row. The simulation results shown in Fig. 3 shows that the rubber fenders achieved better results.

## III. EXPERIMENT

### 1. Experiment Equipments and Measuring Instruments

The experiment was conducted in an ocean environmental flume of the State Key Lab of Coastal and Offshore Engineering (SLCOE), Dalian University of Technology, China. The flume is 40 meters long, 24 meters wide and 1.2 meters deep. A piston type wave maker system designed and constructed by SLCOE is installed at one end of the flume, which can generate multidirectional complex waves of both low-frequency and high-frequency according to different test requirements. Wave absorbers are arranged at the other end of the flume to absorb incoming waves to avoid wave reflection.

In the experiment, the wave data were collected by adopting the DS30 system developed by Beijing Research Institute of Water Conservancy Technology (BRIWT). The system can handle up multipoints of wave surface simultaneously and then process data analyses; the wave measurement instrument spans the range of 35 cm, and the proportional error is less than 0.5%. The measurement of a mooring ship's movement employs the system dedicated to model ship tests with twin CCD optical six-component movement measurement, which is also developed by BRIWT. Such system employs non-contact measurement method to avoid added mass and friction that generated by using the traditional contact one. The system can also be used to simultaneously measure six-component movements, namely surge, sway, heave, pitch, roll and yaw. Besides, the relative error of angular surveying is less than 5%, and the relative error of line displacement is less than 2%.

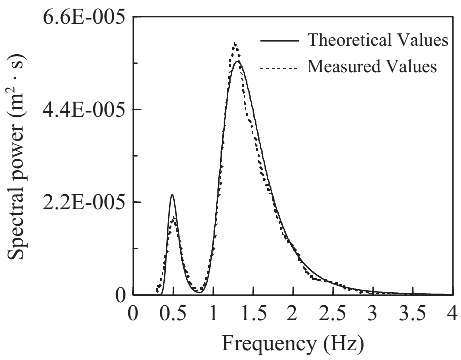


**Fig. 4. Comparison of simulated and measured values of bimodal spectrum waves. (Low-frequency waves: $H_s$ = 0.5 m, $T$ = 16 s; High-frequency waves: $H_s$ = 1.0 m, $T$ = 6 s)**

### 2. Simulations of the Mixed Waves

The water in the flume of the experiment is 0.24 meters deep. The direction of waves is transverse which is the most unfavorable to a mooring ship. In the experiment, the effects of such dynamic factors as currents and wind loads, etc. on the motions of the mooring ship were not taken into account. The bimodal spectral mixed waves were generated by a six-spectrum parameter which was proposed by Ochi and Hubble in 1976 [29]. The low and high frequency parts of the whole spectrum were represented by Pierson-Moscowitz Spectrum with three parameters (significant wave height $H_s$, spectral peak frequency $\omega_m$ and shape parameters $\lambda$) respectively. A six-parameter spectrum can be obtained by combining two three-parameter spectra in both low and high frequency parts of the waves respectively. The equation is given by

$$S(\omega) = \frac{1}{4}\sum_j \frac{\left(\frac{4\lambda_j + 1}{4}\omega_{mj}^4\right)^{\lambda_j}}{\Gamma(\lambda_j)} \frac{H_{sj}^2}{\omega^{4\lambda_j+1}} \exp\left[-\frac{4\lambda_j + 1}{4}\left(\frac{\omega_{mj}}{\omega}\right)^4\right] \quad (1)$$

