Journal of Marine Science and Technology

Volume 21 | Issue 2 Article 6

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Acknowledgements

The research reported herein is partly supported by National Natural Science Foundation of China (grants No. 50978024, 51008018) and grants No. NSC 99-2221-E-032- 020-MY3 of National Science Council, Taiwan. These financial supports are gratefully acknowledged.

Journal of Marine Science and Technology, Vol. 21, No. 2, pp. 149-158 (2013)
DOI: 10.6119/JMST-012-0206-4

VIBRATION SUPPRESSION OF TRAIN-INDUCED MULTIPLE RESONANT RESPONSES OF TWO-SPAN CONTINUOUS BRIDGES USING VE DAMPERS

Ying-Jie Wang[1,2], Jong-Dar Yau[3,4], and Qing-Chao Wei[1]

Key words: high speed trains, two-span continuous bridge, multiple resonances, vibration suppression, visco-elastic (VE) dampers.

ABSTRACT

This paper presents a study of reducing multiple resonant peaks of high-speed trains traveling over a two-span continuous railway bridge using visco-elastic (VE) dampers. The moving train is modeled as a series of two-degree-of-freedom (2 DOF) moving oscillators with identical intervals and the two-span continuous bridge as a Bernoulli-Euler beam with uniform span. Because of closely spaced frequencies of a continuous beam, the first several frequencies of the bridge would be excited to resonance during the passage of moving trains, which may result in multiple resonant peaks appearing on the response of train-bridge system. In order to reduce the multiple resonant peaks of both the high-speed trains and the two-span continuous bridge, VE dampers are installed at the bottom of the bridge deck to absorb the vibrating energy transmitted from the bridge. From the numerical investigations, the inclusion of VE dampers is useful to simultaneously suppress the multiple resonant peaks of train-bridge system and further improves the riding comfort of moving vehicles. Meanwhile, the strategy of arranging the VE dampers on the continuous bridge is also investigated.

I. INTRODUCTION

For railway bridges subjected to high-speed trains, excessive vibration may occur due to a resonance phenomenon caused by periodic wheel-loads of the trains [21]. The resonant vibration of the bridges could not only cause the maintenance and repair costs to be high, but also reduce the riding comfort of passengers. For the sake of better riding comfort of passing trains, the vertical acceleration responses of moving vehicles should be kept within allowable limit; for instance, the limits of 0.10 g (0.98 m/s^2) was suggested in Eurocode [3].

To suppress the resonant vibrations of moving trains and bridges, several schemes through train-bridge system parameters adjustments, implementation of vibration absorbers on bridges and so on have been proposed [7, 8, 13, 14, 16, 20, 23, 24]. Shin *et al.* [14] brought out a new vibration reduction scheme by inserting size-adjusted vehicle(s) in the existing KTX train arrangement and producing out-of-phase loading to suppress the resonance phenomenon. For vibration reduction of railway bridges, the tuned mass damper (TMD) devices have often been adopted as an effective means for suppressing excessive vibration induced by high speed trains [7, 8, 16, 23, 24]. Wang *et al.* [16] studied the effectiveness of the single tuned mass damper (TMD) to suppress the train-induced vibration of simply supported bridges. Then Li *et al.* [7] and Lin *et al.* [8] installed multiple tuned mass dampers (MTMD) to the bridge and discussed the advantages of MTMD over a single TMD. Yau [23] presented a string-type tuned mass damper (STMD) which is composed of a distributed TMD subsystem and a stretched string to mitigate the resonant response of a simple beam due to moving loads. Most of the above researches are based on the simple supported beam, and the first vibration mode contributes almost entirely to the dynamic response of the beam. But for a continuous bridge with closely spaced frequencies, the multiple resonances of the bridge may occur during the passage of trains moving at high speeds [22]. Yau *et al.* [24] developed a wideband multiple tuned mass dampers (MTMD) system to reduce the multiple resonant responses of continuous truss bridges due to moving trains. However, the first disadvantage of the TMD or MTMD is the sensitivity related to the narrow band control and the fluctuation in tuning frequency to the

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controlled frequency of the bridge, and the other limitations are the size of the additional mass (normally about 1% of bridge weight) and the stroke demand of TMD or MTMD.

As the conventional TMD device consisting of a mass connected to structures using a spring and a viscous damper, the mechanism of suppressing structural vibrations is to transfer the vibration energy of the structure to the TMD. From the view of dissipating the vibration energy, Hwang *et al.* [5] conducted the seismic response control of highway bridges installed viscous dampers and proposed design formulations of supplemental linear and non-linear viscous dampers. Recently, Reis *et al.* [13] studied the response of a simply supported beam with viscous damping carrying a moving force by Fourier sine series approach. Yang *et al.* [20] investigated the performance of a tuned mass (TM) (with no damping capability) in suppressing the vibration response of an elastically supported beam to a moving train. The objective of this research is to study the potential availability of using VE dampers to mitigate the multiple resonant peaks of the two-span continuous bridge and moving trains. More specifically, the authors propose the use of *linear* VE dampers as the absorbers attached to the bottom of the continuous bridge for vibration mitigation. In order to achieve this objective, a general formulation of the vehicle-induced bridge vibration controlled with VE dampers is first developed. Then, the feasibility of using VE dampers to reduce the vibration response of the continuous beam and moving trains will be conducted comprehensively. Numerical results indicate that the installation of VE dampers into the bridge can effectively suppress the multiple resonant peaks of the train-bridge system.

II. FORMULATION OF THE PROBLEM

1. Basic Considerations

In the past one decade, much work has been carried out towards analyzing the vehicle-bridge interaction related to the dynamic behavior of bridges, reduction of bridge vibration, and enhancement of the ride comfort [4, 6, 9, 10, 15, 18, 19, 26]. Michaltsos [10] developed a mathematical model to analyze the phenomenon of the bouncing of a vehicle due to an irregularity and determined the critical velocity for which the vehicle loses touch with the road. Stăncioiu *et al.* [15] investigated the dynamics of a two-axle vehicle system travelling along an elastic beam and pointed out that separation may take place at moderate speeds. But for simplicity, the separation of vehicle and beam would be ignored in some researches. Lou [9] derived a finite train-track-bridge coupling element with train and track contact model for analyzing dynamic responses of train, track and bridges. Didn't considering the separation of train and bridge, Ziyaeifar [26] proposed a Maxwell (three-element type) vehicle-bridge-track interaction system and studied the effects of TMD devices in vibration control of bridges and trains. Xia *et al.* [18] built a 3D train-bridge dynamic model to study the dynamic

