ROBUST ADAPTIVE MOTION CONTROL WITH ENVIRONMENTAL DISTURBANCE REJECTION FOR PERTURBED UNDERWATER VEHICLES

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ABSTRACT

This paper addresses the motion control problem of autonomous underwater vehicles (AUVs) perturbed by unknown hydrodynamic coefficients, unmodelled dynamics and environmental disturbances. The proposed algorithm consists of an adaptive subcontroller to tackle the parametric uncertainties and a robust term to vanish the effects of unstructured uncertainties and disturbances. The resulting robust adaptive controller ensures the convergence of tracking error, without any assumption on the upper bound of perturbations in the design procedure. The closed loop stability is shown, using the Lyapunov stability theorem, and verified by various simulations.

I. INTRODUCTION

Motion control of underwater vehicles is one of the main topics in oceanic research and engineering, due to various applications of such vehicles in scientific and military operations [4, 8]. In practice, the dynamic equations of such vehicles are highly nonlinear and the system parameters like hydrodynamic coefficients and added mass are not exactly known. Furthermore, such environmental disturbances as currents and wave's effects cause the desired performance not be achieved. Hence, taking into account the external disturbances, parametric uncertainties, and unmodelled dynamics is necessary in the design procedure.

During the past years, complete, simplified, and linearized models of UVs have been used to develop motion control algorithms. Although using a linearized model facilitates the

designing procedure, the validity of control algorithms depends heavily on the accuracy of the vehicle dynamics. Adopting the linearized dynamics, various linear control techniques such as PID controller [8], LQR and LQG algorithms [6], and linear H_∞ controller [2] have been developed. Some more recent investigations have decoupled the diving plane and steering system states, by ignoring the interaction between the system variables [13, 15]. The decoupled controllers e.g., depth, pitch, and yaw controller can be developed [3, 4, 12], without taking the variation of vehicle roll and yaw angular velocity into account. From a practical viewpoint, such simplicity is obtained at the expense of violating some degrees of freedom and imposing some variables to be zero or constant. Concerning with the complete model, developing some six-degree of freedom controller, based on robust control and intelligent algorithms, has been reported to achieve the desired performance [16, 18]. The previous works suffer from at least one of the following restrictions, (i) the conservative assumptions make the algorithm be applicable for a special class of UVs or for a certain direction of motion, (ii) unmodelled dynamics, parameter variations and environmental disturbances are not incorporated in dynamical equations altogether, (iii) the upper bound of uncertainties and disturbances are known in advance, (iv) robust closed-loop stability and performance is not ensured analytically.

Motivated by the aforementioned drawbacks, this paper concerns with developing an adaptive-based motion controller for UVs to ensure the prescribed robustness properties. In fact, adaptive control is adopted here to tackle the parametric uncertainties, originated by the variations of hydrodynamic coefficients in different operating conditions. Among the Lyapunov-based adaptive design algorithms, backstepping and passivity- based techniques may be commonly applied to special classes of nonlinear systems [10, 11]. Such methods have been developed for special cases of UV maneuverings, as in diving and steering planes [13, 14]. Furthermore, such strategies may be jointed to some intelligent algorithms to compensate the effects of uncertainties [17]. In general, conventional adaptive control may fail in the presence of un-

structured (non-parametric) uncertainties and environmental disturbances [1, 11]. In this paper, a robustifying mechanism is incorporated to ensure the robustness properties of the proposed algorithm. In the design procedure, unstructured uncertainty is incorporated into the nominal model to reflect the existence of unmodelled dynamics and the uncertainty in the matrix of inertia. Moreover, the hydrodynamic coefficients, as parametric uncertainties, are allowed to vary with unknown bound and no pre-assumptions are made on the upper bound and periodicity of environmental disturbances. Compared with the previous motion control algorithms, some specific properties of the proposed method are: (i) dynamical equations may be perturbed by parameter variations, unstructured uncertainties, and external disturbances, altogether, (ii) neither the upper bound nor the periodicity of perturbations is required in the design procedure, (iii) the stability analysis is presented analytically based on the Lyapunov stability theorem.

This paper is organized as follows. In section 2, introducing the system dynamics, the control problem is formulated. Dealing with various kinds of perturbations, two robust adaptive motion control and the stability proofs are given in section 3. The simulation results are presented in section 4, to evaluate the performance of the proposed methods. The concluding remarks are finally given in section 5.

Throughout the paper, for an $n \times 1$ vector V, $\|V\|$ denotes the Euclidean vector norm, and $\|V\|_Q^2 := V^T Q V$, with the weighting matrix Q. Furthermore, $V \in L_\infty[0,\infty)$ if $\|V\| < \infty$, for all $t \in [0\ \infty)$, and $V \in L_2[0,T]$ if $\int_0^T \|V(t)\|^2 dt < \infty$, $T \in [0\ \infty)$.

