



## TRADE-OFF RELATIONSHIP BETWEEN THE HIRE RATES AND EXERCISE PRICES OF PURCHASE OPTIONS IN SHIP CHARTER CONTRACTS: AN OPTION PRICING APPLICATION

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# TRADE-OFF RELATIONSHIP BETWEEN THE HIRE RATES AND EXERCISE PRICES OF PURCHASE OPTIONS IN SHIP CHARTER CONTRACTS: AN OPTION PRICING APPLICATION

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Key words: ship charter, financial lease, purchase options, real options.

## ABSTRACT

Bareboat charters and ship leasing contracts with purchase options are important channels for ship owners and the operators obtaining their ships. This study develops an option pricing model to evaluate the trade-off relationship between the hire rates and the exercise prices of purchase options. Numerical analysis is conducted using four input variables namely the spot ship prices, the volatility of the spot ship prices, the time to maturity of the options and the discount rate. An actual example is also presented. The results show that as the exercise price decreases, it is very likely that this purchase option is exercised; and for this very reason, owners will ask for a higher hire rate in the charters with this kind of purchase option given to charterers, therefore the hire rate difference between the charters with and without this purchase option increases. Evidently it reveals a trade-off relationship between the exercise price and the hire rate. In addition, for a specific exercise price, applying higher spot ship prices, greater volatility, longer time to maturity and higher discount rates all manifest the trade-off relationship between the exercise prices and the charter rates.

## I. INTRODUCTION

To maximise their profits, ship operators, including ship owners and chartered-ship operators, must adopt suitable strategies to obtain ships for operation. Ships can be acquired through different channels, such as signing new ship building

contracts, purchasing second-hand ships from markets and fixing time charters or bareboat charters (in the form of ship leasing contracts) with purchase options. Ship acquisition costs are indicated on income statements as depreciation, interest expenses or charter hire, which have a long-term effect on ship operation costs, and are determining factors of operating profit. Thus, ship operators are continually seeking to procure the lowest-cost ships for operations.

Ship leasing contracts in the form of long-term bareboat charters or time charters, in addition to purchasing second-hand ships or building new vessels, provide shipping companies with an alternative method to secure the required tonnage for operations. In 2008, a total of 36 contracts for 78 vessels used leasing as the method of ship financing (*Marine Money International*, [12]). Ship leasing contracts are either pure leasing deals or sale-and-leaseback deals. Ship leasing, although fundamentally a financing tool with costs comparable to borrowing cost, offers advantages to ship operators (i.e. charterers or lessees), such as fleet flexibility and reduced residual value risk. Ship leasing provides a cheap and low initial cash-out solution for ship financing. Additionally leases can also form part of operators' exit strategy should the market deteriorate.

A plain charter type regulates two major terms, that is the time and hire rate of charters. In addition of these terms, complex and long-term charter types include options such as the charterer's (i.e. lessee's) right to purchase the vessel at various times during the charter, or the charterer's option to extend the charter period. In practice, long-term time charters and bareboat charters typically include a clause enabling the charterers to purchase the ship for a specific price at the expiry date of the charter. Among the 36 lease deals arranged in 2008 reported by *Marine Money International* [12], 6 charters had fixed-price purchase options for the charterers. To include charterer's purchase options in contracts, the hire rate agreed by the owner and charterer should indicate the distinct value of the charter that the other plain charter types do not have. However, this value is typically determined using broad principles or intuitive estimates made by both parties of

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possible ship prices at the time of maturity. Therefore an accurate valuation of these options can enable both contractual parties to reasonably determine the hire rate of the charter.

This option in leasing contracts is one of the real options of financial management. Real options provide rights, without obligation, to holders to make particular business decisions which empower the holders of the options to initiate, abandon, expand, or reduce capital investment. Decisions to implement the action or purchase options require the holders to examine the time to maturity and strike price (also known as the exercise price, hereafter 'strike price' and 'exercise price' are used interchangeably) of the option and the spot prices of the underlying asset at maturity.

The charterer's option to purchase the ship for an established price at the expiry date of charter is a typical European call option, where the holders of the option can only exercise their right at the maturity of the option, that is, at the expiry of the charter. Although infrequently applied, pure American call options are also employed in charters; they allow charterers to exercise their purchase options and terminate the charter at any time before the expiry date. A number of charters with American call options are fixed with an initial lock-up period. An initial lock-up period requires the charterers to charter the vessel until the expiry of the lock-up period before they can exercise their right to purchase the vessel. Purchase options can also be Bermudan-style call options with a limited number of pre-fixed exercise dates. The strike prices of the option are fixed in decreasing numbers in consideration of the decreasing book value of the vessel over time.

In exchange for these options to be fixed into the charters, the holder, as in financial derivatives regimes, typically must pay the writer of the options (i.e., the option seller) some form of premium. However, the purchase option holders, that is, the charterers do not pay a premium to the owners granting the options. Instead, owners granting purchase options to chartered-out vessels, charge higher daily hire rates for the ships fixed to charterers than for ships without purchase options. Furthermore, because of the nature of a call option, purchase options offered to charterers do not provide owners a better guarantee on the residual value of the vessel; they simply eliminate the opportunities for owners to profit from disposing of the ship upon maturity of the options.

