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AN ANALYTICAL SOLUTION FOR LINEAR LONG WAVE REFLECTION BY TWO SUBMERGED RECTANGULAR BREAKWATERS

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AN ANALYTICAL SOLUTION FOR LINEAR LONG WAVE REFLECTION BY TWO SUBMERGED RECTANGULAR BREAKWATERS

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Key words: two rectangular breakwaters, analytical solution, long-wave equation, periodicity of reflection coefficient, total reflection, Bragg resonance.

ABSTRACT

In this paper, an analytical solution for linear long wave reflection by two rectangular breakwaters is explored. A closed-form expression of wave reflection coefficient is obtained which finds two well-known analytical solutions to be its special cases, including wave reflection by a rectangular breakwater given by Mei in 1989 and wave reflection by an infinite step given by Lamb in 1932. It is found that the periodicity of the reflection coefficient as a function of kh existed for a single rectangular breakwater disappears for a pair of breakwaters, and zero reflection phenomenon mostly occurs for symmetrical breakwater structure. It is also shown that the total reflection effect will be enhanced when a new breakwater is added into an existing one or when a single breakwater is decomposed into a pair of breakwaters even if the resulting total sectional area is reduced. Finally, the influence of the width of twin breakwaters to the peak Bragg reflection is studied.

I. INTRODUCTION

The study of the problem of wave field modification by abrupt bathymetric changes was originated from analytical approaches in the early days [5]. By using the linear long wave approximation, Lamb [5] derived the analytical solution for the wave reflection and transmission coefficients over an infinitely long step. The recent study [6] showed that Lamb's solution gives a good agreement to the numerical results based on the full Navier-Stokes equations even for weakly nonlinear dispersive waves, although the transmission coefficient is always overestimated in [5] due to the exclusion of the energy dissipation. After the work by Lamb [5], Jeffreys [3] studied a rectangular breakwater and found that the reflection coefficient is periodical to the ratio of the wavelength and the breakwater length. The rectangular breakwater was also studied by Mei [9], who gave an analytical solution of the wave reflection and transmission coefficients. The periodicity of the reflection coefficient [3] and the complete transmission for some lengths of breakwater [11] can be easily recovered from Mei's theoretical formulas.

For a continuously varying water depth, Kajiura [4] obtained an analytical solution for the case of a continental shelf joint with a parabolic slope, Dean [1] considered the case of a continental shelf joined with a linear slope (both of them can be found in [2]). In Kajiura's and Dean's studies, the water depth within the slope region was assumed to satisfy the power law.

Very recently, Lin and Liu [7] and Liu and Lin [8] obtained two analytical solutions in closed-form of reflection coefficients for linear long waves reflected by a breakwater or a trench of general trapezoidal shape, respectively. Their solutions include the analytical solutions of Lamb [5], Mei [9] and Dean [1] as special cases.

It is worth indicating that, all above mentioned analytical solutions are restricted in one breakwater or trench only. In this paper, we study the reflection of linear long waves by two rectangular breakwaters. A closed-form solution in terms of Bessel functions for reflection coefficient will be given in Section 2. Then in Section 3, two classical analytical solutions for long wave reflection by one rectangular breakwater and by an infinite step respectively are derived by using the present solution. In Section 4, using the present analytical solution, the changing trend of the reflection coefficient is investigated when a new breakwater is added into an existing one or when a single breakwater is decomposed into a pair of breakwaters. Further, the influence of the width of twin breakwaters to the peak Bragg reflection is also analyzed.

II. PROBLEM AND SOLUTION TECHNIQUE

Consider a linear wave train propagating over two sub-

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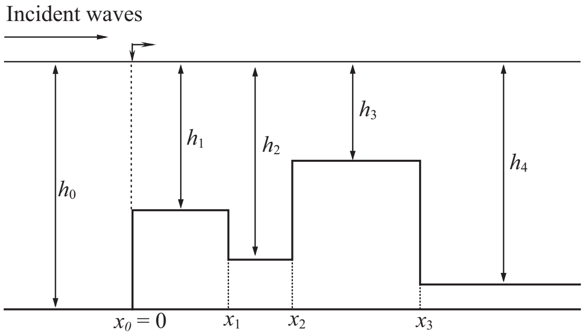

Fig. 1. A sketch of two submerged rectangular breakwaters.

merged rectangular breakwaters and an otherwise flat bottom. The water depths in front of and behind the two breakwaters, however, can be different. See Fig. 1 for a sketch of the system considered. As shown in Fig. 1, the depth $h(x)$ is defined as follows

$$h(x) = \begin{cases} h_0, & x \le x_0, \\ h_1, & x_0 < x \le x_1, \\ h_2, & x_1 < x \le x_2, \\ h_3, & x_2 < x \le x_3, \\ h_4, & x > x_3, \end{cases} \tag{1}$$

where $x_0 = 0$.

