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LINEAR STATE FEEDBACK DESIGN FOR SURGE CONTROL OF AXIAL FLOW COMPRESSOR DYNAMICS

Der-Cherng Liaw, Yun-Hua Huang, and Wen-Ching Chung

Key words: axial flow compressor, Andronov-Hopf bifurcation, surge, stability.

ABSTRACT

Based on Moore and Greitzer's model [15], the surge behavior of axial flow compression system was found to be attributed to the appearance of Andronov-Hopf bifurcationin [12]. In this paper, we focus on the design of control law to prevent and/or delay the occurrence of the bifurcations, which will then suppress the surge behavior. Both types of linear dynamic state feedback and linear washout filter controllers are proposed. The proposed washout design does not require the explicit knowledge of the operating point and keep the original equilibrium characteristic while the dynamic feedback design will change the equilibrium operating points for pre-surge and post-surge regime. In addition, the washout type control scheme can relax the implementation of the control law in the practical applications. Numerical results demonstrate the effectiveness of the proposed designs and the superiority of the washout filter scheme.

I. INTRODUCTION

In the recent years, axial flow compressors was known to play important roles in modern industry due to their potential of high efficiency. In order to achieve high efficient operation of an engine, the air prior to combustion is greatly compressed by compressor [6]. However, when a compressor operates close to its maximum achievable pressure-rise, two types of instabilities might exhibit in axial flow compressor dynamics. One is the so-called ''rotating stall,'' which denotes a dynamic instability and occurs when an asymmetric flow pattern develops in the blade passages of a compressor stage and will cause a drastic pressure drop of the airflow within the compressor. The other is the so-called ''surge behavior,'' which is a large-amplitude and axisymmetric oscillation exhibiting in the overall pumping system. Instead of causing the pressure drop by stall, the surge behavior renders the compressor into suffer violent periodic impingements and eventually damages the compressor. Thus, how to suppress those unstable phenomena becomes a very important issue in improving system maintainability and performance.

Conventionally, a surge (or stall) line is usually drawn to provide a safe operation boundary for the usage of compressors. Such a conservative trade-off unduly restricts the capability of engine. In the recent years, various control schemes have been proposed to allow compressors to operate safely beyond the surge line and thus enhance system efficiency. Among those designs (e.g., [1-3, 7, 8, 10-12]), the backstepping scheme has been proposed to achieve global stability for specific cubic type compressors [7, 8]. It is known that the unstable portion of the compressor's axisymmetric characteristic is hard to measure and the associated system uncertainties are inevitable in practical applications [14]. A robust control scheme was recently proposed to deal with such uncertainties by assuming the axisymmetric characteristic is a specific cubic function [4]. However, those designs were based on the so-called "state feedback scheme," which might affect the operating point of compressor dynamics with respect to the variation of the setting value of the control parameters such as the opening of the throttle. In order to relax such a concern, a washout filter type control law will be proposed in the paper to delay and/or suppress the appearance of surge behavior while simultaneously providing auto equilibrium following with respect to the variation of control setting. For performance comparison, a dynamic feedback control scheme is also proposed in this study to stabilize post-surge dynamics and then used to justify the advantages of auto equilibrium following via washout design.

This paper is organized as follows. In Section II, the mathematical models for describing the nonlinear dynamics of compression system is first recalled. It is followed by the local stability analysis and surge control design. Numerical simulations will then be given in Section IV to demonstrate the success of the proposed control effort. Finally, Section V is devoted to summarize the main conclusions.

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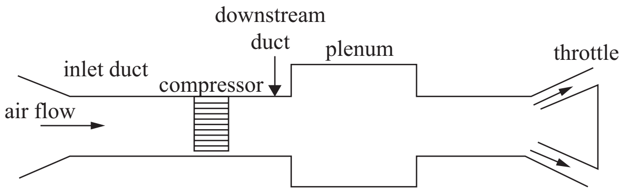

Fig. 1. Schematic diagram of an axial flow compression system.

II. COMPRESSOR DYNAMICS

In this section, we will recall the dynamical model of axial flow compressors from [15] which will then be adopted for the controller design in the sequel.

A compression system is known (e.g., [14, 15]) to be conceptually represented by a series of components including inlet duct, compressor, exit duct, plenum and throttle as shown in Fig. 1. By using the notation of Liaw and Abed [10], a lumped-parameter model of axial flow compressors proposed by Moore and Greitzer [15] is recalled as given by the following equations:

$$\frac{dA}{dt} = \frac{\alpha}{\pi W}\int_0^{2\pi} C_{ss}(\dot{m}_c + WA\sin\theta)\sin\theta\, d\theta, \tag{1}$$

$$\frac{d\dot{m}_C}{dt} = -\Delta P + \frac{1}{2\pi}\int_0^{2\pi} C_{ss}(\dot{m}_c + WA\sin\theta)\, d\theta, \tag{2}$$

$$\frac{d\Delta P}{dt} = \frac{1}{4B^2}\{\dot{m}_c - F(\gamma, \Delta P)\}. \tag{3}$$

In this study, the nondimensional throttle mass flow is taken as

$$F(\gamma, \Delta P) = \gamma(\Delta P)^{1/2}. \tag{4}$$

The axisymmetric compressor characteristic function $C_{ss}(\cdot)$ is often an S-shaped function and can be modeled by a suitable cubic polynomial (e.g., [14, 15]). It is easy to check from (1) that $A = 0$ always results in $dA/dt = 0$. Denote $x^* = [0,\ \dot{m}_c^0,\ \Delta P^0]^T$ an equilibrium point for system (1)-(3) at $\gamma = \gamma^0$. When $A = 0$, it is observed from Eqs. (2)-(3) that we have $\Delta P^0 = C_{ss}(\dot{m}_c^0)$ and $\dot{m}_c^0 = \gamma^0\sqrt{\Delta P^0}$. Note that, there may have equilibrium points of (1)-(3) for which $A \neq 0$ [10].