If the second-hand ship market rises at the time of maturity, charterers can exercise their option to purchase the vessel for profit; however, if the second-hand ship market declines, charterers holding purchase options can simply leave their options. Therefore, for their benefit, owners reasonably require a certain extent of compensation for providing the options. Consequently, they will charge charterers a higher hire rate if a cheaper strike price is fixed in the option. In practice, for charters with charterer's purchase options, owners maintain a trade-off relationship between the daily hire rate fixed to charterers and the strike prices granted in the options to charterers, though the daily hire rate and the strike

price are all subject to negotiation made between both parties. From charterers' perspective, a cheaper strike price granted in the options provides greater option value. Therefore, charterers are willing to pay higher daily hire rates to owners in exchange for holding such options. In conclusion, the premiums of charters with charterer's purchase options are integrated into the daily charter hire rate and paid to owners in monthly hire payments during the duration of the charters. Thus, to calculate the gross premium of these deals, traditional discounting methods or discounted cash flow analysis is required to subtract the sum of charter hire payments with charterer's purchase options from the sum of charters without purchase options.

Few studies have focused on the leasing field. McConnell and Schallheim [7] explored the relationship among various asset leasing contracts, including (1) cancellable operating leases; (2) leases that provide the lessee with the option to extend the lease; (3) leases that provide the lessee with the option to purchase the leased asset at a fixed price at the date of maturity; (4) leases that grant the lessee the right to purchase the leased asset at its 'fair market value' at the date of maturity; (5) leases that grant the lessee the option to purchase the leased asset at a pre-agreed price anytime during the lease; (6) leases that require the lessee to purchase the leased asset at a fixed price at the date of maturity; and (7) leases that contain non-cancellation provisions. They employed a compound option pricing framework to develop a general model for valuing each type of leasing contracts.

Grenadier, Trigeorgis, and Kenyon and Tompaidis [2, 5, 11] recognised leases as a type of transaction that contain embedded options and provide flexibility to lessees. Traditional methods for evaluating this type of asset financing employ discounted cash flow analysis. However, Dixit and Pindyck and Trigeorgis [1, 11] stated that discounted cash flow analysis is inadequate for capital budgeting in contracts that provide varying degrees of ongoing management flexibility like options. Any embedded option for assets, such as ship, provides value in addition to the expected cash flow.

Grenadier [2] used a real options approach to determine the complete term structure of lease rates and presenting a unified framework for pricing a variety of leasing contracts. He developed a sufficiently flexible model using fundamental economic uncertainty and competitive interaction of value-maximising firms as a foundation to establish endogenous processes for rent, supply and asset values. The structure of his model facilitates economic intuition for a wide variety of leasing phenomena. Using a real-options approach in the model, he examined the lease rates for leases such as forward leases, leases with options to renew or cancel, lease insurance contracts, adjustable-rate leases, and leases with payments contingent on asset use.

Trigeorgis [10] discussed the numerical valuation of leasing contracts with various embedded operating options. He proposed a contingent claim analysis (CCA) method for operating lease options and used a CCA-based numerical

analysis method to value leasing contracts with multiple options. Hussain [3] also employed CCA to evaluate different lease contracts and presented that the value of the lease is contingent on the options embedded in the lease contract. He obtained the results of three unique cases by assuming that the value of underlying asset decreases linearly over time in a stochastic environment.

Li [6] examines the advantage and disadvantage of ship leasing as a financing method from the perspectives of both theory-based economists and law and accounting professionals. The result confirmed that economists believe ship leasing offers lessees the advantages of positive tax benefits and an enhanced financial disposition. However, professionals within the shipping industry, based on their experience of the market, have considerable complex and inconsistent views regarding ship leasing. Li also indicated that developments for constructing increasingly sophisticated ship-leasing agreements have been greatly outpaced counterpart empirical research, which is almost nonexistent in the field of applying options to ship leasing.

Rygaard [8] proposed a valuation method for time charter contracts with built-in Bermudan purchase options for chartered ships. He developed a two-factor stochastic model to determine the price of these options by applying techniques from contingent claim analysis, such as dynamic programming. Jørgensen and Giovanni [4] presented a simpler definition of the problem and analysed time charter contracts with American purchase options using a stochastic model for instantaneous time charter rates with time independent parameters. This model is suitable for monotone instantaneous charter rate term structures. The valuation problem was then formulated as a partial differential equation and solved using related numerical techniques. Although number of other academic studies have applied real options analysis in the shipping field, for example, Sødal *et al.* [9] used a real options valuation model with stochastic freight rates to investigate market efficiency and the economics of switching between the dry bulk and the tanker markets in international shipping, none have investigated the purchase options in ship charter contracts.

Nevertheless, previous studies that focused on the evaluation models for the purchase options in charter contracts, the reasonable time to exercise the purchase options from the charterers' perspective and the correlation of the freight rate with the value of charterer's purchase options neither analysed the trade-off relationship between the hire rate and the strike price of purchase options, nor explored the key factors that ship owners must consider when fixing charters. Furthermore, none of these studies was conducted from the ship owners' perspective.

This study addresses this knowledge gap and contributes to purchase option evaluations by employing an option pricing model and using the volatility of ship prices as the subject to simulate the value of purchase options. Additionally we conduct analysis from the ship owners' perspective and explore

ship owners' decisions-making process regarding the trade-off between the hire rate and the strike price of the purchase options fixed by ship owners and charterers of charter contracts. The results of this study should be valuable for ship-brokers, mortgagees and financiers involved in this type of charter contract.

The remainder of this paper is organised as follows: Section 2 introduces the model proposed in this study; Section 3 provides the numerical analysis and implications; and Section 4 presents the conclusions and possible further study directions.

