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PASSIVE FUZZY CONTROLLER DESIGN FOR DISCRETE SHIP STEERING SYSTEMS VIA TAKAGI-SUGENO FUZZY MODEL WITH MULTIPLICATIVE NOISES

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PASSIVE FUZZY CONTROLLER DESIGN FOR DISCRETE SHIP STEERING SYSTEMS VIA TAKAGI-SUGENO FUZZY MODEL WITH MULTIPLICATIVE NOISES

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Key words: passivity theory, Takagi-Sugeno fuzzy model, multiplicative noise, linear matrix inequality.

ABSTRACT

This paper proposes a passive fuzzy controller design for the discrete ship steering system that is represented by the Takagi-Sugeno (T-S) fuzzy model with multiplicative noises. Applying the Lyapunov theory for guaranteeing mean square stability, the sufficient conditions are developed to design the fuzzy controller for the T-S fuzzy model with multiplicative noises. The sufficient conditions derived in this paper belong to the Linear Matrix Inequality (LMI) forms which can be solved by the convex optimal programming algorithm. Besides, the fuzzy controller is carried out by the concept of Parallel Distribution Compensator (PDC). Finally, the simulation results are proposed to show that the strictly input passivity and mean square stability of the closed-loop system can be achieved via the designed fuzzy controller.

I. INTRODUCTION

It's well known that the T-S fuzzy models [2, 6, 8, 9, 11, 13, 15, 19, 20, 23] have become one of the useful control approaches for complex nonlinear systems. It can provide an effective representation of complex nonlinear systems in terms of fuzzy sets, described by a set of IF-THEN rules, which can locally represent linear input-output relations of nonlinear systems. Moreover, its stability analysis and synthesis issue can be transformed to a LMI problem. The LMI technique [1] has emerged as a powerful design tool in areas ranging from control engineering to system identification and structural design even in the T-S fuzzy models. In the point of control systems, recasting the stability analysis and controller design problems as LMI problems is equivalent to finding solutions to the original problems. In the literature, the convex optimal programming algorithm is usually used to solve the LMI problems [1]. Based on the PDC concept, the recasting of stability analysis and design of fuzzy control models to LMI problems was first made in [23]. Employing the PDC technique, some results have been developed to find fuzzy controllers via LMI scheme for the T-S fuzzy models [2, 6, 8, 9, 11, 13, 15, 20]. According to the PDC concept, the passive fuzzy controller design problem is studied in this paper for the discrete nonlinear ship steering system. In order to express better performances, the T-S fuzzy model is employed in this paper to construct the discrete nonlinear ship steering system.

The passivity property [14, 21, 25, 26] and Lyapunov function are usually used to issue stability analysis and synthesis of control systems. The energy of system can be represented by states via Lyapunov function and the system input energy can be represented by passivity properties. The Lyapunov function plays an important role in the stability analysis of nonlinear system described by state-space equations. Using the property of passivity, one can integrate the stability conditions for controlled systems via the Lyapunov theory. The passivity theory provides a useful concept for analyzing the stability of control systems. Based on the passivity theory [14, 21, 25, 26], the purpose of this paper is to deal with the analysis and synthesis problem for the fuzzy controller design of stochastic nonlinear systems which are constructed by the T-S fuzzy models with multiplicative noises. For attenuating the disturbance, the passivity theory is provided to derive the stability condition for designing the fuzzy controller. Some important applications of passivity theory to the control engineering can be referred to [4, 5, 7, 27]. Through applying the passivity theory, the stability sufficient conditions can be derived via the Lyapunov function and can be solved by the LMI technique.

The stability and stabilization problems of stochastic dynamic systems have been attracted much attenuation via stochastic differential equation [12] which can be described by

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Itô's form and Langevin's form. Comparing with the nominal differential equation, the stochastic ones appears a multiplicative noise term to characterize the stochastic behavior of system. In the term, the noise presents the random motions which maybe caused from vibrations, chemical reactions and so on. That means the noises often not only entry additively into a physical plant but also influence multiplicatively components of the plant. Based on stochastic differential equation, many efforts [10, 17, 28] have been proposed to extend the issues from deterministic system to stochastic systems. Due to the T-S fuzzy model has been applied to approximate the nonlinear system. Several stability criteria have been developed for nonlinear stochastic systems via T-S fuzzy model. Specifically, the fuzzy controller design for nonlinear stochastic system was investigated in [3, 16, 24, 29, 30]. According to the T-S fuzzy model with multiplicative noise, this paper intends to develop a fuzzy controller design scheme based on LMI technique such that the passivity property of the closed-loop system can be achieved. In order to demonstrate the effectiveness and applicability of the proposed fuzzy control approach, a discrete nonlinear ship steering system modeled by the T-S fuzzy model with multiplicative noise is introduced in the numerical example. By solving the stability conditions derived in this paper, a passive fuzzy controller can be obtained to stabilize the discrete nonlinear ship steering system.

Notations: The $E[Q(\bullet)]$ denotes the expected value of $Q(\bullet)$. The * denotes the transposed elements or matrices for symmetric position. The $tr(\mathbf{A})$ denotes the summation of the diagonal elements of matrix $\mathbf{A}$. The $\mathbf{I}$ denotes the identity matrix.