II. PROBLEM FORMULATION

In general, two coordinates including earth-fixed frame and body-fixed frame, are used to describe the AUV kinematics. In fact, the position and orientation of the vehicle are described in the earth-fixed coordinate, and the linear and angular velocities are described in the body-fixed frame. The dynamic equations of motion for a six degrees-of-freedom underwater vehicle can be represented in a compact form as [8]

$$M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau$$

$$\dot{\eta} = J(\eta)v \qquad (1)$$

where $v = [u, v, w, p, q, r]^T$ is the vector of linear and angular velocities in body-fixed frame, $\eta = [x, y, z, \phi, \theta, \psi]^T$ denotes the position and orientation in earth-fixed frame, and $J(\eta) \in R^{6\times6}$ represents a transformation matrix between the body-fixed frame and the earth-fixed coordinate. The positive definite inertia matrix $M \in R^{6\times6}$ consists of rigid-body mass and added mass, $C(v) \in R^{6\times6}$ denotes the coriolis and cen-

tripetal matrix, and $D(v) \in R^{6\times6}$ is the matrix of hydrodynamic and damping forces. The vector of control forces and moments is represented by $\tau \in R^6$, and $g(\eta)$ is the vector of buoyancy and gravitational terms.

The equivalent of equation (1) can be represented in body-fixed (global) frame as

$$M_\eta(\eta)\ddot{\eta} + C_\eta(\eta,v)\dot{\eta} + D_\eta(\eta,v)\dot{\eta} + g_\eta(\eta) = \tau_\eta + d_\eta \qquad (2)$$

where $M_\eta = J^{-T}MJ^{-1}$, $C_\eta = J^{-T}(C - MJ^{-1}\dot{J})J^{-1}$, $D_\eta = J^{-T}DJ^{-1}$, $g_\eta(\eta) = J^{-T}g(\eta)$, $\tau_\eta = J^{-T}\tau$, and d_η represents the external disturbance vector.

In practice, the parameters included in $C(v)$ and $D(v)$, e.g., hydrodynamic and damping coefficients, may be changed in various circumstances. On the other hand, in such a complex nonlinear system, model uncertainty inevitably perturb the nominal model. Hence, by defining $H(\eta, \dot{\eta}, v) = C_\eta(\eta, v)\dot{\eta} + D_\eta(\eta, v)\dot{\eta}$, one can write

$$H(\eta,\dot{\eta},v) = H_0(\eta,\dot{\eta},v) + \zeta(\eta,\dot{\eta},v)\Theta_p \qquad (3)$$

where $H_0(\eta, \dot{\eta}, v)$ is the known part, Θ_p denotes an unknown parameter vector, and $\zeta(\eta, \dot{\eta}, v)$ represents a dimensionally compatible matrix.

The uncertainties in the elements of inertia matrix are modelled here by decomposing M generally as

$$M_\eta = M_0 + M_\Delta(t) \qquad (4)$$

in which M_0 denotes the known nominal part and $M_\Delta(t)$ is a norm-bounded unstructured time-varying perturbation with bounded time-derivative, i.e., $\|M_\Delta(t)\| \le \alpha_1$, and $\|\dot{M}_\Delta(t)\| \le \alpha_2$, where α_1, α_2 are two unknown constants. Meanwhile, an uncertain parameter is defined as $\alpha = max\{\alpha_1, \alpha_2\}$.

The motion control objective is to track any given smooth state trajectory η_d, in the presence of unstructured uncertainties, unknown hydrodynamic coefficients and environmental disturbances.

III. ROBUST ADAPTIVE MOTION CONTROL

This section is devoted to design adaptive motion control algorithms which guarantee the robust stability and performance of UVs, in various practical situations.

Consider the dynamic equations of UVs given by (1). To facilitate the design procedure, take an equivalent disturbance vector as $d = d_\eta - g_\eta(\eta)$ and rewrite the dynamic equation (2) as

$$(M_0 + M_\Delta)\ddot{\eta} + H_0(\eta,\dot{\eta},v) + \zeta(\eta,\dot{\eta},v)\Theta_p = \tau_\eta + d \qquad (5)$$

For a given desired vector η_d, define the tracking error $e = \eta_d - \eta$, and the error metric functions $S(t) = \dot{e}(t) + e(t)$ and

$S_r(t) = \ddot{\eta}_d(t) + \dot{e}(t)$.

The applied control input is proposed here as

$$\tau_\eta = KS + M_0 S_r + H_0 + \tau_a + \tau_s \tag{6}$$

where K is a positive definite matrix, τ_a denotes the adaptive subcontroller, and τ_s presents the robust subcontroller. In the following, depending on the characteristics of disturbances, two novel control algorithms are proposed to design τ_a and τ_s. To this end, an adaptive-based H_∞ algorithm is first developed, assuming the external disturbance is belong to L_2 space. Then, removing such hypothesis, a robust adaptive motion control technique is proposed to tackle various kinds of bounded perturbations, without any conservative assumptions.

