



## ANALYSIS OF CONTACT PROBLEM USING IMPROVED FAST MULTIPOLE BEM WITH VARIABLE ELEMENTS LENGTH THEORY

Hai-Lian Gui

*Material Science & Engineering Science College, Taiyuan University of Science and Technology, Taiyuan, P.R. China, guihl2000@yahoo.com.cn*

Qiang Li

*Technology Center, Taiyuan Heavy Industry CO. LTD., Taiyuan, P.R. China*

Qing-Xue Huang

*Material Science & Engineering Science College, Taiyuan University of Science and Technology, Taiyuan, P.R. China*

Guang-Xian Shen

*College of Mechanical Engineering, Yanshan University, Qinhuangdao, P.R. China.*

Follow this and additional works at: <https://jmstt.ntou.edu.tw/journal>



Part of the [Engineering Commons](#)

### Recommended Citation

Gui, Hai-Lian; Li, Qiang; Huang, Qing-Xue; and Shen, Guang-Xian (2013) "ANALYSIS OF CONTACT PROBLEM USING IMPROVED FAST MULTIPOLE BEM WITH VARIABLE ELEMENTS LENGTH THEORY," *Journal of Marine Science and Technology*: Vol. 21: Iss. 1, Article 1.

DOI: 10.6119/JMST-010-0317-1

Available at: <https://jmstt.ntou.edu.tw/journal/vol21/iss1/1>

This Research Article is brought to you for free and open access by Journal of Marine Science and Technology. It has been accepted for inclusion in Journal of Marine Science and Technology by an authorized editor of Journal of Marine Science and Technology.

---

## **ANALYSIS OF CONTACT PROBLEM USING IMPROVED FAST MULTIPOLE BEM WITH VARIABLE ELEMENTS LENGTH THEORY**

### **Acknowledgements**

Financial supports for the project from the National Natural Science Foundation of China (51104104), the Basic Research Project in Shanxi Province (2012021019-3) and Doctor Research Foundation of Taiyuan University of Science and Technology (20102018) are gratefully acknowledged.

# ANALYSIS OF CONTACT PROBLEM USING IMPROVED FAST MULTIPOLE BEM WITH VARIABLE ELEMENTS LENGTH THEORY

Hai-Lian Gui<sup>1</sup>, Qiang Li<sup>2</sup>, Qing-Xue Huang<sup>1</sup>, and Guang-Xian Shen<sup>3</sup>

Key words: fast multipole boundary element method (FM-BEM), coefficient matrix, variable elements length theory, contact problem.

## ABSTRACT

This paper introduces a variable elements length theory to solve the change of contact area in contact problems. It avoids recalculating the whole coefficient matrix. In this theory, element length is changed instead of element number in contact boundary problem. This measure keeps the contact area unchanged and saves computing time. The iterative process is discussed and the error estimate is given. Fast multipole boundary element method (FM-BEM) is improved by the variable elements length theory. The improved FM-BEM is used to solve the contact problems of cubes and strip cold rolling process. The results of improved FM-BEM are compared with the results of traditional FM-BEM and experimental data. Numerical examples clearly demonstrate that the calculation time and accuracy are improved by the improved FM-BEM. This method is suitable for solving contact problems and precision engineering problems.

## I. INTRODUCTION

Contact problem is a common problem in engineering. The traditional numerical method is variational method, finite element method (FEM) and boundary element method (BEM). Variational method is used to analyze elastoplastic frictional contact problems with finite deformation [12], FEM is used to analyze crack-contact interaction in two dimensional incomplete fretting contact problem [3]. However, the results solved by FEM are hard to satisfy the requirement of precision en-

gineering, especially analysis of pressure distribution in contact boundary. The merits of BEM are simple calculation and high accuracy, so it is widely used in contact problems in recent years. Such as small displacement contact problems [1], estimating thermal contact resistance [4], and nonlinear inelastic uniform torsion of composite bars [11]. With the fast multipole method [3, 5, 6, 10] introduced, the new numerical method named fast multipole boundary element method (FM-BEM) is presented and used in engineering problems, such as two dimensional elastics [13], elastoplastic contact with friction [14], thin shell structures [15, 16] and so on. In theory, fast IGMRES(m) method is applied in FM-BEM in order to improve the calculation speed [9]. A dual boundary integral equation is established for large-scale modeling based on FM-BEM [7, 8].

However, the calculation accuracy of traditional FM-BEM is not very precise to solve precision contact problem. This is because that the contact boundary changes in loading step. The results solved in n-th step is not used in (n+1)-th step. There are two methods for solving this problem. One method is the results solved in n-th step are modified and be used in (n+1)-th step. It is difficult to give the modified coefficients accurately and it affects the calculation accuracy. The other method is that the coefficient matrix is recalculated in contact boundary. It saves computing time.