II. SYSTEM DESCRIPTIONS AND PROBLEM FORMULATIONS

Combining linear subsystems, a T-S fuzzy model can be constructed by a set of fuzzy IF-THEN rules that is used in this paper to deal with the stability analysis and synthesis problem of stochastic nonlinear systems. The i-th fuzzy rule of the T-S fuzzy model with multiplicative noises can be described as the following form.

Rule i:
IF $x_1(k)$ is M_{i1} and $x_2(k)$ is M_{i2} and … and $x_p(k)$ is M_{ip}
THEN

$$x(k+1) = \mathbf{A}_i x(k) + \mathbf{B}_{ui} u(k) + \mathbf{B}_{wi} w(k) + \left(\tilde{\mathbf{A}}_i x(k) + \tilde{\mathbf{B}}_{ui} u(k) + \tilde{\mathbf{B}}_{wi} w(k)\right)\beta(k) \tag{1a}$$

$$y(k) = \mathbf{C}_i x(k) + \mathbf{D}_{wi} w(k) \tag{1b}$$

or

$$x(k+1) = \sum_{i=1}^{r} h_i\left(x(k)\right)\left\{\mathbf{A}_i x(k) + \mathbf{B}_{ui} u(k) + \mathbf{B}_{wi} w(k) + \left(\tilde{\mathbf{A}}_i x(k) + \tilde{\mathbf{B}}_{ui} u(k) + \tilde{\mathbf{B}}_{wi} w(k)\right)\beta(k)\right\} \tag{2a}$$

$$y(k) = \sum_{i=1}^{r} h_i\left(x(k)\right)\left\{\mathbf{C}_i x(k) + \mathbf{D}_{wi} w(k)\right\} \tag{2b}$$

where $h_i\left(x(k)\right) = \dfrac{\prod_{j=1}^{p} M_{ij}\left(x_j(k)\right)}{\sum_{i=1}^{r}\prod_{j=1}^{p} M_{ij}\left(x_j(k)\right)}$ and $M_{ij}\left(x_j(k)\right)$ is the grade of membership function of the $x_j(k)$ in M_{ij}, M_{ij} is the fuzzy set; p is the premise variable number; $\mathbf{A}_i$, $\mathbf{B}_{ui}$, $\mathbf{B}_{wi}$, $\tilde{\mathbf{A}}_i$, $\tilde{\mathbf{B}}_{ui}$, $\tilde{\mathbf{B}}_{wi}$, $\mathbf{C}_i$ and $\mathbf{D}_{wi}$ are constant matrices with the compatible dimensions, $x(k) \in \Re^{n_x}$ is the state vector, $u(k) \in \Re^{n_u}$ is the input vector, r is the number of fuzzy rules; $w(k) \in \Re^{n_y}$ is the external disturbance input vector and the noise $\beta(k)$ is a scalar zero mean Gaussian white noise process with the covariance α and $\alpha > 0$. The noise $\beta(k)$ is assumed to satisfy the following properties $E\{\beta(k)\} = 0$, $E\{\beta(k)\beta(k)\} = \alpha$ and $E\{x(k)\beta(k)\} = E\{u(k)\beta(k)\} = E\{w(k)\beta(k)\} = 0$.

Applying the concept of PDC, the fuzzy controller is designed to share the same IF part of the T-S fuzzy model (1). Hence, the fuzzy controller can be represented as follows.

Rule i:
IF $x_1(k)$ is M_{i1} and $x_2(k)$ is M_{i2} and … and $x_p(k)$ is M_{ip}
THEN

$$u(k) = -\mathbf{F}_i x(k) \tag{3}$$

or

$$u(k) = \sum_{i=1}^{r} h_i\left(x(k)\right)\left(-\mathbf{F}_i x(k)\right) \tag{4}$$

Substituting (4) into (2), the closed-loop T-S fuzzy model can be obtained such as

$$\begin{aligned} x(k+1) = {} & \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(x(k)\right)h_j\left(x(k)\right)\Big\{\left(\mathbf{A}_i - \mathbf{B}_{ui}\mathbf{F}_j\right)x(k) + \mathbf{B}_{wi} w(k) \\ & + \left(\left(\tilde{\mathbf{A}}_i - \tilde{\mathbf{B}}_{ui}\mathbf{F}_j\right)x(k) + \tilde{\mathbf{B}}_{wi} w(k)\right)\beta(k)\Big\} \\ = {} & \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(x(k)\right)h_j\left(x(k)\right) \\ & \times\left\{\mathbf{G}_{ij} x(k) + \mathbf{B}_{wij} w(k) + \left(\tilde{\mathbf{G}}_{ij} x(k) + \tilde{\mathbf{B}}_{wij} w(k)\right)\beta(k)\right\} \end{aligned} \tag{5}$$

where

$$\mathbf{G}_{ij} = \frac{\mathbf{A}_i - \mathbf{B}_{ui}\mathbf{F}_j + \mathbf{A}_j - \mathbf{B}_{uj}\mathbf{F}_i}{2}, \tilde{\mathbf{G}}_{ij} = \frac{\tilde{\mathbf{A}}_i - \tilde{\mathbf{B}}_{ui}\mathbf{F}_j + \tilde{\mathbf{A}}_j - \tilde{\mathbf{B}}_{uj}\mathbf{F}_i}{2},$$

$$\mathbf{B}_{wij} = \frac{\mathbf{B}_{wi} + \mathbf{B}_{wj}}{2} \text{ and } \tilde{\mathbf{B}}_{wij} = \frac{\tilde{\mathbf{B}}_{wi} + \tilde{\mathbf{B}}_{wj}}{2}.$$