Theorem 1. For the underwater vehicles described by dynamical Eq. (5), the control input (6), with

$$\tau_a = \tau_{a1} = \zeta \hat{\Theta}_p + \hat{\alpha} \left(\frac{1}{2} S + \hat{\alpha} \frac{SS_r^T S_r}{\|S_r\| \|S\| \hat{\alpha} + \delta e^{-\sigma t}} \right) \tag{7}$$

$$\tau_s = -\frac{1}{2\rho} S \tag{8}$$

and update laws

$$\dot{\hat{\Theta}}_p = \Gamma S \zeta(\eta, \dot{\eta}, v) \tag{9}$$

$$\dot{\hat{\alpha}} = \gamma_\alpha \left(\frac{1}{2} S^T S + \|S_r\| \|S\| \right) \tag{10}$$

where $\Gamma = \Gamma^T < 0$ is the adaptation matrix, and $\gamma_\alpha > 0$ denotes the adaptation gain, solves the motion control problem in the presence of system uncertainties and disturbances. Moreover, δ and σ are two (small) positive constants to provide the smoothness of control input.

Proof. Take a Lyapunov function candidate as

$$V(e, \dot{e}, \tilde{\alpha}, \tilde{\Theta}_p) = e^T K e + \frac{1}{2} S^T M_\eta S + \frac{1}{2\gamma_\alpha} \tilde{\alpha}^2 + \frac{1}{2} \tilde{\Theta}_p^T \Gamma^{-1} \tilde{\Theta}_p \tag{11}$$

where $\tilde{\alpha} = \alpha - \hat{\alpha}$, and $\tilde{\Theta}_p = \Theta_p - \hat{\Theta}_p$ denote the estimation errors. Taking the time derivative of V gives

$$\dot{V} = 2e^T K\dot{e} + S^T (M_\eta \ddot{e} + M_\eta \dot{e}) + \frac{1}{2} S^T \dot{M}_\eta S + \frac{1}{\gamma_\alpha} \tilde{\alpha} \dot{\tilde{\alpha}} + \dot{\hat{\Theta}}_p^T \Gamma^{-1} \tilde{\Theta}_p \tag{12}$$

By $\ddot{e} = \ddot{\eta}_d - \ddot{\eta}$ and dynamical Eq. (5), and replacing control input (6), one can obtain

$$S^T \left(M_\eta \ddot{e} + M_\eta \dot{e} \right)$$

$$= S^T \left((M_0 + M_\Delta) \ddot{\eta}_d - (M_0 + M_\Delta) \ddot{\eta} + (M_0 + M_\Delta) \dot{e} \right)$$

$$= S^T \left(M_0 S_r + M_\Delta S_r + H_0 + \zeta \Theta_p - \tau_\eta - d \right)$$

$$= S^T \left(-KS + M_\Delta S_r + \zeta \Theta_p - \tau_a - \tau_s - d \right) \tag{13}$$

Substituting (13) and subcontrollers (7) and (8) in (12) gives

$$\dot{V} = 2e^T K\dot{e} - S^T KS + S^T M_\Delta S_r + S^T \left(\zeta \Theta_p - \zeta \hat{\Theta}_p \right)$$

$$- \frac{1}{2} \hat{\alpha} S^T S - \hat{\alpha}^2 \frac{S^T SS_r^T S_r}{\|S_r\| \|S\| \hat{\alpha} + \delta e^{-\sigma t}}$$

$$- \frac{1}{2\rho} S^T S - S^T d + \frac{1}{2} S^T \dot{M}_\Delta S - \frac{1}{\gamma_\alpha} \tilde{\alpha} \dot{\hat{\alpha}} - \dot{\hat{\Theta}}_p^T \Gamma^{-1} \tilde{\Theta}_p \tag{14}$$

Applying the equivalence

$$- \frac{1}{2\rho} S^T S - S^T d = -\frac{1}{2\rho} (S + \rho d)^T (S + \rho d) + \frac{1}{2} \rho \|d\|^2$$

into (14) and some manipulations imply that

$$\dot{V} \leq -e^T K e - \dot{e}^T K \dot{e} + \tilde{\alpha} \left(\frac{1}{2} S^T S + \|S_r\| \|S\| \right)$$

$$+ S^T \zeta \tilde{\Theta}_p + \delta e^{-\sigma t} - \frac{1}{2\rho} (S + \rho d)^T (S + \rho d) + \frac{1}{2} \rho \|d\|^2$$

$$- \frac{1}{\gamma_\alpha} \tilde{\alpha} \dot{\hat{\alpha}} - \dot{\hat{\Theta}}_p^T \Gamma^{-1} \tilde{\Theta}_p \tag{15}$$

Now, incorporate the adaptation mechanisms (9) and (10) to obtain

$$\dot{V} \leq -e^T K e - \dot{e}^T K \dot{e} - \frac{1}{2\rho} (S + \rho d)^T (S + \rho d) + \frac{1}{2} \rho \|d\|^2 + \delta e^{-\sigma t} \tag{16}$$

By omitting some strictly negative terms from right hand side of inequality (16), one can conclude

$$\dot{V} \leq -e^T K e + \frac{1}{2} \rho \|d\|^2 + \delta e^{-\sigma t} \tag{17}$$

and

$$\dot{V} \leq -\dot{e}^T K \dot{e} + \frac{1}{2} \rho \|d\|^2 + \delta e^{-\sigma t} \tag{18}$$

which give the following consequents.