In this paper, we focus on linear long-wave reflection by these two submerged rectangular breakwaters and the water surface elevation $\eta(x)$ satisfies the following linear long-wave equation:

$$\frac{d}{dx}\left(h(x) \frac{d\eta(x)}{dx} \right) + \frac{\omega^2}{g}\eta(x) = 0, \tag{2}$$

It is clear that Eq. (2) will degenerate into the Helmholtz equation in any region with constant water depth and the general solution can be expressed as a linear combination of two particular solutions $e^{ik_i x}$ and $e^{-ik_i x}$ with $k_i = \omega / \sqrt{gh_i}$ being the wave number related to water depth h_i, i = 0, 1, 2, 3, 4. Hence if we assume that the incident waves come from the left, then we can take

$$h(x) = \begin{cases} A_I e^{ik_0 x} + A_r e^{-ik_0 x}, & x \le x_0, \\ A_1 e^{ik_1 x} + A_2 e^{-ik_1 x}, & x_0 < x \le x_1, \\ A_3 e^{ik_2 x} + A_4 e^{-ik_2 x}, & x_1 < x \le x_2, \\ A_5 e^{ik_3 x} + A_6 e^{-ik_3 x}, & x_2 < x \le x_3, \\ A_t e^{ik_4 x}, & x > x_3, \end{cases} \tag{3}$$

in which, A_I is the amplitude of incident wave, and A_r and A_t represent the complex amplitudes of the reflected waves and of the transmitted waves, respectively, which together with A_i, i = 1, 2, …, 6, are to be determined.

The continuity of wave elevations and flow fluxes across the common boundaries $x = x_i$, i = 0, 1, 2, 3, requires

$$\eta\big|_{x=x_i^-} = \eta\big|_{x=x_i^+}, \tag{4}$$

$$h_i \frac{d\eta}{dx}\bigg|_{x=x_i^-} = h_{i+1} \frac{d\eta}{dx}\bigg|_{x=x_i^+}, \; i = 0, 1, 2, 3. \tag{5}$$

Eqs. (4)-(5) are equivalent to the following system

$$A_r - A_1 - A_2 = -A_I, \tag{6}$$

$$s_{01}A_r + A_1 - A_2 = s_{01}A_I, \tag{7}$$

$$e^{ik_1x_1}A_1 + e^{-ik_1x_1}A_2 - e^{ik_2x_1}A_3 - e^{-ik_2x_1}A_4 = 0, \tag{8}$$

$$s_{12}e^{ik_1x_1}A_1 - s_{12}e^{-ik_1x_1}A_2 - e^{ik_2x_1}A_3 + e^{-ik_2x_1}A_4 = 0, \tag{9}$$

$$e^{ik_2x_2}A_3 + e^{-ik_2x_2}A_4 - e^{ik_3x_2}A_5 - e^{-ik_3x_2}A_6 = 0, \tag{10}$$

$$s_{23}e^{ik_2x_2}A_3 - s_{23}e^{-ik_2x_2}A_4 - e^{ik_3x_2}A_5 + e^{-ik_3x_2}A_6 = 0, \tag{11}$$

