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EXPONENTIAL STABILITY ANALYSIS FOR NEURAL NETWORKS WITH TIME-VARYING DELAY AND LINEAR FRACTIONAL PERTURBATIONS

Chang-Hua Lien and Ker-Wei Yu

Key words: delayed neural network, global exponential stability, delay-dependent criterion, delay-independent criterion, linear fractional perturbation, linear matrix inequality.

ABSTRACT

In this paper, the global exponential stability and global asymptotic stability for a class of uncertain delayed neural networks (UDNNs) with time-varying delay and linear fractional perturbations are considered. Delay-dependent and delay-independent criteria are proposed to guarantee the robust stability of UDNNs via linear matrix inequality (LMI) approach. Additional nonnegative inequality approach is used to improve the conservativeness of the stability criteria. Some numerical examples are illustrated to show the effectiveness of our results. From the simulation results, significant improvement over the recent results can be observed.

I. INTRODUCTION

The existence of time delays is often a source of oscillation and instability of practical systems. Neural networks has been applied in many mathematical and practical applications, such as approximation, association, diagnosis, decision, generalization, optimization, prediction, and recognition. Many neural networks have been proposed in recent years, such as bidirectional associative memory neural networks [16], cellular neural networks [3], Cohen-Grossberg neural networks [13], and Hopfield neural networks [11]. The delayed neural networks (DNNs) may be used in many areas including the moving images processing and pattern classification. The implementation in hardware for very large scale integration chip, modeling errors, parameters fluctuation, and external disturbance may destory the stability of DNNs. Hence stability of DNNs is very important and significant in practical applications. In practical analysis for uncertain DNNs, it is reasonable to consider the parameters varying in some prescribed intervals or staisfying some classes of parametric uncertainties. DNNs with interval variations are called the interval delayed neural networks (IDNNs) [2, 5, 8, 9, 11, 12, 15]. In [10] and [18], DNNs with linear fractional parametric perturbations have been investigated. IDNNs and DNNs with general structural perturbation in [4] are speical cases of DNNs with linear fractional parametric perturbations. Hence we will consider the stability analysis of DNNs with linear fractional parametric perturbations in this paper.

Depending on whether the stability criterion itself contains the size of delay, criteria for DNN can be classified into two categories, namely delay-independent criteria [2, 5, 9, 12] and delay-dependent criteria [2, 4, 5, 8-10, 15, 18]. Usually the latter is less conservative when the delay is small. In the Lyapunov-based delay-dependent results, the slow-varying constraint $\dot{\tau}(t) < 1$ is usually imposed on the time-varying delay [8, 9, 11, 15]. The constraint will be relaxed and delay-dependent result will be proposed in this paper. In [2, 12], algebraic stability, criteria were proposed based on Halanay inequality, Young's inequality, and Lyapunov functional. It is usually difficult to find feasible solutions for these algebraic criteria. LMI approach is an efficient tool for dealng with these control problems. The LMI problem can be solved quite efficiently by using the toolbox of Matlab [1]. In [4, 5, 7-11, 13-15, 18], stability criteria for DNNs have been proposed via LMI approach. Additional nonnegative inequality approach is used to improve the conservativeness of the obtained results [17]. In this paper, LMI-based delay-dependent and delay-independent criteria are proposed by using new Lyapunov functional. In general, our approach is useful and is easy to generalize to other forms of UDNNs.

The notation used throughout this paper is as follows. For a matrix A, we denote the transpose by A^T, spectral norm by $\|A\|$, minimal (maximal) eigenvalue by $\lambda_{\min}(A)$ ($\lambda_{\max}(A)$),

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symmetric positive (negative) definite by $A > 0$ ($A < 0$). $A \le B$ means that matrix $B - A$ is symmetric positive semi-definite. For a vector x, we denote the Euclidean norm by $\|x\|$. For the state x_t of system, we define $x_t(\theta) := x(t+\theta)$, $\forall \theta \in [-\tau_M, 0]$ and denote its norm by $\|x_t\|_s = \sup_{-\tau_M \le s \le 0} \sqrt{\|x(t+s)\|^2 + \|\dot{x}(t+s)\|^2}$. I denotes the identity matrix. $\underline{n} = \{1, 2, ..., n\}$. $diag[a_i]$ denotes diagonal matrix with the diagonal elements a_i. $diag[a_i]_{i=1}^n$ denotes block diagonal matrix with diagonal row vector a_i.

$$V[\underline{A}, \overline{A}] := \{A = (a_{ij}) \in \Re^{n\times n} \,|\, \underline{A} \le A \le \overline{A}, \text{i.e., } \underline{a}_{ij} \le a_{ij} \le \overline{a}_{ij},\, i,\, j \in \underline{n}\}$$

with $\underline{A} = (\underline{a}_{ij})$ and $\overline{A} = (\overline{a}_{ij})$.

II. PROBLEM FORMULATION

Consider the following uncertain DNN with interval time-varying delay:

$$\dot{x}(t) = -[C + \Delta C]x(t) + [A + \Delta A]y(t) + [B + \Delta B]y(t - \tau(t)) + J,\quad t \ge 0, \tag{1a}$$

$$y(t) = f(x(t)),\quad t \ge 0, \tag{1b}$$

$$x(t) = \phi(t),\qquad t \in [-\tau_M, 0], \tag{1c}$$

where $x(t) = [x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t)]^T$, $n \ge 2$ is the number of neurons in the network, $0 \le \tau(t) \le \tau_M$, $\dot{\tau}(t) \le \tau_D$, $y(t)$ is the output, $J = [J_1 \quad J_2 \quad \cdots \quad J_n]^T$ is the external bias vector, C is a positive diagonal matrix, A is the feedback matrix, B is the delay feedback matrix, and ϕ is the initial continuous function. The linear fractional perturbation matrices ΔC, ΔA, and ΔB are assumed to satisfy the following conditions:

$$\Delta C = M_C \Delta_C(t) N_C,\ \Delta A = M_A \Delta_A(t) N_A,\ \Delta B = M_B \Delta_B(t) N_B, \tag{1d}$$

where

$$\Delta_C(t) = [I - F_C(t)\Theta_C]^{-1} F_C(t),\ \Theta_C \Theta_C^T < I, \tag{1e}$$

$$\Delta_A(t) = [I - F_A(t)\Theta_A]^{-1} F_A(t),\ \Theta_A \Theta_A^T < I, \tag{1f}$$

$$\Delta_B(t) = [I - F_B(t)\Theta_B]^{-1} F_B(t),\ \Theta_B \Theta_B^T < I, \tag{1g}$$

where M_C, M_A, M_B, N_C, N_A, N_B, Θ_C, Θ_A, and Θ_B are some given constant matrices with appropriate dimensions. $F_C(t)$, $F_A(t)$, $F_B(t)$ are some unknown matrices representing the parameter perturbations which satisfy

$$F_C^T(t) F_C(t) \le I,\ F_A^T(t) F_A(t) \le I,\ F_B^T(t) F_B(t) \le I. \tag{1h}$$

The activation functions of DNN (1) given by

$$f(x(t)) = [f_1(x_1(t)) \quad f_2(x_2(t)) \quad \cdots \quad f_n(x_n(t))]^T,$$

are bounded and satisfy the following conditions

$$0 \le \frac{f_i(\xi_1) - f_i(\xi_2)}{\xi_1 - \xi_2} \le L_i,\ \xi_1, \xi_2 \in \Re,\ i \in \underline{n}, \tag{2}$$

where $L_i > 0$, $i \in \underline{n}$, are some positive constants.

Assume $\tilde{x} = [\tilde{x}_1 \quad \tilde{x}_2 \quad \cdots \quad \tilde{x}_n]^T \in \Re^n$ is an equilibrium point of system (1), then we can obtain the following system:

$$\dot{z}(t) = -[C + \Delta C]z(t) + [A + \Delta A]g(z(t)) + [B + \Delta B]g(z(t - \tau(t))), \tag{3}$$

where

$$z(t) = [z_1(t) \quad z_2(t) \quad \cdots \quad z_n(t)]^T = x(t) - \tilde{x},$$

$$g(z(t)) = [g_1(z_1(t)) \quad g_2(z_2(t)) \quad \cdots \quad g_n(z_n(t))]^T,$$

$$g_i(z_i(t)) = f_i(x_i(t)) - f_i(\tilde{x}_i) = f_i(z_i(t) + \tilde{x}_i) - f_i(\tilde{x}_i),$$

$$g_i(0) = 0. \tag{4a}$$

Let $W_j = diag[w_{ji}]$ and $Y_j = diag[y_{ji}]$, j = 1, 2, be two diagonal matrices with $w_{ji}, y_{ji} > 0$. From (2) and (4a), we have

$$0 \le \frac{g_i(z_i(t))}{z_i(t)} \le L_i,\ 0 \le g_i(z_i(t)) z_i(t) \le L_i \cdot z_i^2(t), \tag{4b}$$

$$0 \le g_i^2(z_i(t)) \le L_i \cdot g_i(z_i(t)) z_i(t) \le L_i^2 \cdot z_i^2(t), \tag{4c}$$

$$g^T(z(t)) W_1 g(z(t)) \le g^T(z(t)) \Gamma W_1 z(t),$$
$$g^T(z(t)) \Gamma W_2 z(t) \le z^T(t) \Gamma W_2 \Gamma z(t), \tag{4d}$$

$$g^T(z(t-\tau(t))) Y_1 g(z(t-\tau(t))) \le g^T(z(t-\tau(t))) \Gamma Y_1 z(t-\tau(t)),$$
$$g^T(z(t-\tau(t))) \Gamma Y_2 z(t-\tau(t)) \le z^T(t-\tau(t)) \Gamma Y_2 \Gamma z(t-\tau(t)), \tag{4e}$$

where $\Gamma = diag[L_i]$.

Remark 1. The activation function $f_i(x_i) = 0.5(|x_i + 1| - |x_i - 1|)$ is a general form satisfying (2) with $\Gamma = I$.

Definition 1 [2]. The equilibrium point $\tilde{x}$ of system (1) is said to be the globally exponentially stable (GES) with convergence rate α, if there are two positive constants α and Ψ such that

$$\|x(t) - \tilde{x}\| \leq \Psi \cdot e^{-\alpha t} \text{ for all } t \geq 0.$$

Lemma 1 [10, 18]. Suppose $\Delta(t) = [I - F(t)\Theta]^{-1} F(t)$ with unknown matrix $F(t)$ satisfying $F^T(t)F(t) \leq I$, Θ is a given constant matrix and satisfies $\Theta\Theta^T < I$, then for real matrices H, E and X with $X = X^T$, the following statements are equivalent:

(I) The inequality is satisfied

$$X + H\Delta(t)E + E^T\Delta^T(t)H^T < 0,$$

(II) There exists a scalar $\varepsilon > 0$, such that

$$\begin{bmatrix} X & H & \varepsilon \cdot E^T \\ * & -\varepsilon \cdot I & \varepsilon \cdot \Theta^T \\ * & * & -\varepsilon \cdot I \end{bmatrix} < 0. \tag{5}$$

III. GLOBAL EXPONENTIAL STABILITY ANALYSIS

In this section, we present a delay-dependent criterion for the global exponential stability of system (1) with (2).