For attenuating external disturbance, the passivity theory provides a useful and effective tool to design the controller to achieve the energy constraints for the closed-loop systems. In the passivity theory, the supply rate is an important role to determine kind of energy change. In order to constrain the disturbance energy, a strict input passivity is chosen in this paper. The strict input passivity can be introduced in the following definition.

Definition 1

The system (5) with external disturbance $w(t)$ and output $y(t)$ is called strictly input passive if there exist a scalar $\gamma > 0$ such that

$$E\left\{2\sum_{k=0}^{k_q} y^{\mathrm{T}}(k)\ w(k)\right\} > E\left\{\gamma\sum_{k=0}^{k_q} w^{\mathrm{T}}(k)\ w(k)\right\} \tag{6}$$

for all $k_q \geq 0$. The k_q is the terminal time of control and it is positive. #

Applying the Definition 1, the following section provides sufficient conditions for the T-S fuzzy model with multiplicative noises to be strictly input passive and mean square stable.

III. FUZZY CONTROLLER DESIGN FOR T-S FUZZY MODELS WITH MULTIPLICATIVE NOISES

The fuzzy controller design for T-S fuzzy models with multiplicative noises is developed in this section. The sufficient conditions for guaranteeing the stability and passivity of closed-loop T-S fuzzy models are derived based on the Lyapunov theory and passivity theory. According to the T-S fuzzy model (5) with multiplicative noises, the stability conditions are derived in the following theorem.

Theorem 1

If there exists a positive definite matrix $\mathbf{P} > 0$, feedback gains $\mathbf{F}_{\mathrm{i}}$ and dissipative rate $\gamma > 0$ satisfy the following condition, then the closed-loop T-S fuzzy system (5) is strictly input passive and mean square stable.

$$\mathbf{\Lambda} + \begin{bmatrix} 0 & -\mathbf{C}_{\mathrm{i}}^{\mathrm{T}} \\ * & \gamma\mathbf{I} - \mathbf{D}_{\mathrm{i}}^{\mathrm{T}} - \mathbf{D}_{\mathrm{i}} \end{bmatrix} < 0 \tag{7}$$

where

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{G}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\mathbf{G}_{\mathrm{ij}} - \mathbf{P} + \sigma^2\tilde{\mathbf{G}}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{G}}_{\mathrm{ij}} & \mathbf{G}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\mathbf{B}_{\mathrm{wij}} + \sigma^2\tilde{\mathbf{G}}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{B}}_{\mathrm{wij}} \\ * & \mathbf{B}_{\mathrm{wij}}^{\mathrm{T}}\mathbf{P}\mathbf{B}_{\mathrm{wij}} + \sigma^2\tilde{\mathbf{B}}_{\mathrm{wij}}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{B}}_{\mathrm{wij}} \end{bmatrix} \tag{8}$$

Proof:

Let us choose a Lyapunov function as $V(x(k)) = x^{\mathrm{T}}(k)\mathbf{P}x(k)$ with $\mathbf{P} > 0$. By evaluating the first forward difference of $V(x(k))$ along the trajectory of (5), one can obtain

$$\begin{aligned} E\{\Delta V(x(k))\} &= E\{x^{\mathrm{T}}(k+1)\mathbf{P}x(k+1) - x^{\mathrm{T}}(k)\mathbf{P}x(k)\} \\ &= E\Bigg\{\sum_{\mathrm{i}=1}^{\mathrm{r}}\sum_{\mathrm{j}=1}^{\mathrm{r}} h_{\mathrm{i}}(x(k))h_{\mathrm{j}}(x(k))\Big\{\mathbf{G}_{\mathrm{ij}}x(k) + \mathbf{B}_{\mathrm{wij}}w(k) \\ &\quad + \left[\tilde{\mathbf{G}}_{\mathrm{ij}}x(k) + \tilde{\mathbf{B}}_{\mathrm{wij}}w(k)\right]\beta(k)\Big\}^{\mathrm{T}} \\ &\quad \times\mathbf{P}\Big\{\mathbf{G}_{\mathrm{ij}}x(k) + \mathbf{B}_{\mathrm{wij}}w(k) + \left[\tilde{\mathbf{G}}_{\mathrm{ij}}x(k) + \tilde{\mathbf{B}}_{\mathrm{wij}}w(k)\right]\beta(k)\Big\} \\ &\quad - x^{\mathrm{T}}(k)\mathbf{P}x(k)\Bigg\} \end{aligned} \tag{9}$$

Obviously, Eq. (9) can be rewritten as follows.