In this paper, variable elements length is introduced to solve the contact boundary problem. In Section II, boundary integral equation (BIE) of contact problem is established; in Section III, elements are analyzed in contact boundary. Variable elements length theory is presented and iterative algorithm is given; in section IV, the error analysis of improved FM-BEM is given; in Section V, two numerical examples are given. Through compare the results of improved FM-BEM with traditional FM-BEM and experimental data, it illustrates that improved FM-BEM can improve the computing time and accuracy combined. This method is suitable for solving precision engineering problems.

## II. THE TRANSCEIVER STRUCTURE

Consider a contact problem between body A and body B.

Paper submitted 11/05/09; revised 03/08/10; accepted 03/17/10. Author for correspondence: Hai-Lian Gui (e-mail: guihl2000@yahoo.com.cn).

<sup>1</sup>Material Science & Engineering Science College, Taiyuan University of Science and Technology, Taiyuan, P.R. China.

<sup>2</sup>Technology Center, Taiyuan Heavy Industry CO. LTD., Taiyuan, P.R. China.

<sup>3</sup>College of Mechanical Engineering, Yanshan University, Qinhuangdao, P.R. China.

$\Omega^A$  and  $\Omega^B$  respectively denote the domain of  $A$  and  $B$ ,  $\Gamma^A = \Gamma^{AC} \cup \Gamma^{AD}$  denotes the boundary of  $A$ , where  $\Gamma^{AD}$  and  $\Gamma^{AC}$  respectively denote the non-contact boundary and contact boundary of  $A$ .  $\Gamma^B = \Gamma^{BC} \cup \Gamma^{BD}$  denotes the boundary of  $B$ , where  $\Gamma^{BD}$  and  $\Gamma^{BC}$  respectively denote the non-contact boundary and contact boundary of  $B$ .

Using the weighted residual method, the BIEs of  $A$  and  $B$  are given.

$$c^i u = \int_{\Gamma^A} u_{ij}^{A*} p_j d\Gamma - \int_{\Gamma^A} p_{ij}^{A*} u_j d\Gamma + \int_{\Omega^A} u_{ij}^{A*} b_j d\Omega \quad (1)$$

$$c^i u^i = \int_{\Gamma^B} u_{ij}^{B*} p_j d\Gamma - \int_{\Gamma^B} p_{ij}^{B*} u_j d\Gamma + \int_{\Omega^B} u_{ij}^{B*} b_j d\Omega \quad (2)$$

where  $c_{ij}$  is a geometry function, in smooth boundary,  $c_{ij} = \frac{1}{2} \delta_{ij}$ ,  $u_{ij}^{A*}$  and  $u_{ij}^{B*}$  are components of displacement fundamental solutions,  $p_{ij}^{A*}$  and  $p_{ij}^{B*}$  are components of stress fundamental solutions.

In order to simplify discussion, the body force is defined zero. Discretized boundary of  $\Gamma^A$  and  $\Gamma^B$ , the discrete BIE can be written as

$$c^i u^i = \sum_{\Gamma^{AD}} \int_{\Gamma^{AD}} (u_{ij}^{A*} p_j - p_{ij}^{A*} u_j) d\Gamma + \sum_{\Gamma^{AC}} \int_{\Gamma^{AC}} (u_{ij}^{A*} p_j - p_{ij}^{A*} u_j) d\Gamma \quad (3)$$

$$c^i u^i = \sum_{\Gamma^{BD}} \int_{\Gamma^{BD}} (u_{ij}^{B*} p_j - p_{ij}^{B*} u_j) d\Gamma + \sum_{\Gamma^{BC}} \int_{\Gamma^{BC}} (u_{ij}^{B*} p_j - p_{ij}^{B*} u_j) d\Gamma \quad (4)$$

where  $\Gamma^{AD}$  and  $\Gamma^{AC}$  respectively denote the non-contact boundary and contact boundary of body  $K$  ( $K = A, B$ ).

Let:

$$H_{ij}^{AD} = \int_{\Gamma^{AD}} p_{ij}^{A*} d\Gamma \quad G_{ij}^{AD} = \int_{\Gamma^{AD}} u_{ij}^{A*} d\Gamma$$

$$H_{ij}^{AC} = \int_{\Gamma^{AC}} p_{ij}^{A*} d\Gamma \quad G_{ij}^{AC} = \int_{\Gamma^{AC}} u_{ij}^{A*} d\Gamma$$

$$H_{ij}^{BD} = \int_{\Gamma^{BD}} p_{ij}^{B*} d\Gamma \quad G_{ij}^{BD} = \int_{\Gamma^{BD}} u_{ij}^{B*} d\Gamma$$

$$H_{ij}^{BC} = \int_{\Gamma^{BC}} p_{ij}^{B*} d\Gamma \quad G_{ij}^{BC} = \int_{\Gamma^{BC}} u_{ij}^{B*} d\Gamma$$

Eqs. (3) and (4) can be written as

$$c^i u^i = \sum_{j=1}^{NC} (G_{ij}^{AC} p_j^i + H_{ij}^{AC} u_j^i) + \sum_{j=1}^{ND} (G_{ij}^{AD} p_j^i + H_{ij}^{AD} u_j^i) \quad (5)$$

$$c^i u^i = \sum_{j=1}^{NC} (G_{ij}^{BC} p_j^i + H_{ij}^{BC} u_j^i) + \sum_{j=1}^{ND} (G_{ij}^{BD} p_j^i + H_{ij}^{BD} u_j^i) \quad (6)$$

where NC denotes the element number in contact boundary, ND denotes the element number in non-contact boundary.