Theorem 1. The equilibrium point $\tilde{x}$ of system (1) with (2) and $\tau_D \leq 1$ (resp., $\tau_D > 1$ or unknown) is unique and globally exponentially stable (GES) with convergence rate $\alpha > 0$, if there exist some $n \times n$ positive definite symmetric matrices P, Q_1, Q_2 (resp., $Q_1 = 0$, $Q_2 = 0$), R_1, R_2, S_{22}, a $5n \times 5n$ positive definite symmetric matrix S_{11}, some $n \times n$ positive diagonal matrices V, W_1, W_2, Y_1, Y_2, some matrices $U \in \Re^{n\times n}$, $S_{12} \in \Re^{5n\times n}$, and a positive constant ε, such that the following LMI conditions are satisfied:

$$S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} > 0,\ R_1 > S_{22},\ \Sigma = \begin{bmatrix} \Sigma_1 + \tilde{\Sigma} & \Sigma_2 \\ * & \Sigma_3 \end{bmatrix} < 0, \tag{6}$$

where * is the symmetrical form of matrix,

$$\tilde{\Sigma} = e^{-2\alpha\tau_M} \cdot \{\tau_M \cdot S_{11} + S_{12} \cdot [I \quad -I \quad 0 \quad 0 \quad 0] + [I \quad -I \quad 0 \quad 0 \quad 0]^T S_{12}^T\},$$

$$\Sigma_1 = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} \\ * & \Sigma_{22} & 0 & 0 & \Sigma_{25} \\ * & * & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} \\ * & * & * & \Sigma_{44} & 0 \\ * & * & * & * & \Sigma_{55} \end{bmatrix},$$

$$\Sigma_2 = \begin{bmatrix} \Sigma_{16} & \Sigma_{17} & \Sigma_{18} & \Sigma_{19} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \Sigma_{36} & \Sigma_{37} & \Sigma_{38} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Sigma_{410} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Sigma_{511} \end{bmatrix},$$

$$\Sigma_3 = \begin{bmatrix} \Sigma_{66} & 0 & 0 & \Sigma_{69} & 0 & 0 \\ * & \Sigma_{77} & 0 & 0 & \Sigma_{710} & 0 \\ * & * & \Sigma_{88} & 0 & 0 & \Sigma_{811} \\ * & * & * & \Sigma_{99} & 0 & 0 \\ * & * & * & * & \Sigma_{1010} & 0 \\ * & * & * & * & * & \Sigma_{1111} \end{bmatrix},$$

$$\Sigma_{11} = -PC - C^TP + 2\alpha \cdot P + Q_1 + 2\alpha \cdot \Gamma V + 2\Gamma W_2 \Gamma - e^{-2\alpha\tau_M} \cdot R_2,$$

$$\Sigma_{12} = e^{-2\alpha\tau_M} \cdot R_2,\ \Sigma_{13} = -C^TU^T,\ \Sigma_{14} = PA + \Gamma(W_1 - W_2),$$

$$\Sigma_{15} = PB,\ \Sigma_{16} = PM_C,\ \Sigma_{17} = PM_A,\ \Sigma_{18} = PM_B,$$

$$\Sigma_{19} = -\varepsilon \cdot N_C^T,\ \Sigma_{22} = -e^{-2\alpha\tau_M} \cdot [(1-\tau_D) \cdot Q_1 + R_2] + 2\Gamma Y_2 \Gamma,$$

$$\Sigma_{25} = \Gamma(Y_1 - Y_2),\ \Sigma_{33} = -U^T - U + \tau_M \cdot (R_1 + \tau_M \cdot R_2),$$

$$\Sigma_{34} = UA + V,\ \Sigma_{35} = UB,\ \Sigma_{36} = UM_C,\ \Sigma_{37} = UM_A,$$

$$\Sigma_{38} = UM_B,\ \Sigma_{44} = -2W_1 + Q_2,\ \Sigma_{410} = \varepsilon \cdot N_A^T,$$

$$\Sigma_{55} = -2Y_1 - (1-\tau_D) \cdot e^{-2\alpha\tau_M} \cdot Q_2,\ \Sigma_{511} = \varepsilon \cdot N_B^T,\ \Sigma_{69} = \varepsilon \cdot \Theta_C^T,$$

$$\Sigma_{710} = \varepsilon \cdot \Theta_A^T,\ \Sigma_{811} = \varepsilon \cdot \Theta_B^T,$$

$$\Sigma_{66} = \Sigma_{77} = \Sigma_{88} = \Sigma_{99} = \Sigma_{1010} = \Sigma_{1111} = -\varepsilon \cdot I.$$

Proof. The Lyapunov functional candidate of the system (1) and (2) is given by

$$V_0(z_t) = e^{2\alpha t} \cdot z^T(t)Pz(t) + V_1(z_t) + V_2(z_t), \tag{7a}$$

$$V_1(z_t) = \int_{t-\tau(t)}^{t} e^{2\alpha s} \cdot [z^T(s)Q_1 z(s) + g^T(z(s))Q_2 g(z(s))]ds + \int_{t-\tau_M}^{t} e^{2\alpha s} \cdot (s - (t - \tau_M)) \cdot \dot{z}^T(s)(R_1 + \tau_M \cdot R_2)\dot{z}(s)ds, \tag{7b}$$

$$V_2(z_t) = 2e^{2\alpha t} \cdot \sum_{i=1}^{n} \int_0^{z_i(t)} v_i g_i(s) ds, \tag{7c}$$

where $V = diag[v_1 \cdots v_n]$ and the integral term $\int_0^{z_{i(t)}} v_i g_i(s) ds$ is nonnegative in view of (4b). The time derivatives of $V_0(z_t)$ in (7) along the trajectories of system (3) with (4) satisfy

$$\begin{aligned}\dot{V}_0(z_t) &= e^{2\alpha t} \cdot z^T(t) \cdot 2\alpha P z(t) + e^{2\alpha t} \cdot \dot{z}^T(t) P z(t) \\ &\quad + e^{2\alpha t} \cdot z^T(t) P \dot{z}(t) + \dot{V}_1(z_t) + \dot{V}_2(z_t) \\ &= e^{2\alpha t} \cdot [z^T(t)(-P\bar{C} - \bar{C}^T P + 2\alpha \cdot P) z(t) \\ &\quad + 2z^T(t) P\bar{A} g(z(t)) + 2z^T(t) P\bar{B} g(z(t-\tau(t)))] \\ &\quad + \dot{V}_1(z_t) + \dot{V}_2(z_t),\end{aligned} \tag{8a}$$