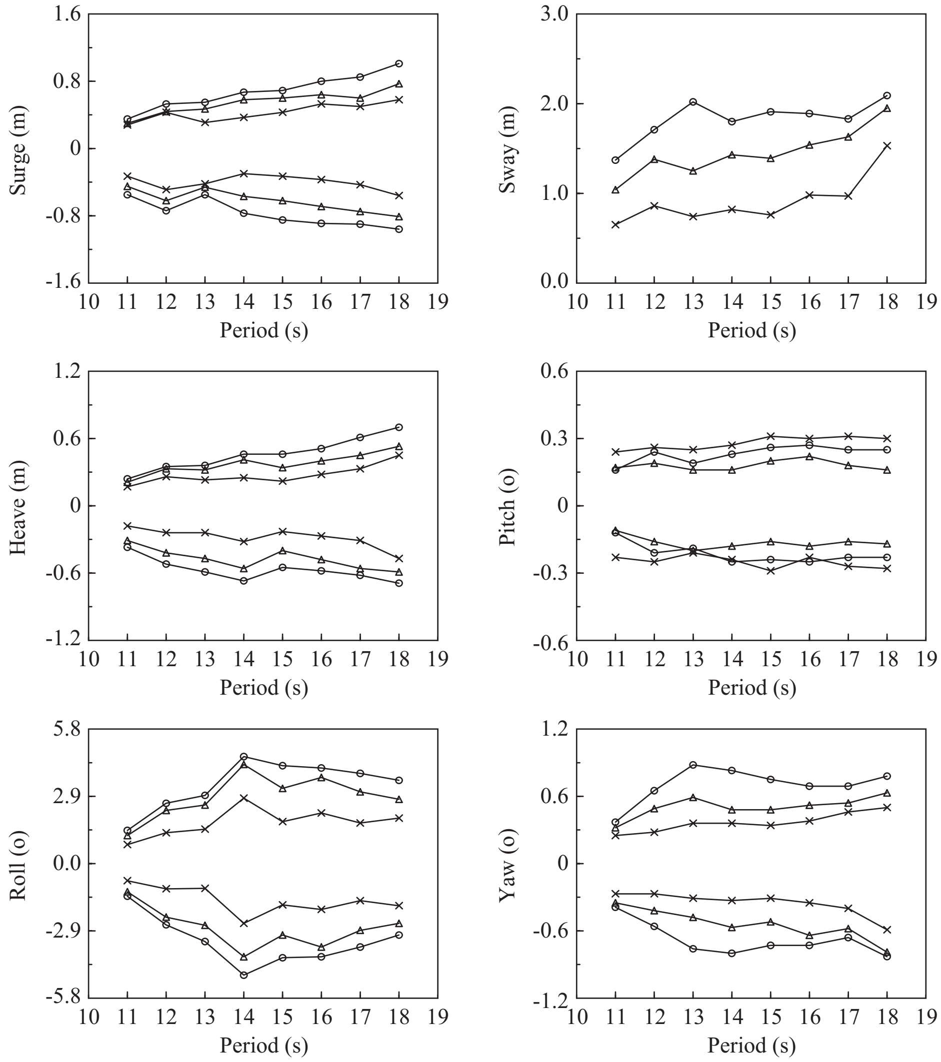


**Fig. 5. Comparison of the ballasted mooring ship's movements in the mixed waves of different energy distribution. (Major contribution of wind saves: —○—; same contribution of wind waves and swells: —△—; major contribution of swells: —×—)**

where $j$ = 1, 2 represents the low- and high-frequency parts. There are totally six parameters in the equation, and each change of a parameter, according to the spectrum shape of actual measurement, may minimize the difference between theoretical and measured values of the spectra. The characteristic parameters of the mixed waves in the experiment consist of the parameters of wind waves as well as the one of swells, which can be referred to in Table 2 and Table 3. From Fig. 4, an illustration of the bimodal spectral mixed waves that generated by employing the method of six-parameter spectrum, the theoretical spectrum is seen better fitted the measured one.