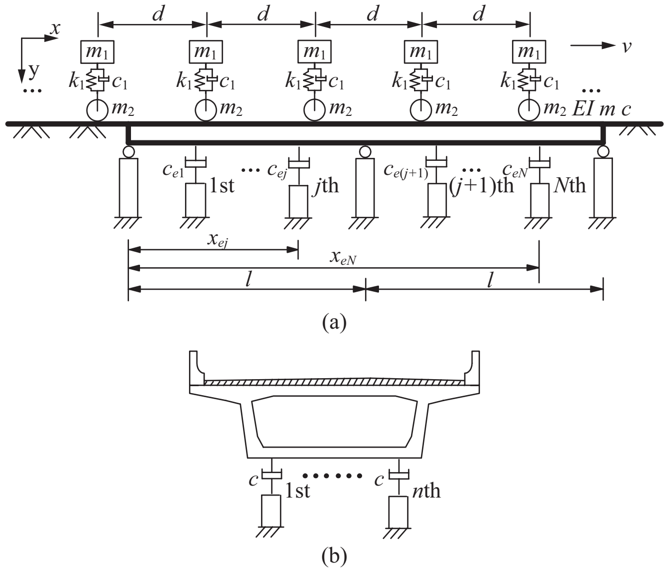


Fig. 1. Train-bridge system model: (a) Successive oscillators moving on a two-span continuous bridge with multiple VE dampers and (b) Installation of VE dampers under bridge deck.

interaction between high-speed trains and bridges. In this model, the vehicle was modeled by the rigid-body dynamics method and the bridge was modeled using the modal superposition technique. Besides, Xia *et al.* [19] also investigated the resonance mechanism and conditions of the train-bridge system through theoretical derivations, numerical simulations, and experimental data analyses. Lee *et al.* [6] also assumed the wheel remained in contact with the bridge and developed 3D dynamic analysis model to predict accurately the dynamic response of a high speed train interacting with a railway bridge. He *et al.* [4] modeled the carriage as 15 DOFs sprung-mass system and the bridge with 3D finite elements, and evaluated the influence of dynamic bridge-train interaction (BTI) on the seismic response of the Shinkansen system in Japan under moderate earthquakes. In this study, the train is assumed to be moving on the continuous bridge without losing contact with it. Bouncing, impact effects and surface roughness of the bridge are not considered.

The interaction dynamics of train-bridge system investigated herein is limited to the vertical vibrations. As shown in Fig. 1(a), the two-span continuous bridge is modeled as a linear elastic Bernoulli-Euler beam with uniform span length (l). In practice, VE dampers can provide efficient energy absorbers for a vibrating structure because of easy maintenance, simple construction, and good performance in vibration mitigation. The external VE dampers are installed between the bottom of bridge deck and the supporting columns. The moving vehicles are simulated as a series of equally spaced 2 DOF moving oscillators and each oscillator consists of a sprung mass (m_1) and an unsprung mass (m_2) interconnected by a spring (k_1) and a dashpot (c_1). Since this paper is focused on vibration suppression of a two-span continuous bridge subjected to moving trains by using VE dampers, the present simplified sprung-mass model may not include the pitching effect of a carriage. Even so, this consideration is acceptable for bridge response because of little mass ratio of train to bridge for high speed rails. The VE dampers are supposed to be installed by connecting the continuous bridge to the *rigid* columns. For the jth VE dampers, the damping constant is denoted as c_{ej} and its location is designated as x_{ej} from the left-hand end of the bridge (see Fig. 1(a)). As the bridge cross-section shown in Fig. 1(b), the jth set of VE dampers is composed of n single VE dampers and the total damping coefficient is denoted as c_{ej}.

2. Equations of Motion

Fig. 1 depicts a sequence of identical 2 DOF moving oscillators with equal intervals d is moving on two-span continuous beam at constant speed v. The equation of motion for the two-span continuous beam subjected to moving vehicles can be written as

$$EI\frac{\partial^4 y(x,t)}{\partial x^4}+m\frac{\partial^2 y(x,t)}{\partial t^2}+c\frac{\partial y(x,t)}{\partial t}=F(x,t) \tag{1}$$

where m is the constant mass per unit length, EI is the flexural rigidity, c the damping of the beam, $y(x, t)$ the displacement of the beam, and $F(x, t)$ is the external force from moving oscillators and the external damping force induced by the VE dampers acting on the beam, and

$$\begin{aligned}F(x,t)=&\sum_{k=1}^{N_v}\Big\{(m_1+m_2)g-m_2\ddot{y}(x_k,t)\\&+c_1\left[\dot{Z}_k(t)-\dot{y}(x_k,t)\right]+k_1\left[Z_k(t)-y(x_k,t)\right]\Big\}\\&\cdot\delta(x-x_k)\cdot\left[H(t-t_k)-H(t-t_k-\Delta t)\right]\\&-\sum_{j=1}^{N}c_{ej}\frac{\partial y(x,t)}{\partial t}\delta(x-x_{ej})\end{aligned} \tag{2}$$

where $(\dot{\bullet})=\partial(\bullet)/\partial(t)$, $H(\cdot)$ = unit step function, and $\delta(\cdot)$ = Dirac's delta function, k = 1, 2, … N_vth moving load on the beam, j = 1, 2, … Nth set of VE dampers attached to the beam, Z_k = vertical displacement of the kth sprung mass, x_k = position of the kth oscillator along the beam, $t_k = (k\text{-}1)dv$ = arriving time of the kth oscillator into the beam, $\Delta t = 2l/v$ = the time of the oscillator passing the beam.

When the kth oscillator runs on the two-span continuous beam, the motion equation of the sprung mass can be written as

$$m_1\ddot{Z}_k(t)+c_1\left[\dot{Z}_k(t)-\dot{y}(x_k,t)\right]+k_1\left[Z_k(t)-y(x_k,t)\right]=0 \tag{3}$$

where $y(x_k, t)$ = vertical displacement of the beam where the kth unsprung mass is located.

III. METHOD OF SOLUTION

To carry out the interaction dynamics of train-bridge system, modal superposition method will be adopted in this study, and the displacement of the beam in mode coordinate can be expressed as

$$y(x,t)=\sum_{n=1}^{\infty}q_n(t)\phi_n(x)\quad(n=1,2\cdots\infty) \tag{4}$$

where $q_n(t)$ is the generalized coordinate associated with the nth natural mode and $\phi_n(x)$ is the nth mode shape function of the beam. The mode shape function of the two-span continuous beam can be divided into two groups, which are symmetrical and asymmetrical, respectively. The circular frequency can be identified as

$$\omega_i=\lambda_i^2\sqrt{\frac{EI}{m}}=\begin{cases}\left(\dfrac{i+1}{2l}\pi\right)^2\sqrt{\dfrac{EI}{m}} & i=1,3,5,\cdots\\[2ex]\left(\dfrac{0.25+i/2}{l}\pi\right)^2\sqrt{\dfrac{EI}{m}} & i=2,4,6,\cdots\end{cases} \tag{5}$$