## II. THE MODEL

Because bareboat charters are the most common charter forms with purchase options in shipping practice, this study uses a bareboat charter case with charterer's purchase options to demonstrate the trade-off relationship between the hire rate and exercise price in ship lease contracts. The premium of the options is represented by the differing sum of the daily hire rates between contracts with and without purchase options. Under the assumption of no arbitrage, ship owners entering into a charter with purchase options are indifferent if the sum of the present values of hire incomes during periods of charters without purchase options equal that of charters with purchase options minus the value of such options, that is,

$$\sum_{j=0}^{n-1} (H_1 \times D_j) \times e^{-rj} = \sum_{j=0}^{n-1} (H_2 \times D_j) \times e^{-rj} + c \quad (1)$$

where  $H_1$  and  $H_2$  are the daily hire rates of charters with and without charterer purchase options, respectively. Obviously  $H_1$  is greater than  $H_2$  because  $H_1$  includes the charterer's additional payment for the purchase option premium.  $D_j$  represents the number of days in the  $j$ th calendar month during the charter period;  $r$  is the applied discount rate; and  $e^{-rj}$  represents the discount factor for converting the monthly payment amount into its present value. Both the practice and the contracts require the charter hire payment to be made monthly or semi-monthly in advance. Therefore, for a particular charter with duration of  $n$  months, the first hire payment must be made at the beginning of the charter, that is, when the time-frame is 0, and the last hire payment by the end of the  $(n-1)$ th month. These monthly payment amounts must be calculated into their present values and summated to determine the present value of the aggregate hire outlay under the respective scenario during the charter period.

In essence, charterer's purchase options are a call option held by charterers. The value of the call option is denoted by  $C$  and largely determined by the strike price of the option, among several other factors. Because the option provides an advantage for charterers, charterers can exercise the option at maturity if they believe the market is favorable, or elect not to if the market is adverse. Thus, according to Eq. (1), we infer

that, in charterers' valuation, the sum of the present values of hire payments under charters without charterer's purchase options should equal that of charters with charterer purchase options plus the fair value of that option. On the other hand, in owners' valuation, the sum of the present values of hire revenues under charters without charterer purchase option should equal that of charters with charterer purchase option minus the fair value of that option.

To analyse the trade-off relationship between the daily hire rate and the exercise price fixed into a charter, this study constructs a valuation model for purchase options. First, we assume that the ship price process follows a geometric Brownian motion process and comprises a constant expected return and a constant variance price change, as described below

$$dS = \mu Sdt + \sigma Sdz \tag{2}$$

Eq. (2) is the most widely used model of price behaviour for a particular underlying asset.  $S$  is the spot price of the underlying asset at time  $t$ , that is, the spot ship price in this study; thus the expected drift rate in  $S$  is assumed to be  $\mu S$  for certain parameters  $\mu$ . This means that in a short interval,  $dt$ , the expected increase in  $S$  is  $\mu Sdt$ . The parameter  $\mu$  is the expected rate of return on the spot ship price. The value of  $\mu$  is determined by the demand/supply condition of the second-hand ship market and the ship depreciation rate. In addition, the spot ship price exhibits volatility as  $\sigma$ , and  $dz$  denotes the underlying uncertainty that drives the model, indicating some incremental value in  $dt$  in a Wiener process. The variability of the percentage return in a short time  $dt$  is reasonably assumed to be the same regardless of the spot price. In other words, an investor is equally as uncertain of the percentage return when the spot price is 50 million U.S. dollars as when it is 10 million U.S. dollars. This implies that the standard deviation of the change in a short time  $dt$  should be proportional to the spot price. The variable  $\sigma$  represents the volatility of the spot price.

Fig. 1 shows three simulated ship values during the vessel's 25 years of life under the assumption of a geometric Brownian motion process, where the general trend of ship value is decreasing because of depreciation in the ship's book value represented by a constant expected drift rate. However, ship values during their life span also fluctuate because of market price volatility.

Under this assumption, by employing Ito's Lemma, we can further infer that the fundamental partial differential equation (PDE) of the option pricing model and the associated boundary conditions are provided by

$$-rC + rC_s S + C_t + \frac{1}{2} C_{ss} \sigma^2 S^2 = 0, \quad 0 \leq S, \quad 0 \leq t \leq T \tag{3}$$

$$C(T) = \max[S_T - K, 0] \tag{4}$$

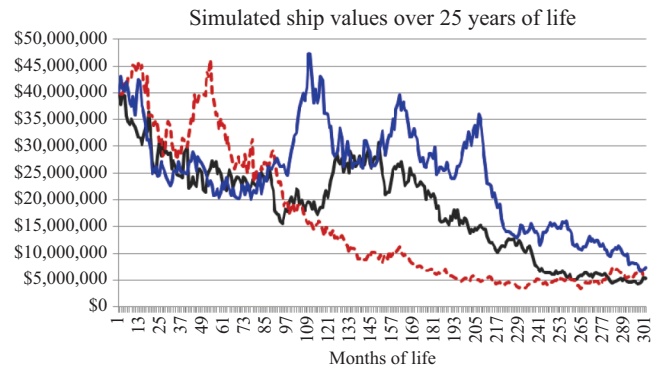


Fig. 1. Three simulations of ship value over the vessel's 25 years of life demonstrating the assumption that ship values follow a geometric Brownian motion process.

where  $C_s$  and  $C_t$  are the first-order partial derivatives of  $C$  with respect to  $S$  and  $t$  respectively,  $C_{ss}$  is the second-order partial derivatives of  $C$  with respect to  $S$ .

We can solve the PDE and obtain a close form solution of the function  $C$  to determine the value of the European-type call option, which is then applied to calculate the value of the charterer's purchase options as follows:

$$C = SN(d_1) - Ke^{-rT} N(d_2) \tag{5}$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{6}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{7}$$

where  $N(\bullet)$  is the cumulative distribution function of the standard normal distribution.  $T$  is the time to maturity if the valuation is conducted at the time  $t = 0$ ,  $S$  is the spot price of the underlying asset,  $K$  is the strike price (also the exercise price),  $r$  is the risk free rate (annual rate, expressed as continuously compounded) and  $\sigma$  is the volatility in the log-returns of the underlying asset.