Using boundary coupling conditions in contact boundary  $u_{ij}^A = u_{ij}^B$  and  $p_{ij}^A + p_{ij}^B = 0$ , BIE can be obtained

$$\begin{pmatrix} \mathbf{H}^{AD} & \mathbf{H}^{AC} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}^{BC} & \mathbf{H}^{BD} \end{pmatrix} \begin{bmatrix} \mathbf{U}^{AD} \\ \mathbf{U}^C \\ \mathbf{U}^{BD} \end{bmatrix} = \begin{pmatrix} \mathbf{G}^{AD} & \mathbf{G}^{AC} & \mathbf{0} \\ \mathbf{0} & -\mathbf{G}^{BC} & \mathbf{G}^{BD} \end{pmatrix} \begin{bmatrix} \mathbf{P}^{AD} \\ \mathbf{P}^C \\ \mathbf{P}^{BD} \end{bmatrix} \quad (7)$$

where  $\mathbf{H}^{KD} = (H_{ij}^{KD})$  and  $\mathbf{H}^{KC} = (H_{ij}^{KC})$  respectively denote displacement coefficient matrix of body  $K$  ( $K = A, B$ ) in non-contact boundary and contact boundary,  $\mathbf{U}^{KD}$  and  $\mathbf{U}^{KC}$  respectively denote displacement matrix of body  $K$  ( $K = A, B$ ) in non-contact boundary and contact boundary,  $\mathbf{G}^{KD} = (G_{ij}^{KD})$  and  $\mathbf{G}^{KC} = (G_{ij}^{KC})$  respectively denote stress coefficient matrix of body  $K$  ( $K = A, B$ ) in non-contact boundary and contact boundary,  $\mathbf{P}^{KD}$  and  $\mathbf{P}^{KC}$  respectively denote stress matrix of body  $K$  ( $K = A, B$ ) in non-contact boundary and in contact boundary.

### III. PROCESSING TECHNIQUES

Iterative calculation is always used in contact problem and the contact boundary may be changed from (n-1)-th step to n-th step. In this state, the displacement coefficient and stress coefficient in contact matrix must be recalculated. At the same time, the results solved in (n-1)-th step are not use in n-th step. This is very waste for solving problems. We must find a method to make sure the contact coefficient matrix is constant. In this paper, variable elements length theory is presented to make sure the contact coefficient matrix in contact boundary is invariant.

Supposed the elements is invariant in contact boundary, the contact boundary change is expressed by element length change. Let the element total number is  $k$  in contact boundary. At (n-1)-th step, the real contact element number is  $m$ , the element length is  $l_i$ ,  $i = 1, 2, \dots, m$ , so the boundary integral in contact boundary is  $d\Gamma_{n-1} = \bigcup_{i=1}^m l_i$ . According to hypothesis, the element number is still  $m$  at n-th step, but the boundary integral in contact boundary is changed into  $d\Gamma_n = \bigcup_{i=1}^m l_i'$ .

The boundary integral term also be changed:  
(n-1)-th step:

$$G_{ij}^{AC_{n-1}} = \int_{\Gamma^{AC_n}} u_{ij}^{A*} d\Gamma_n \quad G_{ij}^{BC_{n-1}} = \int_{\Gamma^{BC_n}} u_{ij}^{B*} d\Gamma_n$$

$$H_{ij}^{AC_{n-1}} = \int_{\Gamma^{AC_n}} p_{ij}^{A*} d\Gamma_n \quad H_{ij}^{BC_{n-1}} = \int_{\Gamma^{BC_n}} p_{ij}^{B*} d\Gamma_n$$

n-th step:

$$G_{ij}^{AC_n} = \int_{\Gamma^{AC_n}} u_{ij}^{A*} d\Gamma_n \quad G_{ij}^{BC_n} = \int_{\Gamma^{BC_n}} u_{ij}^{B*} d\Gamma_n$$

$$H_{ij}^{AC_n} = \int_{\Gamma^{AC_n}} p_{ij}^{A*} d\Gamma_n \quad H_{ij}^{BC_n} = \int_{\Gamma^{BC_n}} p_{ij}^{B*} d\Gamma_n$$

