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RESEARCH ARTICLE

Supply Chain Inventory Model with Markov Chain Demand

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Abstract

This study aims to develop a new one-vendor multiple-buyers integrated inventory model. We believe that our proposed model can forecast the demand of all buyers in the coming future by using data that have already existed and to minimize the total cost-for both buyers and vendors. In recent days, the Markov chain approach has played one of the critical methods of demand forecasting in the Supply Chain Management (SCM) field. The proposed model of this study is to analysis the demand of all buyers in one specific season that was impacted by the demand in the same season from last year. Finally, the results of this article discover the most optimal number of buyers and shipments, and the quantity of demand per period. Furthermore, a sensitivity analysis is also conducted to find out the sensitivity of the new model.

Keywords: Supply chain management (SCM), Markov chain, Demand forecasting

1. Introduction

Supply chain is composed of numerous entities such as manufacturers, vendors and retailers. With the rapid development of the market, focusing on two-layer inventory problem is not enough. That is, multi-echelon inventory problem has now become one of the most significant issues in supply chain management (SCM). To remain competitive, decision makers must cooperate with all members in the supply chain. As a result, more and more researchers began to integrate the whole supply chain rather than just focusing on a single echelon, such as Taebok Kim [12]; who developed a generalized model of a serial multi-echelon supply chain and Yang and Kuo [25]; who developed a three-echelon inventory model to determine optimal joint total profits of the whole supply chain system based on Yang's [26] former study.

The integrated inventory model; one type of mathematical method, is a critical issue of decision makers in determining the quantity of inventories for both vendors and buyers to achieve the system's optimal profits. Only by collaborating all members in the supply chain, could the whole system improve its service level and reduce its total costs by Ben-Daya et al. [3]. By considering equal-sized shipments to the buyer, Lu [18] presented a heuristic approach for the one-vendor multi-buyer integrated inventory case in 1995. He relaxed the assumption of Goyal [7] about completing a batch before starting shipments and investigated a model that allows shipments to occur during production. According to the spirit of Supply Chain Management, stable partnerships have to be established between all Vendors and buyers in systems to ensure the lowest total cost and the optimum profits by Lambert's [14] study. Recently, there are a lot of researchers have used the integrated model to deal with multi-echelon problems. By using a single-vendor multi-buyer integrated production-inventory model, J.K.Jha [11] found out that vendor's holding cost and set up cost show just opposite performance on shipment lot size. Christoph and Teabok [4] developed a multi-vendor single-buyer integrated inventory model to study shipment

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consolidation. Shah and Chaudhari [23] developed an integrated inventory model to study the improvement of deteriorating items. Poonam and Azharuddin [21] attempted to study the ordering strategies for an integrated inventory model with capacity constraint and order size dependent trade credit. In this paper, to satisfy the demand of buyers and minimize the total cost, one Vendor and several buyers in the system are supposed to reach an agreement on inventory planning through this partnership. That is, how high the profits would be, to a large extent, is depending on how much integrated inventory cost could be cut.

Traditional models can only help determine an optimal solution for one of the parties in the system. To address this situation, Banerjee developed the joint economic lot size (JELS) model [2]; which can help determine a joint optimal solution for all parties in the system. To break the barrier, Banerjee developed the JELS model, which can help find a jointly optimal policy for all parties in the system. To minimize the total costs of the whole supply chain system, many scholars today build their models based on the concept of JELS. Leuveano et al. [15] built a new JELS model to minimize joint total cost between vendor and buyer by deciding optimal delivery lot size, number of deliveries, and batch production. Abdelsalam and Elassal [1] considered the JELS problem for multi-echelon supply chain with multi-retailers and single manufacturer and supplier. Sarakhsi et al. [22] studied a single vendor-single buyer supply chain of a single product by using a new JELS model.

The concept of Economic Order Quantity (EOQ) was firstly introduced by Ford W. Harris in 1913 [9]. EOQ is a kind of Fixed Order Quantity Model, which is applicable only when demand for a product is constant over the year and each new order is delivered in full when inventory reaches zero, and is always used to minimize ordering costs and inventory costs of systems by determining the exact quantity of goods per shipment. However, there are varieties of uncertain factors in practice cases. Thus, how to response to those uncertainties is one of the main challenges in supply chain management [17]. This paper will consider two of those uncertain factors, lead time and demand-to make the model more realistic.