When i = 1, 3, 5 … and $x\in[0, 2l]$, the mode shape is expressed as

$$\phi_i(x)=\sin(\lambda_i x) \tag{6}$$

When i = 2, 4, 6 … and $x\in[0, l]$,

$$\phi_i(x)=\frac{\sin(\lambda_i x)}{\sin(\lambda_i l)}-\frac{\sinh(\lambda_i x)}{\sinh(\lambda_i l)} \tag{7}$$

When i = 2, 4, 6 … and $x\in[l, 2l]$,

$$\begin{aligned}\phi_i(x)=&\cos(\lambda_i x-\lambda_i l)-\cosh(\lambda_i x-\lambda_i l)\\&-\left[\sin(\lambda_i x-\lambda_i l)-\sinh(\lambda_i x-\lambda_i l)\right]\cdot\cot(\lambda_i l)\end{aligned} \tag{8}$$

Substituting Eqs. (4) and (2) into Eq. (1), and multiplying by ϕ_n and integrating the resultant equation with respect to x between 0 and $2l$, and then applying the orthogonality conditions, the equation of motion in terms of the generalized displacement $q_n(t)$ is given as:

$$M_n\ddot{q}_n(t)+2\xi_n\omega_n M_n\dot{q}_n(t)+K_n q_n(t)=F_n(t) \tag{9}$$

where ω_n, ξ_n and M_n are the modal frequency, the damping ratio, and the modal mass of the nth mode, respectively, and $K_n(=M_n\omega_n^2)$ means the generalized stiffness of the nth mode. The generalized force $F_n(t)$ is expressed as

$$F_n(t)=\sum_{k=1}^{N_v}\Bigg\{(m_1+m_2)g\phi_n(x_k)-m_2\sum_{i=1}^{\infty}\ddot{q}_i(t)\phi_i(x_k)\phi_n(x_k)$$
$$+\left[k_1Z_k(t)+c_1\dot{Z}_k(t)\right]\phi_n(x_k)$$
$$-\sum_{i=1}^{\infty}\left[k_1q_i(t)+c_1\dot{q}_i(t)\right]\phi_i(x_k)\phi_n(x_k)\Bigg\}$$
$$\cdot\left[H(t-t_k)-H(t-t_k-\Delta t)\right]$$
$$-\sum_{j=1}^{n}\left[c_{ej}\phi_n(x_{ej})\left(\sum_{i=1}^{\infty}\dot{q}_i(t)\phi_i(x_{ej})\right)\right] \tag{10}$$

Subsequently, substituting Eq. (10) into Eq. (9) and moving the terms with $(q_i, \dot{q}_i, \ddot{q}_i, Z_k, \dot{Z}_k)$ to the left side of the differential equation, the nth generalized equation of motion for the continuous beam carrying multiple moving sprung masses is rewritten as

$$M_n\ddot{q}_n(t)+2\xi_n\omega_nM_n\dot{q}_n(t)+K_nq_n(t)$$
$$+m_2\sum_{k=1}^{N_v}\left[\left(\sum_{i=1}^{\infty}\ddot{q}_i(t)\Phi_{ink}\right)\tilde{H}_k\right]$$
$$+c_1\left[\sum_{k=1}^{N_v}\left(\sum_{i=1}^{\infty}\dot{q}_i(t)\Phi_{ink}-\dot{Z}_k(t)\Phi_{nk}\right)\tilde{H}_k\right]$$
$$+k_1\left[\sum_{k=1}^{N_v}\left(\sum_{i=1}^{\infty}q_i(t)\Phi_{ink}-Z_k(t)\Phi_{nk}\right)\tilde{H}_k\right]$$
$$+\sum_{j=1}^{n}c_{ej}\left(\sum_{i=1}^{\infty}\dot{q}_i(t)\Phi_{inj}\right)=(m_1+m_2)g\times\sum_{k=1}^{N_v}\Phi_{nk}\tilde{H}_k \tag{11}$$

where $\Phi_{ink} = \phi_i(x_k)\phi_n(x_k)$, $\Phi_{inj} = \phi_i(x_{ej})\phi_n(x_{ej})$, $\Phi_{nk} = \phi_n(x_k)$ and $\tilde{H}_k=\left[H(t-t_k)-H(t-t_k-\Delta t)\right]$. It is emphasized that as shown in Eq. (11), the inclusion of external VE dampers results in the generalized equations coupling each other. By substituting Eq. (4) into Eq. (3), the motion equation of the kth sprung mass can be described by

$$m_1\ddot{Z}_k(t)+c_1\dot{Z}_k(t)+k_1Z_k(t)-\sum_{i=1}^{\infty}\left[c_1\dot{q}_i(t)+k_1q_i(t)\right]\phi_i(x_k)=0 \tag{12}$$

where Z_k represents the vertical displacement of the kth sprung mass, and $q_i(t)$ the generalized coordinate of the two-span continuous beam. Observing Eqs. (11) and (12), the simultaneous equations show that the responses of Z_k and $q_i(t)$ are linearly coupled, which can be computed using conventional time integration methods without iterations.

As shown in Eqs. (11) and (12), they are coupled with each other. By combining Eqs. (11) and (12), the equations of motion including the two-span continuous beam and all of the sprung masses in modal space are given in matrix form as

Table 1. Properties of the two-span continuous bridge and VE dampers.

l(m)	m(t/m)	EI(kN m^2)	ω_1(Hz)	ω_2(Hz)	c_e(kN s/m)
40	42	2.07×10^8	2.18	3.41	2200

Table 2. Properties and resonant speeds of the 2 DOF moving oscillators.

N_v	d(m)	m_1(t)	m_2(t)	k_1 (kN/m)	c_1 (kN s/m)	v_{res1} (km/h)	v_{res2} (km/h)
16	25	30	2	2600	90	196	306

$$[M]\{\ddot{U}\}+[C]\{\dot{U}\}+[K]\{U\}=\{F\} \tag{13}$$

where $[M]$, $[C]$, $[K]$ denote the mass, damping and stiffness matrices; $(\{U\},\{\dot{U}\},\{\ddot{U}\})$ the vectors of displacement, velocity and acceleration, respectively; and $\{F\}$ represents the vector of exciting forces applying to the dynamic system. As shown in Eq. (13), the displacement vector of $\{U\}$ and the force vector of $\{F\}$ are respectively expressed as follows

$$\{U\}=\left\{\left[q_1, q_2, \cdots q_\infty\right]_{1\times\infty}, \left[Z_1, Z_2, \cdots Z_{N_v}\right]_{1\times N_v}\right\}^T \tag{14}$$

$$\{F\}=\Bigg\{(m_1+m_2)g\cdot\left[\sum_{k=1}^{N_v}\Phi_{1k}\tilde{H}_k, \sum_{k=1}^{N_v}\Phi_{2k}\tilde{H}_k, \cdots, \sum_{k=1}^{N_v}\Phi_{\infty k}\tilde{H}_k\right]_{1\times\infty},$$
$$\left[0, 0, \cdots, 0\right]_{1\times N_v}\Bigg\}^T \tag{15}$$

To obtain simultaneously the dynamic responses of both the two-span continuous beam and moving vehicles, the generalized matrix equation of motion shown in Eq. (13) will be solved using a step-by-step integration method based on the Newmark method. In this study, β = 1/4 and γ = 1/2 are selected, which implies a constant acceleration with unconditional numerical stability [12].