$$e^{ik_3x_3}A_5 + e^{-ik_3x_3}A_6 - e^{ik_4x_3}A_t = 0, \tag{12}$$

$$s_{34}e^{ik_3x_3}A_5 - s_{34}e^{-ik_3x_3}A_6 - e^{ik_4x_3}A_t = 0, \tag{13}$$

with $s_{ij} = \sqrt{h_i / h_j}$, $i, j = 0, 1, 2, 3, 4.$

In matrix form, the above system can be written as

$$\begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ s_{01} & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{ik_1x_1} & e^{-ik_1x_1} & -e^{ik_2x_1} & -e^{-ik_2x_1} & 0 & 0 & 0 \\ 0 & s_{12}e^{ik_1x_1} & -s_{12}e^{-ik_1x_1} & -e^{ik_2x_1} & e^{-ik_2x_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{ik_2x_2} & e^{-ik_2x_2} & -e^{ik_3x_2} & -e^{-ik_3x_2} & 0 \\ 0 & 0 & 0 & s_{23}e^{ik_2x_2} & -s_{23}e^{-ik_2x_2} & -e^{ik_3x_2} & e^{-ik_3x_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{ik_3x_3} & e^{-ik_3x_3} & -e^{ik_4x_3} \\ 0 & 0 & 0 & 0 & 0 & s_{34}e^{ik_3x_3} & -s_{34}e^{-ik_3x_3} & -e^{ik_4x_3} \end{bmatrix} \begin{bmatrix} A_r \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_t \end{bmatrix} = \begin{bmatrix} -A_I \\ s_{01}A_I \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{14}$$

By solving the system (14), the reflection coefficient A_r/A_I can be found to be

$$\frac{A_r}{A_I} = -\frac{z_1 - s_{01}z_2}{z_1 + s_{01}z_2}, \tag{15}$$

where

$$z_1 = -\left(\cos\phi + is_{12}\sin\phi\right)P - \left(\cos\phi - is_{12}\sin\phi\right)Q, \tag{16}$$

$$z_2 = \left(s_{12}\cos\phi + i\sin\phi\right)P - \left(s_{12}\cos\phi - i\sin\phi\right)Q, \tag{17}$$

with

$$P = (1-s_{23})(1+s_{34})e^{i\alpha} + (1+s_{23})(1-s_{34})e^{-i\beta}, \tag{18}$$

$$Q = (1-s_{23})(1+s_{34})e^{-i\alpha} + (1+s_{23})(1+s_{34})e^{i\beta}, \tag{19}$$

$$\alpha = k_3x_2 - k_3x_3 + k_2x_2 - k_2x_1, \tag{20}$$

$$\beta = k_3x_2 - k_3x_3 + k_2x_1 - k_2x_2, \tag{21}$$

$$\phi = k_1x_1. \tag{22}$$

Let λ be the wavelength of the incident waves. By introducting the following three variables

$$M_1 = \frac{x_1}{\lambda}, \tag{23}$$

$$M_2 = \frac{x_2 - x_1}{\lambda}, \tag{24}$$

$$M_3 = \frac{x_3 - x_2}{\lambda}, \tag{25}$$

which represent the wave numbers included in both the breakwater regions and in the middle region between the two breakwaters, respectively, then we have

$$\phi = 2\pi s_{01}M_1, \tag{26}$$

$$\alpha = 2\pi s_{01}s_{12}\left(M_2 - s_{23}M_3\right), \tag{27}$$

$$\beta = -2\pi s_{01}s_{12}\left(M_2 + s_{23}M_3\right). \tag{28}$$

This means that the reflection coefficient $K_R = \left|A_r/A_I\right|$ depends entirely on the three relative wave numbers M_1, M_2 and M_3, and on the four depth ratios s_{01}, s_{12}, s_{23} and s_{34} as well.

III. TWO SPECIAL CASES

In this section, we will show that the analytical solution (15) can reduce into two well-known special cases, namely, waves past a rectangular breakwater with a finite length [9] and a step with an infinite length [5].

When $h_1 = h_2 = h_3$, the obstacle of two breakwaters degenerates into a rectangular breakwater only, which was studied by Mei in [9]. Since $s_{12} = s_{23} = 1$, we have

$$\beta = k_1(x_1 - x_3), \tag{29}$$

$$P = 2(1-s_{34})e^{-ik_1(x_1-x_3)}, \tag{30}$$

$$Q = 2(1+s_{34})e^{ik_1(x_1-x_3)}, \tag{31}$$

$$z_1 = -2(1-s_{34})e^{ik_1x_3} - 2(1+s_{34})e^{-ik_1x_3}, \tag{32}$$

$$z_2 = 2(1-s_{34})e^{ik_1x_3} - 2(1+s_{34})e^{-ik_1x_3}, \tag{33}$$

therefore we have

$$\begin{aligned}\frac{A_r}{A_I} &= -\frac{(1+s_{01})(1-s_{34})e^{ik_1x_3} + (1-s_{01})(1+s_{34})e^{-ik_1x_3}}{(1-s_{01})(1-s_{34})e^{ik_1x_3} + (1+s_{01})(1+s_{34})e^{-ik_1x_3}} \\ &= -\frac{(1-s_{01})(1+s_{43})e^{-ik_1x_3} + (1+s_{01})(1-s_{43})e^{ik_1x_3}}{(1+s_{01})(1+s_{43})e^{-ik_1x_3} - (1-s_{01})(1-s_{43})e^{ik_1x_3}},\end{aligned} \tag{34}$$

which coincides with the reflection coefficient given by Mei in [9] (see pp. 130-131) for this special case, where carefulness must be taken as there is a pen slip in Eq. (4.17).