It is known that two types of nonlinear behavior might exhibit in the axial flow compressor dynamics. One is the so-called ‘‘rotating stall,’’ which is corresponding to the equilibrium solution of (1)-(3) with $A \neq 0$ [10]. The other is the so-called ‘‘surge behavior,’’ which denotes a large amplitude and axisymmetric oscillation appearing in the overall pumping system (e.g., [1, 12]). In this paper, we focus on the control of surge behavior. That is, system (1)-(3) will be restricted on the invariant manifold $A = 0$ and will then be reduced as a two-dimensional system as given by

$$\frac{d\dot{m}_c}{dt} = C_{ss}(\dot{m}_c) - \Delta P, \tag{5}$$

$$\frac{d\Delta P}{dt} = \frac{1}{4B^2}(\dot{m}_c - \gamma\sqrt{\Delta P}). \tag{6}$$

Denote $x^0 = [\dot{m}_c^0,\ \Delta P^0]^T$ an equilibrium point for system (5)-(6) at $\gamma = \gamma^0$, The following two results are recalled from [12] for the stability of system (5)-(6).

Lemma 1: The equilibrium point x^0 of system (5)-(6) is asymptotically stable if the following two conditions hold:

(i) $\dfrac{\gamma^0}{8B^2\sqrt{\Delta P^0}} > C_{ss}^{'}(\dot{m}_c^0)$ and (ii) $1 > C_{ss}^{'}(\dot{m}_c^0)\dfrac{\gamma^0}{2\sqrt{\Delta P^0}}$.

Lemma 2: The system (5)-(6) will undergo an Andronov-Hopf bifurcation at the equilibrium point x^0 if the following two conditions hold:

(i) $C_{ss}^{'}(\dot{m}_c^0) = \dfrac{\gamma^0}{8B^2\sqrt{\Delta P^0}} > 0$ and (ii) $1 > \dfrac{\gamma^0}{2\sqrt{\Delta P^0}} C_{ss}^{'}(\dot{m}_c^0)$.

Note that, the appearance of Andronov-Hopf bifurcation will result in the so-called “surge behavior” (e.g., [12]).

III. FEEDBACK CONTROL DESIGN

In this section, we will study the control problem of surge behavior in axial flow compression system. It is clear from Lemmas 1 and 2 that the conditions for the Hopf bifurcation point given in Lemma 2 will play as a stability boundary for system (5)-(6). Thus, the surge control problem of the system (5)-(6) becomes the stabilization issue of the Hopf bifurcation point. In the following design, the actuation is assumed to be an additive-type of control for the mass flow dynamics, which can be practically implemented in several ways [5]. Here, we consider the case of which the throttle opening is the unique control input.

As discussed in Section II, the system (5)-(6) has two states. One is the mass flow rate and the other is pressure rise of the plenum. In general, the pressure rise will be easily measured by using pressure transducer while the measurement of the mass flow rate will need more efforts for data calibration. By considering the facilitation in practical applications, in the following we will construct the stabilizing controller by using mass flow rate or pressure rise alone as feedback state. Dynamic state feedback control method is first employed in the design of stabilizing control law. It is known that such a design needs the information of the system equilibrium. However, since the explicit knowledge of operating equilibrium cannot be easily obtained, a washout filter aided feedback control method will then be adopted to delay and suppress the occurrence of Andronov-Hopf bifurcation in the compressor system. Such a method is known (e.g., [9]) to be able to provide automatic equilibrium following for compressors with uncertain operating condition. Details are given below.

1. Dynamic State Feedback Control Design

First, we consider to construct dynamic feedback control law by using mass flow rate or pressure rise alone as feedback state for system (5)-(6). Let z be an added dynamic state for feedback control design and $\gamma = \gamma^0 + u$, where $\gamma = \gamma^0$ denotes the setting value of the throttle opening and u is the input for the extra control effort of system stabilization. The closed-loop system of (5)-(6) can then be rewritten as

$$\frac{d\dot{m}_c}{dt} = C_{ss}(\dot{m}_c) - \Delta P, \tag{7}$$

$$\frac{d\Delta P}{dt} = \frac{1}{4B^2}[\dot{m}_c - (\gamma^0 + u)\sqrt{\Delta P}], \tag{8}$$

$$\frac{dz}{dt} = N\dot{m}_c + (1-N)\Delta P - dz. \tag{9}$$

Here, N denotes the switch of choosing the feedback signal and d is the feedback gain of state z. When $N = 1$, the feedback signal is mass flow rate. On the other hand, the feedback signal is pressure rise when $N = 0$.

Denote $x^d = [\dot{m}_c^0 \;\; \Delta P^0 \;\; z^0]^{\mathrm{T}}$ an equilibrium of system (7)-(9) at $\gamma = \gamma^0$, where $\dot{m}_c^0 = \gamma^0\sqrt{\Delta P^0}$, $\Delta P^0 = C_{ss}(\dot{m}_c^0)$, and $z^0 = [N\dot{m}_c^0 + (1 - \mathrm{N})\Delta P^0]/d$.

Let $e = [e_1 \;\; e_2 \;\; e_3]^{\mathrm{T}} = [\dot{m}_c - \dot{m}_c^0 \;\; \Delta P - \Delta P^0 \;\; z - z^0]^{\mathrm{T}}$ with the control

$$u = k_d(z - z^0) = k_d e_3, \tag{10}$$

system (7)-(9) can then be rewritten as

$$\frac{de}{dt} = Ae. \tag{11}$$

Here, k_d denotes the control gain and the Jacobian matrix $\boldsymbol{A}$ is obtained as

$$A = \begin{bmatrix} C_{ss}'(\dot{m}_c^0) & -1 & 0 \\ \frac{1}{4B^2} & \frac{-\gamma^0}{8B^2\sqrt{\Delta P^0}} & \frac{-k_d\sqrt{\Delta P^0}}{4B^2} \\ N & 1-N & -d \end{bmatrix}. \tag{12}$$

The characteristic equation for matrix A can then be calculated from the following equation:

$$\det(\lambda I - A) = \begin{vmatrix} \lambda - C_{ss}'(\dot{m}_c^0) & 1 & 0 \\ -\frac{1}{4B^2} & \lambda + \frac{\gamma^0}{8B^2\sqrt{\Delta P^0}} & \frac{k_d\sqrt{\Delta P^0}}{4B^2} \\ -N & N-1 & \lambda + d \end{vmatrix}$$

$$\triangleq \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \tag{13}$$

where

$$a_1 = d + \beta, \tag{14}$$

$$a_2 = \beta d + \frac{1}{4B^2}[\eta + (1-N)k_d\sqrt{\Delta P^0}], \tag{15}$$

$$a_3 = \frac{1}{4B^2}\{\eta d + k_d\sqrt{\Delta P^0}[-N + (N-1)C_{ss}'(\dot{m}_c^0)]\}. \tag{16}$$

with $\beta = \frac{\gamma^0}{8B^2\sqrt{\Delta P^0}} - C_{ss}'(\dot{m}_c^0)$, and $\eta = 1 - \frac{\gamma^0}{2\sqrt{\Delta P^0}}C_{ss}'(\dot{m}_c^0)$.