However, for contracts with a simple European option, where option holders can only exercise their right at the time of maturity, the daily hire rates for charters with purchase options are subject to (1) the exercise price  $K$ ; (2) the time to maturity  $T$  fixed in the ship charter contract; (3) the spot ship prices  $S$ ; (4) the prevailing annualised risk-free rate  $r$ ; and (5) the relevant volatility of the second-hand ship prices  $\sigma$ . Thus, the value of the purchase option varies according to each contract, as does the hire rate difference between contracts with and without purchase options. Furthermore, as per we suggest in the Introduction Section, a trade-off relationship exists between the hire rate applied to charter contracts with a charterer purchase option and the exercise price.

### III. NUMERICAL ANALYSIS USING EUROPEAN PURCHASE OPTIONS AS AN EXAMPLE

#### 1. A Bareboat Charter Case

To demonstrate the valuation of purchase options, we provide the following example. A charterer entered a bareboat charter party in September 2009 with a ship owner leasing a newbuilding Panamax bulk ship, which was delivered in January 2010, with the following actual excerpted terms:

- .....
- Bareboat charter hire: US\$14,250.- gross per day pro rata for part of a day payable monthly in advance.
  - Period: 5 years plus 1 month at charterers option, .....
  - Charterers have a right to purchase the vessel of the Hull Number JEHI068 at the end of the fifth year after the particular vessel is delivered to Charterers at the price of US\$33.7 million.
- .....

We obtained a spot ship price index of US\$38,350,000 for a 5-year-old ship of a similar type dated on April 12, 2010 from 'Week 15/2010 Weekly Market Report'<sup>1</sup> published by the world's leading ship brokering company 'Banchemo Costa'. Additionally, based on the description of the daily hire rate, the charter period and the exercise price of the charterer purchase options, the value of the purchase option  $C(S = \text{US\$}38,350,000, t = 0)$  held by the charterer is US\$11,752,303.36, and the equivalent hire rate applied to charters without purchase options is US\$7,396.24, according to Eq. (1). Thus a difference of US\$6,853.76, that is,  $H_1 - H_2$ , between the two charters is established.

The spot ship prices of 5-year-old Panamax vessels were obtained from *Shipping Intelligence Weekly*,<sup>2</sup> published by Clarkson Research Services from February 27, 1998, to October 3, 2008, and thereafter until April 9, 2010, from *Weekly Market Report*<sup>1</sup> by Banchemo Costa when Clarkson ceased to publish ship prices. We obtain the annualised volatility ( $\sigma$ ) of the weekly ratios of change from the recorded ship prices, which is 21.9658%. The risk-free rate  $r$  is obtained and set as 2.50%.  $T$  is the 5th years according to the chartering clauses, and  $t$  is set at 0 to imply that the evaluation was conducted on the fixture decision. Under this presumption, we determine the parameters of  $d_1 = 0.7632$ ,  $d_2 = 0.2721$ ,  $N(d_1) = 0.7773$  and  $N(d_2) = 0.6072$ .

We can then construct the simple relationship between the exercise prices of the charterer purchase option  $K$  and the hire rate difference ( $H_1 - H_2$ ). Among these variables, we found that as  $K$  increases, charterers become profitless in exercising their purchase options; thus, the  $H_1 - H_2$  declines. Fig. 2 shows the negative correlation between these two variables.

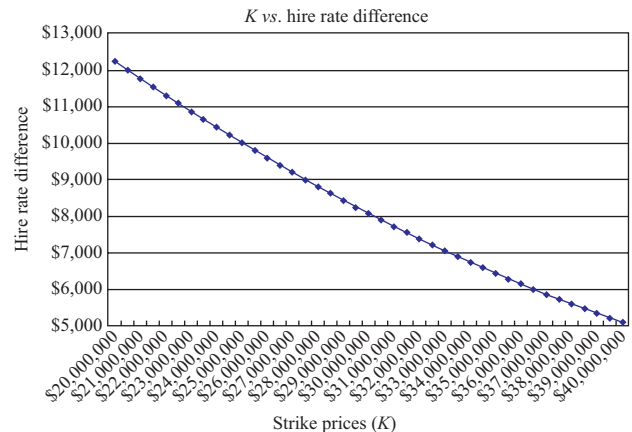


Fig. 2. The trade-off relationship between the exercise prices of the charterer purchase options ( $K$ ) and the hire rate difference ( $H_1 - H_2$ ).

The results also indicate that the decrease in hire rate difference between  $K = \text{US\$}20,000,000$  and  $K = \text{US\$}21,000,000$  is approximately US\$464; whereas the decrease from  $K = \text{US\$}39,000,000$  to  $K = \text{US\$}40,000,000$  is only approximately US\$246. The sensitivity of the hire rate differences rises and the trade-off relationship between  $K$  and the hire rate becomes increasingly evident as the  $K$  amounts reduce. Since, according to our assertion in the Introduction Section, a cheaper strike price granted in the options gives greater option value to charterers, thus we can ascertain that the strike price  $K$ , among other variables, such as  $S$ ,  $\sigma$ ,  $T$  and  $r$ , plays an essential role in determining the hire rate of a charter with charterer purchase options.

#### 2. Sensitivity Analysis

This study then conducted sensitivity analysis based on the following four dimensions: the spot ship price, the volatility of the spot ship price, time to maturity of the options and the discount rate.