Furthermore, if $s_{01} = 1$ or $s_{43} = 1$, the above expression can further degenerate into

$$K_R = \frac{|A_r|}{|A_I|} = \frac{s_{43}-1}{s_{43}+1} = \frac{1-\sqrt{\dfrac{h_3}{h_4}}}{1+\sqrt{\dfrac{h_3}{h_4}}}, \tag{35}$$

or

$$K_R = \frac{|A_r|}{|A_I|} = \frac{s_{01}-1}{s_{01}+1} = \frac{1-\sqrt{\dfrac{h_1}{h_0}}}{1+\sqrt{\dfrac{h_1}{h_0}}}, \tag{36}$$

which are the reflection coefficients for an infinite step given by Lamb in [5].

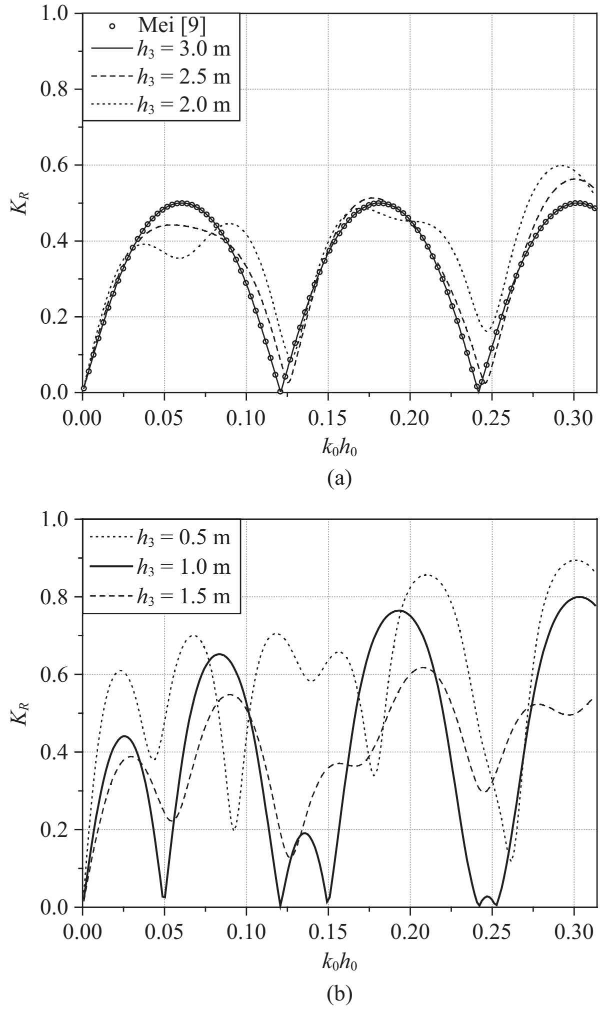

Fig. 2. The changing trend of K_R when the second new breakwater is added to an existing one: x_1 = 45 m, x_2 = 60 m, x_3 = 105 m, h_0 = h_2 = h_4 = 3 m, h_1 = 1 m, and h_3 = 0.5, 1.0, 1.5, 2.0, 2.25, 2.5, 3.0 m.

IV. COMPUTATIONAL RESULTS AND DISCUSSION

As stated in the Introduction, wave reflection by a single submerged rectangular breakwater has been intensively studied by Jeffreys [3], Mei [9] and Newman [11]. In this section, based on the analytical formula (15), we study the influence of the second breakwater to reflection coefficient.

1. Changing Trend of K_R When a New Breakwater Is Added

In this subsection, we investigate the variation of the reflection coefficient when a new breakwater is added to an existing one.