Applying the Routh-Hurwitz stability criteria to (13), we then have the next stabilization result.

Proposition 1: The equilibrium point x^d of system (7)-(9) is asymptotically stabilizable by the dynamic state feedback controller as given in (10) if $a_1 > 0$, $a_3 > 0$, and $a_1a_2 > a_3$.

Consider a special case of which the equilibrium point x^d denotes the Andronov-Hopf bifurcation point. From Lemma 2, we then have $\beta = 0$, $\eta > 0$ and $C_{ss}'(\dot{m}_c^0) > 0$. The values of a_i for $i = 1$, 2 and 3 given in (14)-(16) can be simplified as follows:

$$a_1 = d, \tag{17}$$

$$a_2 = \frac{1}{4B^2}[\eta + (1-N)k_d\sqrt{\Delta P^0}], \tag{18}$$

$$a_3 = \frac{1}{4B^2}\{\eta d + k_d\sqrt{\Delta P^0}[-N + (N-1)C_{ss}'(\dot{m}_c^0)]\}. \tag{19}$$

Thus, the three stabilizability conditions given in Proposition 1 can then be calculated as

(i) $a_1 = d > 0$,

(ii) $a_3 = \frac{1}{4B^2}\{\eta d + k_d\sqrt{\Delta P^0}[-N + (N-1)C_{ss}'(\dot{m}_c^0)]\} > 0$,

(iii) $a_1a_2 - a_3 = \frac{k_d\sqrt{\Delta P^0}}{4B^2}[(1-N)d + N + (1-N)C_{ss}'(\dot{m}_c^0)] > 0.$

Since we have $N = 1$ or $N = 0$, next result follows readily from Proposition 1 and Lemma 2.

Theorem 1: The Andronov-Hopf bifurcation (HB) equilibrium point x^d of system (7)-(9) is asymptotically stabilizable by a dynamic state feedback controller as given in (10) if $d > 0$ and $0 < k_d < \dfrac{\eta d}{\sqrt{\Delta P^0}[N-(N-1)C_{ss}^{'}(\dot{m}_c^0)]}$.

Two special designs can then be obtained from Theorem 1 as given below.

Corollary 1: The Andronov-Hopf bifurcation (HB) equilibrium point x^d of system (7)-(9) is asymptotically stabilizable by a dynamic state feedback controller as given in (10) with $\dot{m}_c$ as solely state feedback signal if $d > 0$ and $0 < k_d < \dfrac{\eta d}{\sqrt{\Delta P^0}}$.

Corollary 2: The Andronov-Hopf bifurcation (HB) equilibrium point x^d of system (7)-(9) is asymptotically stabilizable by a dynamic state feedback controller as given in (10) with ΔP as solely state feedback signal if $d > 0$ and $0 < k_d < \dfrac{\eta d}{C_{ss}^{'}(\dot{m}_c^0)\sqrt{\Delta P^0}}$.

2. Washout Filter Aided Feedback Control Design

Next, we consider to apply the washout filter aided feedback control scheme to stabilize system (5)-(6) at the Hopf bifurcation point. Washout filters are commonly used in control design of power system and aircraft (e.g., [9]). The advantages of using washout filter design include system equilibrium preservation, automatic operating point following and facilitating the design of a robust controller. A washout filter can be a stable high-pass filter with transfer function as given by [9]:

$$G(s) = \frac{y(s)}{v(s)} = \frac{s}{s+d}, \tag{20}$$

with $d > 0$ denote the filter time constant. By using the notation of

$$z(s) \equiv \frac{1}{(s+d)} v(s), \tag{21}$$

the dynamics of the filter in (20) can then be rewritten as

$$\dot{z} = v - dz, \tag{22}$$

along with the output equation

$$y = v - dz. \tag{23}$$

It is clear that we have the steady state of (22) as

$$z^* = \frac{v^*}{d}, \tag{24}$$

and the output $y = 0$. Under this steady state, the input signal v^* is said to have been washed out from the output.

Adopting the design of [9], a linear washout filter aided feedback with measurement of $\dot{m}_c$ or ΔP is added on system (5)-(6). Define the output equation of washout filter as

$$y = N\dot{m}_c + (1-N)\Delta P - dz, \tag{25}$$

where $d > 0$ and $N = 1$ or $N = 0$, the closed-loop system of (5)-(6) with washout filter controller will be in the same form as those given in (7)-(9).

Denote $x^w = [\dot{m}_c^0 \;\; \Delta P^0 \;\; z^0]^{\mathrm{T}}$ an equilibrium of system (7)-(9) at $\gamma = \gamma^0$, where $\dot{m}_c^0 = \gamma^0\sqrt{\Delta P^0}$, $\Delta P^0 = C_{ss}(\dot{m}_c^0)$, and $z^0 = [N\dot{m}_c^0 + (1-N)\Delta P^0]/d$.

Let $e = [e_1 \;\; e_2 \;\; e_3]^{\mathrm{T}} = [\dot{m}_c - \dot{m}_c^0 \;\; \Delta P - \Delta P^0 \;\; z - z^0]^{\mathrm{T}}$.