##### 1) The Spot Ship Prices ( $S$ )

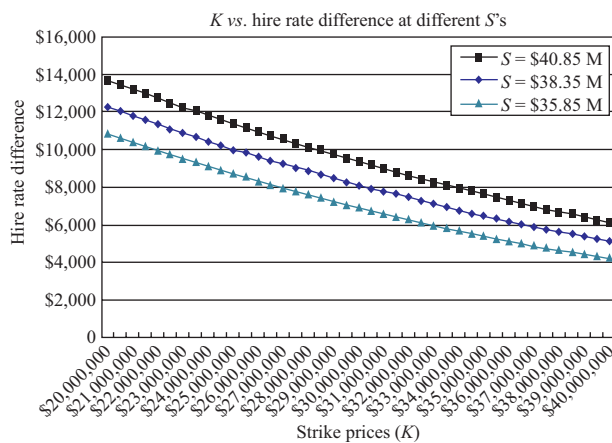
The value of spot ship prices affects charterer's willingness to exercise the purchase options. For example, if the spot ship price at the expiry of the 5-year charter is US\$38,350,000, the charterer can earn a profit of US\$4,650,000 by reselling the vessel after exercising their purchase option at the strike price of US\$33,700,000. If the  $S$  is significantly higher than  $K$ , the charterer obtains a windfall without capital spending by reselling the vessel to the sub-buyer while exercising their purchase option. The profit thus produced enhances the value of the charterer's purchase options  $C$  and should be considered when valuing the hire rate the charterer is willing to pay for in the charter. In this example, the value of the

<sup>1</sup> For more see [www.bancosta.it](http://www.bancosta.it).

<sup>2</sup> For more see [www.clarksons.net](http://www.clarksons.net).

**Table 1. Value of the European-style of charterer purchase option  $C$  depends on spot ship price  $S$  and strike price  $K$ .**

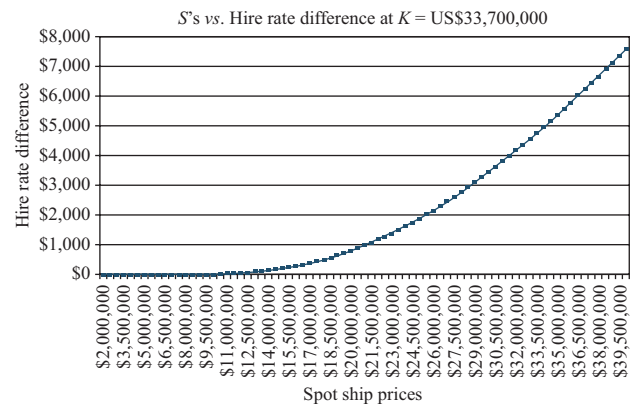
Spot ship prices, $S$	Strike price of purchase option, $K$			
	\$31,000,000	\$32,000,000	\$33,700,000	\$35,000,000
\$2,540,820	\$0	\$0	\$0	\$0
\$2,615,301	\$1	\$0	\$0	\$0
\$2,741,458	\$1	\$1	\$0	\$0
\$2,837,559	\$2	\$1	\$1	\$0
\$33,700,000	\$9,606,755	\$9,114,138	\$8,327,458	\$7,767,499
\$35,850,000	\$11,258,573	\$10,722,626	\$9,861,246	\$9,243,740
\$38,350,000	\$13,274,592	\$12,693,092	\$11,752,303	\$11,072,891
\$40,850,000	\$15,376,464	\$14,754,533	\$13,742,456	\$13,006,794

**Fig. 3. The hire rate difference versus the strike prices of various spot ship prices.**

purchase option  $C$  is US\$11,752,303 using Eq. (5) as described in Section II. If the  $S$  increases to US\$40,850,000, the charterer's resale profit increases to US\$7,150,000, increasing the value of  $C$  to US\$13,742,456 as shown in Table 1. However, if the  $S$  is lower than  $K$ , the charterer simply relinquishes the options.

We also found that with a particular  $K$ , the higher the  $S$  is, the greater the  $(H_1 - H_2)$  becomes; thus, the trade-off relationship between  $K$  and the hire rate is more noticeable. Fig. 3 shows the effect of various spot ship prices on the hire rate differences. As demonstrated by decreasing curves of the three spot ship prices when  $K$  is low, the variance between a high and a low  $S$  is greater than when  $K$  is high. When the  $K$  becomes sufficiently large, the hire rate differences are 0 regardless of how high or low the  $S$  is. As the  $S$  rises, the hire rate difference between contracts with charterer's purchase options and those without increases, that is, the curve with higher  $S$  always exceeds curves with lower  $S$ . This tendency remains even when  $K$  increases to the amount as large as \$300 million where all hire rate differences under three different  $S$ 's are unanimously 0.

Combining Table 1 and Fig. 3, using  $K = \text{US}\$33,700,000$  as an example, there is a curve of  $S = \text{US}\$2,741,458$  renders the

**Fig. 4. The hire rate difference versus spot ship prices at  $K = \text{US}\$33,700,000$ .**

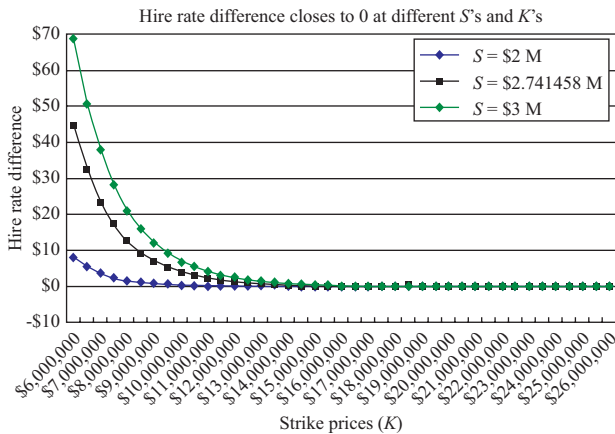
value of charterer's purchase options  $C$  close to 0, and the hire rate differences become extremely fragmented. At this point, whether charterer's purchase options are integrated into the charter contract or not is irrelevant. Fig. 4 displays the curve of hire rate differences under various spot ship prices when  $K$  is fixed at US\$33,700,000. This curve indicates that if  $S$  is between US\$2,000,000 and US\$2,741,458, the hire rate differences remain US\$0.0002 which can almost be omitted in practical operation. This is simply because if the spot ship price is greatly less than the strike price of the option, this option is futile to the holder. Therefore, in our example, the hire rate difference between the charters with and without charterer's purchase options becomes 0 when  $S$  is less than US\$2.5 million.