IV. NUMERICAL INVESTIGATIONS

Fig. 1 shows that a number of identical 2 DOF moving oscillators with equal intervals d are crossing a two-span continuous bridge installed with VE dampers at constant speed v. The properties of the two-span continuous bridge are listed in Table 1, in which ω_1 and ω_2 denote the first two natural frequencies of the bridge. Table 2 shows the properties of the 2 DOF moving oscillators and the first two resonant speeds ($v_{\text{res},i} = \omega_i d$) of the two-span continuous bridge under the moving loads. It was well known that if the acceleration response, rather than the displacement response, of a structure is of concern, the contribution of higher modes has to be included in the computation [11, 25]. From the convergent verification of computed results of a two-span continuous beam under moving train loads presented in Ref. [17], the first 20 modes of shape functions in Eq. (4) are sufficient to compute the acceleration response of a two-span continuous beam. For this reason, the same number of modes will be used in all the examples to follow. Moreover, the time step of 0.001 s and the ending time of $t_{end} = (N_v d + 2l)/v$ are employed to compute the interaction responses of the train-bridge system.

Normally, the VE dampers are usually installed at the place of a structure where the largest response occurs. Thus, the identical VE dampers with total damping coefficient $c_e = 2200$ kN s/m are attached to the midpoint of each span of the two-span continuous beams, i.e., $x_{e1} = l/2$ and $x_{e2} = 3l/2$. In this study, the VE dampers that are concentrically attached to the mid-span of each span of the continuous beam are called “mid-VEDs” in the following examples. In practice, the damping coefficient of the VE dampers for high speed railway bridges is much larger than that for the conventional railway bridges, and this can be achieved by installing several VE dampers at the same place, as shown in Fig. 1(b). For a two-span continuous bridge, the acceleration responses of the first span are larger than those for the second span [1, 2]. So only the acceleration response of the first span will be considered herein.

In order to verify the control effectiveness of using mid-VEDs to reduce the train-induced resonance of a continuous beam, let us first consider the two-span continuous beam with a damping ratio ξ = 1% and compute the acceleration response at the midpoint of the first span under the action of a series of 2 DOF moving oscillators at the first two resonant speeds v_{res1} and v_{res2}.

Figs. 2(a) and (b) show the time-history responses of the midpoint acceleration (a_{mid}) of the first span of the two-span continuous beam with and without VE dampers at v_{res1} and v_{res2}, respectively. As a train travels over a bridge without VE dampers at a resonant speed, it is obvious that the response of the bridge is generally built up as there are more loads passing the bridge [21, 25]. This is known as ‘resonance phenomenon’ for train-induced response of railway bridges. But once the mid-VEDs are installed at the bridge, the acceleration amplitude of the bridge is mitigated and limited at a certain value. As indicated in Figs. 2(a) and (b), the VE dampers can effectively reduce the resonant responses of the beam because the VE dampers absorb mostly the vibrating energy of the beam induced by moving loads.

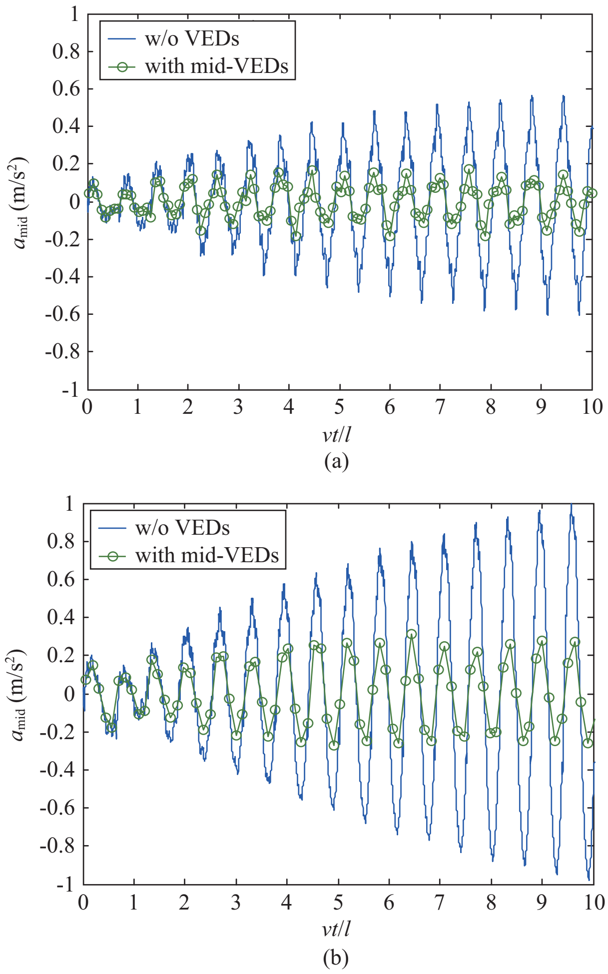


Fig. 2. Time history responses of beam with and without VEDs at resonant speeds: (a) at the first resonant speed of v_{res1} and (b) at the second resonant speed of v_{res2}.

1. Suppression Effect of Mid-VEDs on The Undamped Beam

To investigate the control effectiveness of mid-VEDs, the dynamic resposne of an undamped two-span continuous beam due to successive moving vehicles will be investigated first. The three-dimensional (3D) plots for the maximum acceleration responses (a_{max}) along the beam axis (x/l) against moving speeds (v) for the vibrating beam with/without mid-VEDs have been drawn in Figs. 3(a) and (b), respectively. Such a 3D plot will be called a_{max}-v-x/l 3D plot in the following examples.

As can be seen from Fig. 3(a), when the running speed of the moving vehicles coincides with any of the resonant and sub-resonant speeds of the vibration modes, the maximum acceleration responses of the beam will be significantly amplified. Another observation depicted in Fig. 3(a) is that, except the resonant peaks associated with the first two natural frequencies, i.e., at v_{res1} and v_{res2}, there exist some sub-resonant peaks, such as $v_{res3,\,sub3}$, which is equal to the third sub-resonant speed of the 3rd mode that has been excited, i.e.,

$$v_{res3,\,sub3} = \frac{\omega_3 d}{3} = \frac{8.72 \times 25}{3} = 73 \text{ m/s} = 262 \text{ km/h}$$

Compared the computed results in Figs. 3(a) and (b), when the mid-VEDs are installed at the midpoint of each span of the undamped beam, the acceleration amplitudes of the beam at all the moving speeds have been reduced significantly, especially at the resonant speeds.