Here, we choose the control input u for system (7)-(9) be given by $u = k_w y$. Then we have

$$\begin{aligned} u &= k_w(N\dot{m}_c + (1-N)\Delta P - dz) \\ &= k_w\{N(\dot{m}_c - \dot{m}_c^0 + \dot{m}_c^0) \\ &\quad + (1-N)[(\Delta P - \Delta P^0 + \Delta P^0) - d(z - z^0 + z^0)]\} \\ &= k_w[Ne_1 + N\dot{m}_c^0 + (1-N)e_2 + (1-N)\Delta P^0 - de_3 - dz^0] \\ &= k_w[Ne_1 + (1-N)e_2 - de_3], \end{aligned} \tag{26}$$

where k_w denotes the control gain. It is clear from (26) that the control effort will be independent of the system equilibrium. That's why the washout filter design can keep the original open-loop system equilibria with respect to the different setting value $\gamma = \gamma^0$ of the throttle even after adding control input. Note that, although a washout-like controller was proposed in [1] for suppressing surge behavior, however, that design cannot provide the property as given in (26) since the extra control effort in [1] is a nonlinear function of linear washout state and full system state. Thus, the proposed control law in [1] will change the original open-loop system equilibria for different setting value of the throttle after adding control.

By using the control input as given in (26), system (7)-(9) can then be rewritten as $\dot{e} = Le$,
where

$$L = \begin{bmatrix} C_{ss}^{'}(\dot{m}_c^0) & -1 & 0 \\ \dfrac{1 - k_w N\sqrt{\Delta P^0}}{4B^2} & \dfrac{-r^0 - 2(1-N)k_w\Delta P^0}{8B^2\sqrt{\Delta P^0}} & \dfrac{dk_w\sqrt{\Delta P^0}}{4B^2} \\ N & 1-N & -d \end{bmatrix}. \tag{27}$$

The characteristic equation for matrix L can then be calculated from the following equation:

$$\det(\lambda I - L)$$
$$= \begin{vmatrix} \lambda - C_{ss}^{'}(\dot{m}_c^0) & 1 & 0 \\ \dfrac{-1 + k_w N\sqrt{\Delta P^0}}{4B^2} & \lambda + \dfrac{\gamma^0 + 2(1-N)k_w \Delta P^0}{8B^2\sqrt{\Delta P^0}} & \dfrac{-dk_w}{4B^2}\sqrt{\Delta P^0} \\ -N & N-1 & \lambda + d \end{vmatrix}$$
$$\triangleq \lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0, \tag{28}$$

where

$$b_1 = d + \beta + \frac{(1-N)k_w\sqrt{\Delta P^0}}{4B^2}, \tag{29}$$

$$b_2 = \beta d + \frac{\eta}{4B^2} - \frac{k_w\sqrt{\Delta P^0}}{4B^2}[(1-N)C_{ss}^{'}(\dot{m}_c^0) + N], \tag{30}$$

$$b_3 = \frac{d\eta}{4B^2}. \tag{31}$$

Applying the Routh-Hurwitz stability criteria to (28), we then have the next stabilization result.

Proposition 2: The equilibrium point x^w of system (7)-(9) is asymptotically stabilizable by the washout filter aided feedback controller as given in (26) if $b_1 > 0$, $b_3 > 0$, and $b_1 b_2 > b_3$.

Consider a special case of which the equilibrium point x^w denotes the Andronov-Hopf bifurcation point. From Lemma 2, we then have $\beta = 0$ and $\eta > 0$ and $C_{ss}^{'}(\dot{m}_c^0) > 0$. The values of b_i for i =1, 2 and 3 given in (29)-(31) can be simplified as follows:

$$b_1 = d + \frac{(1-N)k_w\sqrt{\Delta P^0}}{4B^2}, \tag{32}$$

$$b_2 = \frac{\eta}{4B^2} - \frac{k_w\sqrt{\Delta P^0}}{4B^2}[(1-N)C_{ss}^{'}(\dot{m}_c^0) + N], \tag{33}$$

$$b_3 = \frac{d\eta}{4B^2}. \tag{34}$$

The three stabilizability conditions given in Proposition 2 can hence be calculated as (i) $d > 0$,

$$\text{(ii)}\ d + \frac{(1-N)k_w\sqrt{\Delta P^0}}{4B^2} > 0, \tag{35}$$

$$\text{(iii)}\ \frac{k_w(1-N)\sqrt{\Delta P^0}}{4B^2}\{\frac{\eta}{4B^2} - \frac{k_w\sqrt{\Delta P^0}}{4B^2}[(1-N)C_{ss}^{'}(\dot{m}_c^0) + N]\}$$
$$- \frac{dk_w\sqrt{\Delta P^0}}{4B^2}[(1-N)C_{ss}^{'}(\dot{m}_c^0) + N] > 0, \tag{36}$$

Next result follows readily from Proposition 2 and Lemma 2.

Theorem 2: The Andronov-Hopf bifurcation (HB) equilibrium point x^w of system (7)-(9) is asymptotically stabilizable by the washout filter aided feedback controller as given in (26) if $d > 0$ and the two conditions given in (35)-(36) hold.

Two special designs can then be obtained from Theorem 2 as given below.

Corollary 3: The Andronov-Hopf bifurcation (HB) equilibrium point x^w of system (7)-(9) is asymptotically stabilizable by a washout filter aided feedback controller as given in (26) with $\dot{m}_c$ as solely state feedback signal if $d > 0$ and $k_w < 0$.

Corollary 4: The Andronov-Hopf bifurcation (HB) equilibrium point x^w of system (7)-(9) is asymptotically stabilizable by a washout filter aided feedback controller as given in (26) with ΔP as solely state feedback signal if (i) $d > 0$,

$$\text{(ii)}\ d + \frac{k_w\sqrt{\Delta P^0}}{4B^2} > 0 \ \text{and}$$

$$\text{(iii)}\ \frac{k_w\sqrt{\Delta P^0}}{4B^2}[\frac{\eta}{4B^2} - C_{ss}^{'}(\dot{m}_c^0)(d + \frac{k_w\sqrt{\Delta P^0}}{4B^2})] > 0\ .$$

Remark 1: It is observed from Corollary 4 that we can have $k_w < 0$ for large enough value of B and $k_w > 0$ for small enough value of B to make conditions (ii) and (iii) easily hold. Two examples are given in Section IV to demonstrate the design.