## IV. RESULTS AND DISCUSSION

### 1. The Effects of Energy Distribution of Mixed Waves on the Motions of a Mooring Ship

According to the different ratios of energy distribution in low and high frequency parts, mixed waves can be divided into three types: major contribution of wind waves, same contribution of wind waves and swells, and major contribution of swells. According to Huang *et al*. [10] and Zou [30], the energy $E$ of the mixed waves is superposed in the low and high frequency parts, that is, the overall energy of the mixed waves is

$$E = \frac{1}{4}\rho g H^2 = \frac{1}{4}\rho g (H_1^2 + H_2^2) \tag{2}$$

where $\rho$ is the density of seawater, $g$ the acceleration due to gravity, $H$ the combined wave height of the mixed waves, and $H_1$, $H_2$ the wave heights of the mixed waves in the low and high frequency parts respectively. Thus, even with the same overall energy, the different types of mixed waves have considerable differences in energy distributions of low and high frequency.

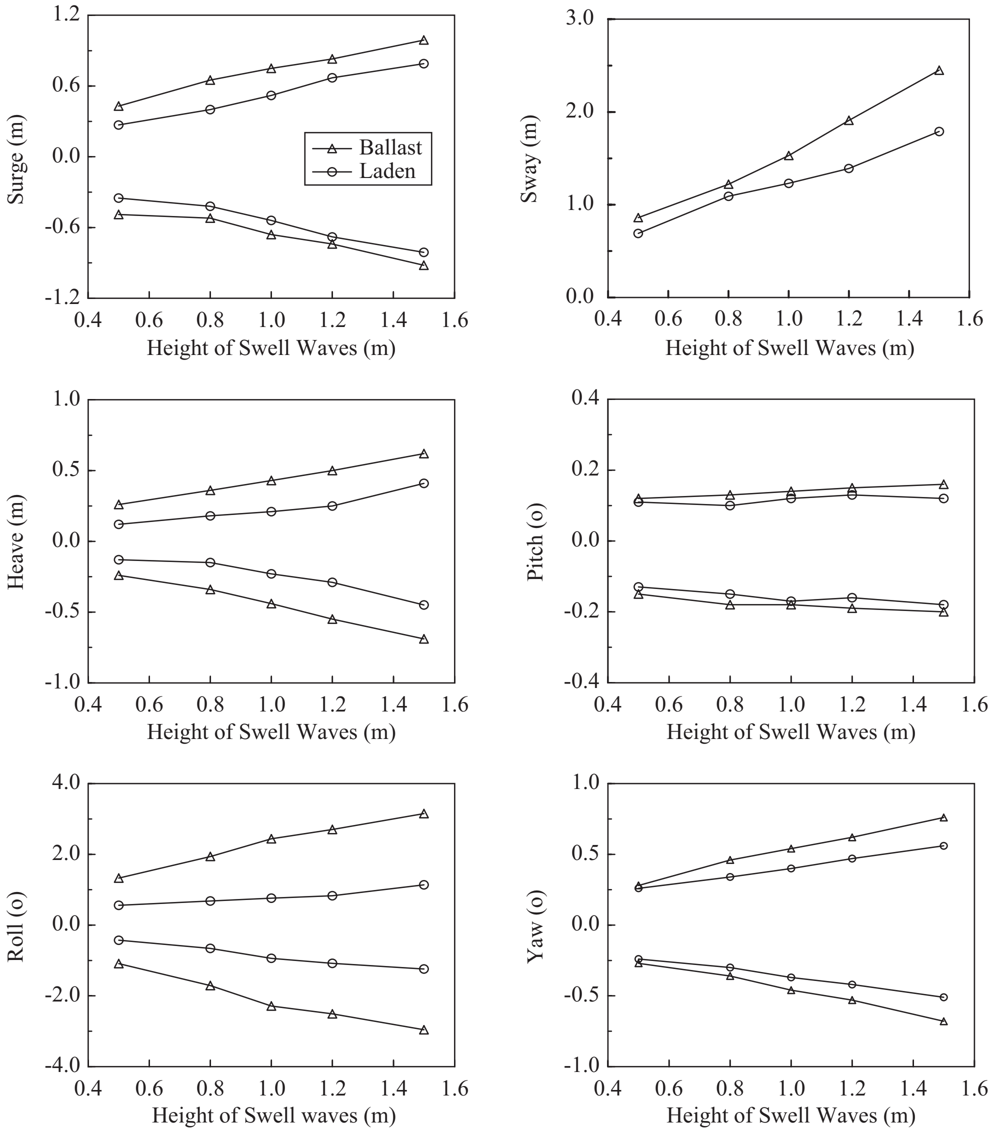


**Fig. 6. Changes of the mooring ship's movements with the increasing wave height of swells on the premise of invariant wind waves in the mixed waves.**