where $\bar{C} = C + \Delta C$, $\bar{A} = A + \Delta A$, $\bar{B} = B + \Delta B$. With $\tau_D \le 1$, $Q_1 > 0$, $Q_2 > 0$, or $\tau_D > 1$, $Q_1 = 0$, $Q_2 = 0$, the time derivative of $V_1(z_t)$ is bounded by

$$\begin{aligned}\dot{V}_1(z_t) &= e^{2\alpha t}[z^T(t) Q_1 z(t) + g^T(z(t)) Q_2 g(z(t)) \\ &\quad - (1 - \dot{\tau}(t)) \cdot e^{-2\alpha\tau(t)} \cdot z^T(t-\tau(t)) Q_1 z(t-\tau(t)) \\ &\quad - (1 - \dot{\tau}(t)) \cdot e^{-2\alpha\tau(t)} \cdot g^T(z(t-\tau(t))) Q_2 g(z(t-\tau(t))) \\ &\quad + \tau_M \cdot \dot{z}^T(t)(R_1 + \tau_M \cdot R_2) \dot{z}(t) \\ &\quad - \int_{t-\tau_M}^{t} e^{2\alpha(s-t)} \cdot \dot{z}^T(s)(R_1 + \tau_M \cdot R_2) \dot{z}(s) ds \\ &\quad - \tau_M \cdot \int_{t-\tau_M}^{t} e^{2\alpha(s-t)} \cdot \dot{z}(s)^T R_2 \dot{z}(s) ds] \\ &\le e^{-2\alpha t}[z^T(t) Q_1 z(t) + g^T(z(t)) Q_2 g(z(t)) \\ &\quad - (1 - \tau_D) \cdot e^{-2\alpha\tau_M} \cdot z^T(t-\tau(t)) Q_1 z(t-\tau(t)) \\ &\quad - (1 - \tau_D) \cdot e^{-2\alpha\tau_M} \cdot g^T(z(t-\tau(t))) Q_2 g(z(t-\tau(t))) \\ &\quad + \tau_M \cdot \dot{z}^T(t)(R_1 + \tau_M \cdot R_2) \dot{z}(t) \\ &\quad - e^{-2\alpha\tau_M} \cdot \int_{t-\tau(t)}^{t} \dot{z}(s)^T R_1 \dot{z}(s) ds \\ &\quad - \tau_M \cdot e^{-2\alpha\tau_M} \cdot \int_{t-\tau(t)}^{t} \dot{z}(s)^T R_2 \dot{z}(s) ds].\end{aligned} \tag{8b}$$

From condition in (4c), the time derivative of $V_2(z_t)$ is bounded by

$$\begin{aligned}\dot{V}_2(z_t) &= 4\alpha \cdot e^{2\alpha t} \sum_{i=1}^{n} \int_0^{z_i(t)} v_i g_i(s) ds + 2e^{2\alpha t} \sum_{i=1}^{n} v_i g_i(z_i(t)) \dot{z}_i(t) \\ &\le 4\alpha \cdot e^{2\alpha t} \sum_{i=1}^{n} \int_0^{|z_i(t)|} v_i L_i s ds + 2e^{2\alpha t} \sum_{i=1}^{n} v_i g_i(z_i(t)) \dot{z}_i(t) \\ &= e^{2\alpha t} \cdot [2\alpha \cdot z^T(t) \Gamma V z(t) + 2g^T(z(t)) V \dot{z}(t)].\end{aligned} \tag{8c}$$

Define

$$Z^T(t) = [z^T(t) \quad z^T(t-\tau(t)) \quad \dot{z}^T(t) \quad g^T(z(t)) \quad g^T(z(t-\tau(t)))].$$

By Leibniz-Newton formula and LMI (6), the following additional nonnegative inequality can be introduced:

$$\begin{aligned}0 &\le e^{-2\alpha\tau_M} \cdot \int_{t-\tau(t)}^{t} \begin{bmatrix} Z(t) \\ \dot{z}(s) \end{bmatrix}^T \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} \begin{bmatrix} Z(t) \\ \dot{z}(s) \end{bmatrix} ds \\ &= e^{-2\alpha\tau_M} \cdot \{\tau(t) \cdot Z^T(t) S_{11} Z(t) + 2Z^T(t) S_{12}[z(t) - z(t-\tau(t))] \\ &\quad + \int_{t-\tau(t)}^{t} \dot{z}^T(s) S_{22} \dot{z}(s) ds\} \\ &\le e^{-2\alpha\tau_M} \cdot \{\tau_M \cdot Z^T(t) S_{11} Z(t) + 2Z^T(t) S_{12}[z(t) - z(t-\tau(t))] \\ &\quad + \int_{t-\tau(t)}^{t} \dot{z}^T(s) S_{22} \dot{z}(s) ds\}.\end{aligned} \tag{8d}$$

From (3), we have

$$\begin{aligned}&-\dot{z}^T(t)(U^T + U)\dot{z}(t) \\ &\quad + \dot{z}^T(t) U[-\bar{C} z(t) + \bar{A} g(z(t)) + \bar{B} g(z(t-\tau(t)))] \\ &\quad + [\bar{C} z(t) + \bar{A} g(z(t)) + \bar{B} g(z(t-\tau(t)))]^T U^T \dot{z}(t) = 0.\end{aligned} \tag{8e}$$

