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MIMO TRANSMISSION IN SPATIALLY CORRELATED FADING

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Key words: multiple-input, multiple-output (MIMO) system, ergodic capacity, moment-generating function (MGF), correlated-Nakagami- m , correlated-Rician fading.

ABSTRACT

A multiple-input, multiple-output (MIMO) system transmission over a channel characterized by spatially correlated fading is addressed in this article. Two scenarios that involved using well-known statistical fading distributions, correlated-Rician and correlated-Nakagami- m fading, were deployed in analysis. The moment-generating function obtained using a stochastic process was applied to determine the ergodic, or average, channel capacity, thus by passing the difficulty traditionally associated with calculating the probability density function of the signal-to-noise ratio. The results of several numerical and Monte-Carlo comparisons are offered for validating the accuracy of the theoretically derived formulae. Furthermore, many plots were designed by combining different numbers of transmitters and receivers, $N_T \times N_R$, for comparison. The discussion addresses the nonlinearity of increments in channel capacity regarding different numbers of transmitter or receiver antennas and different correlation coefficients. The fading parameter is still the dominant factor, whether the correlated-Nakagami- m or correlated-Rician distributed fading is adopted as the channel model for the MIMO system, thus governing the average capacity.

I. INTRODUCTION

Using multiple antenna elements at both the transmitter and receiver ends promotes the overall system capacity and increases link robustness of a multiple-input multiple-output (MIMO) system [16]. A MIMO system, which increases reliability and spectral efficiency and mitigates the considerable degradation that can result from channel fading and mul-

tiuser interference in a wireless environment, is a smart technology for systems requiring substantial bandwidth at high data rates, such as 3G and 4G (third generation and fourth generation) wireless systems, wireless local area networks, and wireless broadband access [5, 19]. However, the channel correlation, or the propagation environment, has been proven to affect the channel capacity of a MIMO system; that is, capacity increases as signal correlation decreases [3, 12]. Furthermore, the authors of [22] described how the potential capacity gain is highly dependent on multipath richness because a fully correlated MIMO radio channel offers only one subchannel; whereas, a completely decorrelated radio channel potentially offers multiple subchannels, depending on the antenna configuration.

The stochastic model developed in [9], which focuses on the narrowband channels and uses the correlation matrices at the base station and mobile station, yielded results from numerous single-input, multiple-output, and MIMO radio channels that have been applied to examine the capacity of MIMO radio systems. The authors in [7] determined that the optimal transmission strategy for a MIMO system is a single user, with beam-forming (BF) and covariance feedback; the results showed that the optimal transmission strategy was generated in the direction of the eigenvectors of the transmit-correlation matrix. The authors of [10] claimed that the exact bit error rate derived for various modulation schemes in a correlated-Rayleigh MIMO channel. In [2], the authors investigated the capacity performance of a multiple-antenna system that was designed to switch among different MIMO transmission schemes, including statistical BF, double space-time transmit diversity, and spatial multiplexing over spatially correlated channels. The exact closed-form expression of the moment-generating function (MGF) of the mutual information for MIMO channels, accompanied by both interference in the presence of correlated Rayleigh and cochannel interference, was derived by the authors of [6]. The linear scaling of throughput achieved in a MIMO system was challenged by the authors of [21], who described how linear scaling can be achieved in wireless networks by using only receiver antenna arrays, without a multiple transmit antenna.

Regarding the differences between this investigation and some other relevant publications, the authors of [17] considered only the level-crossing rate and average fade duration in

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their discussion of the channel correlation that occurs in a correlated-Nakagami- m channel; their study covered only certain combinations of transmitter and receiver antenna numbers (2×2 , 4×4 , and 6×6 only) and did not explore correlated-Rician fading. Conference paper [20] considers only the different fading parameters involved in evaluating the performance of a MIMO maximal ratio combining (MRC) system over a Nakagami- m environment; these results were based on immature work. In [23], the authors claimed that they derived several ergodic capacity upper- and lower-boundary formulae with Meijer G-function for C-MIMO and D-MIMO in a Nakagami- m channel. By contrast, the authors of [13] proposed an interference-aware, user-selected scheme for an uplink multiuser MIMO system in a multicell environment; although the study confirmed that intercell interference definitely degrades the system throughput, it did not discuss the correlation phenomena or correlated-Rician fading nor obtain the closed form.

In the literature, our study found motivation, first, from the incompleteness of these basic studies, and, second, from the reflection, diffraction, and scattering that exist in spatial communication systems. The propagation channel exhibits substantial multipath-fading behavior, especially over multiple antenna environments. The final motivation for our study was the assumption that the transmission paths were insufficiently separated or that the number of receiver terminals was inadequate [18]. Thereafter, our analysis method adopted the much simpler MGF calculation, by obtaining the probability density function (pdf) of the SNR at the output of the MRC, and our study began evaluating the performance of a MIMO system operating over two well-known statistical distributions: correlated-Nakagami- m and correlated-Rician fading. These two statistical models have been shown, through certified experimentation, to be the most suitable for characterizing the fading channel in multipath environments [11, 15]. Nevertheless, the main contribution of this study is the effect of the correlation of the transmitting and receiving branches, which were considered as having an arbitrary correlation coefficient. This study examines a MIMO system with a BF receiver in combination with an MRC operating over a correlated fading channel, in both different transmitter and different receiver antenna environments. The impact of this correlation on the performance of a MIMO system is analyzed using the correlated-Rician and correlated-Nakagami- m distributions. Furthermore, theoretical and numerical analysis are performed using different transmission and receiver antenna numbers during the MIMO signaling. Finally, the effectiveness of the derived formulae are validated through the Monte-Carlo computer results. The remainder of this report is organized as follows: Section II presents a MIMO system with a BF combination and the statistical models of a fading channel, followed by the derivations of the MGF of the spatially correlated fading. Section III presents the results of evaluating the ergodic capacity performance of the MIMO system. Section IV presents a numerical manifestation, based on the analytical

results reported in Section III, and a brief discussion. Finally, Section V concludes the report.