IV. SIMULATION RESULTS

In the following, numerical simulations will be given to demonstrate the proposed designs given in Section III. Here, the axisymmetric compressor characteristic $C_{ss}(\dot{m}_c)$ is adopted from [10] as given by

$$C_{ss}(\dot{m}_c) = 1.56 + 1.5(\dot{m}_c - 1) - 0.5(\dot{m}_c - 1)^3. \tag{37}$$

In addition, we set $B = 2$ with initial point $x_0 = [1.49, 1.8, 0]$. By using the code AUTO, the bifurcation diagram for the open-loop system (5)-(6) with respect to the varied setting of the throttle opening is depicted in Fig. 2, where the solid-line denotes the stable equilibrium point and the dotted-line denotes the unstable operating point, respectively. The Andronov-Hopf bifurcation is found to appear at $\gamma = 1.2449$ as

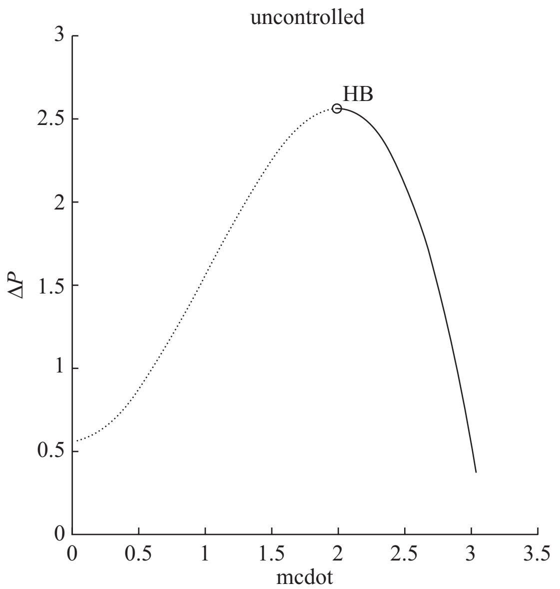

Fig. 2. Bifurcation diagram of system (5)-(6) for B = 2 with respect to the setting of the throttle opening.

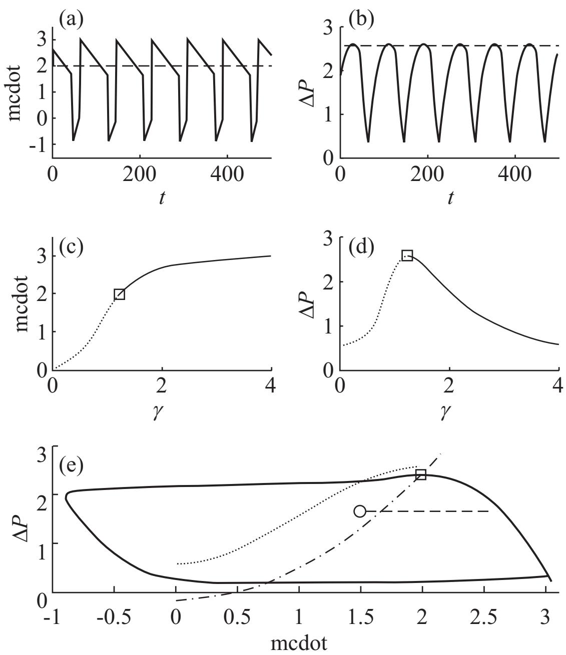

Fig. 3. Time responses and bifurcation diagram for open-loop system with B = 2.

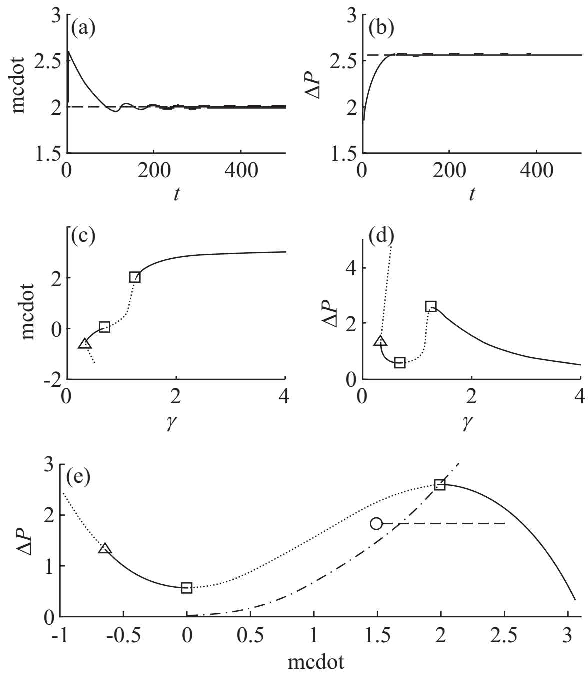

Fig. 4. Time responses and bifurcation diagram for dynamic state feedback control system with feedback signal $\dot{m}_c$ with B = 2, d = 3 and k_d = 1.5.

depicted by the circle point in Fig. 2. Deep surge behavior is observed for the open-loop system at γ= 1.2449 as depicted in Fig. 3, where Figs. 3(a) and 3(b) show the time responses obtained by using code MATLAB and Figs. 3(c) and 3(d) depict the bifurcation diagram with respect to the variation of the throttle opening obtained by using code AUTO, respectively. Note that, the dashed line denotes the equilibrium point in Figs. 3(a) and 3(b). In addition, in Figs. 3(c)-3(e), the solid line stands for stable system equilibria, the dotted-line stands for unstable system equilibria, the dashed-line stands for the time response, the dash-dotted line stands for the throttle function, the square box denotes the Hopf bifurcation point and the circle box denotes the initial point. The Hopf bifurcation is found at $\dot{m}_c = 1.9919$ and ΔP^0 = 2.5599. Moreover, the equilibrium point is found to be stable for γ > 1.2449.

The stabilizing results by using mass flow rate as feedback signal as given in Corollaries 1 and 3 are obtained as depicted in Figs. 4 and 5. The control parameters for the stabilization designs are selected as d = 3, k_d = 1.5 and k_w = -5, respectively. It is observed from those figures that the Hopf bifurcation point of the compression system becomes asymptotically stable by using two different control schemes, which agree with the results given in Corollaries 1 and 3.