At first, we fix x_1 = 45 m, x_2 = 60 m, x_3 = 105 m, $h_0 = h_2 = h_4$ = 3 m, h_1 = 1 m, and let h_3 take 0.5 m, 1.0 m, 1.5 m, 2.0 m, 2.5 m and 3.0 m, respectively. It is clear that when h_3 = 3.0 m, the second breakwater will disappear and the structure with a pair of rectangular breakwaters degenerates into a single one. For k_0h_0 varying from 0 to $\pi/10$, computational results of reflection coefficient for all six cases calculated by formula (15) are displayed in Fig. 2(a)-(b).

It can be seen from Fig. 2(a)-(b) that when h_3 = 3.0 m, the reflection coefficient K_R coincides with the analytical solution by Mei [9] which is periodic with respect to k_0h_0. However, when h_3 takes 0.5 m, 1.0 m, 1.5 m, 2.0 m, 2.5 m respectively, the periodicity of the reflection coefficient cannot be observed, this means that the periodicity of the reflection coefficient for a single breakwater no longer remains if another new breakwater is added to an existing one, no matter the size of the new breakwater is small or big. In addition, the phenomenon of zero reflection coefficient is observed only for two cases with h_3 = 3.0 m and h_3 = 1.0 m in which the breakwater structure is symmetrical, which is similar to the finding revealed by Xie *et al*. [12] for a rectangular breakwater with two scour trenches. It is also found that as h_3 decreases from 3.0 m to 0.5 m, both the maximal reflection coefficient and the total reflection (i.e., the area under the reflection coefficient curve) for $0 < k_0h_0 < \pi/10$ increase since the size of the second new breakwater becomes larger and larger.

Secondly, we fix x_1 = 30 m, x_2 = 60 m, $h_0 = h_2 = h_4$ = 4 m, $h_1 = h_3$ = 2 m, and let $x_3 - x_2$ take 0 m, 2 m, 4 m, 6 m, 8 m, 10 m, 15 m, 30 m and 60 m, respectively. For k_0h_0 varying from 0 to $\pi/10$, computational results of reflection coefficient for all nine cases calculated by formula (15) are displayed in Fig. 3(a)-(b) together with Mei's result [9] for $x_3 - x_2 = 0$. As expected, the present analytical model reproduces Mei's results [9] with periodicity of the reflection coefficient. When $x_3 - x_2$ takes 2 m, 4 m, 6 m, 8 m, 10 m, 15 m, 30 m and 60 m, respectively, the periodicity of the reflection coefficient does not appear anymore due to the existence of the second breakwater. Again, the phenomenon of zero reflection appears only for symmetrical structure with $x_3 - x_2$ = 0 m and 30 m. Further, the total reflections for all cases with $x_3 - x_2 \neq 0$ exceed the total reflection produced by the single breakwater with $x_3 - x_2 = 0$. As $x_3 - x_2$ increases from 2 m to 60 m, the size of the second breakwater becomes larger and larger, thus the total reflection becomes more and more significant.

2. Changing Trend of K_R When One Breakwater Is Divided Into Two

In this subsection, we investigate the changing trend of the reflection coefficient when one breakwater is divided into two under the assumption that the total sectional area of the composite breakwaters keeps a fixed value.

Firstly, we fix $h_0 = h_2 = h_4$ = 4 m, $h_1 = h_3$ = 2 m, x_3 = 90 m and $x_2 - x_1$ = 30 m, then let x_1 take 0 m, 1 m, 2 m, 3 m, 4 m, 5 m, 10 m, 15 m, 20 m, 25 m and 30 m, respectively. The condition $x_2 - x_1$ = 30 m means that the total sectional area of one or two breakwaters keeps the same value 120 m^2. When x_1 = 0, the structure with two breakwaters degenerates into a single breakwater, so the result of the reflection