$$\begin{pmatrix} \mathbf{G}^{AD} & \mathbf{G}^{AC_n} & \mathbf{0} \\ \mathbf{0} & -\mathbf{G}^{BC_n} & \mathbf{G}^{BD} \end{pmatrix} = \begin{pmatrix} \mathbf{G}^{AD} & \mathbf{G}^{AC_0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{G}^{BC_0} & \mathbf{G}^{BD} \end{pmatrix} + \begin{pmatrix} \mathbf{0} & \mathbf{G}^{AC_{n-0}} & \mathbf{0} \\ \mathbf{0} & -\mathbf{G}^{BC_{n-0}} & \mathbf{0} \end{pmatrix} \quad (13)$$

Supposed the initial coefficient matrix are:

$$H_{ij}^{KC_0} = \int_{\Gamma^{KC_0}} p_{ij}^{K*} d\Gamma_0$$

$$G_{ij}^{KC_0} = \int_{\Gamma^{KC_0}} u_{ij}^{K*} d\Gamma_0$$

where  $H_{ij}^{KC_0}$  ( $K = A, B$ ) denotes the component of displacement coefficient in contact boundary at initial step,  $G_{ij}^{KC_0}$  ( $K = A, B$ ) denotes the component of stress coefficient in contact boundary at initial step.

According to the variable elements length theory, the n-th step can be written as:

$$H_{ij}^{KC_n} = \int_{\Gamma^{KC_n}} p_{ij}^{K*} d\Gamma_n = \int_{\Gamma^{KC_0}} p_{ij}^{K*} d\Gamma_0 + \int_{\Gamma^{KC_{n-0}}} p_{ij}^{K*} d\Gamma_{n-0}$$

$$= H_{ij}^{KC_0} + H_{ij}^{KC_{n-0}} \quad (8)$$

$$G_{ij}^{KC_n} = \int_{\Gamma^{KC_n}} u_{ij}^{K*} d\Gamma_n = \int_{\Gamma^{KC_0}} u_{ij}^{K*} d\Gamma_0 + \int_{\Gamma^{KC_{n-0}}} u_{ij}^{K*} d\Gamma_{n-0}$$

$$= G_{ij}^{KC_0} + G_{ij}^{KC_{n-0}} \quad (9)$$

where  $H_{ij}^{KC_n}$  ( $K = A, B$ ) denotes the component of displacement coefficient in contact boundary at n-th step,  $G_{ij}^{KC_n}$  ( $K = A, B$ ) denotes the component of stress coefficient in contact boundary at n-th step.

So in Eqs. (5) and (6), the coefficient matrix in contact boundary can be written as:

$$\sum_{j=1}^N H_{ij}^{KC_n} u^j = \sum_{j=1}^N H_{ij}^{KC_0} u^j + \sum_{j=1}^N H_{ij}^{KC_{n-0}} u^j \quad (10)$$

$$\sum_{j=1}^N G_{ij}^{KC_n} p^j = \sum_{j=1}^N G_{ij}^{KC_0} p^j + \sum_{j=1}^N G_{ij}^{KC_{n-0}} p^j \quad (11)$$

$$\begin{pmatrix} \mathbf{H}^{AD} & \mathbf{H}^{AC_n} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}^{BC_n} & \mathbf{H}^{BD} \end{pmatrix} = \begin{pmatrix} \mathbf{H}^{AD} & \mathbf{H}^{AC_0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}^{BC_0} & \mathbf{H}^{BD} \end{pmatrix} + \begin{pmatrix} \mathbf{0} & \mathbf{H}^{AC_{n-0}} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}^{BC_{n-0}} & \mathbf{0} \end{pmatrix} \quad (12)$$

Combined with fast multipole method, equation system can be solved in Krylov space, the displacement and stress in boundary can be obtained.

#### IV. ERROR ANALYSIS

Variable elements length theory is used in FM-BEM, called improved FM-BEM. It is not only reduce the computational complexity of coefficient matrix, but also avoids the error accumulation. After using the variable elements length theory, the error analysis about displacement is given as follow.

**TH1:** To the Dirichlet problem of three dimensional Laplace equation, there is:

$$u(\bar{y}) = \frac{1}{4\pi} \int_{\Gamma} \frac{\sigma(\bar{x})}{|\bar{x} - \bar{y}|} ds_{\bar{x}}, \quad \bar{y} \in R^3, \quad (14)$$