Ouyang and Wu [20] maintained that shorter lead time could reduce the safety stock, improve the whole system's service level and help the company have much stronger competitive. Chandra and Grabis [6] indicated that short lead time could enhance the service level and lower inventory level effectively. Nevertheless, since lead time consists of order preparation, order transit, supplier lead time, delivery time and set-up time [24], it is really hard for researchers to estimate lead time and the demand during the lead time precisely. In order to make models much more realistic, more and more researchers begin to use normal distribution lead time or assume that the demand during led time is normally distributed. For instance, M.A.Hoque [10]; developed a vendor-buyer integrated production inventory model with normal distribution of lead time to find out the optimal solution of the model.

Numerous studies have assumed that the demand of buyers is constant over time. However, in practice, demands of buyers can easily be effected by other factors. That is, if researchers want to develop a realistic model, they have to consider fluctuating demand. Kocer [13] proposed a modified Markov Chain model to forecast intermittent demand. This paper will also adopt Markov Chain method to forecast the future demand. Markov Chain was first developed and introduced by the Russian mathematician Andrey Andreevich Markov [19]. His papers on Markov chains adopted the theory of determinants (of finite square matrices), and focused heavily on what are in effect finite stochastic matrices [5]. A rough description of the conception of Markov Chain is introduced in the follows.

Suppose that for any nX1 probability vector X_0, X_t in the equation $X_t = A^t X_0$ (where A is a nxn transition matrix, all elements of which are all positive and the sum of each line is 1) would tend to a constant matrix X as t tends to positive infinite (AX = X). For example, supposed that there is a 3 × 3 probability vector A, all elements of which meet the following condition: $a + b + c = d + e + f = g + h + i = 1$, and a 3 × 1 probability vectorX_t, where the sum of all elements (x_t, $y_t andz_t$) equals to 1. a, b and c represent incidences of three different conditions of the event in the next period while the condition of this period is x. Similarly, d, e and f represent possibilities of three different conditions of the event while the condition of this period is y. g, h and i represent possibilities of three different conditions of the event while the condition of this period is z. $x_k, y_k andz_k$ present the incidences of three different conditions respectively in period t. Just like the process presented in appendix 1, with t approaches infinite, X_t tends to a constant probability vector and the equation $X_t = AX_{t-1}$ becomes $X_t = AX_t$. This process is called as Markov Chain.

Nowadays, Markov Chain has already been widely applied to numerous fields that have something to do with probability, such as statistics, biology and economics. In this study, we used a Markov chain to determine the possibility of the constant increase in demand by ρ, no change in demand, and the decrease in demand by ρ in a specific season.

Combining integrated the inventory model, concepts of SCM, JELS and EOQ, and the spirit of Markov chain, this paper would build a new model to find out the best strategy for decision makers to gain the optimal profit, and to determine the exact value of the total profit of the system.

2. Model formulation

Regarding the number of buyers, number of shipments of buyer j per period in season k and demand per period for all buyers in season k these three elements as three decision variables of the system, this section is going to set up the one-vendor multiple-buyers integrated inventory model with the spirit of Markov chain, deducing the general function of the profit of the whole system.

2.1. The definition of the symbols

The following are the definition of all 25 notations used in the proposed model:

m; Number of buyers
j; A subscript used to represent different buyers
k; A subscript used to represent different seasons in one year
S; Setup cost per lot ($/lot)
D; Demand per year ($D = \sum_{k=1}^{4} D_k$, units/year)
D_k; Demand for all buyers in season k($D_k = \sum_{j=1}^{m} d_{jk}$)
d_{jk}; Demand for buyer j in season k
P; Production per season (units/season)
n_{jk}; Number of shipments of buyer j per period in season k (times/period)
q_j; Number of goods per shipment of buyer j (units/shipment)
Q_k; Demand per period for all buyers in season k($Q_k = \sum_{j=1}^{m} n_{jk}q_j$)
H_v; Vendor's holding cost per unit per season ($/unit/season)
H_{bj}; Buyer j's holding cost per unit per season ($/unit/season)
C_o; Ordering cost per order ($/order)
C_s; Subcontracting cost per unit ($/unit)
C_d; Disposing cost per unit ($/unit)
C_t; Transportation cost per shipment ($/shipment)
P_c; Production cost
S_p; Sale price
L; Leading time
F_j; The standard variance of buyer j's sales volume during the leading time
ρ; The percentage by which the demand in season k might increase or decrease
α_k; Possibility of the actual demand in season k increasing by ρ
β_k; Possibility of the actual demand in season k is exactly same as the expected demand
γ_k; Possibility of the actual demand in season k decreasing by ρ