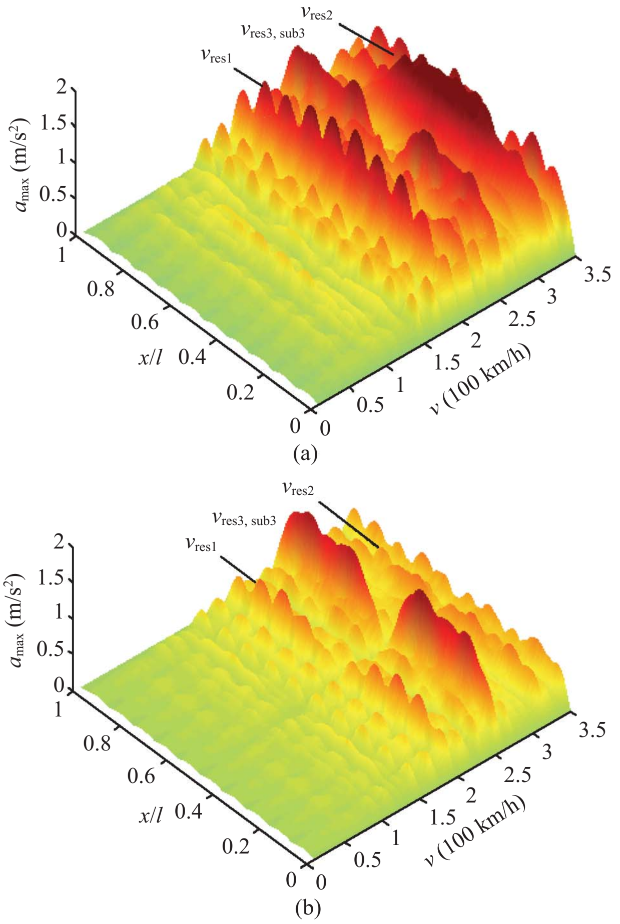


Fig. 3. a_{max}-v-x/l 3D plots of the undamped beam: (a) without VEDs and (b) with mid-VEDs.

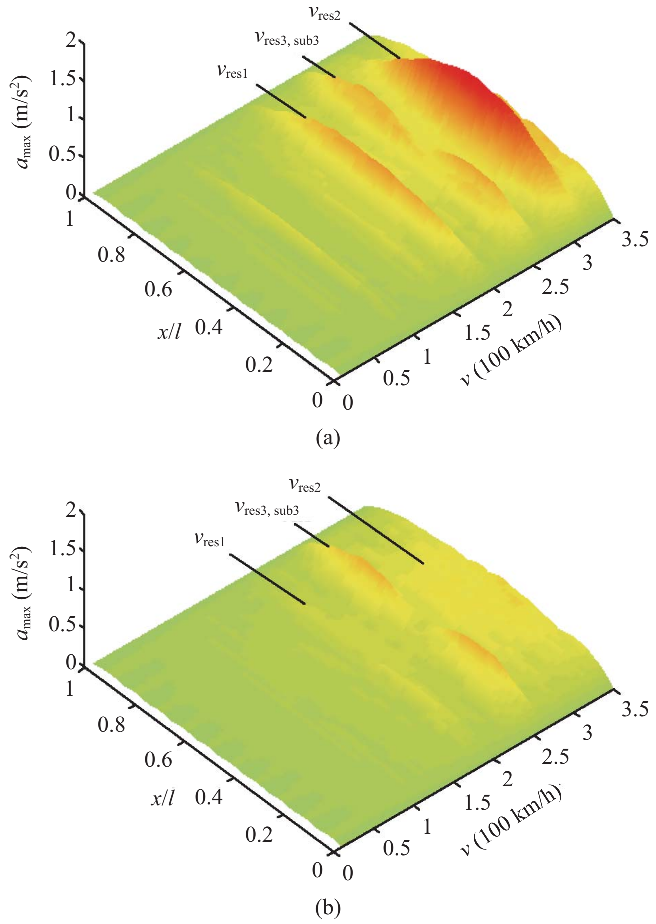


Fig. 5. a_{max}-v-x/l 3D plots of the damped beam. (a) without VEDs and (b) with mid-VEDs.

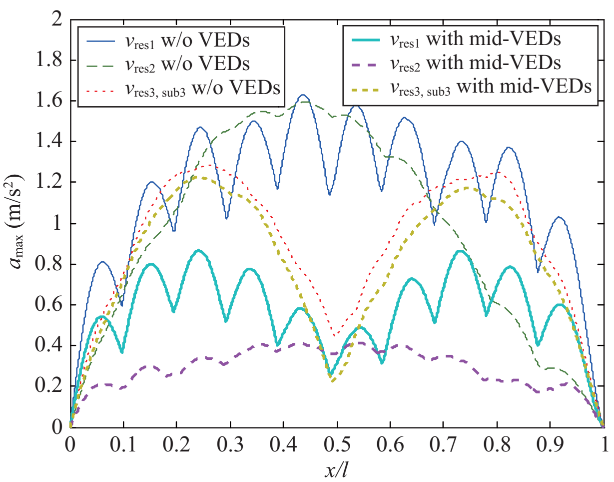


Fig. 4. a_{max}-x/l plot of the undamped beam.

For the purpose of illustration, the maximum accelerations (a_{max}) computed along the beam axis (x/l) have been plotted at the resonant and sub-resonant speeds in Fig. 4. Such a relationship is denoted as a_{max}-x/l plot in the following. The a_{max}-x/l plot indicates that the maximum accelerations response of the undamped beam could be suppressed observably by the mid-VEDs at resonant speeds, i.e., at v_{res1} and v_{res2}, but is lightly reduced at sub-resonant speed, i.e., at $v_{res3, sub3}$.

2. Applications of Mid-VEDs to Vibration Reduction of The Damped Beam

However, the effect of beam damping has not yet been considered in section 4.1. The purpose of the following section is to investigate such an effect on the resonant and sub-resonant acceleration response of the beam with mid-VEDs. A damping ratio $\xi = 1\%$ is assumed for the two-span continuous beam. Then, the a_{max}-v-x/l 3D plots of the damped beam with/without mid-VEDs have been drawn in Figs. 5(a) and (b), respectively.