II. MIMO SYSTEM MODELS

A narrowband MIMO system, considered to employ N_T transmitter and N_R receiver antennas, is equipped with the receiver signal vector $Y_R \in \mathcal{C}^{N_R \times 1}$ and the transmit signal vector $X_T \in \mathcal{C}^{N_T \times 1}$, respectively. The transmitted signal power is constricted as the value equivalent to the number of transmitter antennas; that is, $P_T = \mathbb{E}[\|X_T\|^2] = N_T$, and the MIMO channel complex matrix $\mathbf{H} \in \mathcal{C}^{N_R \times N_T}$ is characterized as the spatially corrected Rician distribution with the Rician gain factor K_R , which is represented as

$$\mathbf{H} = \tilde{\mathbf{H}}(1/K_R + 1)^{0.5} + \bar{\mathbf{H}}(K_R/K_R + 1)^{0.5} \quad (1)$$

where $\tilde{\mathbf{H}}$ and $\bar{\mathbf{H}}$ correspond to the non-LOS (line of sight) and the LOS components of the channel models' matrix \mathbf{H} . The aforementioned non-LOS channel matrix can be decomposed as the combination of transmit and receive spatial correlation matrices; that is, the non-LOS channel matrix component is expressed as $\tilde{\mathbf{H}} = \mathbf{M}_R^{1/2} \mathbf{Z} \mathbf{M}_S^{1/2}$, where $\mathbf{Z} \in \mathcal{C}^{N_R \times N_T}$ includes i.i.d. (independent and identically distributed) Gaussian entries with zero mean and unit variance; that is, $\mathbf{Z} \sim CN_{M_R M_S}(0, \sigma_{M_R M_S})$, where $\sigma_{M_R M_S}$ is the covariance matrix of the random matrix \mathbf{Z} , and where \mathbf{M}_R and \mathbf{M}_S denote the matrices of the receiving and transmitting spatial correlations, respectively. In addition, the n th eigenvalue of \mathbf{M}_R and \mathbf{M}_S are designated as $\lambda_i^{M_R}$, $i = 0, \dots, N_T - 1$, and $\lambda_i^{M_S}$, $i = 0, \dots, N_T - 1$, respectively. By using the eigenvalue decomposition method, the spatial-correlation matrices can be rewritten in advance as

$$\mathbf{M}_R = U_{M_R} \Lambda_{M_R} U_{M_R}^\dagger \quad (2)$$

and

$$\mathbf{M}_S = U_{M_S} \Lambda_{M_S} U_{M_S}^\dagger \quad (3)$$

, respectively, where the symbol \dagger denotes conjugate transpose and where U_{M_R} and U_{M_S} contain the corresponding entries of the eigenvectors with respect to the related eigenvalues, while Λ_{M_R} is a diagonal matrix with eigenvalues that are obtained from spatial-correlation matrices in the main diagonal elements.

Thereafter, components, characterized as noncoherent or coherent, of the channel models' matrix are placed in the equation in relation to the transmitting channel \mathbf{H} and to the spatial correlation. Once construction of the propagating channel scenario is completed, the received signal intensity, $\mathbf{Y}_R \in \mathcal{C}^{N_R \times 1}$, for each spatially correlated channel fading en-

environment experienced can be expressed as

$$\mathbf{Y}_R = \sqrt{E_s/N_T} \mathbf{H} \mathbf{X}_T + \mathbf{N} \quad (4)$$

where E_s is the symbol energy, $\mathbf{X}_T \in \mathbb{C}^{N_T \times 1}$ denotes the transmit signal energy which is constrained in $\mathbb{E} \left[\left| \mathbf{X}_T \mathbf{X}_T^\dagger \right|^2 \right] = N_T \mathbf{I}_{N_T}$, and $\mathbf{N} \in \mathbb{C}^{N_R \times 1}$ denotes the transmit signal energy which is constrained in the additive Gaussian noise vector with zero-mean and covariance $\mathbb{E} [\mathbf{N} \mathbf{N}^\dagger] = N_0 \mathbf{I}_{N_R}$, where $N_0/2$ denotes the double-sided power spectral density of the AWGN (additive white Gaussian noise). The SNR at the receiver is defined as $\gamma_s = E_s / N_0$.

III. EVALUATION OF ERGODIC CAPACITY

This section analyzes the channel capacity of a MIMO system with a BF transmission scheme used over a double-sided (transmitter and receiver) correlated channel. Following, first, a stochastic process for calculating the MGF, two scenarios of spatial correlation were, then, assumed for the ergodic channel capacity evaluation. Generally, the ergodic capacity can be calculated directly from the information capacity, which is expressed as

$$C_{pdf}^{BF} = \mathbb{E} [\iota(S, y)] \quad (5)$$

where subscript pdf depends on the fading channel model assumed and $\iota(S, y)$ represents the instantaneous mutual information between the input vector and output y of the MIMO signaling scheme. However, obtaining the closed-form smoothly by simply substituting the terms of mutual information into (5) is difficult. There is an easier alternative for calculating the MGF of a random spatially correlated matrix to determine its ergodic capacity. Thus, according to the system model described in the previous section, the mutual information can be naturally expressed as $\iota(S, y) = \log_2(\mathbf{I} + \mathbf{H} \mathbf{Q} \mathbf{H}^\dagger / N_0)$, where \mathbf{H} has been defined in (1) and \mathbf{I} and \mathbf{Q} denote the unit matrix and input covariance matrix of a MIMO system, respectively. An assumption is made, designating all of the power components to be constrained for \mathbf{Q} . \mathbf{Q} is an element crucial to the maximization of system performance because there are a high number of essential pieces of information included in it. For instance, if the distance between antennas is adequate, the arrival of angle of the in/out signals is suitable. Although the MIMO system employs a BF with an MRC receiver, the covariance matrix, \mathbf{Q}_{BF} , has now replaced it, and the instantaneous mutual information can be implicitly decomposed as $\mathbf{Q}_{BF} = U_{M_s} \Lambda_{BF} U_{M_R}$, where U_{M_s} and U_{M_R} are defined in Section II and $\Lambda_{BF} = \text{Diag}[E_s, 0, \dots, 0]$. Hence, by substituting the channel matrix into the mutual information