Fig. 5 shows the comparison of the mooring ship's motion responses under the influence of the mixed waves of three different types with the same energy. Seen from the figure, the values of surge, sway, heave, yaw of the mooring ship increase with the increasing energy in the low-frequency part of the mixed waves, except that the value of pitch is almost unchanged. The mooring ship's main modes of motion the under the function of transverse waves—sway and roll, which are the most obvious movements along with the changes of energy distribution in the low-frequency part of the mixed waves. When the low-frequency energy is doubled, the increase of sway and roll will almost double as well.

## 2. The Effects of Energy Distribution of Mixed Waves on the Motions of a Mooring Ship

Fig. 5 also indicates that the periodic changes of swells of the mixed waves have effects on the motion responses of the mooring ship. As shown in the graphs, the value of surge will gradually increase with the periodic increase of swells under the ballasted condition; the value of sway will increase with the periodic increase of swells, and its first peak value will occur during 12~13 s. At this moment, the ratio of the period of swells and the natural period of the mooring ship's roll is 1.11 to 1.20; the value of heave will increase with the periodic increase of swells; the great peak value of the ship's roll appears when the period of swells is 14 s. The value of the roll will rapidly decrease along with the periodic increase (or decrease) of swells before and after the peak value. Just then, the ratio of the period of swells and the natural period of the mooring ship's roll is 1.29; the peak value of yaw emerges when the wave period is during 13~14 s, and then the ratio of the period of swells and the natural period of the mooring ship's roll is 1.20 to 1.29; the value of pitch doesn't change much with the periodic change of swells.