As showed in Figs. 5(a) and (b), the mid-VEDs can effectively suppress the vibration of the damped beam, especially at the resonant speeds. The phenomenon of the damped beam with mid-VEDs subjected to moving trains is similar to the one of the undamped beam. Then, the maximum accelerations (a_{max}) computed along the beam axis (x/l) have been

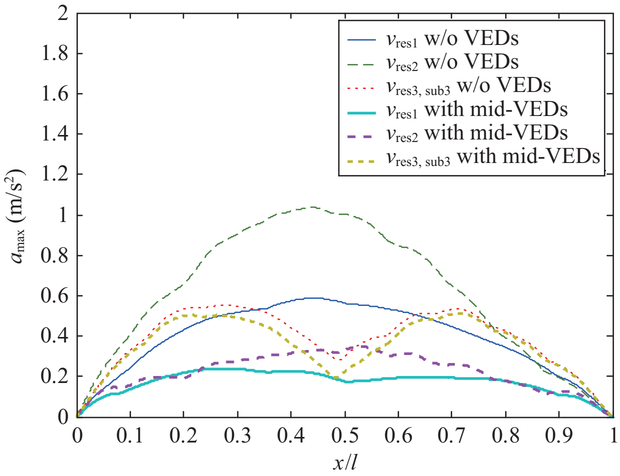


Fig. 6. a_{max}-x/l plot of the damped beam.

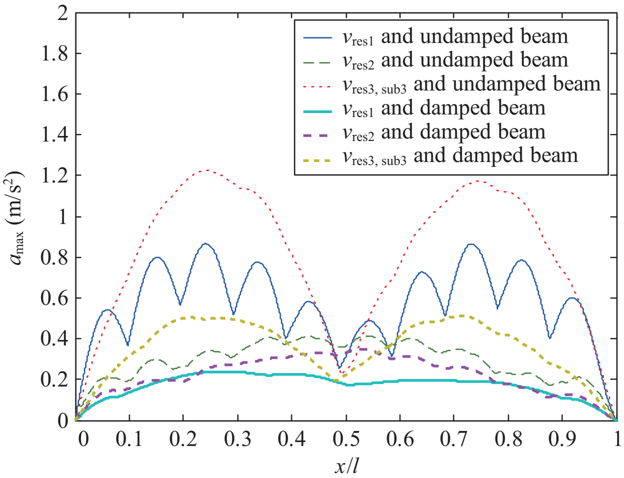


Fig. 7. Comparison of a_{max}-x/l plot at resonant and sub-resonant speeds.

plotted at the resonant and sub-resonant speeds in Fig. 6. The a_{max}-x/l plot indicates that the maximum accelerations response of the damped beam could be suppressed observably by the mid-VEDs at resonant speeds, i.e., at v_{res1} and v_{res2}, but is lightly reduced at sub-resonant speed, i.e., at $v_{res3,\,sub3}$.

As can be seen from Figs. 3(b) and 5(b), once the damping of the beam is included, all the resonant and sub-resonant peaks of maximum accelerations of the beam with mid-VEDs will be significantly reduced. To investigate the effect of structural damping to the vibration of the continuous beam with mid-VEDs, the maximum accelerations (a_{max}) of the damped and undamped beam computed along the beam axis (x/l) at the resonant and sub-resonant speeds have been plotted in Fig. 7. It is noted that the structural damping could reduce the maximum accelerations of the beam noticeably. Another observation from the acceleration responses is that the maximum acceleration amplitudes associated with higher modes need not occur at the midpoint of the beam, i.e., at the sub-resonant speed of $v_{res3,\,sub3}$.

Table 3. Parameters of the M-VEDs.

M-VEDs	Position (x)	damping coefficients
I	$l/4$, $l/2$, $3l/4$	$c_e/3$, $c_e/3$, $c_e/3$
II	$l/4$, $l/2$, $3l/4$	$c_e/4$, $c_e/2$, $c_e/4$

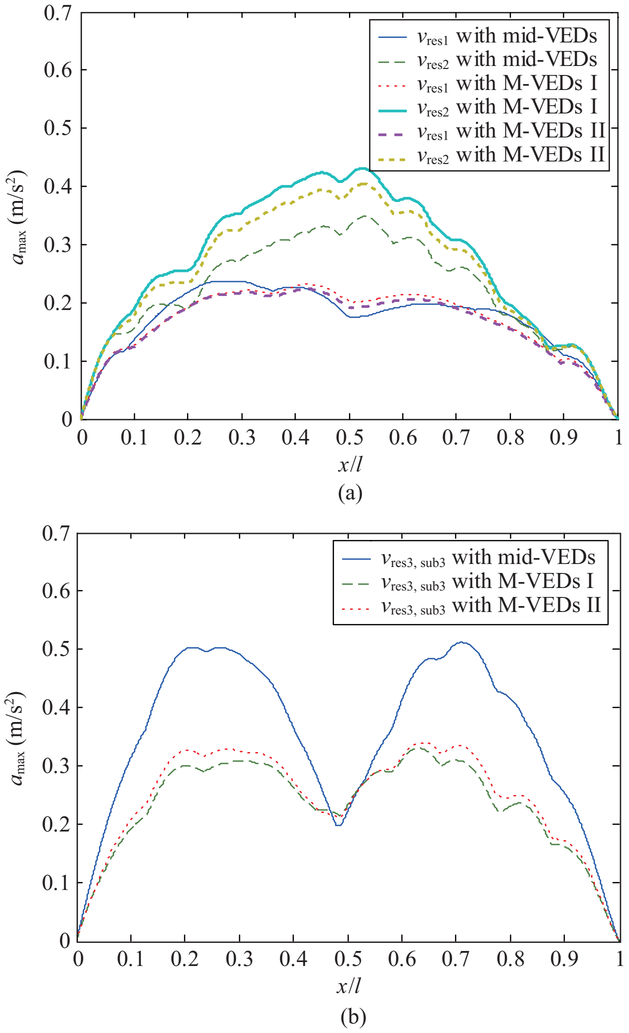


Fig. 8. Comparison of a_{max}-x/l plots with mid-VEDs and M-VEDs: (a) at the resonant speeds of v_{res1} and v_{res2} and (b) at the sub-resonant speed of $v_{res3,\,sub3}$.

3. Application of M-VEDs to Vibration Reduction of The Damped Beam

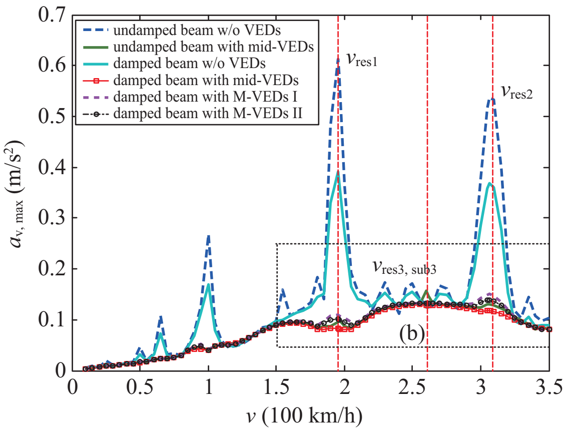
When the sub-resonances associated with higher modes are excited, the maximum acceleration response of the beam need not occur at the midpoint of the beam [11, 17, 25], as shown in Fig. 5(b). To suppress the maximum acceleration amplitudes occurring at one-quarter and three-quarters of the span, the VE dampers are divided into three groups and equipped at three locations, which are $l/4$, $l/2$, and $3l/4$, along each span of the continuous beam, respectively. Here, this arrangement of dividing the VE dampers into several multiple sub-VED units is named as “M-VEDs”. With the same total external damping of c_e, two types of M-VEDs are designed and listed in Table 3. Then, the maximum accelerations (a_{max}) of the damped beam with mid-VEDs and M-VEDs computed along the beam axis (x/l) at resonant and sub-resonant speeds have been drawn in Figs. 8(a) and (b), respectively.