formula $\iota(S, \lambda)$, the new mutual information becomes

$$\begin{aligned} \iota(S, \lambda) &= \log_2(\mathbf{I}_{BF} + \mathbf{H} \mathbf{Q}_{BF} \mathbf{H}^\dagger / N_0) \\ &= \log_2 \left[(\mathbf{I}_{BF} + (E_s / N_0) \mathbf{M}_R^{1/2} \mathbf{Z} \mathbf{M}_S^{1/2} \mathbf{Q}_{BF} (\mathbf{M}_S \mathbf{Z} \mathbf{M}_R)^{1/2}) \right] \end{aligned} \quad (6)$$

where \mathbf{I}_{BF} is a unit matrix with same dimension as the MRC, E_s/N_0 represents the SNR, and \mathbf{M}_R , \mathbf{Z} , and \mathbf{M}_S have been defined in the previous section. By including the covariance matrix \mathbf{Q}_{BF} and matrices combined with eigenvectors, (6) can be rewritten as (see the Appendix)

$$\begin{aligned} \iota(S, \lambda) &= \log_2 \left[\mathbf{I}_{BF} + (E_s / N_0) \mathbf{M}_R^{1/2} \mathbf{Z} \mathbf{M}_S^{1/2} \mathbf{Q}_{BF} (\mathbf{M}_S \mathbf{Z} \mathbf{M}_R)^{1/2} \right] \\ &= \log_2 \left[\mathbf{I}_{BF} + \left| \lambda_{\max}^{M_s} \gamma_s \zeta \zeta^\dagger \right| \right] \end{aligned} \quad (7)$$

where γ_s is defined in (4); $\lambda_{\max}^{M_s}$ indicates a maximum value selected from the eigenvalues found by the matrix \mathbf{M}_S ; that is, $\lambda_{\max}^{M_s} = \max(\lambda_i^{M_s})$, $i = 0, 1, \dots, N_R - 1$; ζ is defined as $\zeta = [(\lambda_1^{M_R})^{0.5} z_1, \dots, (\lambda_{N_R}^{M_R})^{0.5} z_{N_R}]^T$, in which z_i , $i = 0, 1, \dots, N_R - 1$, represents the i th entry of first column generated from matrix \mathbf{Z} , and z_i 's denotes i.i.d. zero-mean and unit variance complex Gaussian random variables. Note that by using a simple operation in linear algebra, $|\mathbf{I}_u + \mathbf{X} \mathbf{Y}| = |\mathbf{I}_v + \mathbf{Y} \mathbf{X}|$, where $\mathbf{X} \in \mathbb{C}^{u \times v}$, and $\mathbf{Y} \in \mathbb{C}^{v \times u}$, after decomposing the matrices and substituting them back into (7), which becomes

$$\iota(S, y) = \log_2(1 + \eta \Omega) \quad (8)$$

where $\eta = \sum_{i=0}^{N_R-1} \lambda_i^{M_R} \varepsilon_i$, $\Omega = \gamma_s \lambda_{\max}^{M_s} / 2$, and where $\eta/2 = \zeta^\dagger \zeta$, and ζ has been defined in (7) as the received intensity vectors of the MIMO system. After the pdf of η is obtained, the mutual information in (8) can be calculated. Moreover, the pdf of η , which is the combination of random variables ε_i , $i = 0, 1, \dots, N_R - 1$, must be calculated. Thus, an evaluation of the pdf for ε_i is necessary.

As previously mentioned, it is difficult to calculate the channel capacity simply through the substitution of the pdf of receiving intensity directly into the mutual information formula. However, an alternative method can be adopted to produce the definition of MGF. The first step to calculating of the average channel capacity is to determine the MGF of mutual information by incorporating (8) into the MGF equation, which can be determined as

$$\begin{aligned} M(\phi) &= \mathbb{E} \left[\exp(\phi \cdot \iota(S, y) / \log_2 e) \right] \\ &= \mathbb{E} \left[\log_2 \left| \mathbf{I}_t + \mathbf{Q}_t \right| \right] \end{aligned} \quad (9)$$

Because the channel information is considered to be previously unknown, it is necessary to search out a conditional pdf for calculating the ergodic capacity results; that is, the channel side information that has not been involved in the scenario of this study. The two types of statistical distribution included in this study are Rician and Nakagami- m distributions; these two statistical models are used to evaluate the channel capacity for a MIMO system over correlated-transmission environments.

1. Spatially Correlated Rician Channels

To calculate the equation of mutual information shown in (8), it is necessary to obtain the pdf of ε_i and η first. We consider a BF with an MRC applied to a MIMO system and, then, refer back to (8); a MIMO system operating over an indoor environment is considered in this subsection. Based on our assumption, it is known that η forms a central quadratic distribution as ε_i 's belongs to i.i.d. exponentially distributed R.V. (special cases of Rayleigh distributed). In the condition of LOS considered in the analysis, by adopting the general form of the results from [2] as the pdf of η and expressed as

$$f(\eta) = \sum_{i=0}^{N_R-1} A_i P(\lambda_i^{M_R}, \eta) = \sum_{i=0}^{N_R-1} \prod_{m=0, m \neq i}^{N_R-1} \left(\frac{\lambda_i^{M_R}}{\lambda_i^{M_R} - \lambda_m^{M_R}} \right) \frac{\exp(-\eta/2\lambda_i^{M_R})}{2\lambda_i^{M_R}} \quad (10)$$