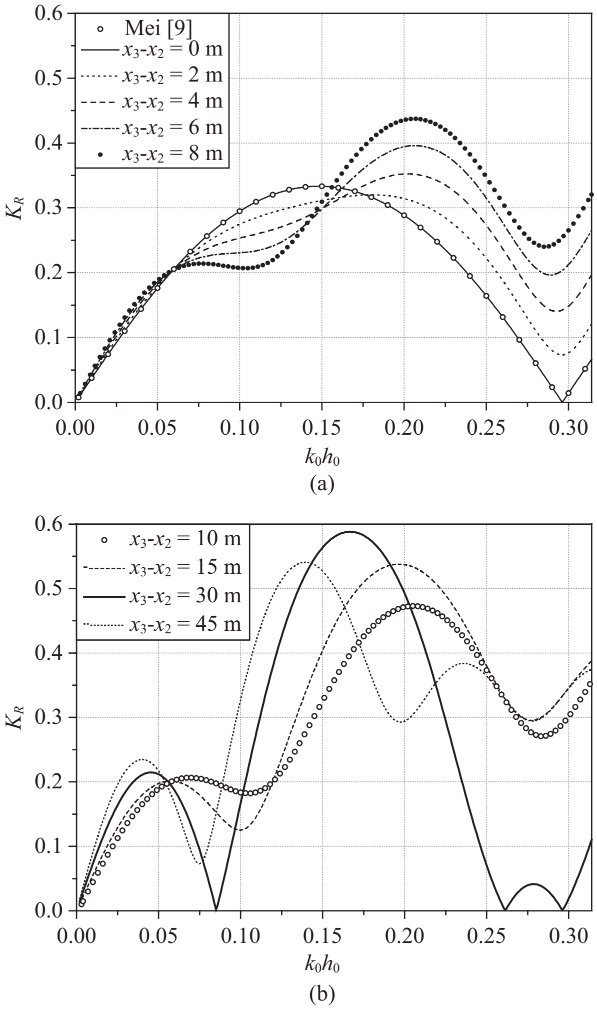

Fig. 3. Influence of the width of the second breakwater to K_R: x_1 = 30 m, x_2 = 60 m, $h_1 = h_3$ = 2 m, $h_0 = h_2 = h_4$ = 4 m, and $x_3 - x_2$ = 2, 4, 6, 8, 10, 15, 30, 45, 60 m.

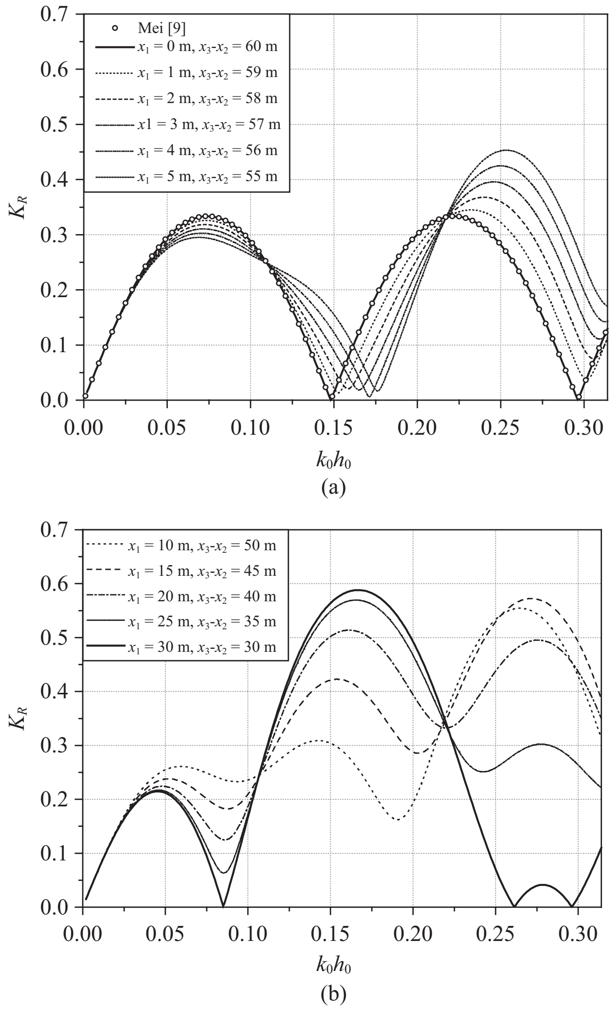

Fig. 4. Changing trend of K_R when one breakwater is divided into two: $h_0 = h_2 = h_4$ = 4 m, $h_1 = h_3$ = 2 m, x_3 = 90 m, $x_2 - x_1$ = 30 m, and x_1 = 0, 1, 2, 3, 4, 5, 10, 15, 20, 25, 30 m.

coefficient K_R by the present model agrees with Mei's result [9], see Fig. 4(a). For all other cases with x_1 = 0 m, i.e., when the single rectangular breakwater is divided into two, the corresponding total reflection by composite breakwaters exceeds the total reflection by the single breakwater, see Fig. 4(a)-(b), though the total sectional area of the composite breakwaters equals to that of the single breakwater.