Next, we consider to use ΔP as the feedback signal. It is clear from Figs. 6 and 7 that the surge behavior shown in Fig. 3 can be suppressed by using either dynamic state feedback controller with d = 3 and k_d = 1.5 or washout filter control with d = 0.1 and k_w = 0.1, which agree with those results given in Corollaries 2 and 4. As stated in Corollary 4, we can have negative control value of k_w for stabilizing the system operation with large B-parameter. An example is obtained as shown in Fig. 8 with B = 200, d = 3 and k_w = -0.1.

Comparison of timing responses at post-surge regime for γ = 1.244, Hopf bifurcation point and pre-surge regime for γ = 1.6 with mass flow rate or pressure rise of plenum as feedback signal are obtained as depicted in Figs. 9-14. Note that, the control gains for those numerical results are chosen as same as those for Figs. 4-7. It is clear from those timing

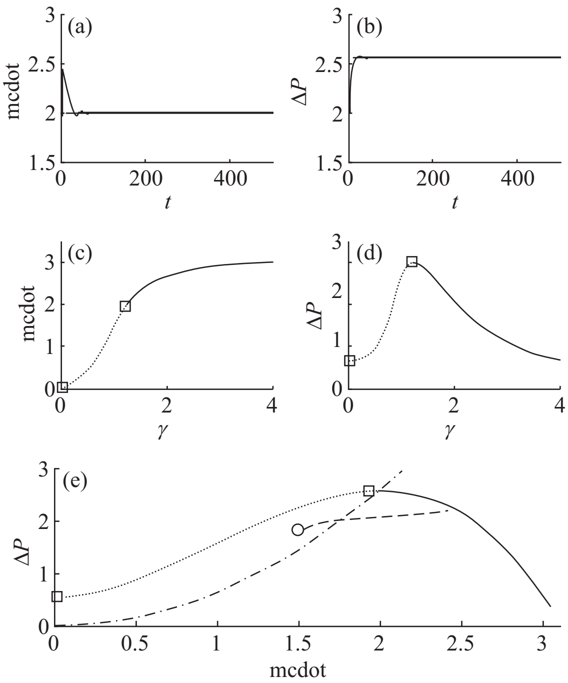

Fig. 5. Time responses and bifurcation diagram for washout filter control system with feedback signal $\dot{m}_c$ with B = 2, d = 3 and k_w = -5.

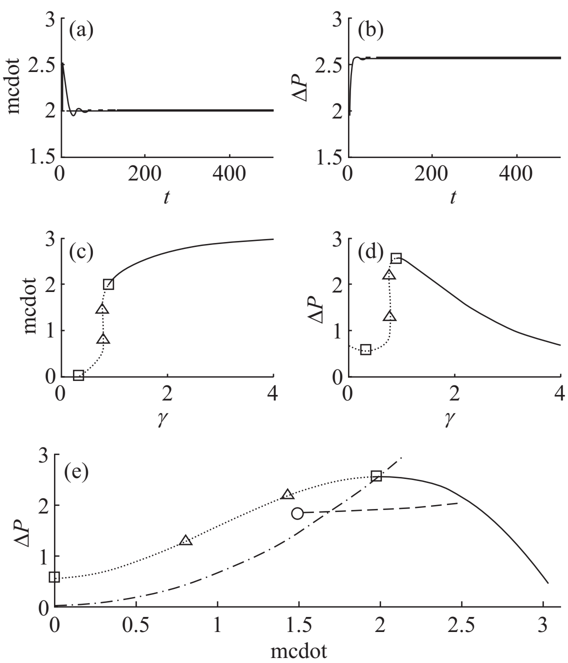

Fig. 6. Time responses and bifurcation diagram for dynamic state feedback control system with feedback signal ΔP with B = 2, d = 3 and k_d = 1.5.

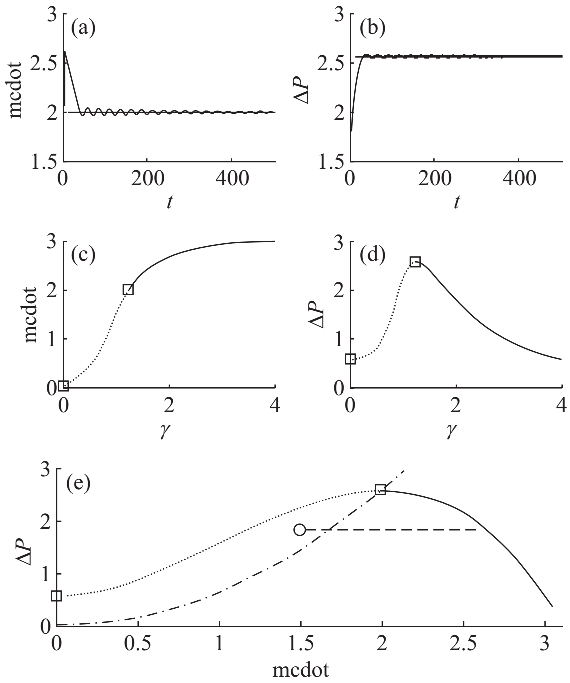

Fig. 7. Time responses and bifurcation diagram for washout filter control system with feedback signal ΔP with B = 2, d = 0.1 and k_w = 0.1.

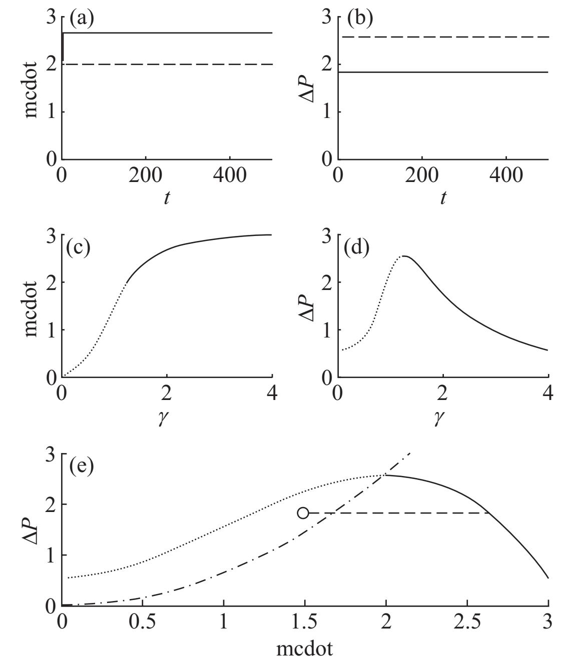

Fig. 8. Time responses and bifurcation diagram for washout filter control system with feedback signal ΔP with B = 200, d = 3 and k_w = -0.1.

trajectories that the design by using washout filter type control laws will provide auto system equilibrium following with respect to the different setting value of the throttle opening at both post-surge and pre-surge regime while it is not true by using dynamic state feedback stabilizing controller.