$$\begin{aligned} E\{\Delta V(x(k))\} &= E\Bigg\{\sum_{\mathrm{i}=1}^{\mathrm{r}}\sum_{\mathrm{j}=1}^{\mathrm{r}} h_{\mathrm{i}}(x(k))h_{\mathrm{j}}(x(k)) \\ &\quad \times\Big[x^{\mathrm{T}}(k)\left(\mathbf{G}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\mathbf{G}_{\mathrm{ij}} - \mathbf{P} + \sigma^2\tilde{\mathbf{G}}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{G}}_{\mathrm{ij}}\right)x(k) \\ &\quad + x^{\mathrm{T}}(k)\left(\mathbf{G}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\mathbf{B}_{\mathrm{wij}} + \sigma^2\tilde{\mathbf{G}}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{B}}_{\mathrm{wij}}\right)w(k) \\ &\quad + w^{\mathrm{T}}(k)\left(\mathbf{B}_{\mathrm{wij}}^{\mathrm{T}}\mathbf{P}\mathbf{G}_{\mathrm{ij}} + \sigma^2\tilde{\mathbf{B}}_{\mathrm{wij}}\mathbf{P}\tilde{\mathbf{G}}_{\mathrm{ij}}^{\mathrm{T}}\right)x(k) \\ &\quad + w^{\mathrm{T}}(k)\left(\mathbf{B}_{\mathrm{wij}}^{\mathrm{T}}\mathbf{P}\mathbf{B}_{\mathrm{wij}} + \sigma^2\tilde{\mathbf{B}}_{\mathrm{wij}}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{B}}_{\mathrm{wij}}\right)w(k)\Big]\Bigg\} \\ &= x^{\mathrm{T}}(k)\mathbf{G}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\mathbf{G}_{\mathrm{ij}}x(k) + x^{\mathrm{T}}(k)\mathbf{G}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\mathbf{B}_{\mathrm{wij}}w(k) \\ &\quad + w^{\mathrm{T}}(k)\mathbf{B}_{\mathrm{wij}}^{\mathrm{T}}\mathbf{P}\mathbf{G}_{\mathrm{ij}}x(k) + w^{\mathrm{T}}(k)\mathbf{B}_{\mathrm{wij}}^{\mathrm{T}}\mathbf{P}\mathbf{B}_{\mathrm{wij}}w(k) \\ &\quad + \sigma^2x^{\mathrm{T}}(k)\tilde{\mathbf{G}}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{G}}_{\mathrm{ij}}x(k) + \sigma^2x^{\mathrm{T}}(k)\tilde{\mathbf{G}}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{B}}_{\mathrm{wij}}w(k) \\ &\quad + \sigma^2w^{\mathrm{T}}(k)\tilde{\mathbf{B}}_{\mathrm{wij}}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{G}}_{\mathrm{ij}}x(k) + \sigma^2w^{\mathrm{T}}(k)\tilde{\mathbf{B}}_{\mathrm{wij}}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{B}}_{\mathrm{wij}}w(k) \\ &\quad - x^{\mathrm{T}}(k)\mathbf{P}x(k) \end{aligned} \tag{10}$$

Arranging the Eq. (10), one has

$$\begin{aligned} &E\{\Delta V(x(k))\} \\ &= E\left\{\sum_{\mathrm{i}=1}^{\mathrm{r}}\sum_{\mathrm{j}=1}^{\mathrm{r}} h_{\mathrm{i}}(x(k))h_{\mathrm{j}}(x(k))\begin{bmatrix} x(t) \\ w(t) \end{bmatrix}^{\mathrm{T}}\mathbf{\Lambda}\begin{bmatrix} x(t) \\ w(t) \end{bmatrix}\right\} \end{aligned} \tag{11}$$

where $\mathbf{\Lambda}$ is defined in (8). Integrating both side of (11) form 0 to k_q with zero initial condition, then one has

$$E\left\{V\left(x\left(k_q\right)\right)\right\} = E\left\{\sum_{k=0}^{k_q}\Delta V(x(k))\right\} \tag{12}$$

For nonzero external disturbance, i.e., $w(t) \neq 0$, one can define a performance function such as

$$J_D \le E\left\{\sum_{k=0}^{k_q}\left(\gamma w^{\mathrm{T}}(k)w(k) - 2y^{\mathrm{T}}(k)w(k)\right) + V\left(x(k)\right)\right\}$$
$$= E\left\{\sum_{k=0}^{k_q}\left(\gamma w^{\mathrm{T}}(k)w(k) - 2y^{\mathrm{T}}(k)w(k) + \Delta V\left(x(k)\right)\right)\right\}$$
$$\equiv E\left\{\sum_{k=0}^{k_q} L(x,w,k)\right\} \tag{13}$$

where

$$L(x\ ,w\ ,k) = \gamma w^{\mathrm{T}}(k)w(k) - 2y^{\mathrm{T}}(k)w(k) + \Delta V\left(x(k)\right) \tag{14}$$