3. Changing Trend of K_R When One Breakwater Is Excavated Into Two

In this subsection, we further investigate the changing trend of reflection coefficient when one breakwater is decomposed into two by excavating some part in the middle of the original breakwater.

We fix x_1 = 30 m, x_2 = 60 m, x_3 = 90 m, $h_0 = h_4$ = 4 m, $h_1 = h_3$ = 2 m, and let h_2 take 2.0 m, 2.5 m, 3.0 m and 4.0 m, then we obtain four composite breakwaters as shown in Fig. 5(a). Using the analytical formula (15), the reflection coefficient for all four cases are calculated and plotted in Fig. 5(b). When h_2 = 2.0 m, the submerged structure is only a single breakwater, the result of the reflection coefficient calculated by the present analytical model coincides with the solution by Mei [9]. When h_2 = 2.5 m, 3.0 m and 4.0 m, the single breakwater has been decomposed into two with some part of the original breakwater being excavated, as a result, the periodicity of the reflection coefficient for a single breakwater now disappears. The phenomenon of zero reflection happens in all four cases may due to the symmetricalness of the breakwater structure. It can be seen that not only the maximal reflection coefficient but also the total reflection produced by each of composite breakwaters has been greatly enhanced although the total sectional area of each of composite breakwaters in all three cases is even less than the sectional area of the original single breakwater.

4. Influence of Width of Breakwaters to Bragg Resonance

In this subsection, we always assume that $h_0 = h_2 = h_4$, $h_1 = h_3$ and $x_3 - x_2 = x_1$, then the composite breakwaters

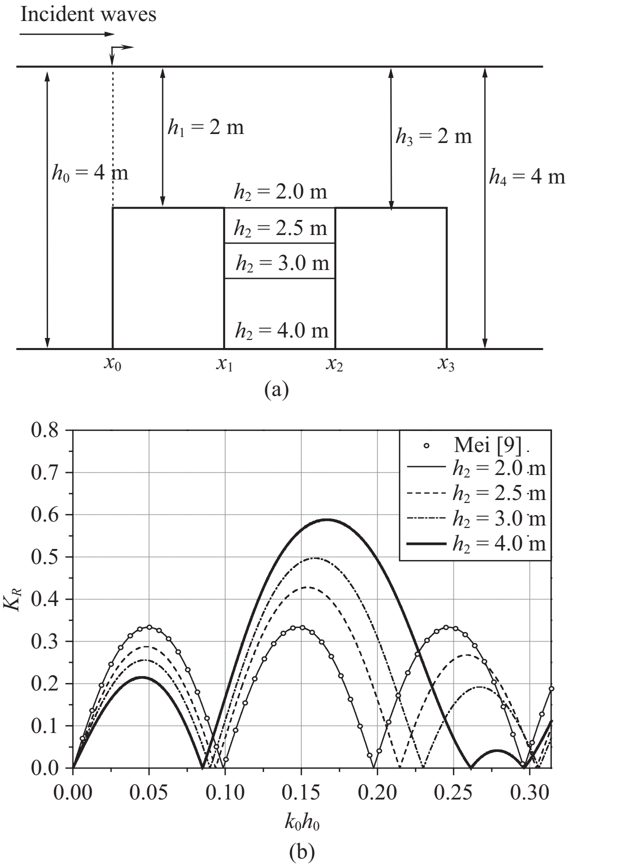

Fig. 5. Changing trend of reflection coefficient when one breakwater is excavated into two: $h_0 = h_4 = 4$ m, $h_1 = h_3 = 2$ m, $x_1 = 30$ m, $x_2 = 60$ m, $x_3 = 90$ m, and $h_2 = 2.0, 2.5, 3.0, 4.0$ m.