V. CONCLUSIONS

Based on the Moore and Greitzer's model for axial flow compressor, both dynamic state feedback control and washout filter type control were proposed to stabilize the surge behavior of the compressor dynamics. The stabilizing

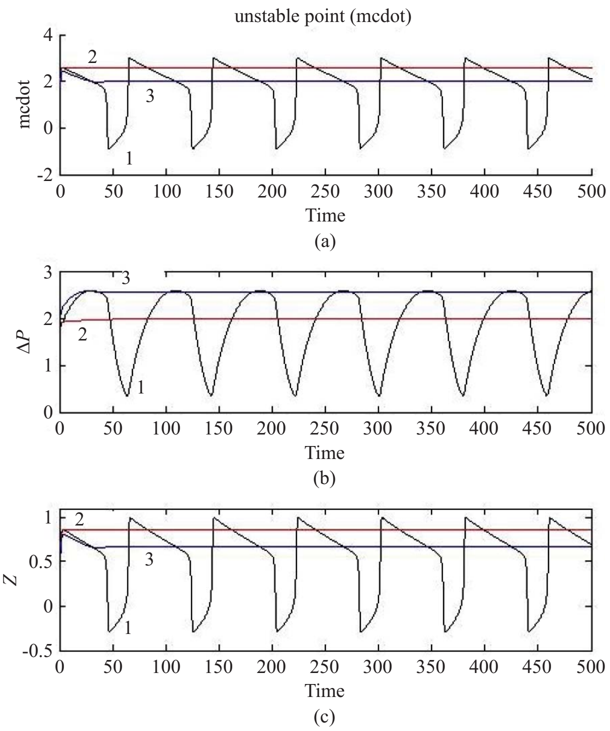

Fig. 9. Comparison of timing responses at post-surge regime with feedback signal $\dot{m}_c$: 1 denotes the open-loop system, 2 denotes the dynamic state feedback control system and 3 denotes the washout filter control system.

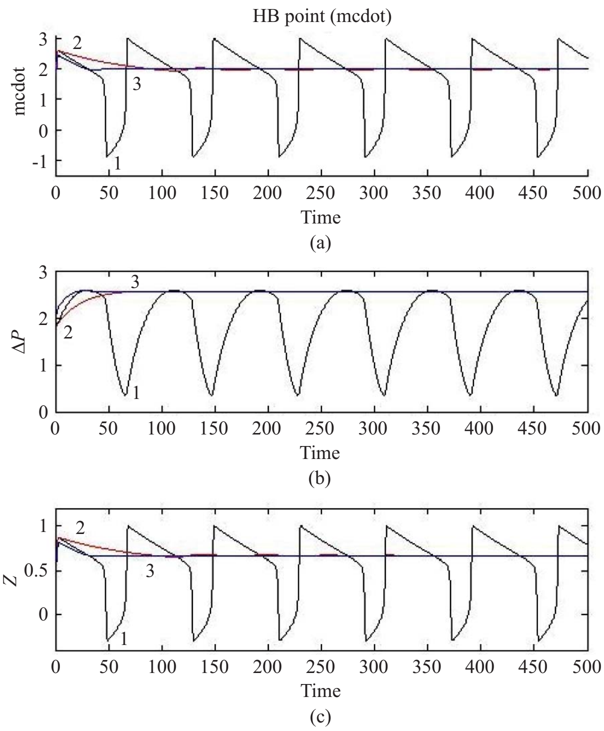

Fig. 11. Comparison of timing responses at Hopf bifurcation point with feedback signal $\dot{m}_c$: 1 denotes the open -loop system, 2 denotes the dynamic state feedback control system and 3 denotes the washout filter control system.

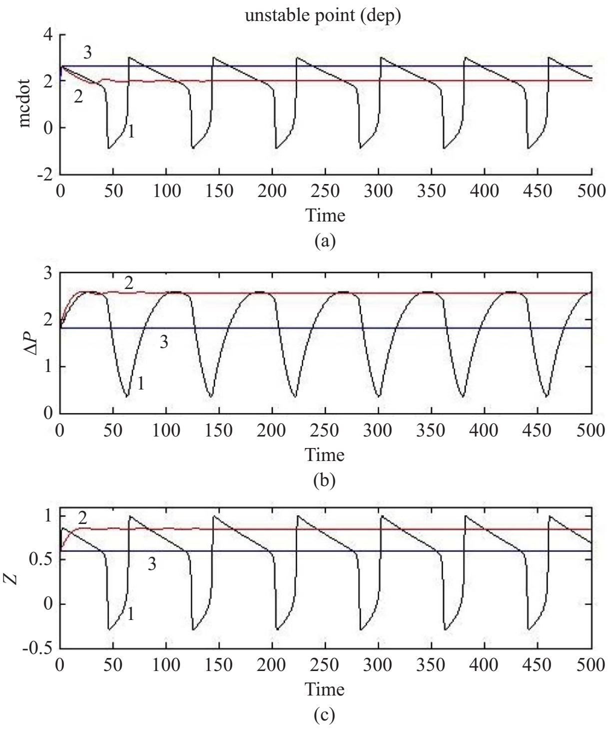

Fig. 10. Comparison of timing responses at post-surge regime with feedback signal ΔP: 1 denotes the open-loop system, 2 denotes the dynamic state feedback control system and 3 denotes the washout filter control system.