Substituting (2b) and (11) into (14), one has

$$L(x\ ,w\ ,k) \le \sum_{\mathrm{i}=1}^{\mathrm{r}}\sum_{\mathrm{j}=1}^{\mathrm{r}} h_{\mathrm{i}}\left(x(k)\right)h_{\mathrm{j}}\left(x(k)\right)$$
$$\times\begin{bmatrix} x(k) \\ w(k)\end{bmatrix}^{\mathrm{T}}\left(\boldsymbol{\Lambda} + \begin{bmatrix} 0 & -\mathbf{C}_{\mathrm{i}}^{\mathrm{T}} \\ * & \gamma\mathbf{I} - \mathbf{D}_{\mathrm{i}}^{\mathrm{T}} - \mathbf{D}_{\mathrm{i}}\end{bmatrix}\right)\begin{bmatrix} x(k) \\ w(k)\end{bmatrix} \tag{15}$$

If the condition (7) of Theorem 1 is satisfied, then one can deduce that $\boldsymbol{\Lambda} + \begin{bmatrix} 0 & -\mathbf{C}_{\mathrm{i}}^{\mathrm{T}} \\ * & \gamma\mathbf{I} - \mathbf{D}_{\mathrm{i}}^{\mathrm{T}} - \mathbf{D}_{\mathrm{i}}\end{bmatrix} < 0$ and $L(x, w, k) < 0$. From (13), the inequality $L(x, w, k) < 0$ implies

$$J_D < 0 \tag{16}$$

or

$$E\left\{2\sum_{k=0}^{k_q} y^{\mathrm{T}}(k)w(k)\right\} > E\left\{\sum_{k=0}^{k_q}\gamma w^{\mathrm{T}}(k)w(k)\right\} \tag{17}$$

for all nonzero external disturbance. Since (17) is equivalent to (6), the system is strictly input passive.

Next, it is necessary to show that the system is mean square stable. According to (15), if the condition (7) is held, i.e., $\boldsymbol{\Lambda} + \begin{bmatrix} 0 & -\mathbf{C}_{\mathrm{i}}^{\mathrm{T}} \\ * & \gamma\mathbf{I} - \mathbf{D}_{\mathrm{i}}^{\mathrm{T}} - \mathbf{D}_{\mathrm{i}}\end{bmatrix} < 0$, thus one has $L(x, w, k) < 0$. By assuming $w(k) = 0$, one can find $\Delta V(x(k)) < 0$ from (14) due to $L(x, w, k) < 0$, one has

$$E\left\{\Delta V\left(x(k)\right)\right\} < 0 \tag{18}$$

Based on the Lemma 6.1 of [7], one can find that the system is mean square stable driven by control law (4) under the case of $E\{\Delta V(x(k))\} < 0$. The proof of this theorem is completed.

#

The stability condition derived in this theorem cannot be calculated by convex optimization algorithm directly. So, the condition of Theorem 1 must be converted into LMI problems for finding the solutions of fuzzy controllers. For this reason, the Schur complement [1] is employed to convert the above condition into the LMI form in the following theorem.

Theorem 2

If there exists a positive definite matrix $\mathbf{P} > 0$, feedback gains $\mathbf{F}_{\mathrm{i}}$ and dissipative rate $\gamma > 0$ satisfy the following conditions, then the T-S fuzzy system (5) is strictly input passive and mean square stable.

$$\begin{bmatrix} -\mathbf{X} & -\mathbf{X}\mathbf{C}_{\mathrm{i}}^{\mathrm{T}} & \mathbf{R}_{\mathrm{ij}}^{\mathrm{T}} & \sigma\tilde{\mathbf{R}}_{\mathrm{ij}}^{\mathrm{T}} \\ * & \gamma\mathbf{I} - \mathbf{D}_{\mathrm{i}}^{\mathrm{T}} - \mathbf{D}_{\mathrm{i}} & \mathbf{B}_{w\mathrm{ij}}^{\mathrm{T}} & \sigma\tilde{\mathbf{B}}_{w\mathrm{ij}}^{\mathrm{T}} \\ * & * & -\mathbf{X} & 0 \\ * & * & * & -\mathbf{X}\end{bmatrix} < 0 \tag{19}$$

where

$$\mathbf{R}_{ij} = \left(\frac{\mathbf{A}_{\mathrm{i}}\mathbf{X} - \mathbf{B}_{u\mathrm{i}}\mathbf{Y}_{\mathrm{j}} + \mathbf{A}_{\mathrm{j}}\mathbf{X} - \mathbf{B}_{u\mathrm{j}}\mathbf{Y}_{\mathrm{i}}}{2}\right)$$
$$\tilde{\mathbf{R}}_{ij} = \left(\frac{\tilde{\mathbf{A}}_{\mathrm{i}}\mathbf{X} - \tilde{\mathbf{B}}_{u\mathrm{i}}\mathbf{Y}_{\mathrm{j}} + \tilde{\mathbf{A}}_{\mathrm{j}}\mathbf{X} - \tilde{\mathbf{B}}_{u\mathrm{j}}\mathbf{Y}_{\mathrm{i}}}{2}\right)$$
$$\mathbf{X} = \mathbf{P}^{-1},\ \mathbf{Y}_{\mathrm{i}} = \mathbf{F}_{\mathrm{i}}\mathbf{X} \tag{20}$$