By the inequality in [4], we have

$$\begin{aligned}&-\tau_M \cdot e^{-2\alpha\tau_M} \cdot \int_{t-\tau(t)}^{t} \dot{z}(s)^T R_2 \dot{z}(s) ds \\ &\le -\tau(t) \cdot e^{-2\alpha\tau_M} \cdot \int_{t-\tau(t)}^{t} \dot{z}(s)^T R_2 \dot{z}(s) ds \\ &\le -e^{-2\alpha\tau_M} \cdot \left[\int_{t-\tau(t)}^{t} \dot{z}(s) ds\right]^T R_2 \left[\int_{t-\tau(t)}^{t} \dot{z}(s) ds\right] \\ &= -e^{-2\alpha\tau_M} \cdot (z(t) - z(t-\tau(t)))^T R_2 (z(t) - z(t-\tau(t))).\end{aligned} \tag{8f}$$

From (4d) and (4e), we have

$$g^T(z(t))\Gamma W_1 z(t) - g^T(z(t))W_1 g(z(t)) \ge 0,$$

$$z^T(t)\Gamma W_2 \Gamma z(t) - g^T(z(t))\Gamma W_2 z(t) \ge 0, \tag{9a}$$

$$g^T(z(t-\tau(t)))\Gamma Y_1 z(t-\tau(t)) - g^T(z(t-\tau(t)))Y_1 g(z(t-\tau(t))) \ge 0,$$

$$z^T(t-\tau(t))\Gamma Y_2 \Gamma z(t-\tau(t)) - g^T(z(t-\tau(t)))\Gamma Y_2 z(t-\tau(t)) \ge 0, \tag{9b}$$

From the inequality $R_1 > S_{22}$ in (6) and conditions (8)-(9), we have

$$\begin{aligned}
&\dot V_0(z_t) + 2e^{2\alpha t}\cdot[g^T(z(t))\Gamma W_1 z(t) - g^T(z(t))W_1 g(z(t)) \\
&\quad + z^T(t)\Gamma W_2 \Gamma z(t) - g^T(z(t))\Gamma W_2 z(t)] \\
&\quad + 2e^{2\alpha t}\cdot[g^T(z(t-\tau(t)))\Gamma Y_1 z(t-\tau(t)) \\
&\quad - g^T(z(t-\tau(t)))Y_1 g(z(t-\tau(t)))] \\
&\quad + 2e^{2\alpha t}\cdot[z^T(t-\tau(t))\Gamma Y_2 \Gamma z(t-\tau(t)) \\
&\quad - g^T(z(t-\tau(t)))\Gamma Y_2 z(t-\tau(t))] \\
&\le e^{2\alpha t}\cdot Z^T\cdot \bar\Sigma_1 \cdot Z + \int_{t-\tau(t)}^{t} \dot z^T(s)(S_{22}-R_1)\dot z(s)ds \\
&\le e^{2\alpha t}\cdot Z^T\cdot \bar\Sigma_1\cdot Z,
\end{aligned} \tag{10}$$

where

$$\Sigma_1 = \begin{bmatrix} \bar\Sigma_{11} & 0 & -\bar C^T U & P\bar A + \Gamma W & P\bar B + \Gamma Y \\ * & \Sigma_{22} & 0 & 0 & 0 \\ * & * & \Sigma_{33} & U\bar A + V & U\bar B \\ * & * & * & \Sigma_{44} & 0 \\ * & * & * & * & \Sigma_{55} \end{bmatrix} + \tilde\Sigma, \tag{11}$$

$$\bar\Sigma_{11} = -P\bar C - \bar C^T P + 2\alpha\cdot P + Q_1 + 2\alpha\cdot\Gamma V + 2\Gamma W_2\Gamma - e^{-2\alpha\tau_M}\cdot R_2,$$

Σ_{22}, Σ_{33}, Σ_{44}, Σ_{55}, and $\tilde\Sigma$ have been defined in (6). From (5), the matrix in (11) can be rearranged as

$$\bar\Sigma_1 = \begin{bmatrix} \Sigma_{11} & 0 & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} \\ * & \Sigma_{22} & 0 & 0 & 0 \\ * & * & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} \\ * & * & * & \Sigma_{44} & 0 \\ * & * & * & * & \Sigma_{55} \end{bmatrix} + \tilde\Sigma$$

$$+\begin{bmatrix} E_C & E_A & E_B \end{bmatrix}\begin{bmatrix} \Delta_C(t) & 0 & 0 \\ 0 & \Delta_A(t) & 0 \\ 0 & 0 & \Delta_B(t) \end{bmatrix}\begin{bmatrix} H_C \\ H_A \\ H_B \end{bmatrix}$$

$$+\begin{bmatrix} H_C \\ H_A \\ H_B \end{bmatrix}^T\begin{bmatrix} \Delta_C(t) & 0 & 0 \\ 0 & \Delta_A(t) & 0 \\ 0 & 0 & \Delta_B(t) \end{bmatrix}^T\begin{bmatrix} E_C & E_A & E_B \end{bmatrix}^T, \tag{12}$$

where

$$E_C = \begin{bmatrix} M_C^T P & 0 & M_C^T U^T & 0 & 0 \end{bmatrix}^T,\ H_C = \begin{bmatrix} -N_C & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$E_A = \begin{bmatrix} M_A^T P & 0 & M_A^T U^T & 0 & 0 \end{bmatrix}^T,\ H_A = \begin{bmatrix} 0 & 0 & 0 & N_A & 0 \end{bmatrix},$$

$$E_B = \begin{bmatrix} M_B^T P & 0 & M_B^T U^T & 0 & 0 \end{bmatrix}^T,\ H_B = \begin{bmatrix} 0 & 0 & 0 & 0 & N_B \end{bmatrix}.$$