Similarly, as the  $K$  increases, a corresponding higher  $S$  causes  $C$  and the hire rate differences to near or equal 0. When spot ship price is US\$2,000,000, the hire rate difference becomes US\$0.0089 when  $K$  equals US\$16,500,000. However, the same difference occurs when  $K$  equals US\$23,500,000 and US\$26,000,000 for spot ship prices at US\$2,741,458 and US\$3,000,000, respectively. Fig. 5 shows that, for higher  $K$ 's, a higher  $S$  renders hire rate differences close or equal 0.

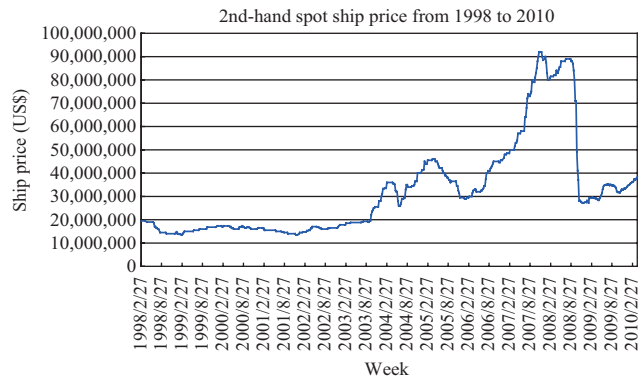
The trend of the second-hand Panamax spot ship prices from February 1998 to April 2010 is shown in Fig. 6. During

**Table 2. Value of the European-style of charterer purchase option, depends on spot ship price  $S$  and volatility of ship prices  $\sigma$ .**

Spot ship prices, $S$	Volatility of the second-hand ship prices, $\sigma$			
	10.00%	21.9658%	30.00%	40.00%
\$33,700,000	\$5,231,257	\$8,327,458	\$10,450,981	\$13,031,896
\$35,850,000	\$6,930,712	\$9,861,246	\$11,988,281	\$14,607,748
\$38,350,000	\$9,089,191	\$11,752,303	\$13,855,297	\$16,498,190
\$40,850,000	\$11,382,376	\$13,742,456	\$15,797,512	\$18,444,336



**Fig. 5. Hire rate difference close to 0 at various  $S$ 's and  $K$ 's.**



**Fig. 6. The historical trend of spot ship prices. (Data source: Clarkson's Shipping Intelligence Weekly and Banchemo Costa's Weekly Market Reports from February 1998 to April 2010.)**

the recorded period, the price peak was at US\$92,000,000 in December 2007 and the trough at US\$13,500,000 in December 2001. Because both parties refer to the prevailing spot ship price as an indicator for negotiating the strike price of the charterer's purchase option when fixing contracts, therefore, charterers tend to negotiate with owners at time of relative market trough, like time of the 4th quarter of 2001, for purchase options to be exercised a number of years later even though the strike price of the options is fixed marginally higher than the prevailing spot price. However, if the contract is fixed at the time of a relative market peak, charterers will request that strike prices are fixed at a discount to the prevailing spot prices. On the contrary, owners have to ask for the strike price to be fixed at as higher amount as possible if they enter into this type of charters in the time of a relative market trough. Because with this effort, when the market rises and charterers exercise their option to purchase the vessel, owners, though are not able to enjoy the market by reselling the vessel in their account, can at least obtain a better residual value of the ship.

2) The Volatility ( $\sigma$ )

As discussed in the previous sub-section, under a particular volatility, higher spot prices increase the value of options. In addition to spot prices, the volatility applied also affects the value of the options substantially. Table 2 shows  $C$ 's of charterer's purchase options under different volatilities according

to the option price model previously described. We deliberately included three other hypothetical volatilities,  $\sigma = 10\%$ ,  $\sigma = 30\%$  and  $\sigma = 40\%$ , in the table. As shown, for a particular spot ship price, higher volatility produces higher  $C$ .

In the meantime, as shown in Fig. 7, the higher the volatility is, the flatter the curve becomes, indicating that the sensitivity is low if the volatility is high, and vice versa. For a fixed volatility, as the strike price increases, the value of the charterer's purchase option  $C$  decreases, thereby decreasing the hire rate difference ( $H_1 - H_2$ ). We once again confirm a negative relationship between the strike price and the hire rate difference at a given volatility of the spot ship prices. Conversely, with a particular  $K$ , the volatility, the value of the charterer's purchase option and hire rate difference show positive relationships with each other. Therefore, under a specific  $K$ , the higher the volatility, the greater the hire rate difference is; thus, the trade-off relationship between  $K$  and the hire rate is more noticeable. For a particular hire rate difference, higher volatility can reflect higher  $K$  according to the Fig. 7, indicating that higher volatility provides more opportunity for  $S$  to increase. Thus, in a more volatile market, charterers are more willing to accept and owners can ask for higher  $K$ 's. This finding is demonstrated by the results shown in Table 2.