As shown above, m, j, k, S, D, D_k, d_{jk}, P, n_{jk}, q_j, Q_k, H_v, H_{bj}, C_o, C_s, C_d, C_t, P_c, S_p, L, F, ρ, α_k, β_k and γ_k are all 25 symbols which would be used in the model.

2.2. Assumptions

In order to establish a feasible model, several assumptions have to be introduced:

1. The production in one period is greater than the demand ($p \geq \sum_{j=1}^{m} d_{jk}$).
2. The ordering cost per order and transportation cost per shipment for each buyer are the same.
3. The number of the shipments in one period is equal for each buyer, while the size of which might be different.
4. Holding cost and ordering cost are equal for each buyer.
5. The quantity of safety stock is equal for each buyer.
6. The system only consists of a single type of item.
7. The time costs generated by subcontracting could be ignored.
8. The total demand for all buyers of the whole year is divided into 4 parts. That is, $D = \sum_{k=1}^{4}\sum_{j=1}^{m} d_{jk}$, where d_{jk} is the demand for buyer j in season k.
9. All buyers' sales volume during the leading time is normally distributed.
10. All buyers have the same standard variance (F) during the leading time
11. Transportation cost will be borne by buyers.
12. m, n_{jk} and Q_k are decision variables.
13. $n_k \geq 1$; $Q_k \geq 0$; $Q_k \leq D_k$; $m \geq 1$; n_k, m and D_k/Q_k are integers.

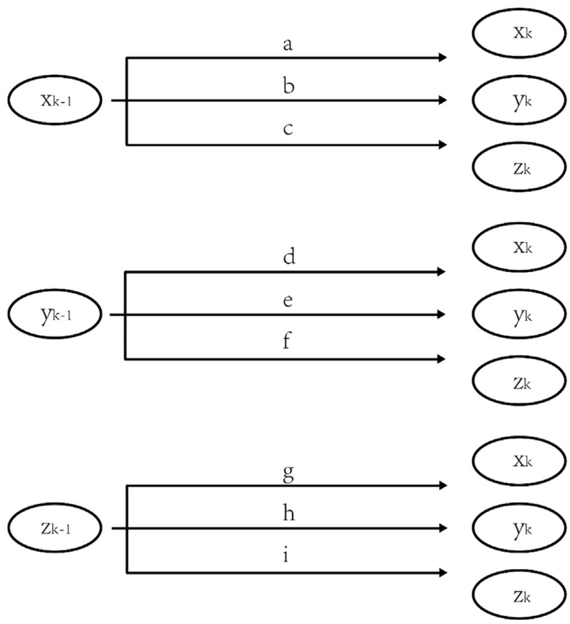


Fig. 1. The schematic diagram of Markov Chain.

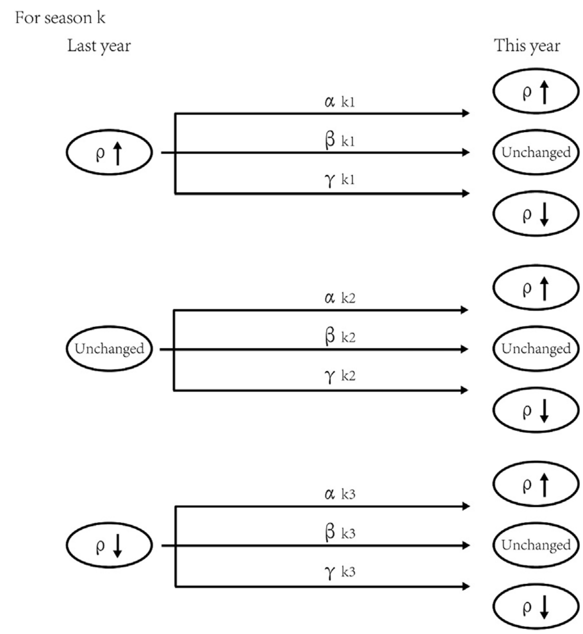


Fig. 2. The schematic diagram of Markov Chain applying in the model.