(i) The boundedness of disturbance signal d implies that there exists a $\bar{d} > 0$ such that $\|d\| \leq \bar{d}$. By inequality (17), one can obtain $\dot{V} \leq -\lambda_K \|e\|^2 + \frac{1}{2}\rho\bar{d}^2 + \delta$, where λ_K is the minimum eigenvalue of K. Choosing $\lambda_K > \frac{\rho\bar{d}^2 + 2\delta}{2\varepsilon^2}$ for any small $\varepsilon > 0$, there exists a $\kappa > 0$ such that $\dot{V} \leq -\kappa\|e\|^2 < 0$ for all $\|e\| > \varepsilon$. Thus, there is a $T > 0$ such that $\|e\| \leq \varepsilon$ for all $t \geq T$. This implies that the tracking error $e(t)$ is uniformly ultimately bounded [11], and all the closed-loop signals are also bounded.

(ii) Taking the inequality (18) into account and following a procedure, similar to that given in (i), the boundedness of $\dot{e}$ is also concluded.

(iii) Integrating the inequality (17) from $t = 0$ to $t = T$ yields

$$\int_0^T \|e(t)\|_K^2\, dt + V\left(e(T), \dot{e}(T), \tilde{\alpha}(T), \tilde{\Theta}_p(T)\right)$$

$$\leq V\left(e(0), \dot{e}(0), \tilde{\alpha}(0), \tilde{\Theta}_p(0)\right) + \frac{\delta}{\sigma}\left(1 - e^{-\sigma t}\right) + \frac{1}{2}\rho\int_0^T \|d(t)\|^2\, dt$$

$$\tag{19}$$

for all $0 \leq T < \infty$. This implies that for any disturbance, belongs to L_2 space, $e(t)$ is square-integrable which together with the boundedness property of $e(t)$ and $\dot{e}$, achieved in (i) and (ii), the Barbalat's lemma (see the appendix) [11], ensures the convergence of tracking error $e(t)$, despite the system uncertainties and environmental disturbances.

Remark 1. The exponential terms, incorporated in adaptive subcontroller (7), are to avoid chattering and discontinuity of control input, without violating the convergence property of tracking error and closed-loop stability.

Remark 2. Choosing a smaller $\rho > 0$ provides the system with faster response. This may be obtained at the expense of larger control effort. In fact, there exists a trade-off between the value of subcontroller gain ρ and the magnitude of control input τ_s.

In some practical situations, belonging the wave disturbances to L_2 space, implicitly assumed in Theorem 1, may be not satisfied. On the other hand, the uncertainty in hydrodynamic and damping matrices, taken as parametric variations in (3), cannot reflect all aspects of perturbations. Hence, Eq. (3) is modified as

$$H(\eta, \dot{\eta}, \nu) = H_0(\eta, \dot{\eta}, \nu) + \zeta(\eta, \dot{\eta}, \nu)\Theta_p + H_\Delta(\eta, \dot{\eta}, \nu) \tag{20}$$

where H_Δ denotes the unstructured uncertainty. The impor-

tance of robust stability and performance in various operation conditions of UVs motivates developing a more general motion control algorithm, relaxing any conservative assumption. As a preliminary step to design such a controller, the vector of lumped uncertainty is defined as

$$\Delta(\eta, \dot{\eta}, \nu) = H_\Delta(\eta, \dot{\eta}, \nu) - d \tag{21}$$

with $\|\Delta\| \leq \beta$, where β is an unknown parameter.

Theorem 2. Consider the dynamical equation of perturbed underwater vehicles, described by (5). The control law (6), formed by $\tau_s = 0$ and adaptive subcontroller

$$\tau_a = \tau_{a1} + \hat{\beta}^2 \frac{S}{\|S\|\hat{\beta} + \delta e^{-\sigma t}} \tag{22}$$

with adaptation mechanisms (9), (10), and

$$\dot{\hat{\beta}} = \gamma_\beta \|S\| \tag{23}$$

where $\hat{\beta}$ denotes the estimated value of β, and $\gamma_\beta > 0$ is the adaptation gain, ensures the convergence of tracking error despite the perturbations.

Proof. Choose the Lyapunov function

$$U(e, \dot{e}, \tilde{\alpha}, \tilde{\Theta}_p, \tilde{\beta}) = V(e, \dot{e}, \tilde{\alpha}, \tilde{\Theta}_p) + \frac{1}{2\gamma_\beta}\tilde{\beta}^2 \tag{24}$$

where $\tilde{\beta} = \beta - \hat{\beta}$ is the parameter estimation error.