Once the pdf of η has been determined, its MGF can be obtained by definition; and, by substituting (10) into (9), the MGF can be evaluated in advance, which is expressed as

$$\begin{aligned} M(\phi) &= \int_0^\infty (1 + \Omega\eta)^\phi f(\eta) d\eta \\ &= \int_0^\infty (1 + \Omega\eta)^\phi \sum_{i=0}^{N_R-1} \prod_{m=1, m \neq i}^{N_R-1} \left(\frac{\lambda_i^{M_R}}{\lambda_i^{M_R} - \lambda_m^{M_R}} \right) \frac{\exp(-\eta/2\lambda_i^{M_R})}{2\lambda_i^{M_R}} \\ &= \sum_{i=1}^{N_R-1} \prod_{i=1, m \neq i}^{N_R-1} \left(\frac{\lambda_i^{M_R}}{\lambda_i^{M_R} - \lambda_m^{M_R}} \right) \frac{1}{2\lambda_i^{M_R}} \int_0^\infty (1 + \Omega\eta)^\phi \exp(-\eta/2\lambda_i^{M_R}) d\eta \end{aligned} \quad (11)$$

where $\Omega = \gamma_s \lambda_{\max}^{M_S} / 2$. The integral part in the previous equation can be solved by general formula; thereafter, the closed-form of (10) can be determined and obtained as

$$\begin{aligned} M(\phi) &= \sum_{i=0}^{N_R-1} \prod_{m=1, m \neq i}^{N_R-1} \left(\frac{\lambda_i^{M_R}}{\lambda_i^{M_R} - \lambda_m^{M_R}} \right) \left(\frac{1}{2\lambda_i^{M_R}} \right) \cdot \Omega^{-\phi} \cdot \mu^{-\phi-1} \\ &\quad \cdot e^{\frac{\mu}{\Omega}} \cdot \Gamma\left(\phi + 1, \frac{\mu}{\Omega}\right) \end{aligned} \quad (12)$$

where $\Gamma(x, y)$ denotes the incomplete beta function, and inte-

gral formulae expressed as [4]

$$\int_0^\infty (x + \beta)^\nu \exp(-\mu x) dx = \mu^{-\nu-1} \cdot e^{\beta\mu} \cdot \Gamma(\nu + 1, \beta\mu) \quad (13)$$

and,

$$\int_0^\infty \left(1 + \frac{x}{\beta}\right)^\nu \cdot \exp(-\mu x) dx = \beta^{-\nu} \cdot \mu^{-\nu-1} \cdot e^{\beta\mu} \cdot \Gamma(\nu + 1, \beta\mu) \quad (14)$$

have been employed in calculating the MGF from (11) to yield the novel result of (12), with variables designated as $\phi = \nu$ and $\mu = 1/2\lambda_i$.

The first step in determining the MGF of a random stochastic is the standard procedure for obtaining an ergodic capacity; following this rule, the original ergodic capacity can be obtained by the formula

$$C_{Ric}^{BF} = \log_2 e \cdot \left. \frac{\partial M(\phi)}{\partial \phi} \right|_{\phi=0} \quad (15)$$

where, now, the derivative of MGF, $\partial M(\phi)/\partial \phi$, in the previous equation can be calculated as

$$\begin{aligned} \frac{\partial M(\phi)}{\partial \phi} &= \frac{\partial}{\partial \phi} \left[\sum_{i=0}^{N_R-1} \prod_{m=1, m \neq i}^{N_R-1} \left(\frac{\lambda_i^{M_R}}{\lambda_i^{M_R} - \lambda_m^{M_R}} \right) \left(\frac{1}{2\lambda_i^{M_R}} \right) \cdot e^{\frac{\mu}{\Omega}} \cdot B(\phi) \cdot A(\phi) \right] \\ &= \sum_{n=0}^{N_R-1} \prod_{m=1, m \neq i}^{N_R-1} \left(\frac{\lambda_i^{M_R}}{\lambda_i^{M_R} - \lambda_m^{M_R}} \right) \left(\frac{1}{2\lambda_i^{M_R}} \cdot e^{\frac{\mu}{\Omega}} \right) \\ &\quad \times \left[B(\phi) \cdot \frac{\partial A(\phi)}{\partial \phi} + \frac{\partial B(\phi)}{\partial \phi} \cdot A(\phi) \right] \end{aligned} \quad (16)$$

where $A(\phi) = \Gamma(\phi + 1, \mu/\Omega)$, and $B(\phi) = \Omega^{-\phi} \cdot \mu^{-\phi-1}$. According to the formula of the derivative of incomplete gamma function, it can be expressed as

$$\begin{aligned} \frac{\partial^n \Gamma(a, z)}{\partial a^n} &= \Gamma^{(n)}(a) - \sum_{k=0}^{\infty} \frac{(-1)^{n-k} \Gamma(n+1, -(a+k) \cdot \log(z))}{(a+k)^{n+1} \cdot k!}, \\ n &\in N \end{aligned} \quad (17)$$

Calculating the first order derivative, let $n = 1$ be in the last equation; hence, the partial derivative of $A(\phi)$ is easily obtained as [4]

$$\frac{\partial A(\phi)}{\partial \phi} = \Gamma(\phi + 1) - \sum_{k=0}^{\infty} \frac{(-1)^{1-k} \Gamma(2, -(\phi + 1 + k) \cdot \log(\mu/\Omega))}{(\phi + 1 + k)^2 \cdot k!} \quad (18)$$

and the partial derivative of $B(\phi)$ can also be determined as

$$\frac{\partial B(\phi)}{\partial \phi} = \frac{\partial}{\partial \phi} [\Omega^{-\phi} \cdot \mu^{-\phi-1}] = \frac{1}{\mu} (-\phi)(\Omega \cdot \mu)^{-\phi-1} \quad (19)$$