Supposed  $u_h$  denote the approximate solution,

$$u_h(\bar{y}) = \frac{1}{4\pi} \int_{\Gamma_h} \frac{\sigma_h(\bar{x})}{|\bar{x} - \bar{y}|} ds_{\bar{x}}$$

where  $\sigma_h(\bar{x}) \in V_h(\Gamma_h)$ ,  $\bar{y} \in R^3$ . The distance from point  $\bar{y}$  to boundary  $\Gamma$  is  $d(\bar{y}, \Gamma) \geq \delta > 0$ ,  $h$  is sufficiently small, the error estimate can be written as:

$$|u(\bar{y}) - u_h(\bar{y})| \leq \frac{Cd(\bar{y}, \Gamma) + C}{(d(\bar{y}, \Gamma))^2} \{h^{k+1} \|\sigma\|_{0,\Gamma} + h^{m+2} \|\sigma\|_{m+1,\Gamma} + \|u_0 - \tilde{u}_{0k}\|_{0,\Gamma} + h^{\frac{1}{2}} \|u_0 - \tilde{u}_{0k}\|_{\frac{1}{2},\Gamma}\} \quad (15)$$

$$|\partial^\alpha u(\bar{y}) - \partial^\alpha u_k(\bar{y})| \leq \frac{Cd(\bar{y}, \Gamma) + C}{(d(\bar{y}, \Gamma))^{\alpha+2}} \{h^{k+1} \|\sigma\|_{0,\Gamma} + h^{m+2} \|\sigma\|_{m+1,\Gamma} + \|u_0 - \tilde{u}_{0k}\|_{0,\Gamma} + h^{\frac{1}{2}} \|u_0 - \tilde{u}_{0k}\|_{\frac{1}{2},\Gamma}\} \quad (16)$$

Because the error of boundary element method (BEM) comes from discrete elements and boundary function approximation, it is not relationship with the number of elements and nodes. So the calculation accuracy is not affected by variable elements length theory.

When the boundary precision is satisfied, the above error estimate can be written as:

$$|u(\bar{y}) - u_h(\bar{y})| \leq C \left( \sum_{l=0}^{m+2} \frac{1}{(d(\bar{y}, \Gamma))^{l+1}} \right) h^{2m+3} \|\sigma\|_{m+1,\Gamma} \quad (17)$$

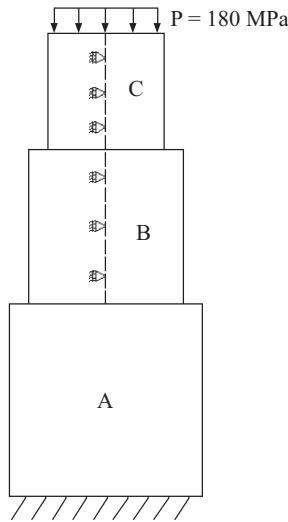


Fig. 1. Computing model of contact problem.

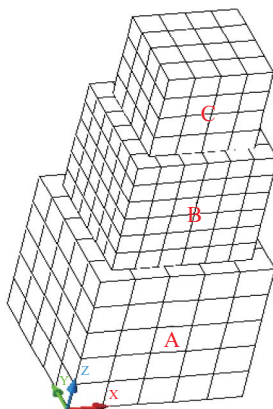


Fig. 2. Discrete model of contact problem.

$$|\partial^\alpha u(\bar{y}) - \partial^\alpha u_h(\bar{y})| \leq C \left( \sum_{l=0}^{m+2} \frac{1}{(d(\bar{y}, \Gamma))^{l+|\alpha|}} \right) h^{2m+3} \|\sigma\|_{m+1, \Gamma} \quad (18)$$

Through the error estimate, we can know that the variable elements length theory effects the calculation accuracy only  $O(h^{2m+3})$ . But the elements are always subdivided in contact boundary, the elements length are very small, so the change of elements have even smaller impact to accuracy.

## V. NUMERICAL EXAMPLES

### 1. Contact of Three Elastic Cubes

Consider the contact problem of three elastic cubes. The side length of cubes are respectively 120 mm, 100 mm, 80 mm. The computing model and discrete model show as Figs. 1 and 2. Body A is fixed constraint on the bottom surface and body C is forced a uniform load  $P = 100 \text{ MPa}$  on the upper surface. For three bodies, the Young's modulus is  $E = 210 \text{ GPa}$ , the

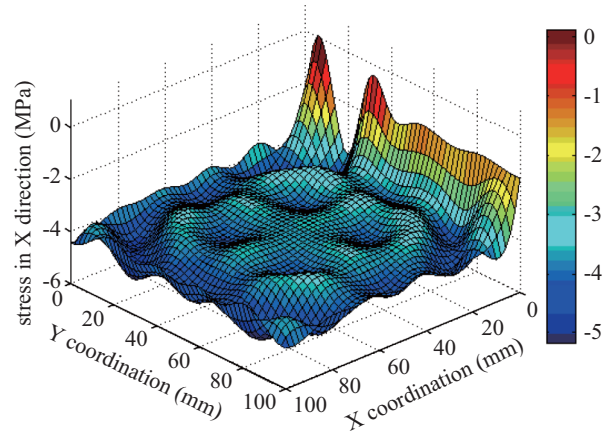


Fig. 3. Computed result of contact problem using traditional FM-BEM.