Furthermore, the curves show that variance of hire rate differences between various volatilities is trivial when  $K$  is small, and becoming diverging with increasing amounts of  $K$ . The



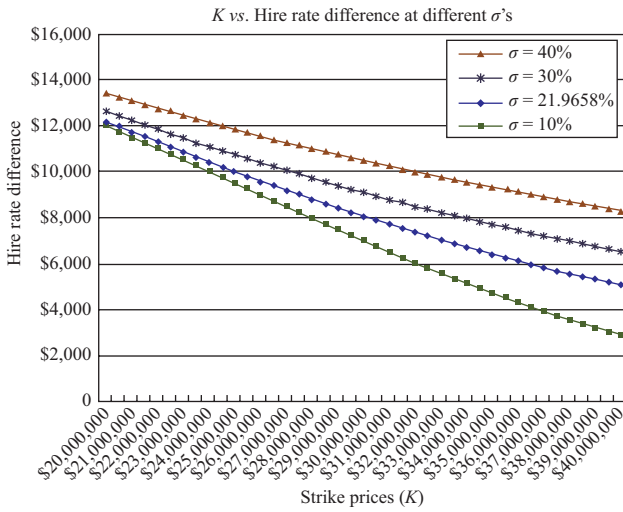


Fig. 7. The hire rate difference versus the strike prices in various volatilities.

decrease in hire rate difference under greater volatility is smaller than that under lower volatilities. This phenomenon also supports the inference that charterers can accept higher  $K$  when volatility is high. Furthermore, this also indicates that as charterers, decision-makers prefer a highly volatile market of spot ship prices compared to a market with low volatility.

### 3) The Time to Maturity ( $T$ )

As shown in Fig. 8, the time to maturity,  $T$ , is set as 5 years according to the clause in the example, where  $T = 5 - 0$ . Additionally, two other hypothetical scenarios of  $T = 3$  and  $T = 8$  years of time to maturity of the charterer’s purchase option are included without altering other variables. As a general trend, given a designated  $K$ , the longer the time to maturity of the option, the higher the required hire rate difference ( $H_1 - H_2$ ). Conversely, for a specific  $T$ , as the  $K$  increases, the hire rate difference decreases. Therefore, the trade-off relationship between  $K$  and the hire rate is negative and becomes increasingly evident as  $T$  increases. In other words, charterers are willing to pay a higher charter hire rate for charters with purchase options if the time to maturity  $T$  of the options is longer.

Furthermore, for a particular hire rate difference, charterers can accept a higher  $K$  when  $T$  is longer. This finding contradicts the traditional accounting concept which states that the book value of the asset decreases because of depreciation and that the longer the  $T$  is, the lower the book value of the vessel becomes. However, this study also found that the longer the  $T$ , the higher the  $K$  is accepted. This inference can be because, for assets such as houses and buildings, the asset’s market value does not decrease over time; instead, the value increases because of inflation. Charterers holding purchase options can appreciate this merit because the longer time to maturity provides them with more time to wait as the ship’s market value increases because of inflation. When the time is appropriate

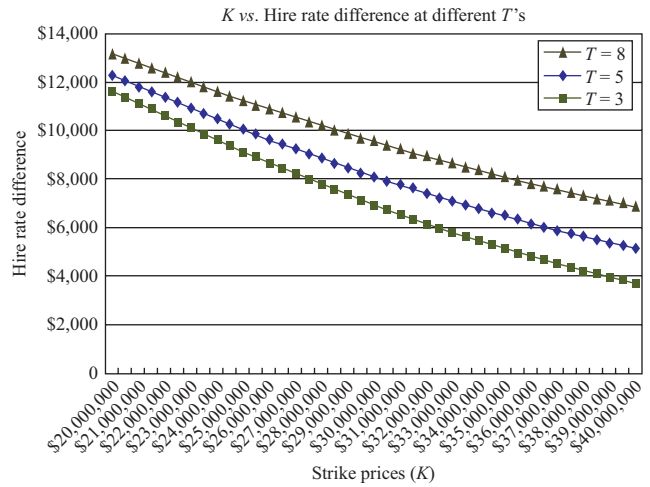


Fig. 8. The hire rate difference versus the strike prices in different terms of time to maturity.

and the market value is high, charterers can obtain a profit from the resale of the ship after exercising their purchase options. This also explains why, for a given hire rate difference, greater  $T$  can yield higher  $K$  compared to that of smaller  $T$ .

According to our calculations, the curves converge to an extremely narrow range at  $K = \text{US}\$14,000$ , and as  $K$  decreases, the hire rate difference increases. Eventually, the curves completely converge at appropriately  $K = \text{US}\$ 0.005$ , where the hire rate difference of  $H_1 - H_2$  is  $\text{US}\$22,365$  per day, and the value of  $C$  is close to the spot ship price in our example of  $\text{US}\$38,350,000$  for all three  $T$ ’s. This result implies that under the said three times to maturity, the equivalent hire rate applied to the contract without purchase options is approximately  $\text{US}\$-8,115.11$  per day, if the hire rate is fixed at  $\text{US}\$14,250$  to contracts with purchase options, as shown in the examples provided in the Sub-Section 3.1. With this result, we can interpret that when the amount of  $K$  decreases and nears to 0, the holder of the option does not have to pay any penny to acquire the vessel at the maturity of the option, and he can obtain a profit by reselling the vessel at the spot price. Therefore, regardless of different times to maturity applied in our example, the value of charterer’s purchase option  $C$  is close or equal to  $S$  and the three curves converge at the left side when  $K$  amount nears to 0.