By substituting (18) and (19) back into (15), the ergodic capacity of a MIMO system over spatially correlated Rician fading, C_{Ric}^{BF} , can be obtained using (15) and expressed as

$$\begin{aligned} C_{Ric}^{BF} &= \log_2 e \cdot \left. \frac{\partial M(\phi)}{\partial \phi} \right|_{\phi=0} \\ &= \log_2 e \cdot \sum_{n=0}^{N_R-1} \prod_{m=1, m \neq i}^{N_R-1} \left(\frac{\lambda_i^{M_R}}{\lambda_i^{M_R} - \lambda_m^{M_R}} \right) \left(\frac{1}{2\lambda_i^{M_R}} \right) \cdot e^{\frac{\mu}{\Omega}} \\ &\quad \times B(0) \cdot \left(\Gamma(1) - \sum_{k=0}^{\infty} \frac{(-1)^{1-k} \Gamma(2, -(1+k) \cdot \log(\mu/\Omega))}{(1+k)^2 \cdot k!} \right) \end{aligned} \quad (20)$$

where $B(0)$ now becomes $B(0) = \mu^{-1}$.

2. Spatially Correlated Nakagami- m Channels

With spatially correlated Nakagami- m channels, the intensity of the transmitted signals is expressed as, ε_i , $i = 0, \dots, N_R - 1$, and the transmitted signals are assumed to be random correlated-Nakagami- m variables [8]. An average of the transmitted signals' power is then normalized according to the fading parameters of the Nakagami- m fading and represented as $\sigma_{N_R} = E[Z_{N_R}^2]/m_i$, $i = 0, \dots, N_R - 1$, where $m_0 \leq m_2 \leq \dots \leq m_{N_R-1}$ represent the fading parameters of the corresponding correlated fading channels and $E[\cdot]$ is the expectation operator. The squared value of the intensity Z_i is distributed in a gamma distribution; that is, $\zeta_i = \varepsilon_i^2$ and $\zeta_i \sim \text{Gamma}(z; m_i, \sigma_i)$. Moreover, ζ_i becomes a special case with an Erlang distribution when all fading parameters, m_i , are designated as integer numbers; that is, the pdf of ζ_i is expressed as

$$f_{\zeta_i}(z, m_i, \sigma_i) = \frac{z^{m_i-1} \cdot \exp(-z/\sigma_i)}{\Gamma(m_i) \sigma_i^{m_i}} u(z) \quad (21)$$

where $u(z)$ is the unit step function. Because the BF with the MRC is adopted as the receiver scheme, the summation of all multiple input at the MIMO receiver is written as $\eta = \sum_{i=0}^{N_R-1} \zeta_i$ in which the pdf can be obtained as [8]

$$f(\eta) = \sum_{i=0}^{N_R-1} \sum_{k=1}^{m_i} \Delta_{N_R}(i, k, m_u, \sigma_u, l_q) \cdot f_{\zeta_i}(z, m_i, \sigma_i) \quad (22)$$

where $u = 0, \dots, N_R - 1$, $q = 1, \dots, N_R - 1$, and $\Delta_{N_R}(i, k, m_u, \sigma_u, l_q)$ is a recursive formula expressed as

$$\begin{aligned} &\Delta_{N_R}(n, m_n - k, m_u, \sigma_u, l_q) \\ &= \frac{1}{k} \sum_{\substack{j, u, q=0 \\ n \neq u}}^{N_R-1} \frac{m_u}{\sigma_n^j} \left(\frac{1}{\sigma_n} - \frac{1}{\sigma_u} \right)^{-j} \times \Delta_{N_R}(n, m_u - k + j, m_u, \sigma_u, l_q) \end{aligned} \quad (23)$$

Because the general form of the pdf of a squared correlated-Nakagami- m distribution is with the results from [8], the detail is not rewritten here. Thus, by substituting the pdf of received intensity shown in (21) into the definition of the MGF shown in (11), the MGF of a Nakagami- m spatial correlation channel of a MIMO system becomes

$$\begin{aligned} M(\phi) &= \int_0^{\infty} (1 + \Omega \eta)^\phi f(\eta) d\eta \\ &= \int_0^{\infty} (1 + \Omega \eta)^\phi \sum_{i=0}^{N_R-1} \sum_{k=1}^{m_i} \Delta_{N_R}(i, k, m_u, \sigma_u, l_q) \\ &\quad \times \frac{\eta^{(m_i-1)}}{\sigma_i^{m_i} (m_i - 1)!} \cdot \exp(-\eta/\sigma_i) \\ &= \sum_{i=1}^{N_R} \sum_{k=1}^{m_i} \Delta_{N_R}(i, k, m_u, \sigma_u, l_q) \\ &\quad \times \frac{1}{\sigma_i^{m_i} (m_i - 1)!} \int_0^{\infty} (1 + \Omega \eta)^\phi \cdot \eta^{(m_i-1)} \cdot \exp(-\eta/\sigma_i) d\eta \end{aligned} \quad (24)$$

The completion of the integral in the previous equation can be solved using the equivalent formula expressed as [4]

$$\int_0^{\infty} e^{-gx} \cdot x^{h-1} \cdot (1+ax)^{-\nu} dx = a^{-h} \Gamma(h) \psi(h, h+1-\nu; g/a) \quad (25)$$

where $\psi(i, j; k) = \int_0^{\infty} e^{-kt} t^{\alpha-1} (1+t)^{j-i-1} / \Gamma(i) dt$ is the confluent hypergeometric function [23]; thus, by changing the variables and letting $g = (1/\sigma_i)$, $h = m_i$, $a = \Omega$, and $\nu = -\phi$, (25) then becomes

$$\begin{aligned} M(\phi) &= \sum_{i=0}^{N_R-1} \sum_{k=1}^{m_i} \Delta_{N_R}(i, k, m_u, \sigma_u, l_q) \\ &\quad \times \frac{1}{\sigma_i^{m_i} (m_i - 1)!} \cdot \Omega^{-m_i} \cdot \Gamma(m_i) \cdot \psi \left(m_i, m_i + 1 - \phi; \frac{1}{\Omega \sigma_i} \right) \end{aligned} \quad (26)$$