From conditions (1e)-(1g), we have

$$\begin{bmatrix} \Delta_C(t) & 0 & 0 \\ 0 & \Delta_A(t) & 0 \\ 0 & 0 & \Delta_B(t) \end{bmatrix}$$

$$= \begin{bmatrix} [I - F_C(t)\Theta_C]^{-1}F_C(t) & 0 & 0 \\ 0 & [I - F_A(t)\Theta_A]^{-1}F_A(t) & 0 \\ 0 & 0 & [I - F_B(t)\Theta_B]^{-1}F_B(t) \end{bmatrix}$$

$$= \left\{ I - \begin{bmatrix} F_C(t) & 0 & 0 \\ 0 & F_A(t) & 0 \\ 0 & 0 & F_B(t) \end{bmatrix}\begin{bmatrix} \Theta_C & 0 & 0 \\ 0 & \Theta_A & 0 \\ 0 & 0 & \Theta_B \end{bmatrix} \right\}^{-1}$$

$$\cdot\begin{bmatrix} F_C(t) & 0 & 0 \\ 0 & F_A(t) & 0 \\ 0 & 0 & F_B(t) \end{bmatrix},$$

where

$$\begin{bmatrix} F_C(t) & 0 & 0 \\ 0 & F_A(t) & 0 \\ 0 & 0 & F_B(t) \end{bmatrix}^T\begin{bmatrix} F_C(t) & 0 & 0 \\ 0 & F_A(t) & 0 \\ 0 & 0 & F_B(t) \end{bmatrix} \le I.$$

By using Lemma 1, LMI condition $\Sigma < 0$ in (6) will imply $\bar\Sigma_1 < 0$ in (10). By the S-procedure of [6] with conditions (8)-(10) and $\bar\Sigma_1 < 0$, there exists a positive constant $\rho > 0$ such that

$$\dot V_0(z_t) \le -\rho\cdot e^{2\alpha t}\cdot \|z(t)\|^2.$$

From the condition $\dot{V}(z_t) \le 0$, we have

$$V_0(z_t) \le V_0(z_0),$$

where

$$\begin{aligned} V_0(z_0) &= z^T(0)Pz(0) \\ &\quad + \int_{-\tau(t)}^{0} e^{2\alpha s}\cdot[z^T(s)Q_1 z(s) + g^T(z(s))Q_2 g(z(s))]ds \\ &\quad + \int_{-\tau_M}^{0} e^{2\alpha s}\cdot(s+\tau_M)\cdot\dot{z}^T(s)(R_1+\tau_M\cdot R_2)\dot{z}(s)ds \\ &\quad + 2\sum_{i=1}^{n}\int_0^{z_i(0)} v_i g_i(s)ds \\ &\le [\lambda_{\max}(P)+\tau_M\cdot\lambda_{\max}(Q_1)+\tau_M\cdot\lambda_{\max}(\Gamma Q_2\Gamma) \\ &\quad + \tau_M^2\cdot\lambda_{\max}(R_1+\tau_M\cdot R_2)+\lambda_{\max}(\Gamma V)\cdot\|z_0\|_s^2 \\ &= \delta_1\cdot\|z_0\|_s^2, \end{aligned}$$

with

$$\begin{aligned} \delta_1 &= \lambda_{\max}(P)+\tau_M\cdot\lambda_{\max}(Q_1)+\tau_M\cdot\lambda_{\max}(\Gamma Q_2\Gamma) \\ &\quad + \tau_M^2\cdot\lambda_{\max}(R_1+\tau_M\cdot R_2)+\lambda_{\max}(\Gamma V). \end{aligned}$$

On the other hand, we have

$$V_0(z_t) \ge e^{2\alpha t}\cdot z^T(t)Pz(t) \ge \lambda_{\min}(P)\cdot e^{2\alpha t}\cdot\|z(t)\|^2.$$

Consequently, we have

$$\|z(t)\| = \|x(t)-\tilde{x}\| \le \sqrt{\frac{\delta_1}{\lambda_{\mathrm{mim}}(P)}}\cdot\|z_0\|_s\cdot e^{-\alpha t},\ t\ge 0.$$

From Definition 1, this implies that the equilibrium point $\tilde{x}$ of system (1) is globally exponentially stable with convergence rate α. Next we will prove the uniqueness of the equilibrium point $\tilde{x}$, i.e., the equilibrium point $\tilde{z} = [0\ \cdots\ 0]^T$ of (3). Assume $\tilde{z}$ is an equilibrium point of the system (3). Then we have

$$-\bar{C}\tilde{z} + \bar{A}g(\tilde{z}) + \bar{B}g(\tilde{z}) = 0.$$

Multiplying both sides of preceding equation by $2\tilde{z}^T P$, we have

$$\tilde{z}^T(-P\bar{C} - \bar{C}^T P)\tilde{z} + 2\tilde{z}^T P\bar{A}g(\tilde{z}) + 2\tilde{z}^T P\bar{B}g(\tilde{z}) = 0.$$

From (9) and $\tilde{\Sigma}$ in (6), we have

$$\begin{aligned} &\tilde{z}^T[-P\bar{C} - \bar{C}^T P + 2\alpha\cdot P + 2\alpha\cdot\Gamma V + Q_1 - e^{-2\alpha\tau_M}\cdot(1-\tau_D)\cdot Q_1 \\ &+ 2\Gamma(W_2+Y_2)\Gamma]\tilde{z} + 2\tilde{z}^T[P\bar{A}+P\bar{B}]g(\tilde{z}) \\ &+ 2\tilde{z}^T[\Gamma(W_1-W_2)+\Gamma(Y_1-Y_2)]g(\tilde{z}) + g^T(\tilde{z})[-2W_1-2Y_1]g(\tilde{z}) \\ &+ g^T(\tilde{z})[Q_2 - e^{-2\alpha\tau_M}\cdot(1-\tau_D)\cdot Q_2]g(\tilde{z}) \ge 0, \\ &[\tilde{z}^T\ \ \tilde{z}^T\ \ 0\ \ g^T(\tilde{z})\ \ g^T(\tilde{z})]\tilde{\Sigma}[\tilde{z}^T\ \ \tilde{z}^T\ \ 0\ \ g^T(\tilde{z})\ \ g^T(\tilde{z})]^T \ge 0. \end{aligned}$$