14. Suppose that the demand in season k would be influenced by the demand in the same season last year. For example, if the practical demand in season k last year, compared with the forecasting demand, increased by ρ, the possibility for the demand in the same season this year of increasing by ρ would be α_{k1}, remaining unchanged would be β_{k1} and decreasing by ρ would be γ_{k1}. By parity of reasoning, all situations can be represented as the following matrix and Fig. 2:

$$\text{For season k:} \begin{bmatrix} \alpha_{k1} & \beta_{k1} & \gamma_{k1} \\ \alpha_{k2} & \beta_{k2} & \gamma_{k2} \\ \alpha_{k3} & \beta_{k3} & \gamma_{k3} \end{bmatrix} [\alpha_k \ \beta_k \ \gamma_k] = [\alpha_k \ \beta_k \ \gamma_k]$$

As shown above, these are the 14 assumptions on which the new model is built.

2.3. Basic model

The vendor's inventory level against time is shown as follows. The step-by-step derivation of the function of vender's inventory is given in Appendix 2.

From Fig. 3, the area of the triangle LKO in one period in season k is given by

$$=\frac{Q_k}{2P}\left(Q_k=\sum_{j=1}^{m} n_{jk}q_j\right) \tag{1}$$
$$=\frac{Q_k{}^2}{2P}$$

The area of the rectangle LONM in one period in season k is given by

$$=\left(Q_k=\sum_{j=1}^{m} n_{jk}q_j\right)\sum_{j=1}^{m}\left(\frac{Q_k}{d_{jk}}-\frac{Q_k}{P}-\frac{Q_k}{d_{jk}n_{1k}}+\frac{Q_k d_{1k}}{d_{jk}Pn_{1k}}\right)$$
$$=Q_k\sum_{j=1}^{m}\left(\frac{Q_k}{d_{jk}}-\frac{Q_k}{P}-\frac{Q_k}{d_{jk}n_{1k}}+\frac{Q_k d_{1k}}{d_{jk}Pn_{1k}}\right)$$
$$=\frac{Q_k{}^2}{P}\sum_{j=1}^{m}\frac{Pn_{1k}-d_{jk}n_{1k}-P+d_{1k}}{d_{jk}n_{1k}} \tag{2}$$

The area of rectangles (The darker part in the schematic diagram shown in Fig. 3) for each buyer in one period in season k is as follows:

For buyer j, the area of the rectangle

$$=\frac{Q_k d_{jk}}{n_{jk}D_k}\left(\frac{Q_k}{n_{jk}D_k}(n_{jk}-1)+\frac{Q_k}{n_{jk}D_k}(n_{jk}-2)+\frac{Q_k}{n_{jk}D_k}(n_{jk}-3)\right.$$
$$\left.+\ldots+\frac{Q_k}{n_{jk}D_k}\right)+n_{jk}\left(\frac{Q_k d_{jk}}{n_{jk}D_k}\left(\frac{Q_k}{n_{1k}D_k}(n_{1k}-1)\right.\right.$$
$$\left.\left.-\frac{Q_k}{n_{jk}D_k}(n_{jk}-1)+\frac{Q_k d_{1k}}{Pn_{1k}D_k}-\sum_{i=1}^{j}\frac{Q_k d_{ik}}{Pn_{ik}D_k}\right)\right)$$

$$=\frac{Q_k{}^2 d_{jk}}{D_k{}^2}\left(\frac{n_{jk}-1}{2n_{jk}}+\frac{n_{1k}-1}{n_{1k}}-\frac{n_{jk}-1}{n_{jk}}+\frac{d_{1k}}{Pn_{1k}}-\sum_{i=1}^{j}\frac{d_{ik}}{Pn_{ik}}\right)$$