Proof:

Using the Schur complement [1], the inequality (19) can be written as follows.

$$\begin{bmatrix} -\mathbf{X} + \mathbf{R}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{X}^{-1}\mathbf{R}_{\mathrm{ij}} + \sigma^2\tilde{\mathbf{R}}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{X}^{-1}\tilde{\mathbf{R}}_{\mathrm{ij}} & \mathbf{R}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{X}^{-1}\mathbf{B}_{w\mathrm{ij}} - \mathbf{X}\mathbf{C}_{\mathrm{i}}^{\mathrm{T}} + \sigma^2\tilde{\mathbf{R}}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{X}^{-1}\tilde{\mathbf{B}}_{w\mathrm{ij}} \\ * & \gamma\mathbf{I} - \mathbf{D}_{\mathrm{i}}^{\mathrm{T}} - \mathbf{D}_{\mathrm{i}} + \mathbf{B}_{w\mathrm{ij}}^{\mathrm{T}}\mathbf{X}^{-1}\mathbf{B}_{w\mathrm{ij}} + \sigma^2\tilde{\mathbf{B}}_{w\mathrm{ij}}^{\mathrm{T}}\mathbf{X}^{-1}\tilde{\mathbf{B}}_{w\mathrm{ij}}\end{bmatrix} < 0 \tag{21}$$

By setting $\mathbf{P} = \mathbf{X}^{-1}$ and $\mathbf{Y}_{\mathrm{i}} = \mathbf{F}_{\mathrm{i}}\mathbf{X}$, the inequality (21) can be arranged in the following form.

$$\begin{bmatrix} \mathbf{P}^{-1}\mathbf{G}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\mathbf{G}_{\mathrm{ij}}\mathbf{P}^{-1} - \mathbf{P}^{-1} + \sigma^2\mathbf{P}^{-1}\tilde{\mathbf{G}}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{G}}_{\mathrm{ij}}\mathbf{P}^{-1} & \mathbf{P}^{-1}\mathbf{G}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\mathbf{B}_{w\mathrm{ij}} - \mathbf{P}^{-1}\mathbf{C}_{\mathrm{i}}^{\mathrm{T}} + \sigma^2\mathbf{P}^{-1}\tilde{\mathbf{G}}_{\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{B}}_{w\mathrm{ij}} \\ * & \gamma\mathbf{I} - \mathbf{D}_{\mathrm{i}}^{\mathrm{T}} - \mathbf{D}_{\mathrm{i}} + \mathbf{B}_{w\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\mathbf{B}_{w\mathrm{ij}} + \sigma^2\tilde{\mathbf{B}}_{w\mathrm{ij}}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{B}}_{w\mathrm{ij}}\end{bmatrix} < 0 \tag{22}$$

Pre- and post-multiplying the inequality (22) by $diag\{\mathbf{P}, \mathbf{I}\}$, one can obtain the following inequality.

$$\mathbf{\Lambda} + \begin{bmatrix} 0 & -\mathbf{C}_i^T \\ * & \gamma\mathbf{I} - \mathbf{D}_i^T - \mathbf{D}_i \end{bmatrix} < 0 \tag{23}$$

It is easy to find that (23) is equivalent to (7). One can conclude that if the condition (19) of Theorem 2 is satisfied then the condition (7) of Theorem 1 is also held. Thus, if the condition (19) is satisfied then the closed-loop T-S fuzzy system (5) is strictly input passive and mean square stable. #

Based on the condition of Theorem 2, the feasible solutions can be obtained via LMI technique by using MATLAB LMI-Toolbox. And then, the fuzzy controller can be designed via PDC technique with state feedback gains. Hence, the proposed fuzzy controller design for T-S fuzzy models with multiplicative noises can be achieved by solving the LMI conditions (19). In the following section, the proposed design method is applied to design a passive fuzzy controller for the nonlinear ship steering system.

IV. FUZZY CONTROLLER DESIGN FOR DISCRETE NONLINEAR SHIP STEERING SYSTEM

In this section, the proposed technique for T-S fuzzy model with multiplicative noise is employed to design the fuzzy controller to achieve stability and passivity performances. In the late 1950s, Nomoto has established the response ship steering movement mathematical model from control theory's viewpoint [18]. Furthermore, referring to [22], the second order Nomoto model in [18] can be simplified into first order model. The simplified Nomoto model can describe the great rudder angle ship steering characteristic and express the unstable degree of ship. The Nomoto's first-order ship steering system can be expressed such as

$$T\ddot{\psi} + H(\dot{\psi}) = K\delta \tag{24}$$

where

$$H(\dot{\psi}) = \alpha_0 + \alpha_1\dot{\psi} + \alpha_2\dot{\psi}^2 + \alpha_3\dot{\psi}^3 \tag{25}$$

and α_i(i = 0, 1, 2, 3) is the coefficients of Norrbin. The mathematical model (24) describes that the rudder angle $\delta(s)$ is the system input and the heading angle $\psi(s)$ is the system output of dynamic systems. For ship hull with symmetrical, one has $\alpha_0 = \alpha_2 = 0$. Besides, $\alpha_1 = 1$ expresses the ships for the heading stabilization, $\alpha_1 = -1$ expresses the ships for the heading instability; and α_3 can be determined by the spiral test [22]. Using the characteristics $H(\dot{\psi})$ (25) to replace the nonlinear term $\dot{\psi}$ of (24), the corresponding nonlinear ship motion model can be obtained as follows.