### 4) The discount rate ( $r$ )

The discount rate affects the value of the purchase options and the difference between the hire rates  $H_1 - H_2$  applied to contracts with charterer purchase options  $H_1$  and those without  $H_2$ . For a particular discount rate  $r$ , such as  $r = 2.5\%$  in our demonstration, the hire rate difference decreases as  $K$  increases. To further observe the effect of  $r$ , in addition to  $r = 2.5\%$ , we deliberately included two additional scenarios by assuming  $r = 1\%$  and  $5\%$ , respectively. The curves in Fig. 9 show the relationship between the  $K$ ’s and hire rate

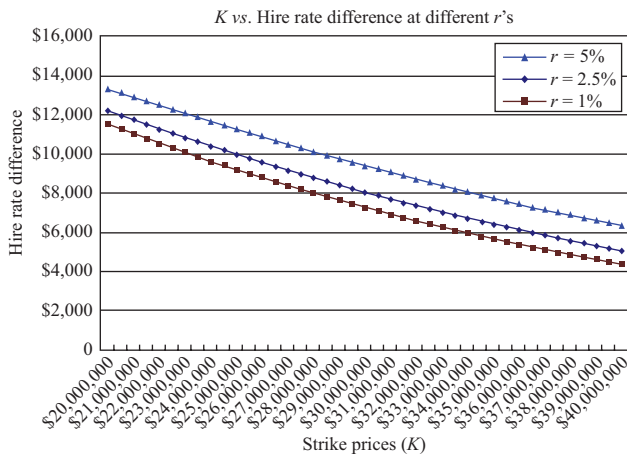


Fig. 9. The hire rate difference versus the strike prices under various discount rates.

differences under three  $r$ 's. Those curves show that, given a designated  $K$ , the higher the discount rate applied, the greater the hire rate difference ( $H_1 - H_2$ ) is. Therefore, we can conclude from the above that, in addition to the negative relationship between  $K$  and the hire rate, this trade-off relationship between the two variables becomes increasingly evident as  $r$  increases. In other words, charterers are willing to pay a higher charter hire rate for charters with purchase options if the discount rate  $r$  applied is higher. These curves also indicate that for a particular hire rate difference, the higher  $r$  yields higher  $K$ , implying that charterers can accept a higher strike price if  $K$  is discounted by a higher rate, thereby producing the same present value as that of lower strike price and discount rate combinations.

We have also observed from our calculations that the curves converged towards both sides. At the left side, the curves converged at  $K$  closes to 0, approximately 0.005, where the value of  $C$  is close to the spot ship price of US\$38,350,000 ( $C \approx S$ ) and  $H_1 - H_2 = \text{US}\$22,365$ . This finding coincides with the finding we obtained in the calculation of different times to maturity; likewise, regardless of the discount rates applied in our example, the value of charterer's purchase option  $C$  is close or equal to  $S$  and the three curves converge at the left side when  $K$  amount nears to 0. At the right side, the curves converged at  $K = \text{US}\$1,009,352,675$ , where both the values of  $C$  and  $H_1 - H_2$  were 0. This is because when the amount of  $K$  increases to an extremely large number, the chance of this option to be exercised becomes slim and regardless of the discount rates applied the values of charterer's purchase option in our example,  $C$ , becomes worthless.

#### IV. CONCLUSION

This study explores the trade-off relationship between the hire rates and the exercise prices of purchase options in ship charter contracts. The value of European-type purchase options is paid by charterers through a higher hire rate agreed

between the contractual parties. In addition to the exercise price of the purchase option, this value is affected by the prevailing spot ship prices, the volatility of the ship prices, the time to maturity and the discount rate. This study developed a European option pricing model for valuing purchase options. The numerical results indicate that higher spot ship prices increase the value of charterer purchase options. Additionally, for a particular spot ship price used in calculation, higher volatility of the spot prices produces a higher value of the option. This implies that higher volatility provides greater opportunity for spot prices to increase than lower volatility does; and thus, higher volatility increases the value of the option.

As a general trend, with a designated strike price, the longer the time to maturity of the option, the higher the required hire rate difference between the hire rate in contracts with charterer purchase option and that of contracts without charterer purchase options is. Furthermore, for a given  $K$ , as the discount rate increases, the required hire rate difference also increases. Therefore, for a given  $K$ , higher spot ship prices, higher volatility, longer time to maturity of the option and higher discount rates manifest the trade-off relationship between exercise prices ( $K$ ) and hire rates.

The above findings that we obtain in this study are meaningful to those shipping practitioners in the following prospects: (1) a rational and theoretical model is developed and can be employed in determining a reasonable hire difference between charters with and without charterer's European-style purchase option when they fix their bareboat or time charters of the same nature; (2) the influence of four major variables on the hire difference is presented for those practitioners' reference in evaluating the relevant hire rates. With combination of these four major variables, owners as well as charterers can calculate a reasonable hire rate applied to the charters with and without charterer's purchase options. For example, if the spot ship price is much higher than the agreed strike price, the volatility and the discount rate are high, and the time to maturity is long, then the owner will have to ask for and the charterer agree to pay a higher charter rate for charters with charterer's European-style purchase option because the option is more valuable to the charterer. Conversely, in the condition that the spot ship price is much lower than the agreed strike price, both the volatility and the discount rate are low, and the time to maturity is short, then the owner will probably ask for and the charterer agree to pay about the same charter rate for charters with charterer's purchase option as the one without the option. The option in the latter case is simply less valuable to charterers. Other inferences under different combination of the four variables can also be obtained on the basis of the theorem of the model we present in this research. The deadlock in negotiation of charter rates between owners and charterers for charters with charterer's European-style purchase option can thus be greatly avoidable.

Further study can be extended to the valuation of Bermudan-type call options where charterers holding the options are

provided multiple pre-fixed time points to exercise their right to purchase the vessel at fixed strike prices. Conversely, future studies can also examine the valuation of European-type put options where owners holding the options are guaranteed their vessel's residual value after termination of the charter by exercising their right to sell the vessel to charterers at fixed strike prices.

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