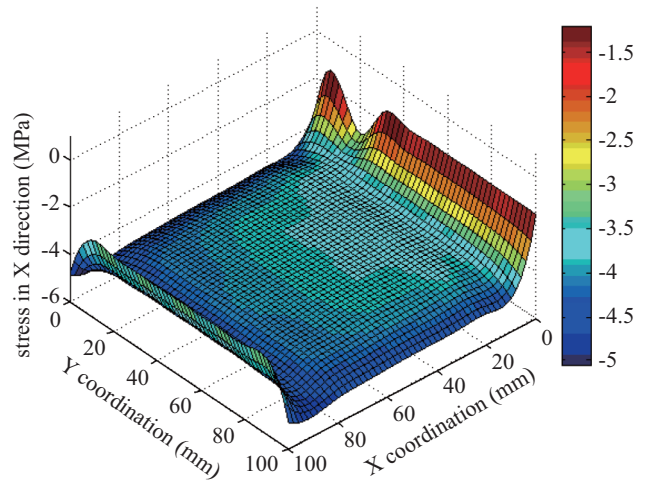


Fig. 4. Computed result of contact problem using improved FM-BEM.

Poisson's ratio is  $\nu = 0.3$ , the frictional coefficient is  $f = 0.3$ , the contact limited is 0.001 mm.

This model is solved by traditional FM-BEM and improved FM-BEM. All computations are run on a Windows XP computer equipped with one 2.80 GHz Intel Pentium 4 unit and 512 MB of core memory. We mainly discuss the stress and pressure distribution in contact boundary. The calculation time of these methods are respectively 19.42 seconds and 9.33 seconds.

In order to illustrate the stable and calculation accuracy, stress and pressure distribution are discussed about body B. Figs. 3 and 4 show that the stress in X direction solved by traditional FM-BEM and improved FM-BEM. Figs. 5 and 6 show that the pressure distribution on the upper surface in Z direction. From the figures, we know that the stress and pressure continuity obtained by improved FM-BEM is better than traditional FM-BEM. The result of traditional FM-BEM has uneven distribution in center on the surface. This is because that the iterative error reduced using variable elements length. The coefficient matrix is constant in non-contact boundary.

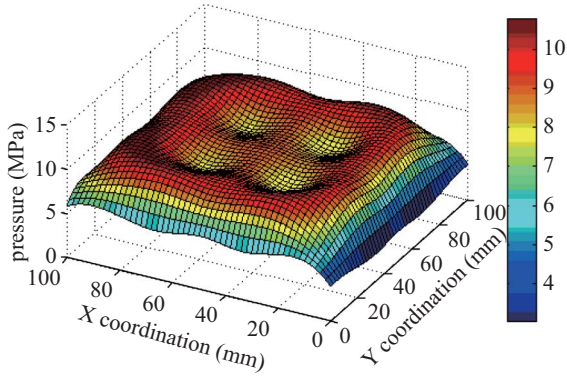


Fig. 5. Calculation result of traditional FM-BEM.

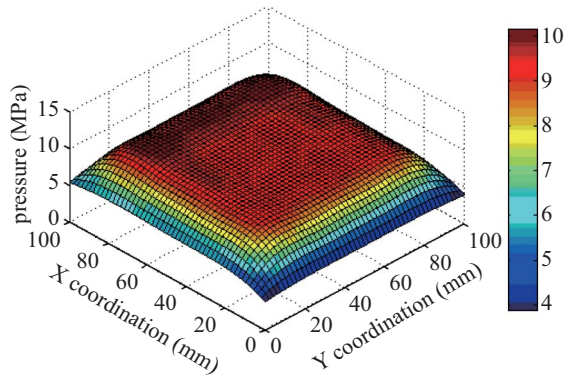


Fig. 6. Calculation result of improved FM-BEM.

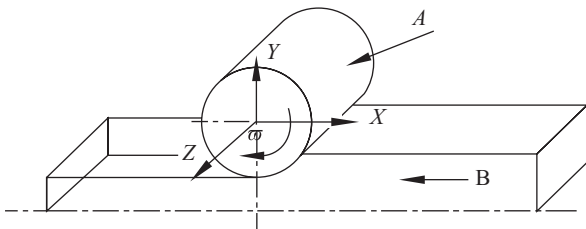


Fig. 7. Calculation model of cold rolling process.

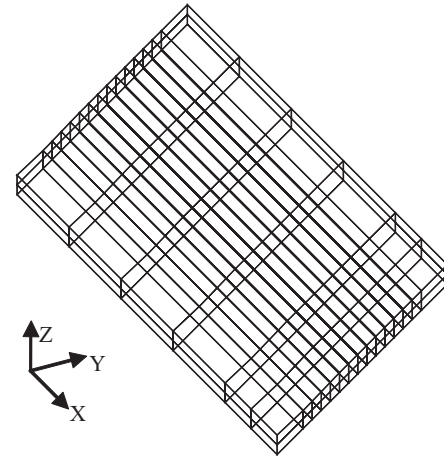


Fig. 8. Element division of strip.