Differentiating U implies that

$$\dot{U} = \dot{V} + \frac{1}{\gamma_\beta}\tilde{\beta}\dot{\tilde{\beta}} \tag{25}$$

Taking the Eqs. (12) and (20) into account, and then substituting τ_η from (6), one can obtain

$$\dot{U} = 2e^T K\dot{e} + S^T\left(M_0 S_r + M_\Delta S_r + H_0 + \zeta\Theta_p - \tau_\eta + H_\Delta - d\right)$$

$$+ \frac{1}{2}S^T \dot{M}_\eta S - \frac{1}{\gamma_\alpha}\tilde{\alpha}\dot{\hat{\alpha}} - \dot{\hat{\Theta}}_p^T \Gamma^{-1}\tilde{\Theta}_p - \frac{1}{\gamma_\beta}\tilde{\beta}\dot{\hat{\beta}}$$

$$\leq 2e^T K\dot{e} - S^T KS + \alpha\|S_r\|\|S\| + S^T \zeta\Theta_p$$

$$+ S^T\Delta - S^T\tau_a + \frac{1}{2}\alpha S^T S - \frac{1}{\gamma_\alpha}\tilde{\alpha}\dot{\hat{\alpha}} - \dot{\hat{\Theta}}_p^T \Gamma^{-1}\tilde{\Theta}_p - \frac{1}{\gamma_\beta}\tilde{\beta}\dot{\hat{\beta}}$$

$$\tag{26}$$

Replacing adaptive subcontroller (22) yields

$$\dot{U} \leq -e^T K e - \dot{e}^T K \dot{e} + \delta e^{-\sigma t} - \hat{\beta}^2 \frac{S^T S}{\|S\|\hat{\beta} + \delta e^{-\sigma t}} + S^T \Delta - \frac{1}{\gamma_\beta} \tilde{\beta}\dot{\hat{\beta}}$$

$$\leq -e^T K e - \dot{e}^T K \dot{e} + \delta e^{-\sigma t} - \hat{\beta}\|S\| + \delta e^{-\sigma t} + \beta\|S\| - \frac{1}{\gamma_\beta} \tilde{\beta}\dot{\hat{\beta}} \quad (27)$$

Now, incorporate the update law (23) to obtain

$$\dot{U} \leq -e^T K e - \dot{e}^T K \dot{e} + 2\delta e^{-\sigma t} \qquad (28)$$

Following a procedure, similar to the proof of Theorem 1, implies that the goal of motion control is achieved, despite the unstructured and parametric uncertainties, and environmental disturbances.

Remark 3. In order to design a six-degree of freedom controller, decoupling the equations of motion is avoided here to take into account all the possible interactions between system variables in the design procedure. In general, such decomposition restricts the application of UVs to a certain operating conditions such as moving in diving plane or travelling in a fixed depth [4, 15].

Remark 4. Unlike some previous works [2, 5, 14], almost all kinds of uncertainties and disturbances due to the various applications of underwater vehicles in different conditions, are taken into account by theorems 1 and 2, without any pre-assumption on the periodicity or the bound of such perturbations.

Remark 5. From a practical point of view, the limitations on choosing small sampling time and imperfect implementation of adaptation mechanism (23) may cause the estimated value $\hat{\beta}$ increase without bound. On the other hand, $\hat{\beta}$ has direct impact on control law (22) and may cause instability. Hence, an effective modification is proposed here to alleviate this practical drawback, as follows.

Substitute the update law (23) in theorem 2, with

$$\dot{\hat{\beta}} = \eta \|S\| \qquad (29)$$

where

$$\eta = \begin{cases} \gamma_\beta & if \ \|e\| > \varepsilon \\ 0 & otherwise \end{cases}$$

Such modification ensures that all the signals and states of the closed loop system are bounded and the norm of tracking error is robustly converged to a (small) prescribed bound $\varepsilon > 0$. In fact, this modification acts as a projection algorithm and therefore the stability analysis can be followed,
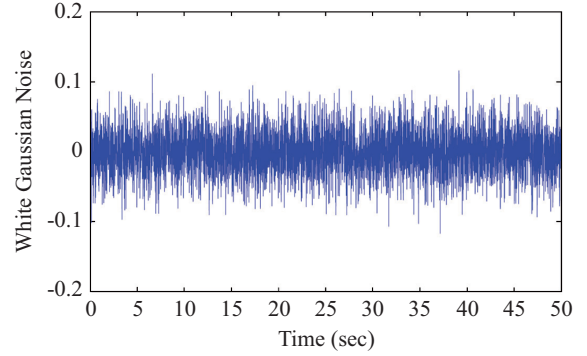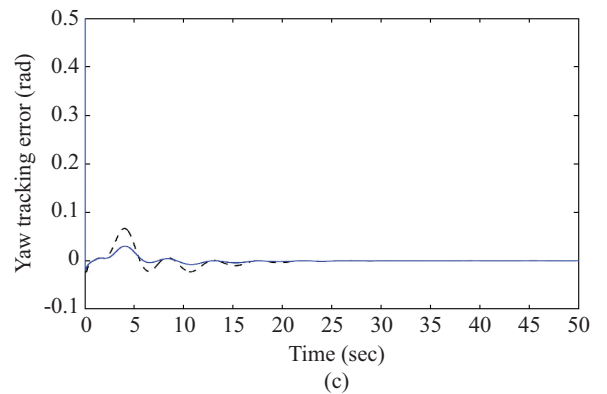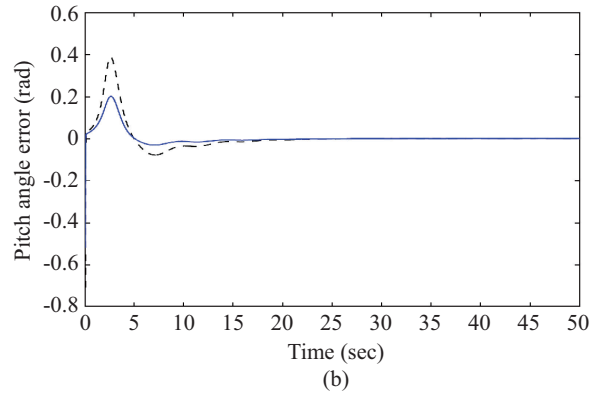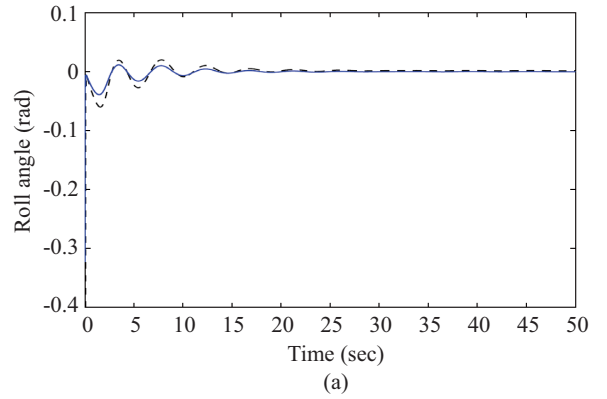