Again, the formula for differentiation of confluent hypergeometric function is applied [14], which is expressed as

$$\frac{d^n}{dz^n} \psi(\alpha, r; z) = \frac{(\alpha)_n}{(r)_n} \cdot \psi(\alpha + 1, r + 1; z) \quad (27)$$

where $(\alpha)_n$ is the Pochhammer symbol. Consequently, the derivative of the MGF can be obtained by incorporating (21) into (24) and expressed as

$$\begin{aligned} \frac{\partial \mathbf{M}(\phi)}{\partial \phi} &= \sum_{i=1}^{N_R} \sum_{k=1}^{m_i} \Delta_{N_R}(i, k, m_u, \sigma_u, l_q) \times \frac{1}{\sigma_i^{m_i} (m_i - 1)!} \cdot \Omega^{-m_i} \cdot \Gamma(m_i) \\ &\quad \cdot \frac{\partial}{\partial \phi} \left[\psi \left(m_i, m_i + 1 - \phi; \frac{1}{\Omega \sigma_i} \right) \right] \\ &= \sum_{i=1}^{N_R} \sum_{k=1}^{m_i} \Delta_{N_R}(i, k, m_u, \sigma_u, l_q) \frac{1}{\sigma_i^{m_i} (m_i - 1)!} \cdot \Omega^{-m_i} \cdot \Gamma(m_i) \\ &\quad \cdot \frac{m_i}{m_i + 1 - \phi} \cdot \psi \left(m_{i+1}, m_i + 2 - \phi; \frac{1}{\Omega \sigma_i} \right) \end{aligned} \quad (28)$$

where $u = 0, \dots, N_R - 1$ and $q = 1, \dots, N_R - 1$. The MGF of correlated-Nakagami- m fading for receiver antennas be followed by analyzed steps, similar to those of the Rician case, and, then, the ergodic capacity of a MIMO system over the spatially correlated Nakagami- m channel, C_{Nak}^{BF} , can finally be evaluated as

$$\begin{aligned} C_{Nak}^{BF} &= \log_2 e \cdot \left. \frac{\partial \mathbf{M}(\phi)}{\partial \phi} \right|_{\phi=0} \\ &= \log_2 e \cdot \sum_{i=0}^{N_R-1} \sum_{k=1}^{m_i} \Delta_{N_R}(i, k, m_u, \sigma_u, l_q) \\ &\quad \times \frac{1}{\sigma_i^{m_i} (m_i - 1)!} \cdot \Omega^{-m_i} \cdot \Gamma(m_i) \cdot \frac{m_i}{m_i + 1} \cdot \psi \left(m_i + 1, m_i + 2; \frac{1}{\Omega \sigma_i} \right) \end{aligned} \quad (29)$$

where $\Delta_{N_R}(\cdot)$ has been defined in (22).

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, numerical results are provided for certifying the validity of the derived theoretical formulae for a MIMO system over spatially correlated channels. Moreover, the simulation results for the statistical properties of the capacity of various Nakagami- m MIMO channels are also presented for purpose of accuracy. Because the main focus of this article is the spatially correlated channel, the Rician factor is always set as a fixed value of $K = 3.6$. However, if a radio system is operating over a Rician fading channel, the higher the values of the Rician factor are, then the higher the performance of the radio system will be. Therefore, the capacities shall be not evaluated as a parameter of the Rician factor K in this article. For simplicity, however, without loss of generality, correlation coefficients, generated by the Gaussian correlation model of an equally spaced linear array with an arbitrary correlation

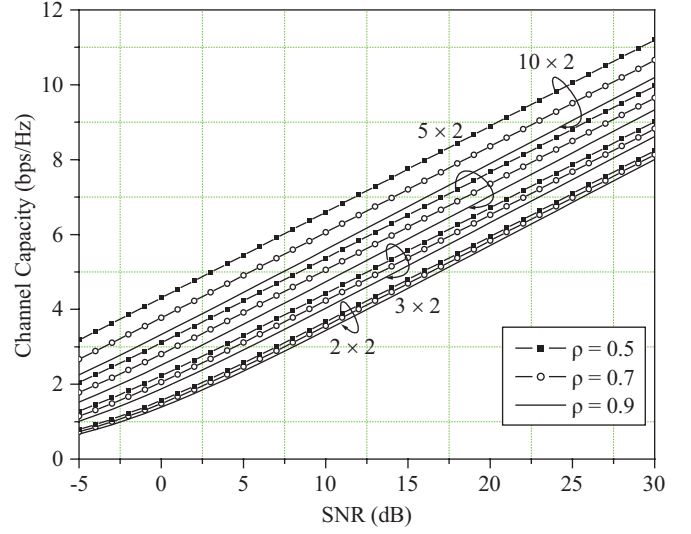


Fig. 1. SNR vs. channel capacity for correlated-Rician fading with different $N_R \times N_T$ number and correlation coefficients $K = 3.6$ dB.