$$T\ddot{\psi} + \alpha_1\dot{\psi} + \alpha_3\dot{\psi}^3 = K\delta \tag{26}$$

Referring to [31], the discrete type of (26) can be obtained with simple definition. Using the same definitions, let define $\ddot{\psi} = r(k+1)$ and $\dot{\psi} = r(k)$, where ψ is the heading angle. Then, one can directly find the discrete model of (26) as follows.

$$Tr(k+1) + \alpha_3 r^3(k) + \alpha_1 r(k) = K\delta \tag{27}$$

where r is the yaw angular velocity. Selecting $\delta = u$, $x_1(k) = \psi$ and $x_2(k) = r$, the Eq. (27) can be rewritten as state space equation. In addition, let us consider the external disturbance effect on the ship and stochastic behaviors of system. Thus, the discrete ship steering model (27) can be rewritten with as follows.

$$x_1(k+1) = x_2(k) + 0.1w(k) + \left(0.033x_2(k) + 0.01w(k)\right)\beta(k) \tag{28a}$$

$$x_2(k+1) = \frac{-1}{T}\left(\alpha_1 x_2(k) + \alpha_3 x_2^3(k)\right) + \frac{K}{T}u(k) + \left(-0.0001x_2(k) + 0.00016u(k)\right)\beta(k) \tag{28b}$$

In this example, it is assumed that the length of the ship is 126 m, the width of the ship is 20.8 m, the loaded draft is 8.0 m, square coefficient is 0.681, the ship speed is 7.2 *m*/*s* and the parameters of state Eq. (28) are given as $T = 261.73s$, $K = 0.42s^{-1}$, $\alpha_1 = 1$, $\alpha_3 = 30$. In this example, the max angle and max change rate of rudder angle have been considered in the revised paper. The working range of max angle is constrained between ±90°. And the max change rate of rudder angle is constrained between ± 2. Thus, employing the fuzzy modeling technique expressed in [20], the T-S fuzzy model with multiplicative noise for the ship steering system (28) can be described as follows with three fuzzy rules.

$$x(k+1) = \sum_{i=1}^{3}\bar{h}_i\left\{\mathbf{A}_i x(k) + \mathbf{B}_{ui}u(k) + \mathbf{B}_{wi}w(k) + \left(\tilde{\mathbf{A}}_i x(k) + \tilde{\mathbf{B}}_{ui}u(k) + \tilde{\mathbf{B}}_{wi}w(k)\right)\beta(k)\right\} \tag{29a}$$

$$y(k) = \sum_{i=1}^{3}\bar{h}_i\left\{\mathbf{C}_i x(k) + \mathbf{D}_{wi}w(k)\right\} \tag{29b}$$

where

$$\mathbf{A}_1 = \mathbf{A}_3 = \begin{bmatrix} 0 & 1 \\ 0 & -0.4623 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ 0 & -0.0038 \end{bmatrix},$$

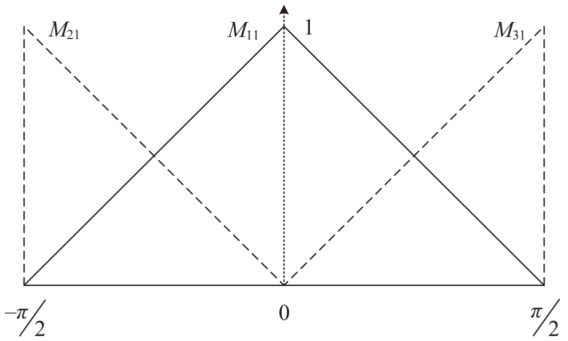

Fig. 1. The membership function of state $x_2(k)$.

$$\mathbf{B}_{u1}=\mathbf{B}_{u2}=\mathbf{B}_{u3}=\begin{bmatrix}0\\0.0016\end{bmatrix},\ \mathbf{B}_{w1}=\mathbf{B}_{w2}=\mathbf{B}_{w3}=\begin{bmatrix}0.1\\0\end{bmatrix},$$

$$\tilde{\mathbf{A}}_1=\tilde{\mathbf{A}}_2=\tilde{\mathbf{A}}_3=\begin{bmatrix}0 & 0.0333\\0 & -0.0001\end{bmatrix},\ \tilde{\mathbf{B}}_{w1}=\tilde{\mathbf{B}}_{w2}=\tilde{\mathbf{B}}_{w3}=\begin{bmatrix}0.01\\0\end{bmatrix},$$

$$\tilde{\mathbf{B}}_{u1}=\tilde{\mathbf{B}}_{u2}=\tilde{\mathbf{B}}_{u3}=\begin{bmatrix}0\\0.0001605\end{bmatrix},\ \mathbf{C}_1=\mathbf{C}_2=\mathbf{C}_3=\begin{bmatrix}1 & 0\end{bmatrix},$$

and

$$\mathbf{D}_1=\mathbf{D}_2=\mathbf{D}_3=1.$$

The membership functions of the T-S fuzzy system are presented in Fig. 1. Through applying the proposed design technique, the fuzzy controller can be obtained to guarantee the considered system achieving mean square stability and the passivity performance. For starting analyzing and designing, one can first select the supply rate γ = 1.1985 and σ = 1. Solving the sufficient condition (19) via MATLAB LMI-Toolbox, the solution of common positive definite matrix **P** can be obtained as follows.