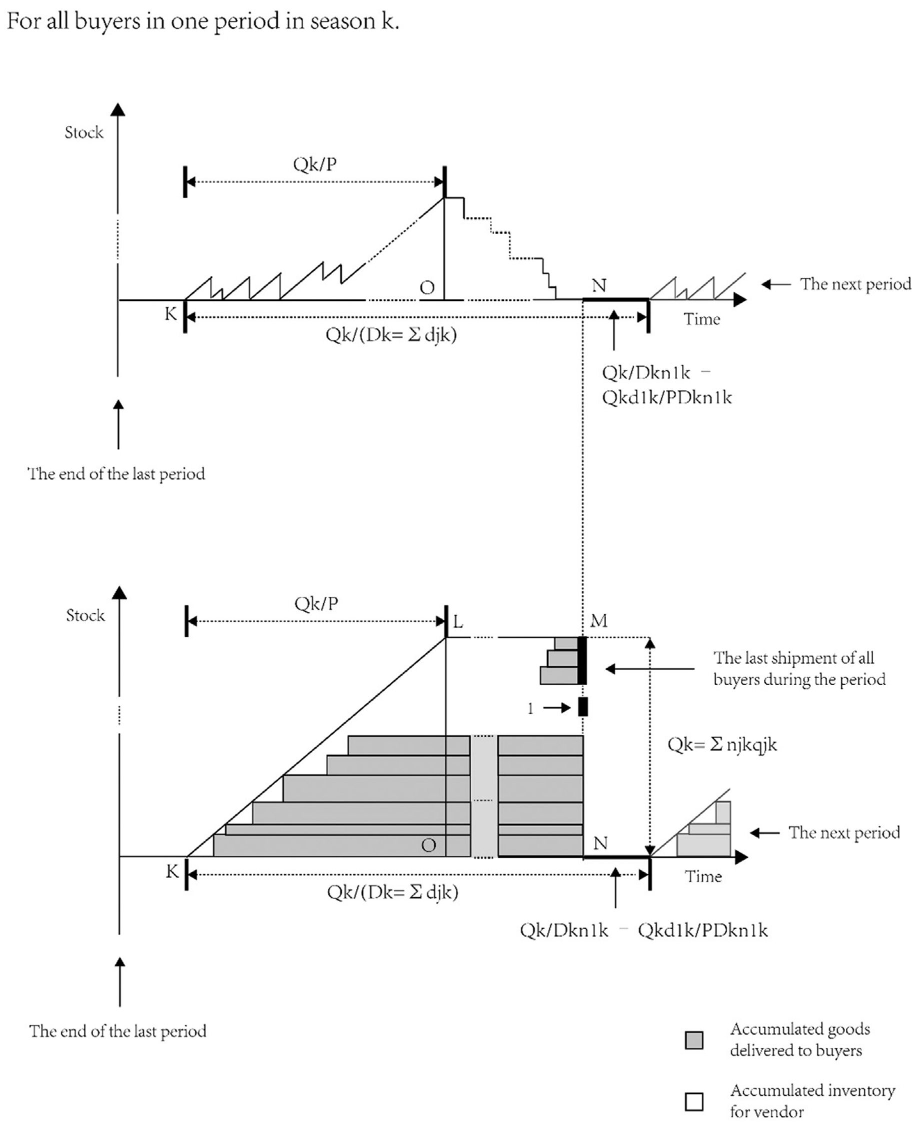


Fig. 3. This schematic diagram shows vendor's inventory level against time. The grey part in the first diagram shows the level of accumulated inventory for vendors while the darker part in the second diagram represents the quantity of goods delivered to buyers.

$$=\frac{Q_k{}^2 d_{jk}}{2D_k{}^2P}\left(\frac{2n_{jk}\left(P(n_{1k}-1)+d_{1k}-n_{1k}\sum_{i=1}^{j}\frac{d_{ik}}{n_{ik}}\right)-Pn_{1k}\left(n_{jk}-1\right)}{n_{1k}n_{jk}}\right)$$

The sum of the area of rectangles for all buyers in one period in season k is given by:

$$=\frac{Q_k^2}{2D_k^2P}\left(\sum_{j=1}^{m} d_{jk}\frac{2n_{jk}\left(P(n_{1k}-1)+d_{1k}-n_{1k}\sum_{i=1}^{j}\frac{d_{ik}}{n_{ik}}\right)-Pn_{1k}(n_{jk}-1)}{n_{1k}n_{jk}}\right) \tag{3}$$