coefficient, were adopted [1]. The correlation matrix, followed by the linear array, has a Toeplitz form constructed of correlation elements, ρ_{ij} , $i, j = 0, \dots, N_R - 1$, which is revealed as

$$\rho_{ij} = \exp[-0.5\eta(i-j)^2(d/\lambda)^2], \quad i, j = 0, \dots, N_R - 1 \quad (30)$$

where $\eta \cong 21.4$ is a coefficient chosen from setting this correlation model equal to the Bessel correlation model [1] with a -3 dB point, d/λ as the normalized distance between two neighboring branches, and λ as the wavelength of the carrier frequency. The simulation is performed to validate the performance results of the derived equations for the MIMO schemes. A simplified relationship, defined using (30), is adopted to configure the correlation model, and the Monte-Carlo of the rural-area channel model is repeated for 10 runs each, with 10^4 samples generated from gamma-distribution to obtain reliable statistics. The parameter d/λ is applied to determine the threshold level of correlation. Values of $d/\lambda = 0, 0.1, 0.2$, and ∞ are employed in this numerical analysis, with $d/\lambda = 0$ and $d/\lambda \rightarrow \infty$ representing two extreme conditions, the fully correlated and uncorrelated branches, respectively. The results shown in Fig. 1 depict different scenarios, each with a distinct number of transmitter and receiver antennas; that is, plots worked out from four deploying types, $N_R \times N_T$ equal to $2 \times 2, 3 \times 2, 5 \times 2$, and 10×2 , correspondingly. Moreover, curves in Fig. 1 also show the spatial correlation effect, $\rho = 0.5, 0.7, 0.9$, and, as expected, the ergodic capacity decreasing with the increasing correlation effect. In addition, the increments in channel capacity are nonlinear with an increase of receiver antennas. At the conditions of SNR = 15 dB and $\rho = 0.7$, the channel capacity increases about 0.68 bps/Hz from $N_R \times N_T = 2 \times 2$ to $N_R \times N_T = 3 \times 2$, however, it is just increasing about 1.01 bps/Hz from $N_R \times N_T = 5 \times 2$ to

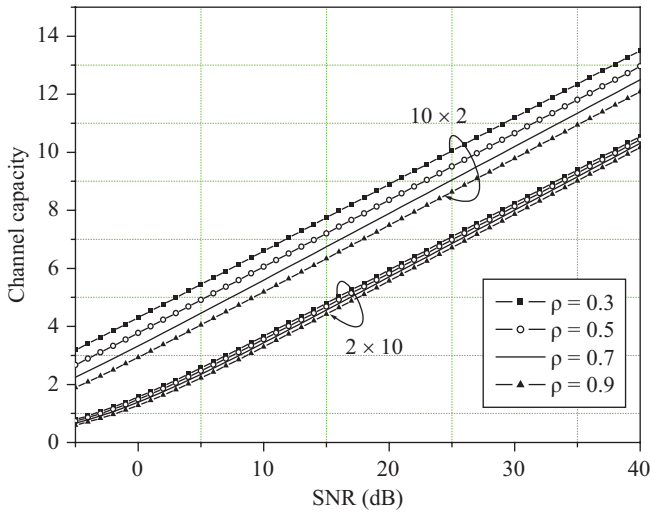


Fig. 2. SNR vs. channel capacity for correlated-Rician fading to show the characteristics of different $N_R \times N_T$.

$N_R \times N_T = 10 \times 2$. In other words, the increment in channel capacity has been strictly constrained when the spatial correlation fading occurs in the MIMO system. The outcomes of the investigation regarding the effect of varying the numbers of transmitter and receiver antennas are illustrated in Fig. 2, with each scenario accompanied by a different number of transmitter and receiver antennas; that is, 10×2 and 2×10 are applied for comparison. With different correlation coefficient values, the curves denoting channel capacity show the 10×2 scenario outperforming the others; but, the 2×10 scenario, nevertheless, has correlation coefficients.

Furthermore, we want to discuss the results from the cases with different fading parameters of correlated-Nakagami- m shown in Fig. 3 and Fig. 4, in which $m = 2$ and $m = 3$ are assigned, respectively. The simulated plots are marked with circle symbols for comparison, and both the curves obtained theoretically and numerically are similarly matched. To distinguish them from those in the Rician case, the correlations are designated as $\rho = 0.4, 0.6$, and 0.8 . There is a trend toward superior performance as the value of the correlation coefficient becomes higher. This trend largely matches the results from the case of a MIMO system over a correlated-Rician channel. Another pertinent fact is that the fading parameter of a Nakagami- m distribution is still dominant in increasing the average capacity; that is, the penalty of the average channel capacity approximately one order of 10 when the correlation coefficient is reduced from $\rho = 0.8$ to 0.4 with the condition of $N_R \times N_T = 2 \times 4$. However, at the same condition of SNR = 40 dB and $\rho = 0.6$, the average capacity increases proportionately from about 9.2 bps/Hz to 468.48 bps/Hz as the fading parameter is adjusted from $m = 2$ (Fig. 3) to $m = 3$ (in Fig. 4).

V. CONCLUSION

In this study, two cases of spatially correlated fading

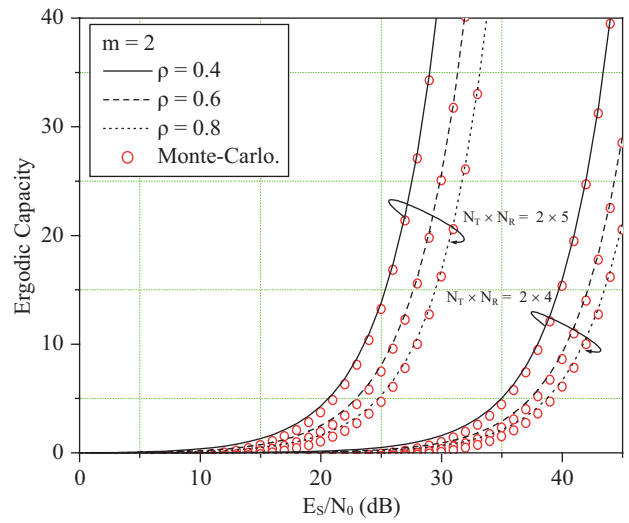


Fig. 3. SNR vs. ergodic capacity for correlated-Nakagami- m fading with different $N_T \times N_R$ numbers and correlation coefficients when $m = 2$.