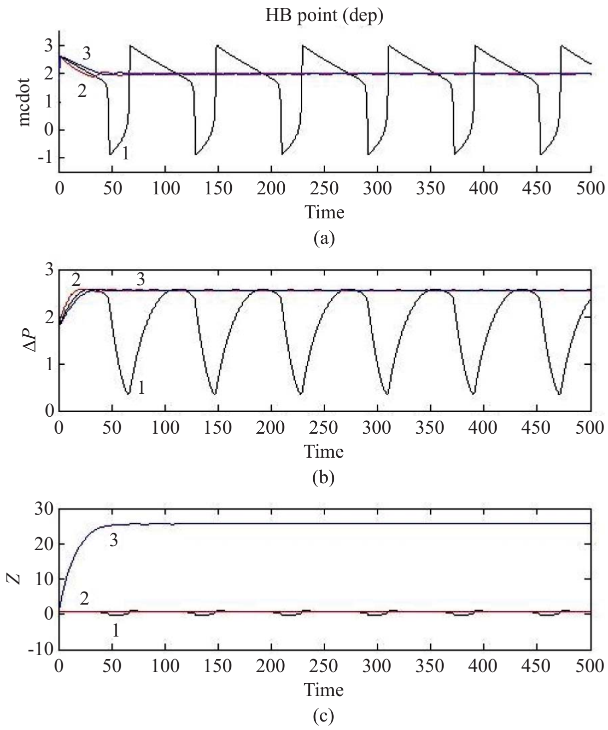

Fig. 12. Comparison of timing responses at Hopf bifurcation point with feedback signal ΔP: 1 denotes the open-loop system, 2 denotes the dynamic state feedback control system and 3 denotes the washout filter control system.

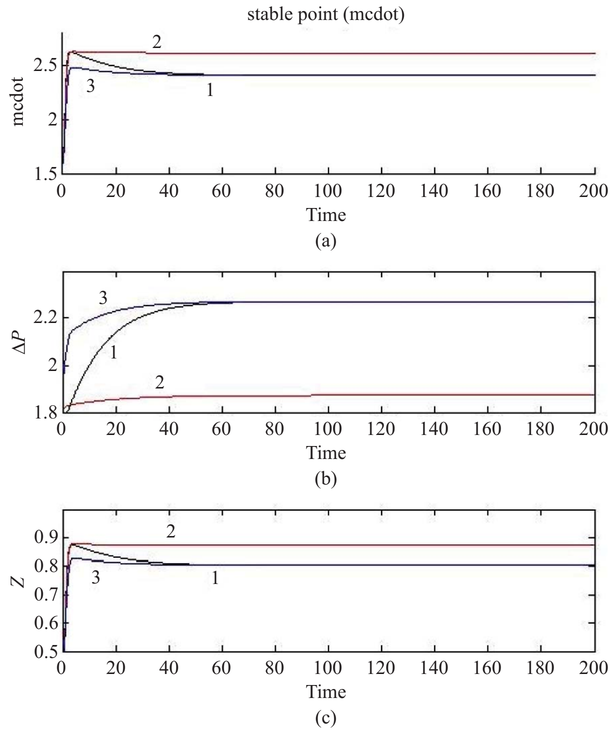

Fig. 13. Comparison of timing responses at pre-surge regime with feedback signal $\dot{m}_c$: 1 denotes the open loop system, 2 denotes the dynamic state feedback control system and 3 denotes the washout filter control system.

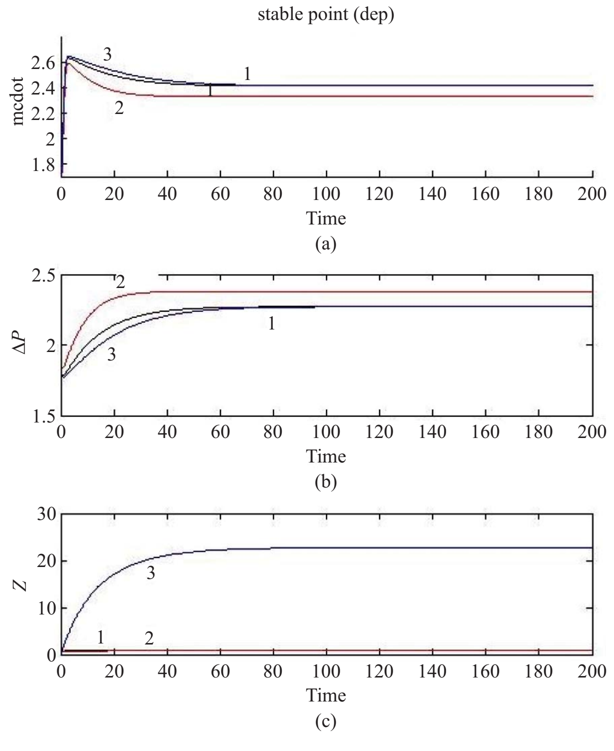

Fig. 14. Comparison of timing responses at pre-surge regime with feedback signal ΔP: 1 denotes the open-loop system, 2 denotes the dynamic state feedback control system and 3 denotes the washout filter control system.

conditions at the Andronov-Hopf bifurcation point were also derived by using mass flow rate or pressure rise of plenum as solely feedback signal. Numerical results did demonstrate the success of the proposed design and the superiority of the washout filter type design. Although both designs can provide the system stabilization, it is clear from the numerical simulations that only washout filter type control law will give the benefit of auto system equilibrium following. In fact, the design of the washout filter type control law does not require the explicit knowledge of system equilibrium. This will give a good hand in the practical implementation of control law. Moreover, although the pressure rise of plenum is easier to measure than mass flow rate, it was also found in this study that the control of surge behavior might be abruptly different for high rotor speed and low rotor speed operation of compressor by using pressure rise of plenum as solely feedback input.

NOMENCLATURE

A	amplitude of the first angular mode of rotating wave
ΔP	nondimensional pressure rise within the plenum
$\dot{m}_c$	nondimensional compressor mass flow rate
θ	circumferential coordinate
B	Greitzer *B*-parameter, proportional to rotor speed
α	a geometry-related constant
W	scaling parameter for normalized velocities
C_{ss}	nondimensional axisymmetric compressor characteristic
C'_{ss}	first derivative of C_{ss} function
F	nondimensional throttle function
γ	control parameter of throttle function

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