According to Fig. 3, vendor's average inventory is given by:

$$=\frac{Eq.(1)+Eq.(2)-Eq.(3)}{Q_k/D_k}$$

$$=\frac{D_k}{Q_K}\left(\frac{Q_k^2}{2P}+\frac{Q_k^2}{P}I_A-\frac{Q_k^2}{2D_k^2P}I_B\right) \tag{4}$$

$$=\frac{Q_k}{2PD_k}\left(D_k^2+2D_k^2I_A-I_B\right)$$

where

$$I_A=\sum_{j=1}^{m}\frac{Pn_{1k}-d_{jk}n_{1k}-P+d_{1k}}{d_{jk}n_{1k}} \tag{5}$$

$$I_B=\sum_{j=1}^{m} d_{jk}\frac{2n_{jk}\left(P(n_{1k}-1)+d_{1k}-n_{1k}\sum_{i=1}^{j}\frac{d_{ik}}{n_{ik}}\right)-Pn_{1k}(n_{jk}-1)}{n_{1k}n_{jk}} \tag{6}$$

2.3.1. Vendor's total cost

In this model, vendor's total cost in season k includes the setup cost, holding cost, subcontracting cost and disposing cost.

$$\text{Setup cost in season k}=S\frac{\sum_{j=1}^{m}d_{jk}}{Q_k} \tag{7}$$

According to Eq. (4), vendor's holding cost in season k is given by

$$=H_v\frac{Q_k}{2PD_k}\left(D_k^2+2D_k^2I_A-I_B\right)\frac{D_k}{Q_k}$$

$$=\frac{H_v}{2P}\left(D_k^2+2D_k^2I_A-I_B\right) \tag{8}$$

$$\text{Vendor's subcontracting cost in season k}=\rho C_s\alpha_k D_k \tag{9}$$

$$\text{Vendor's disposing cost in season k}=\rho C_d\gamma_k D_k \tag{10}$$

Since vendor's total cost in season k includes the setup, holding cost, subcontracting cost and disposing cost, vendor's total cost in season k is obtained as follows:

$$=\mathrm{Eq.}(7)+\mathrm{Eq.}(8)+\mathrm{Eq.}(9)+\mathrm{Eq.}(10)$$

$$=S\frac{D_k}{Q_k}+\frac{H_v}{2P}\left(D_k^2+2D_k^2I_A-I_B\right)+\rho C_s\alpha_k D_k+\rho C_d\gamma_k D_k \tag{11}$$

Vendor's total cost in one year is

$$=\sum_{k=1}^{4}\left(S\frac{D_k}{Q_k}+\frac{H_v}{2P}\left(D_k^2+2D_k^2I_A-I_B\right)+\rho C_s\alpha_k D_k+\rho C_d\gamma_k D_k\right) \tag{12}$$

2.3.2. Buyers' total cost

Since buyers' total cost in season k includes holding cost, ordering cost, and transportation cost, the process of deduction could be shown as follows.

Every buyer's inventory level against time is illustrated in the following schematic diagram.

Since buyers' safety stock is normally distributed, as demonstrated to Fig. 4,

$$\text{buyer j's average inventory}=\frac{q_j}{2}+F\sigma\sqrt{L} \tag{13}$$

where σ is the confidence level.

$$\text{Buyer j's holding cost}=H_{bj}\left(\frac{q_j}{2}+F\sigma\sqrt{L}\right) \tag{14}$$

Buyers' holding cost in season

$$k=\sum_{j=1}^{m}\left(H_{bj}\left(\frac{q_j}{2}+F\sigma\sqrt{L}\right)\right) \tag{15}$$