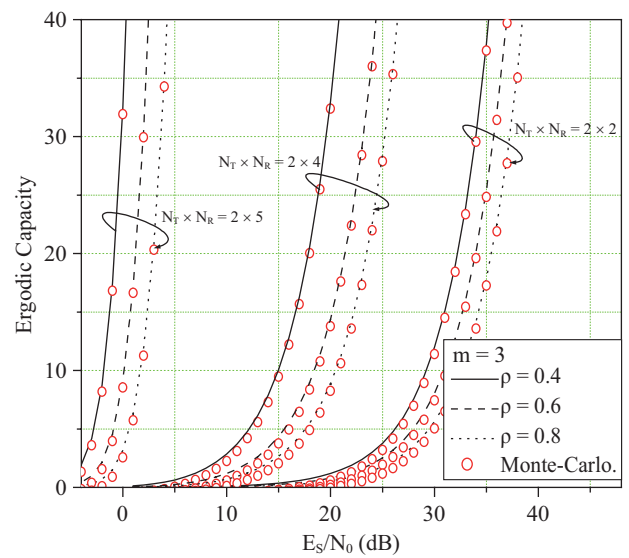


Fig. 4. SNR vs. ergodic capacity for correlated-Nakagami- m fading with different $N_T \times N_R$ numbers and correlation coefficients when $m = 3$.

channels were assumed for evaluating the ergodic capacity of a MIMO system. Both the numerical and Monte-Carlo simulations applied to various scenarios with distinct numbers of antennas at the transmitting and receiving ends yielded markedly similar results. Of particular relevance are the results demonstrating superior performance in the situations combining a high number of receiver antennas with a spatially correlated channel. Furthermore, this study determined how increments in channel capacity manifest a nonlinearity to different totals of transmitter or receiver antennas and correlation coefficients. Moreover, the fading parameter is still the dominant factor for increasing the average capacity, when the

correlated-Nakagami- m environment is established as the channel model for a MIMO system.

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APPENDIX A

This appendix shows the calculations for deriving the mutual information. First, the spatial correlation matrices shown in (2) and (3) are substituted into the mathematical representation of mutual information, which is expressed as

$$\begin{aligned} \iota(S, y) &= \log_2 \left[\mathbf{I}_{BF} + (1/N_0) \mathbf{M}_R^{1/2} \mathbf{Z} \mathbf{M}_S^{1/2} \mathbf{Q}_{BF} (\mathbf{M}_S \mathbf{Z} \mathbf{M}_R)^\dagger \right] \\ &= \log_2 \left[\mathbf{I}_{BF} + (1/N_0) (\mathbf{U}_{M_R} \Lambda_{M_R} \mathbf{U}_{M_R}^\dagger)^{1/2} \right. \\ &\quad \times \mathbf{Z} (\mathbf{U}_{M_S} \Lambda_{M_S} \mathbf{U}_{M_S}^\dagger)^{1/2} \\ &\quad \times \mathbf{Q}_{BF} (\mathbf{U}_{M_S} \Lambda_{M_S} \mathbf{U}_{M_S}^\dagger)^{\dagger/2} \\ &\quad \left. \times \mathbf{Z} (\mathbf{U}_{M_R} \Lambda_{M_R} \mathbf{U}_{M_R}^\dagger)^{\dagger/2} \right] \end{aligned} \quad (\text{A-1})$$

The MIMO receiver adopts the BF with MRC diversity as the receiver scheme; thus, by employing the diagonal of the input covariance matrix, the final equation can be calculated as

$$\begin{aligned} \iota(S, y) &= \log_2 \left[\mathbf{I}_{BF} + (1/N_0) \Lambda_{M_R}^{1/2} \mathbf{Z} \Lambda_{M_S}^{1/2} \Lambda_{BF} \Lambda_{M_S}^{\dagger/2} \Lambda_{M_R}^{\dagger/2} \right] \\ &= \log_2 \left[\mathbf{I}_{BF} + (1/N_0) \Lambda_{M_R}^{1/2} \mathbf{Z} \Lambda_{M_S}^{1/2} \text{Diag}[E_s, 0, \dots, 0] \Lambda_{M_S}^{\dagger/2} \mathbf{Z}^{\dagger/2} \Lambda_{M_R}^{\dagger/2} \right] \end{aligned} \quad (\text{A-2})$$

The set of all power components, which were constrained to optimize the mutual information, was maximized; and, hence, the mutual information can be obtained as

$$\iota(S, y) = \text{Arg} \underset{\Lambda_Q \rightarrow \text{tr}(\Lambda_Q) = E_s}{\text{Max}} \left[\log_2 \left[\mathbf{I}_{BF} + (E_s / N_0) \lambda_i^{M_S} (\Sigma^T)^\dagger \Sigma \right] \right] \quad (\text{A-3})$$

where $\Sigma = \left[\sqrt{\lambda_0^{M_R}} \zeta_0, \dots, \sqrt{\lambda_{N_R-1}^{M_R}} \zeta_{N_R-1} \right]$ and $\lambda_i^{M_S}$ and $\lambda_i^{M_R}$, $i = 0 \dots \dots N_R - 1$, are eigenvalues of matrix \mathbf{M}_S and \mathbf{M}_R , respectively. Therefore, Σ is substituted back into (A-3), which can be expressed as

$$\begin{aligned} \iota(S, y) &= \log_2 \left[\mathbf{I}_{BF} + (E_s / N_0) \lambda_n^{M_S} (\lambda_0^{M_R} \zeta_0^2 + \lambda_1^{M_R} \zeta_1^2 + \dots + \lambda_{N_R-1}^{M_R} \zeta_{N_R-1}^2) \right] \\ &= \log_2 \left[\mathbf{I}_{BF} + (E_s / N_0) \sum_{i=0}^{N_R-1} \lambda_{\max}^{M_S} \zeta_i^2 \right] \end{aligned} \quad (\text{A-4})$$

Letting the variables be $\Omega = \gamma_0 \lambda_{\max}^{M_S} / 2$, $\eta = \sum_{n=0}^{N_R-1} \lambda_n^{M_R} \varepsilon_n$, the proof is finally completed, and equation (A-4) becomes as shown in (8).

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