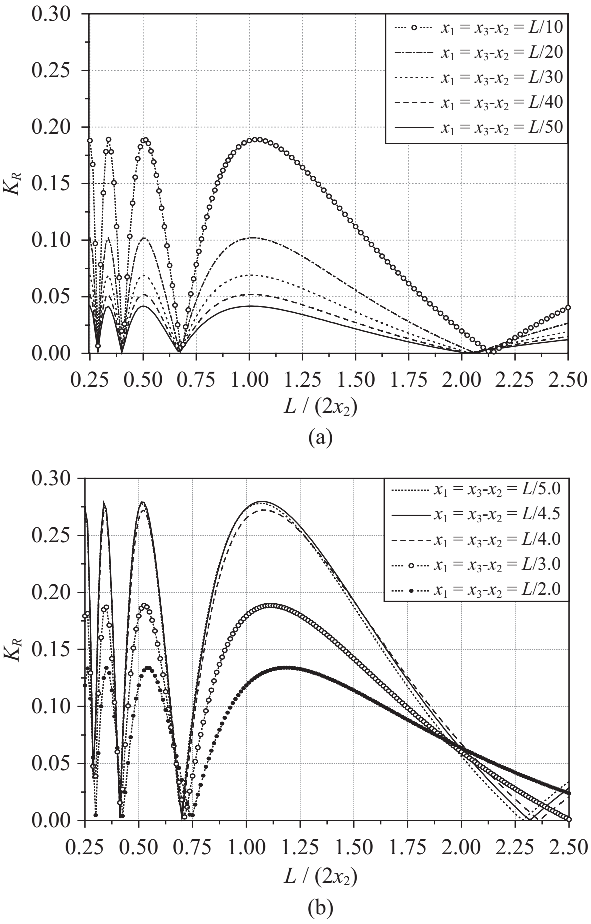

Fig. 6. Changing trend of the peak Bragg reflection: $h_1 = h_3 = 2$ m, $h_0 = h_2 = h_4 = 4$ m, $k_0h_0 = 0.2$, i.e., $L = 40\pi$, $x_1 = x_3 - x_2 = L/50, L/40, \ldots, L/2$.

becomes twin breakwaters with same size which can be regarded as an artificial periodic sandbars with the wavelength (or distance) being x_2. According to the original Bragg law in optics and Miles' theory [10] for wave Bragg reflection, the peak Bragg reflection occurs at $2x_2/L$ being positive integers, where L is the wavelength of incident waves. It is clear that the magnitude of the peak Bragg reflection will be affected by the width of the twin breakwaters, i.e., x_1 or $x_3 - x_2$.

To see the changing trend of the magnitude of the peak Bragg reflection against the width of the twin breakwaters, we fix $h_0 = h_2 = h_4 = 4$ m, $h_1 = h_3 = 3$ m, $k_0h_0 = 0.2$ (i.e., $L = 40\pi$), then we let x_1 and $x_3 - x_2$ vary from $L/50$ to $L/2$. By using the analytical solution (15), reflection coefficients against $L/2x_2$ in (0.25, 2.5) for 10 cases with $x_1 = x_3 - x_2$ ranged from $L/50$ to $L/2$ are calculated and the results are presented in Fig. 6(a)-(b). It can be seen that for all 10 cases, as expected, the peak Bragg reflections do occur at $2x_2/L$ = 1, 1/2, 1/3 and 1/4, i.e., $2x_2/L$ = 1, 2, 3 and 4. When $x_1 = x_3 - x_2 = L/50$, the magnitude of the peak Bragg reflection is 0.0417. Then as the width of the twin breakwaters increases to $x_1 = x_3 - x_2 = L/4.5$, the magnitude of the peak Bragg reflection reaches its maximal value 0.28. As the width of the twin breakwaters further increases, the magnitude of the peak Bragg reflection begins to decline. By the way, because all the twin breakwaters are symmetrical, the phenomenon of zero reflection can be always observed.

V. CONCLUSIONS

In this paper, the reflection of linear long waves by two rectangular breakwaters is studied analytically. An explicit expression of the reflection coefficient in a closed-form is obtained. The new solution is simple but can be reduced into two well-known analytical solutions [5] and [9] for special cases.

Based on the present analytical solution, the variation of reflection coefficient is firstly investigated when a new breakwater is added to an existing one. It is found that, once the second breakwater is present, the periodicity of the reflection coefficient existing for a single breakwater disappears, and the total reflection produced by the composite breakwaters exceeds that produced by the single breakwater. Secondly, the changing trend of reflection coefficient is investigated when a single breakwater is decomposed into two breakwaters, it is found that the total reflection will be enhanced after the single breakwater is decomposed even if the sectional area of the resulting composite breakwaters is less than the area of the original breakwater. Finally, by comparing the computing results of the peak Bragg reflection, it is found that the width of breakwaters will affect the magnitude of the peak Bragg reflection.

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