Fig. 1. The zero mean white Gaussian noise.

Fig. 2. Time response of output signals in case 1, when $\rho = 1$ (− −) and $\rho =$ 0.3 (—), (a) Roll angle, (b) Pitch angle error, (c) Yaw tracking error.
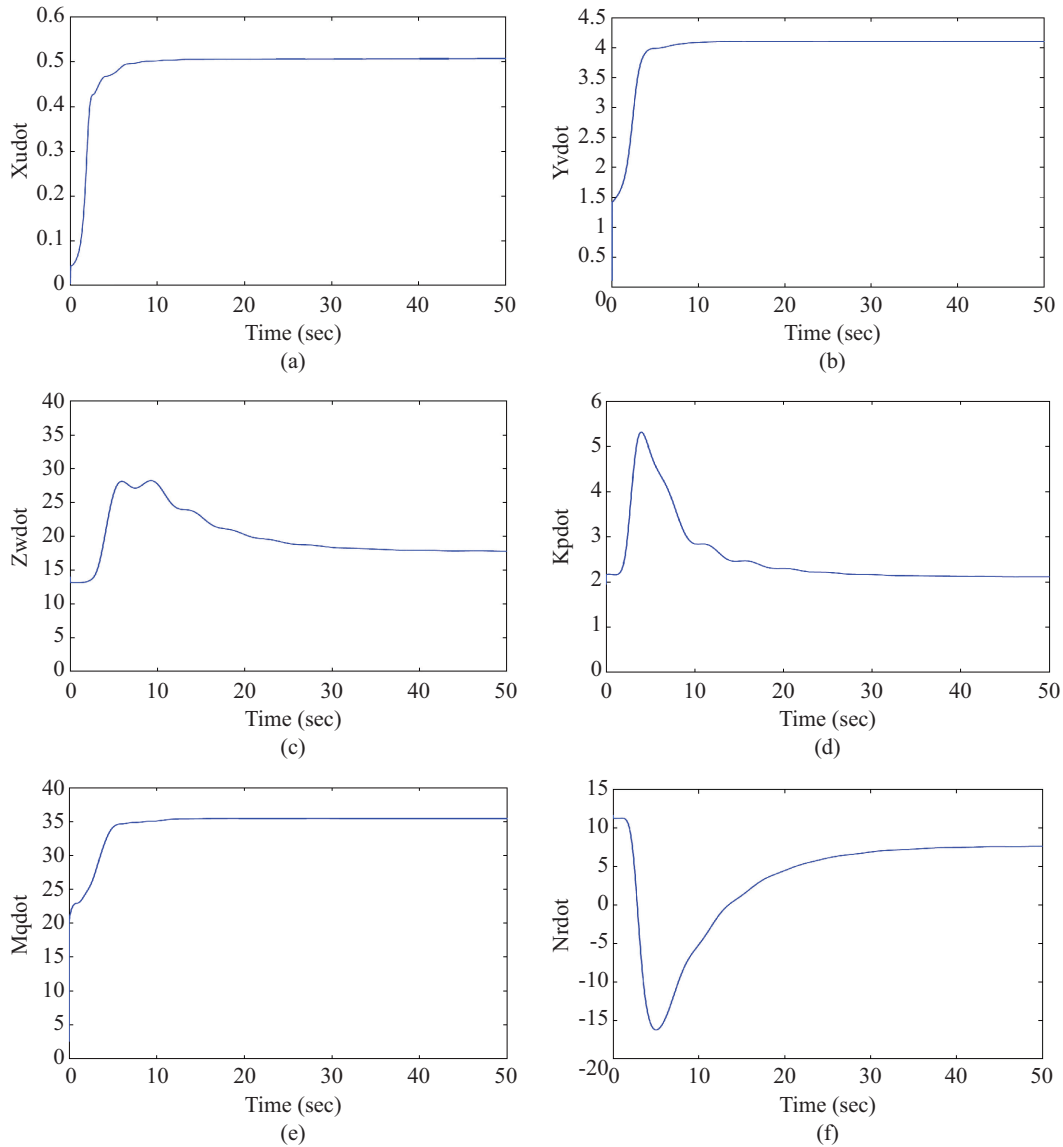
Fig. 3. Convergence of hydrodynamic coefficients, using $\rho = 0.3$ in Theorem 1, (a) $\dot{X}_u$, (b) $\dot{Y}_v$, (c) $\dot{Z}_w$, (d) $\dot{K}_p$, (e) $\dot{M}_q$, (f) $\dot{N}_r$.