This implies

$$[\tilde{z}^T\ \ \tilde{z}^T\ \ 0\ \ g^T(\tilde{z})\ \ g^T(\tilde{z})]\bar{\Sigma}_1[\tilde{z}^T\ \ \tilde{z}^T\ \ 0\ \ g^T(\tilde{z})\ \ g^T(\tilde{z})]^T \ge 0,$$

where $\bar{\Sigma}_1$ is defined in (11). Note that the condition $\Sigma < 0$ in (6) is equivalent to $\bar{\Sigma}_1 < 0$ in (11), this will imply the result $\tilde{z} = g(\tilde{z}) = [0\ \ \cdots\ \ 0]^T$. Hence the equilibrium point $\tilde{z} = [0\ \ \cdots\ \ 0]^T$ is unique i.e., $\tilde{x}$ is the unique equilibrium point of uncertain DNN (1). This completes the proof. □

By setting $\alpha = 0$ and $R_1 = R_2 = U = S = 0$ in Theorem 1, we may obtain the following delay-independent asymptotic stability condition (independent of τ_M) for system (1) with (2).

Corollary 1. The equilibrium point $\tilde{x}$ of system (1) with (2) and $\tau_D \le 1$ (resp., $\tau_D > 1$ or unknown) is unique and globally asymptotically stable (GAS), if there exist some $n \times n$ positive definite symmetric matrices P, Q_1, Q_2 (resp., $Q_1 = 0$, $Q_2 = 0$), some $n \times n$ positive diagonal matrices W_1, W_2, Y_1, Y_2, and a positive constant ε, such that the following LMI conditions are satisfied:

$$\hat{\Sigma} = \begin{bmatrix} \hat{\Sigma}_{11} & 0 & \hat{\Sigma}_{13} & \hat{\Sigma}_{14} & \hat{\Sigma}_{15} & \hat{\Sigma}_{16} & \hat{\Sigma}_{17} & \hat{\Sigma}_{18} & 0 & 0 \\ * & \hat{\Sigma}_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \hat{\Sigma}_{33} & 0 & 0 & 0 & 0 & 0 & \hat{\Sigma}_{39} & 0 \\ * & * & * & \hat{\Sigma}_{44} & 0 & 0 & 0 & 0 & 0 & \hat{\Sigma}_{410} \\ * & * & * & * & \hat{\Sigma}_{55} & 0 & 0 & \hat{\Sigma}_{58} & 0 & 0 \\ * & * & * & * & * & \hat{\Sigma}_{66} & 0 & 0 & \hat{\Sigma}_{69} & 0 \\ * & * & * & * & * & * & \hat{\Sigma}_{77} & 0 & 0 & \hat{\Sigma}_{710} \\ * & * & * & * & * & * & * & \hat{\Sigma}_{88} & 0 & 0 \\ * & * & * & * & * & * & * & * & \hat{\Sigma}_{99} & 0 \\ * & * & * & * & * & * & * & * & * & \hat{\Sigma}_{1010} \end{bmatrix} < 0, \tag{13}$$

where

$$\hat{\Sigma}_{11} = -PC - C^T P + Q_1 + 2\Gamma W_2 \Gamma,\ \hat{\Sigma}_{13} = PA + \Gamma(W_1 - W_2),$$

$$\hat{\Sigma}_{14} = PB,\ \hat{\Sigma}_{15} = PM_C,\ \hat{\Sigma}_{16} = PM_A,\ \hat{\Sigma}_{17} = PM_B,$$

$$\hat{\Sigma}_{18} = -\varepsilon \cdot N_C^T,\ \hat{\Sigma}_{22} = -[(1-\tau_D)\cdot Q_1] + 2\Gamma Y_2 \Gamma,\ \hat{\Sigma}_{24} = \Gamma(Y_1 - Y_2),$$

$$\hat{\Sigma}_{33} = -2W_1 + Q_2,\ \hat{\Sigma}_{39} = \varepsilon \cdot N_A^T,\ \hat{\Sigma}_{44} = -2Y_2 - (1-\tau_D)\cdot Q_2,$$

$$\hat{\Sigma}_{410} = \varepsilon \cdot N_B^T,\ \hat{\Sigma}_{58} = \varepsilon \cdot \Theta_C^T,\ \hat{\Sigma}_{69} = \varepsilon \cdot \Theta_A^T,\ \hat{\Sigma}_{710} = \varepsilon \cdot \Theta_B^T,$$

$$\hat{\Sigma}_{55} = \hat{\Sigma}_{66} = \hat{\Sigma}_{77} = \hat{\Sigma}_{88} = \hat{\Sigma}_{99} = \hat{\Sigma}_{1010} = -\varepsilon \cdot I.$$

Remark 2. In Corollary 1 with $\alpha = 0$ and $R_1 = R_2 = U = S = 0$, the obtained result is delay-independent of τ_M. Hence the result "$\tau_M < \infty$" can be guaranteed when the LMI (13) is feasible.

IV. NUMERICAL EXAMPLES

Example 1. Consider the UDNNs in (1) with (2) and the following parameters: (Example 2 of [10])

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ A = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1.5 \end{bmatrix},\ B = \begin{bmatrix} -2 & 0.5 \\ 0.5 & -2 \end{bmatrix},$$

$$M_C = M_A = M_B = \begin{bmatrix} 0 & 0 \\ -0.1 & -0.1 \end{bmatrix},$$

$$N_C = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix},\ N_A = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix},\ N_B = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0 \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.8 \end{bmatrix},\ \Theta_C = \Theta_A = \Theta_B = 0.3. \tag{14}$$

With $\tau_D = 0.2$, LMI conditions in Corollary 1 have a feasible solution:

$$P = \begin{bmatrix} 0.8112 & 0.1937 \\ 0.1937 & 0.9452 \end{bmatrix},\ Q_1 = \begin{bmatrix} 0.3581 & 0.0991 \\ 0.0991 & 0.2138 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 2.9551 & -0.6585 \\ -0.6585 & 3.1723 \end{bmatrix},\ V = \begin{bmatrix} 0.8847 & 0 \\ 0 & 0.8847 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 2.3825 & 0 \\ 0 & 1.9513 \end{bmatrix},\ W_2 = \begin{bmatrix} 0.1643 & 0 \\ 0 & 0.0368 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 0.6111 & 0 \\ 0 & 0.165 \end{bmatrix},\ Y_2 = \begin{bmatrix} 0.3178 & 0 \\ 0 & 0.0491 \end{bmatrix},\ \varepsilon = 0.2989.$$

The system (1) with (2), (14), and $\tau_D = 0.2$, is asymptotically stable and the equilibrium point $\tilde{x}$ is unique. With $\tau_D = 1$, LMI conditions in Corollary 1 are not feasible. Hence Theorem 1 should be used to show delay-dependent results.