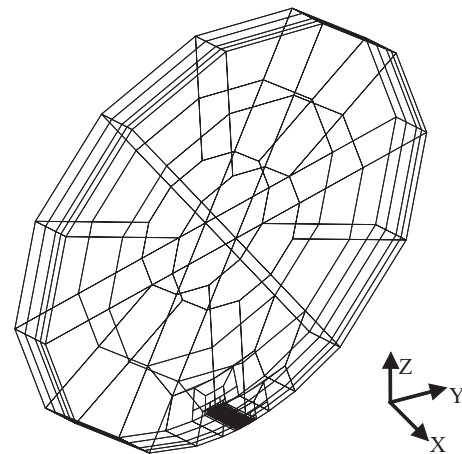


Fig. 9. Element division of work roll.

## 2. Contact Problem in Strip Cold Rolling Process

In this case, variable elements length theory is applied in engineering problems, such as the strip cold rolling process. Consider the simplified model as Fig. 7, A is work roll and B is strip. Supposed the zero displacement constraints are forced in the thick and horizontal surface, no tension force, namely the former and latter tension is zero. Friction condition obeys the Coulomb friction law, and the friction coefficient is constant. The calculation parameters of rolling process are given in Table 1.

Figs. 8 and 9 show the element division of strip and work roll. In contact boundary, elements are divided according to the rule of fast algorithm, namely strip elements in contact boundary are divided into  $l$  levels,  $l = 1, 2, \dots$ . Work roll elements are divided into  $ll$  levels. The interaction of particles

Table 1. Conditions of rolling process.

Calculation parameters	Parameter value
Diameter ( $D$ )	750 mm
Strip width ( $B$ )	1250 mm
Strip thickness ( $H$ )	2 mm
Depress ratio ( $\varepsilon$ )	20%
Shear friction coefficient ( $m$ )	0.7
Young's modulus ( $E$ )	210 GPa
Yield stress ( $\sigma_s$ )	250 MPa
Sliding friction coefficient ( $f$ )	0.2
Yield hardening coefficient ( $h$ )	0.005
Poisson ratio ( $\nu$ )	0.3
Angle incremental ( $\Delta\theta$ )	0.002Ra
Rolling speed ( $v$ )	0.240 $m \cdot s^{-1}$

are  $Z_d^c$ , which  $d$  represents the number of element,  $c$  represents the levels of element.

The rolling process is simulated by four-high reversing cold





Fig. 10. The laboratory mill.



Fig. 11. The working status of mill.

rolling mill in Taiyuan University of science and technology. Fig. 10 shows that the laboratory four-high reversing cold rolling mill, Fig. 11 is the working status of mill.

Through solved by improved FM-BEM, the displacement in Y direction shows in Fig. 12. Compared the results solved by improved FM-BEM with the results of FEM and experimental data measured in laboratory mill, it is concluded that the results solved by improved FM-BEM is more close to the experimental data than FEM and improved FM-BEM is more suitable for solving contact problems. It can provide stable and accuracy simulation for engineering problems. Fig. 13 shows the three dimensional image about displacement in Y direction solved by improved FM-BEM.

## VI. CONCLUSIONS

Based on fast multipole boundary element method (FM-BEM), variable elements length theory is presented to solve the contact problem. In this theory, the number of elements is supposed constant, the element length is changed to illustrate the contact boundary change. The coefficient matrix of displacement and stress are only calculated in contact boundary

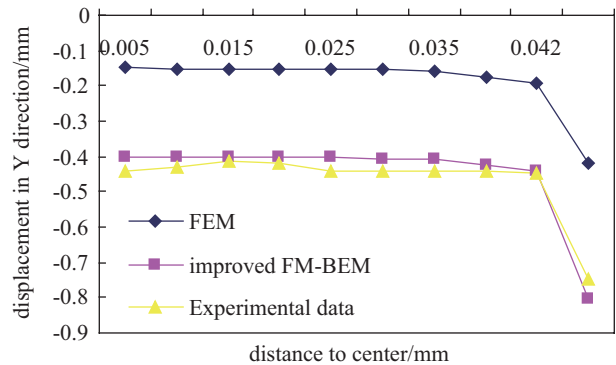


Fig. 12. Displacement in Y direction.