As showed in Fig. 8(b), after the installation of M-VEDs I and II, the maximum acceleration amplitudes occurring at one-quarter and three-quarters of the span associated with the sub-resonant speed of $v_{res3,\ sub3}$ have been suppressed significantly. On the contrary, the maximum acceleration amplitudes associated with the resonant speed of v_{res2} is larger than that with mid-VEDs, shown in Fig. 8(a). This can be attributed that the dissipating force induced by the sub-VEDs placed at the midpoint of the span of the M-VEDs is smaller than that by the mid-VEDs. Thus, for the M-VEDs, the sub-VEDs placed at the positions along the beam span appear different degree of influence on suppressing the resonant vibration of the beam. This computed results indicate that the sub-VEDs placed at $l/2$ would dominate the vibration reduction of resonant responses of the two-span continuous beam at v_{res1} and v_{res2}, and the sub-VEDs placed at $l/4$ and $3l/4$ can suppress the sub-resonant vibrations of the beam at $v_{res3,\ sub3}$.

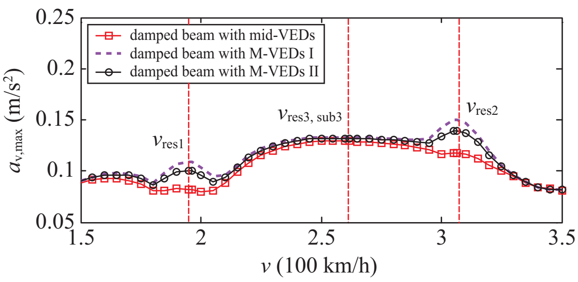
4. Maximum Acceleration Response of Moving Oscillators

To study the acceleration response of moving vehicles running on the two-span continuous beam, let us consider the aforementioned 6 cases. Case (1): undamped beam without VE dampers; Case (2): undamped beam with mid-VEDs; Case (3): damped beam without VE dampers; Case (4): damped beam with mid-VEDs; Case (5): damped beam with M-VEDs I; Case (6): damped beam with M-VEDs II. Then, the maximum accelerations ($a_{v,max}$) of the moving vehicles running on the two-span continuous beam against various speeds (v) with 6 cases have been plotted in Figs. 9(a) and (b). For the purpose of observation, Fig. 9(b) is the locally enlarged drawing of the specific range shown in Fig. 9(a). Such a relationship is denoted as $a_{v,max}$-v plot in this paper.

As can be seen from Fig. 9(a), for the undamped and damped beam without VE dampers (Case 1 and Case 3), there exist multiple resonant peaks in the $a_{v,max}$-v plot, especially at the resonant speed of v_{res1} and v_{res2}. With the installation of the mid-VEDs at the midpoint of each span of the undamped and damped beam (Case 2 and Case 4), the multiple resonant peaks are suppressed simultaneously and the riding comfort of moving vehicles is improved as well. Compared the Case 1 with the Case 3, the structural damping provides much improvement for riding comfort of the moving oscillators. But after the installation of mid-VEDs, the effect of suppression of the structural damping becomes less significant, as depicted in Case 2 and Case 4. This can be attributed that most of the vibration energy transmitted from the vibrating beam are dissipated by the mid-VEDs. When the M-VEDs I and II are considered and installed on the continuous beam (Case 5 and



(a)



(b)

Fig. 9. Comparison of $a_{v,max}$-v plot for the moving oscillators: (a) global drawing of $a_{v,max}$-v plot and (b) local drawing of $a_{v,max}$-v plot.

Case 6), the multiple resonant peaks of the moving vehicles are also suppressed (compared with Case 3). Obviously, both the mid-VEDs and the M-VEDs are effective to suppress the multiple resonant peaks of train-bridges systems.

As shown in Fig. 9(b), the control performance of both the M-VEDs I and II is not so well as that of the mid-VEDs. One of the reasons is that the total damping coefficient of the sub-VEDs of the M-VEDs I and II arranged at the mid-spans are smaller than that of mid-VEDs. The maximum acceleration amplitudes of the moving vehicles at the resonant speed of v_{res2} are much larger than that at the resonant speed of v_{res1}, this is determined by the vibration of the beam with mid-VEDs and M-VEDs (see Fig. 8(a)). To obtain a better control performance of moving vehicles, the damping of the sub-VEDs at $l/2$ in the M-VEDs can be designed larger than that at other positions of $l/4$ and $3l/4$. Even so, as the previous results shown in Fig. 8(b), with the installation of the M-VEDs I or II, the maximum acceleration amplitudes occurring at one-quarter and three-quarters of the span associated with the sub-resonant speed of $v_{res3,\ sub3}$ have been suppressed significantly in comparison with the mid-VEDs.

V. CONCLUSIONS

In this paper, the control effectiveness of using multiple sets of VE dampers to mitigate the multiple resonant responses of a moving train running on a two-span continuous beam was investigated. From the numerical studies, the following conclusions are reached:

(1) The VE dampers can effectively suppress the multiple resonant peaks of two-span continuous railway bridges and subsequently has the ability to control the acceleration amplitude for running vehicles. The strategy of vibration reduction proposed herein is simple and robust, which should find applications in areas where multiple resonant peaks are a problem of major concern.

(2) With the installation of mid-VEDs at the midpoint of each span of either the undamped beam or the damped beam, the maximum acceleration responses of the beam can be suppressed noticeably at the resonant speeds of v_{res1} and v_{res2}, but slightly reduced at sub-resonant speed of $v_{\text{res3, sub3}}$. Meanwhile, the multiple resonant peaks at the resonant speeds of v_{res1} and v_{res2} for the train-bridge system can be mitigated and the riding comfort of the moving train is improved as well.

(3) The structural damping can reduce the resonant and sub-resonant peaks of maximum acceleration responses of the beam significantly, especially for the sub-resonant peaks associated with higher modes. But it has less effect on improving riding comfort of moving vehicles in comparison with the control effectives of mid-VEDs. This can be attributed that the mid-VEDs dissipate most of the vibration energy transmitted from the vibrating beam that can excite to the moving vehicles over it.