similar to that of conventional projection methods in the literature [10].

IV. SIMULATION STUDY

In order to illustrate the validity of the proposed motion control algorithms, the simulation results are presented here, using the nominal parameters of the vehicle, as given by Fossen [7]. In this study, the performance of the methods, developed by theorems 1 and 2, are evaluated by incorporating various kinds of uncertainties and disturbances into the six degree of freedom model. To this end, two cases are considered here as follows.

Case I. Uncertain hydrodynamic parameters and white Gaussian noise. The performance of the developed algorithm in

theorem 1 is evaluated in the presence of the zero mean white noise, depicted in Fig. 1. Choosing $\rho = 1$ and $\rho = 0.3$, Fig. 2 show that the output regulation is achieved, without steady state error. In other words, roll stabilization, regulation of the desired pitch, corresponding to the desired depth $z = 8$ m, and $\psi = 20°$ as the desired yaw angle are all satisfied despite the parametric uncertainties and disturbance input. Moreover, Fig. 2 show that choosing a lower attenuation level ρ can more effectively override the effect of external disturbances. In order to illustrate the convergence of unknown system parameters, using the proposed adaptive control algorithm, the absolute values of the some hydrodynamic coefficients, normalized by density value, are shown in Fig. 3. In fact, the convergence of such parameters which correspond to diagonal elements of matrix, concisely satisfies the robust performance of the algorithm.
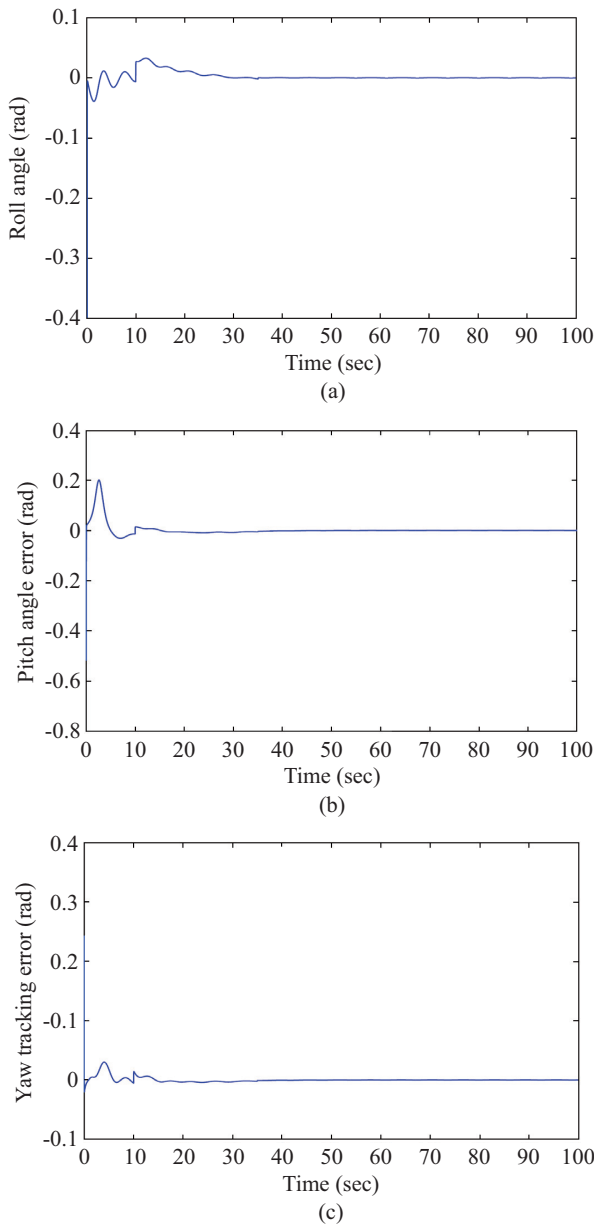
(a)

(b)

(c)

Fig. 4. Time history of system outputs, in the presence of biased sine disturbance from $t = 10$ sec, (a) Roll angle, (b) Pitch regulation error, (c) Yaw tracking error.