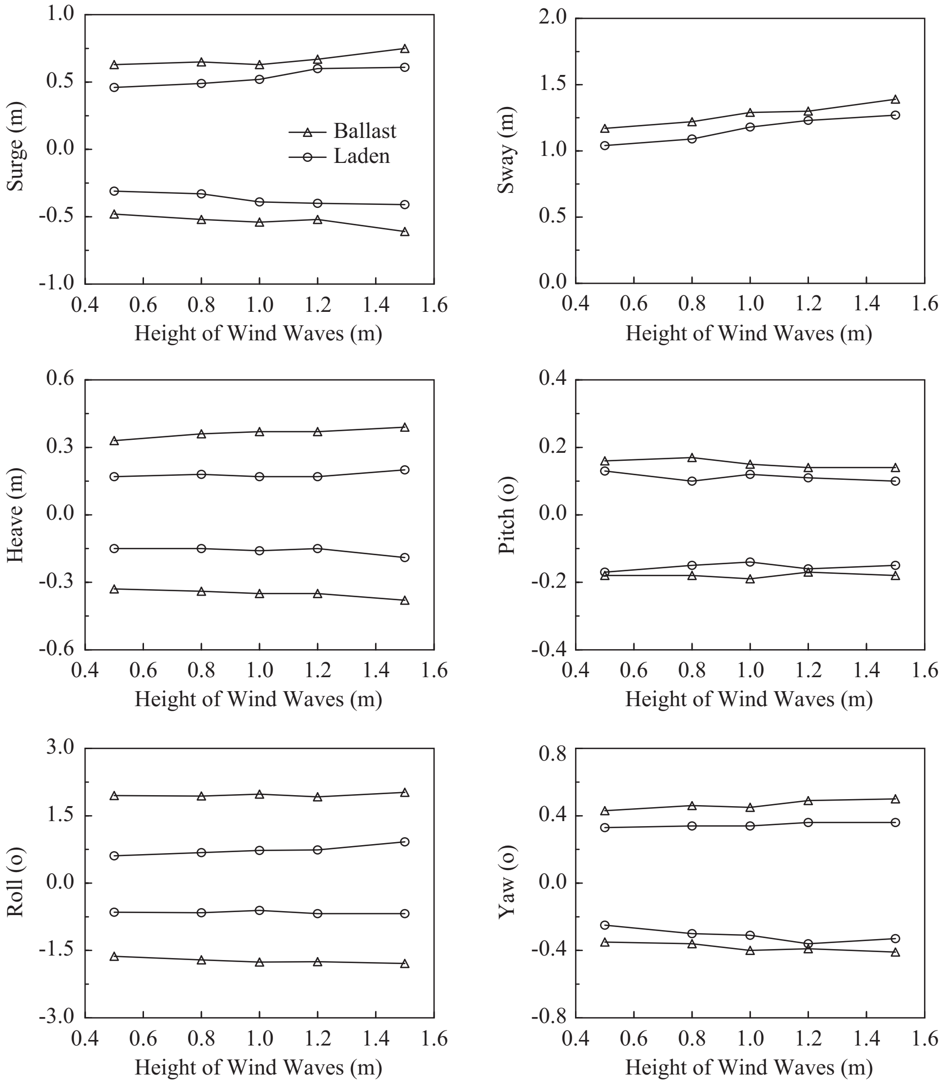


**Fig. 7. Changes of the mooring ship's movements with the increasing wave height of wind waves on the premise of invariant swells in the mixed waves.**

### 3. The Effects of the Wave Height Changes of Swells of the Mixed Waves on the Motions of the Mooring Ship

As shown in Fig. 6, when the wind waves the mixed waves is certain, the changes of wave heights of swells have ultimately led to changes in movements of the mooring ship. Seen from the figure, no matter under what loading conditions, the mooring ship's values of surge, sway, heave, roll and yaw all increase along with the increase of the swell wave heights except that the pitch is less affected. The sway and roll of the mooring ship are affected the greatest by the changes of wave heights of swells. The values of sway and roll increased over 2 times when the wave height of swells increases from 0.5 m to 1.5 m; the values of surge, heave and yaw are all more than doubled.

### 4. The Effects of the Wave Height Changes of Wind Waves of the Mixed Waves on the Motions of the Mooring Ship

As shown in Fig. 7, when the swells the mixed waves is certain, the changes of wave heights of wind waves have ultimately resulted in changes in movements of the mooring ship. Seen from the figure, under the effect of mixed waves, the movements of the mooring ship were less affected by the changes in the wave heights of the high-frequency wind waves. When the wave height of the wind waves increases from 0.5 m to 1.5 m, the rate of the increase of sway is about 20%, which was affected the greatest by the increase of wind heights of wind waves; the values of surge, heave, roll and yaw increased slightly along with the increase of the wave heights of wind waves, and value of pitch was not affected much by the changes of wave heights of wind waves.

## V. CONCLUSION

On the basis of the analyses of the experiment results of the effect of energy distribution of the mixed waves and the changes of wave conditions on the motion responses of the moored ship, it can be seen that:

- When the energy of mixed waves is certain, the movements of the mooring ship will generally increase along with the increase of wave energy in the low-frequency part of the mixed waves;
- When the loading conditions are certain, the surge and heave of the mooring ship will increase along with the periodic increase of swells in mixed waves, the peak values of sway, roll, and yaw are in proportion to the natural period of the mooring ship's roll;
- When the conditions of wind waves in the mixed waves are definite, most movements of the mooring ship will rapidly increase along with the increase of the wave height of swells;
- When the conditions of swells in the mixed waves are definite, the changes of wave height of wind waves will have less effects on the movements of the mooring ship;
- The pitch of the mooring ship will be influenced little by changes of the transverse mixed waves.

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