(4) With the same total damping constant of VE dampers, the M-VEDs can develop its control effectiveness in simultaneously mitigating the multiple resonant peaks of the train-bridge system. Although the control performance of the M-VEDs in reducing the resonant responses of the moving train is not so well as that of the mid-VEDs, the proposed M-VEDs behave a better vibration control in mitigating the resonant peaks at one-quarter and three-quarter span of the bridge traveled by the train with the specific resonant speed, such as the sub-resonant speed of $v_{\text{res3, sub3}}$ in this study. This concludes that if the vibration control of a continuous bridge were of concern, the M-VEDs would be useful in reducing the train-induced multiple resonant responses of the bridge, in which the resonant peak may not occur at the mid-span.

ACKNOWLEDGMENTS

The research reported herein is partly supported by *National Natural Science Foundation* of China (grants No. 50978024, 51008018) and grants No. NSC 99-2221-E-032-020-MY3 of *National Science Council*, Taiwan. These financial supports are gratefully acknowledged.

REFERENCES

1. Ayre, R. S., Ford, G., and Jacobsen, L. S., "Transverse vibration of a two-span beam under the action of a moving constant force," *Journal of Applied Mechanics*, Vol. 17, No. 1, pp. 1-12 (1950).
2. Ayre, R. S. and Jacobsen, L. S., "Transverse vibration of a two-span beam under the action of a moving alternating force," *Journal of Applied Mechanics*, Vol. 17, No. 3, pp. 283-290 (1950).
3. European Committee for Standardization, *EUROCODE 1: Basis of Design and Actions on Structures, Part 3: Traffic Loads on Bridges*, ENV 1991-3 (1995).
4. He, X. W., Kawatani, M., Hayashikawa, T., and Matsumoto, T., "Numerical analysis on seismic response of Shinkansen bridge-train interaction system under moderate earthquakes," *Earthquake Engineering and Engineering Vibration*, Vol. 10, No. 1, pp. 85-97 (2011).
5. Hwang, J. S. and Tseng, Y. S., "Design formulations for supplemental viscous dampers to highway bridges," *Earthquake Engineering and Structural Dynamics*, Vol. 34, No. 13, pp. 1627-1642 (2005).
6. Lee, Y. S. and Kim, S. H., "Structural analysis of 3D high-speed train-bridge interactions for simple train load models," *Vehicle System Dynamics*, Vol. 48, No. 2, pp. 263-281 (2010).
7. Li, J. Z., Su, M. B., and Fan, L. C., "Vibration control of railway bridges under high-speed trains using multiple tuned mass dampers," *Journal of Bridge Engineering*, Vol. 10, No. 3, pp. 312-320 (2005).
8. Lin, C. C., Wang, J. F., and Chen, B. L., "Train-induced vibration control of high-speed railway bridges equipped with multiple tuned mass dampers," *Journal of Bridge Engineering*, Vol. 10, No. 4, pp. 398-414 (2005).
9. Lou, P., "Finite element analysis for train-track-bridge interaction system," *Archive of Applied Mechanics*, Vol. 77, No. 10, pp. 707-728 (2007).
10. Michaltsos, G. T., "Bouncing of a vehicle on an irregularity: a mathematical model," *Journal of Vibration and Control*, Vol. 16, No. 2, pp. 181-206 (2010).
11. Museros, P. and Alarcón, E., "Influence of the second bending mode on the response of high-speed bridges at resonance," *Journal of Structural Engineering*, Vol. 131, No. 3, pp. 404-415 (2005).
12. Newmark, N. M., "A method of computation for structural dynamics," *Journal of Engineering Mechanics*, Vol. 85, No. 3, pp. 67-94 (1959).
13. Reis, M. and Pala, Y., "Dynamic response of a beam supported with damper under moving load," *Structural Engineering and Mechanics*, Vol. 31, No. 4, pp. 477-480 (2009).
14. Shin, J. R., An, Y. K., Sohn, H., and Yun, C. B., "Vibration reduction of high-speed railway bridges by adding size-adjusted vehicles," *Engineering Structures*, Vol. 32, No. 9, pp. 2839-2849 (2010).
15. Stăncioiu, D., Ouyang, H., and Mottershead, J. E., "Dynamics of a beam and a moving two-axle system with separation," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, Vol. 222, No. 10, pp. 1947-1956 (2008).
16. Wang, J. F., Lin, C. C., and Chen, B. L., "Vibration suppression for high-speed railway bridges using tuned mass dampers," *International Journal of Solids and Structures*, Vol. 40, No. 2, pp. 465-491 (2003).
17. Wang, Y. J., Wei, Q. C., and Yau, J. D., "Interaction responses of train loads moving over a two-span continuous beam," *International Journal of Structural Stability and Dynamics*, Vol. 13, No. 1, pp. 1350002 (18 pages) (2013).
18. Xia, H. and Zhang, N., "Dynamic analysis of railway bridge under high-speed trains," *Computers & Structures*, Vol. 83, No. 23-24, pp. 1891-1901 (2005).
19. Xia, H., Zhang, N., and Guo, W. W., "Analysis of resonance mechanism and conditions of train-bridge system," *Journal of Sound and Vibration*, Vol. 297, No. 3-5, pp. 810-822 (2006).
20. Yang, Y. B., Lin, C. L., and Yau, J. D., "Vibration reduction of elastically supported beams under moving loads by tuned mass devices," *The IES Journal Part A: Civil & Structural Engineering*, Vol. 1, No. 1, pp. 55-67 (2008).
21. Yang, Y. B., Yau. J. D., and Hsu, L. C., "Vibration of simple beams due to trains moving at high speeds," *Engineering Structures*, Vol. 19, No. 11, pp. 936-944 (1997).
22. Yau, J. D., "Resonance of continuous bridges due to high speed trains," *Journal of Marine Science and Technology*, Vol. 9, No. 1, pp. 14-20 (2001).

23. Yau, J. D., "Train-induced vibration control of simple beams using string-type tuned mass dampers," *Journal of Mechanics*, Vol. 23, No. 4, pp. 329-340 (2007).
24. Yau, J. D. and Yang, Y. B., "A wideband MTMD system for reducing the dynamic response of continuous truss bridges to moving train loads," *Engineering Structures*, Vol. 26, No. 12, pp. 1795-1807 (2004).
25. Yau, J. D. and Yang, Y. B., "Vertical accelerations of simple beams due to successive loads traveling at resonant speeds," *Journal of Sound and Vibration*, Vol. 289, No. 1-2, pp 210-228 (2006).
26. Ziyaeifar, M., "Vibration control in train-bridge-track systems," *Vehicle System Dynamics*, Vol. 46, No. 5, pp. 355-384 (2008).