$$\mathbf{P}=\begin{bmatrix}0.8565 & -0.0764\\-0.0764 & 0.9374\end{bmatrix} \tag{30}$$

And then the fuzzy controller can be stated as

$$u(t)=-\sum_{i=1}^{3}h_i\left(x(k)\right)\mathbf{F}_i x(k) \tag{31}$$

where $\mathbf{F}_1$ = [–5.1125 338.9103], $\mathbf{F}_2$ = [–5.2045 56.036] and $\mathbf{F}_3$ = [–5.2048 338.9234].

For emphasizing the important as considering the disturbance and stochastic behaviors effect on system, the fuzzy controller designed by [31] is applied to stabilize the system (28). The fuzzy controller in [31] was designed with nine fuzzy rules for system (28) without external disturbance and multiplicative noise. In this example, the external disturbance

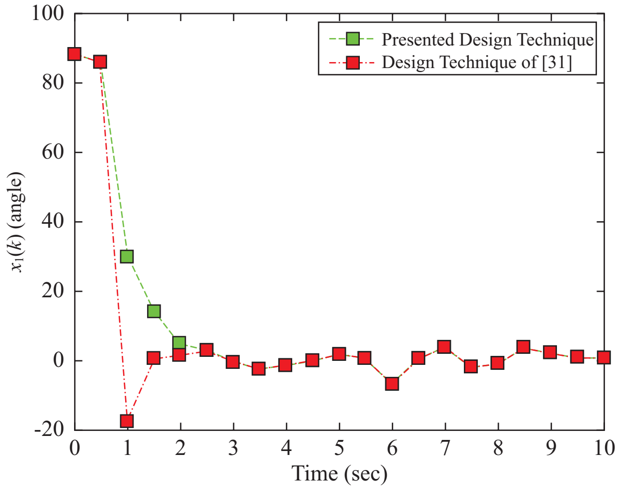

Fig. 2. Responses of the state $x_1(k)$.

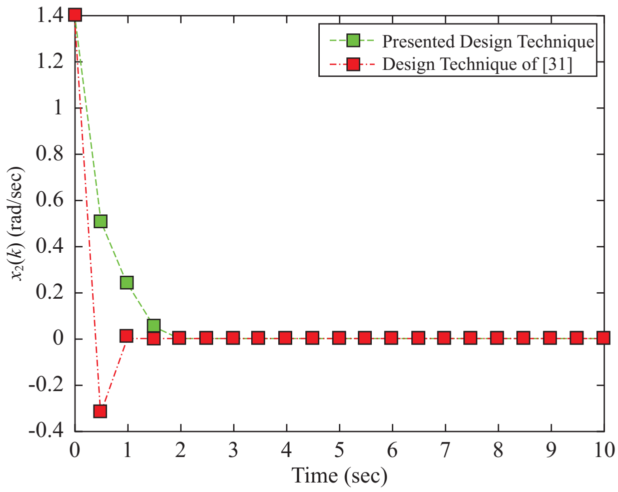

Fig. 3. Responses of the state $x_2(k)$.

$w(k)$ is chosen as a zero mean white noise with variance 0.1 and initial condition is chosen as $x(0) = [88° \ \ 1.4]^T$. From the simulation results, in Figs. 2 and 3, one can find that both of controllers designed by this paper and [31] can stabilize the system (28). The controller in [31] has great robustness in stabilizing the system that is caused by nine fuzzy rules. However, from [4], the difficulty in stability analysis and synthesis of fuzzy model is increasing with raising fuzzy rules. Besides, from the simulation results, the over shoot of states in (28) with fuzzy controller designed by [31] are bigger than that driven by (31). Hence, one can find that the proposed design technique can provide improvement for previous works. And, the discrete nonlinear ship steering system (28) can be stabilized by the passive fuzzy controller (31).

V. CONCLUSIONS

The attenuation of the external disturbance energy and the controller design problems for the discrete T-S fuzzy model with multiplicative noises have been studied in this paper. For attenuation performance, the passivity theory was employed to derive stability condition based on the Lyapunov function. The stochastic behavior of control system was discussed by considering the multiplicative noise term which can be analyzed in sense of mean square stability. In order to apply convex optimal programming algorithm to solve the proposed fuzzy controllers, the stability conditions must be converted into the LMI forms. By solving the derived LMI stability conditions, the fuzzy controller can be carried out by the concept of PDC. Finally, the simulation results showed that the discrete nonlinear ship steering system can be controlled to be strictly input passive and mean square stable via the designed fuzzy controllers.

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