Table 1. Some comparisons for system (1) with (2) and (14).

Some upper bounds of the time delay for the stability of system (1) with (2) and (14)		
Results	[10]	Our results
$\tau_D = 0$ (Constant)	$\tau_M < \infty$	$\tau_M < \infty$ (GAS)
$\tau_D = 0.5$	$\tau_M = 2.9653$	$\tau_M < \infty$ (GAS)
$\tau_D = 0.9$	$\tau_M = 0.8629$	$\tau_M < \infty$ (GAS)
$\tau_D = 1$ or unknown	Not provided	$\alpha = 0,\ \tau_M = 5000000$ (GAS)
		$\alpha = 0.1,\ \tau_M = 215$ (GES)

With $\alpha = 0.1$, $\tau_D = 1$, $\tau_M = 215$, LMI conditions in Theorem 1 still have a feasible solution. The system (1) with (2), (14), $\tau_D = 1$, and $\tau_M = 215$, is exponentially stable with convergence rate $\alpha = 0.1$. In order to show the improvement, we summarize some comparisons in Table 1.

Example 2. Consider the UDNNs in (1) with (2) and the following parameters: (Example 1 of [14])

$$C = \begin{bmatrix} 1.2769 & 0 & 0 & 0 \\ 0 & 0.6231 & 0 & 0 \\ 0 & 0 & 0.923 & 0 \\ 0 & 0 & 0 & 0.448 \end{bmatrix},$$

$$A = \begin{bmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.086 & -0.3824 & -0.5785 \\ -0.1311 & 0.3253 & -0.9534 & -0.5015 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.022 \\ 0.0474 & -0.9164 & 0.036 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} 0.1137 & 0 & 0 & 0 \\ 0 & 0.1279 & 0 & 0 \\ 0 & 0 & 0.7994 & 0 \\ 0 & 0 & 0 & 0.2368 \end{bmatrix} = \begin{bmatrix} L_1 & 0 & 0 & 0 \\ 0 & L_2 & 0 & 0 \\ 0 & 0 & L_3 & 0 \\ 0 & 0 & 0 & L_4 \end{bmatrix},$$

$$M_C = M_A = M_B = N_C = N_A = N_B = 0,\ \Theta_C = \Theta_A = \Theta_B = 0. \tag{15}$$

Some comparisons of proposed results are shown in Table 2.

With $f_i(x_i) = L_i \times 0.5(|x_i + 1| - |x_i - 1|)$, i = 1, 2, 3, 4, $\tau(t)$ = 50, J = 0, $x(t) = [10 \quad -10 \quad 5 \quad -5]^T,\ t \in [-50, 0]$, the system state trajectories are shown in Fig. 1, where 0 is the unique equilibrium point.

Table 2. Some comparisons for system (1) with (2) and (15).

Some upper bounds of the time delay for the stability of system (1) with (2) and (15)			
Results	[7]	[14]	Our results
$\tau_D = 0$	$1 \le \tau(t) \le 3.5841$	$1 \le \tau(t) \le 3.8363$	$\tau_M < \infty$ (GAS)
$\tau_D = 0.5$	$1 \le \tau(t) \le 2.5802$	$1 \le \tau(t) \le 2.7299$	$\tau_M < \infty$ (GAS)
$\tau_D = 0.9$	$1 \le \tau(t) \le 2.2736$	$1 \le \tau(t) \le 2.3811$	$\tau_M < \infty$ (GAS)
$\tau_D = 1$ or unknown	$1 \le \tau(t) \le 2.2393$	$1 \le \tau(t) \le 2.3114$	$\alpha = 0$, $\tau_M = 12016155$ (GAS)
			$\alpha = 0.1$, $\tau_M = 229$ (GAS)

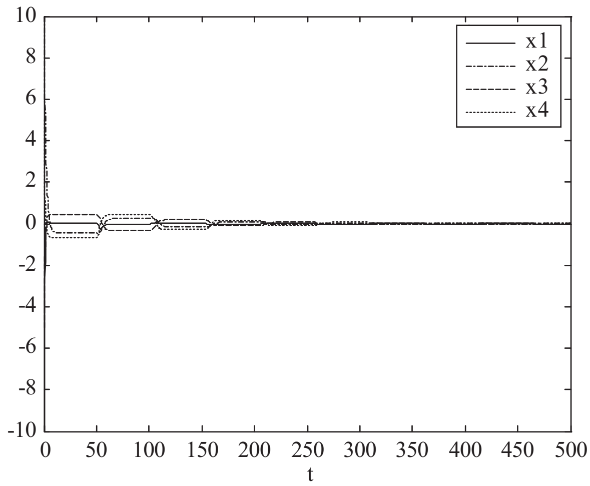

Fig. 1. The system state trajectories of DNN (1) with (15).

V. CONCLUSIONS

In this paper, the global stability and uniqueness of equilibrium point for a class of uncertain delayed neural networks with time-varying delay and linear fractional perturbations has been investigated. Based on the LMI approach and some proposed additional nonnegative inequalities, some delay-dependent and delay-independent criteria have been proposed to guarantee the exponential stability and asymptotic stability of UDNN, respectively. Some numerical examples by using the proposed results have shown great improvement over recent published results.

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