Case II. Mass uncertainty, unmodelled dynamics, and biased sinusoidal disturbance. Although the nominal equations of motion have been determined in the literature, but robust performance is obtained provided that various kinds of perturbations in real world applications are considered. In this case, the variation of inertia matrix is also taken into account. To this end, consider the unstructured uncertainty about 20% of the nominal parameters, in the presence of sinusoidal disturbance $sin0.2t$, biased by 0.5, from $t = 10$ sec. As shown in Fig. 4, the desired regulation performance goal is met by the proposed robust adaptive control algorithm, considering almost all kinds of possible uncertainties and
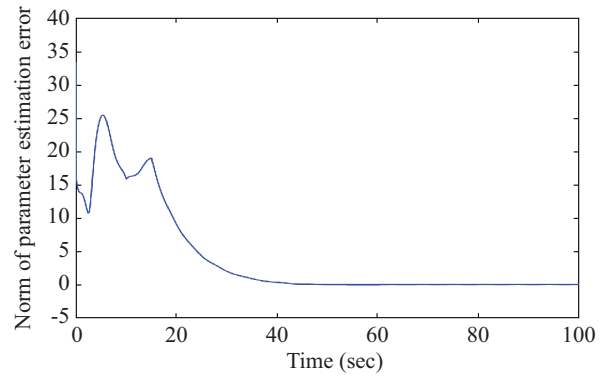

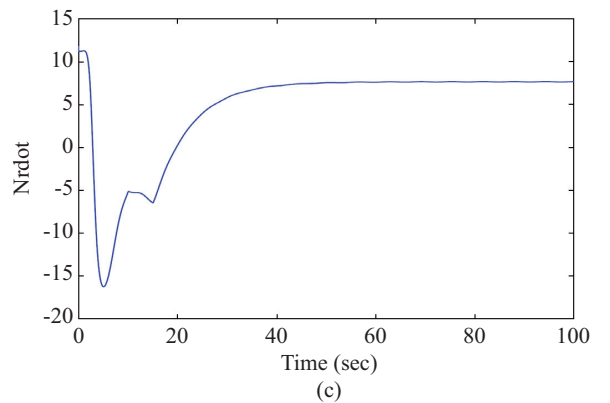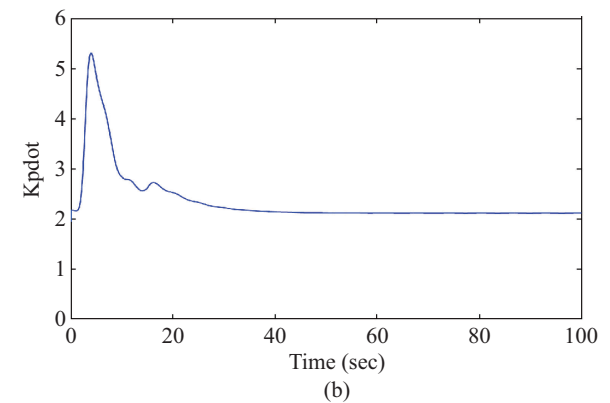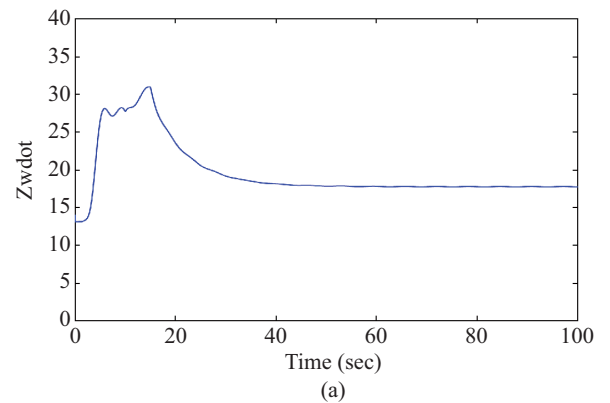**Fig. 5. The norm of parameter estimation error in case 2.**

(a)

(b)

(c)

Fig. 6. Hydrodynamic parameter adjustment in the presence of biased sine disturbance from $t = 10$ sec, (a) $\dot{Z}_w$, (b) $\dot{K}_p$, (c) $\dot{N}_r$.

environmental disturbances. The convergence of system parameters is shown by Figs. 5 and 6 which respectively illustrates the norm of estimation error and the hydrodynamic parameter adjustment.

V. CONCLUSION

Robust adaptive strategy is proposed here for motion control of underwater vehicles. Depending on the characteristics of the system uncertainties and disturbances, two control algorithms are developed to ensure the robust stability and performance. Using the Lyapunov stability theorem, the closed loop stability is guaranteed without any pre-assumption on the bound of perturbations. Various simulation results are also presented to verify the effectiveness of the methods.

APPENDIX

One of the results of Barbalat's lemma, used in the stability proof in this paper, is as [9, 11]

Lemma: If $e, \dot{e} \in L_\infty$ and $\dot{e} \in L_2$, then $e \rightarrow 0$ as $t \rightarrow \infty$.

ACKNOWLEDGMENTS

NOMENCLATURE

Symbol	Description
θ	Pitch angle
φ	Roll angle
ψ	Yaw angle
p	Pitch angular velocity
q	Roll angular velocity
r	Yaw angular velocity
u	Surge velocity
v	Sway velocity
w	Heave velocity
x	Position in x-direction
y	Position in y-direction
z	Position in z-direction

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