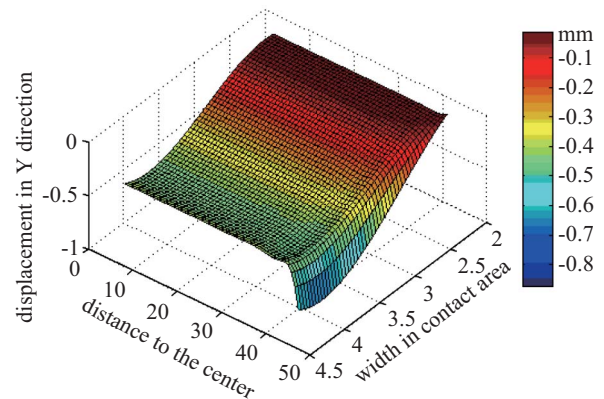


Fig. 13. Displacement in Y direction.

instead of calculating whole coefficient matrix. Combined variable elements length theory with FM-BEM, two numerical example about contact problems are solved by improved FM-BEM. Through analysis of the results, the calculation time and accuracy are improved. The continuity of stress and pressure are also stable than FEM. So it illustrates that variable elements length theory is in favor of contact problems and improved FM-BEM is more suitable for solving precision engineering problems.

## ACKNOWLEDGMENTS

Financial supports for the project from the National Natural Science Foundation of China (51104104), the Basic Research Project in Shanxi Province (2012021019-3) and Doctor Research Foundation of Taiyuan University of Science and Technology (20102018) are gratefully acknowledged.

## REFERENCES

- Blázquez, A. and París, F., "On the necessity of non-conforming algorithms for 'small displacement' contact problems and conforming discretizations by BEM," *Engineering Analysis with Boundary Elements*, Vol. 33, No. 2, pp. 184-190 (2009).
- Cheng, H., Greengard, L., and Rokhlin, V., "A fast adaptive multipole algorithm in three dimensions," *Journal of Computational Physics*, Vol. 6,



- pp. 229-270 (1997).
3. Gill, J., Divo, E., and Kassab, A. J., "Estimating thermal contact resistance using sensitivity analysis and regularization," *Engineering Analysis with Boundary Elements*, Vol. 33, No. 1, pp. 54-62 (2009).
  4. Giner, E., Tur, M., Vercher, A., and Fuenmayor, F. J., "Numerical modeling of crack-contact interaction in 2D incomplete fretting contacts using X-FEM," *Tribology International*, Vol. 42, No. 9, pp. 1269-1275 (2009).
  5. Greengard, L., "Spectral integration and two-point boundary value problems," *Society for Industrial and Applied Mathematics*, Vol. 28, pp. 1071-1080 (1991).
  6. Greengard, L. and Rokhlin, V., "A Fast algorithm for particle simulations," *Journal of Computational Physics*, Vol. 73, pp. 325-348 (1987).
  7. Liu, Y. J., "A new fast multipole boundary element method for solving large-scale two-dimensional elasto static problems," *International Journal for Numerical Methods in Engineering*, Vol. 65, pp. 863-881 (2006).
  8. Liu, Y. J. and Shen, L., "A dual BIE approach for large-scale modeling of 3-D electrostatic problems with the fast multipole boundary element method," *International Journal for Numerical Methods in Engineering*, Vol. 71, pp. 837-855 (2007).
  9. Mu, Y. F. and Yu, C. X., "Fast IGMRES(m) method based on the frictional contact FM-BEM and its convergence," Second International Conference on Innovative Computing, Information and Control, ICICIC '07, Kumamoto, Japan, p. 505 (2007).
  10. Rokhlin, V., "Rapid solution of integral equations of classical potential theory," *Journal of Computational Physics*, Vol. 60, pp. 187-207 (1985).
  11. Sapountzakis, E. J. and Tsipiras, V. J., "Nonlinear inelastic uniform torsion of composite bars by BEM," *Computers and Structures*, Vol. 87, Nos. 3-4, pp. 151-166 (2009).
  12. Wan, J., Tang, G. J., and Li, D. K., "Shape design sensitivity analysis of elastoplastic frictional contact problems with finite deformation," *Chinese Journal of Theoretical and Applied Mechanics*, Vol. 41, No. 4, pp. 503-517 (2009).
  13. Yamada, Y. and Hayami, K., "A multipole boundary element method for two dimensional elastics," *Technical Reports*, Department of Mathematical Engineering and Information Physics, Faculty of Engineering, The University of Tokyo (1995).
  14. Yu, C. X. and Shen, G. X., "Program-iteration pattern Fast Multipole BEM for elasto-plastic contact with friction," *Chinese Journal of Computational Mechanics*, Vol. 25, No. 1, pp. 65-71 (2008).
  15. Zhao, L. B. and Yao, Z. H., "The fast multipole-BEM about 3-D elastic problem applied in thin shell structure," *Postdoctoral Report in Tsinghua University*, pp. 20-43 (2003).
  16. Zhao, L. B. and Yao, Z. H., "Fast multipole BEM for 3-D elasto static problems with application for thin structures," *Tsinghua Science and Technology*, Vol. 10, pp. 67-75 (2005).