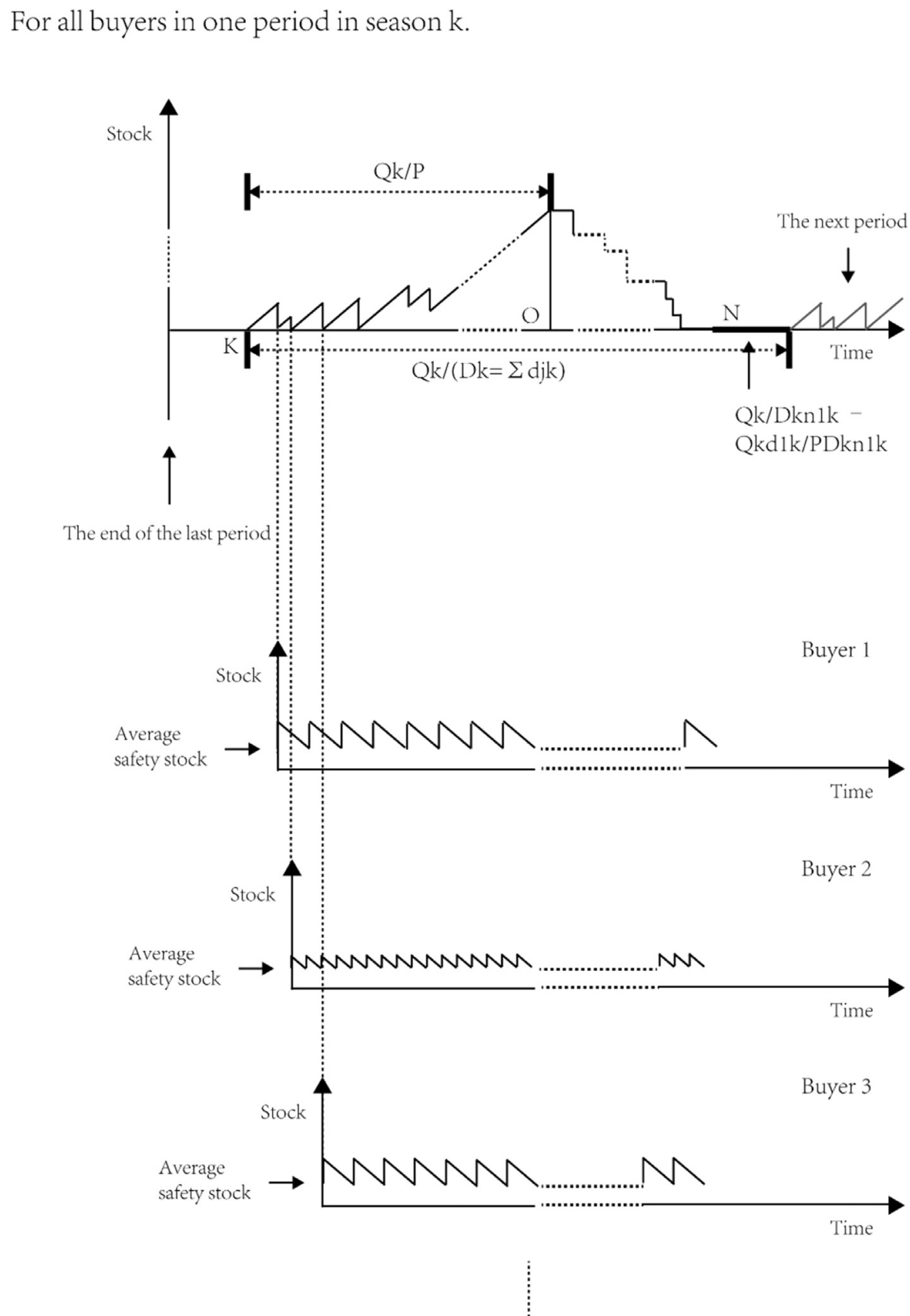


Fig. 4. This schematic diagram shows buyers' inventory level against time respectively.

$$\text{Buyers' ordering cost in season k} = \frac{C_o m D_k}{Q_k} \tag{16}$$

$$\text{Transportation cost in season k} = C_t \frac{D_k}{Q_k} \sum_{j=1}^{m} n_{jk} \tag{17}$$

Since buyers' total cost in season k consists of holding cost, ordering cost, transportation cost, buyers' total cost in season k

$$= \text{Eq.}(15) + \text{Eq.}(16) + \text{Eq.}(17)$$

$$= \sum_{j=1}^{m} H_{bj}\left(\frac{q_j}{2} + F\sigma\sqrt{L}\right) + \frac{C_o m D_k}{Q_k} + C_t \sum_{j=1}^{m} n_{jk} \tag{18}$$

Buyers' total cost in one year

$$= \sum_{k=1}^{4}\left(\sum_{j=1}^{m}\left(H_{bj}\left(\frac{q_j}{2} + F\sigma\sqrt{L}\right)\right) + \frac{C_o m D_k}{Q_k} + C_t \frac{D_k}{Q_k}\sum_{j=1}^{m} n_{jk}\right) \tag{19}$$

Table 1. The incidence of three different conditions in season k.

	Season 1			Season 2		
The condition of the last season	α_k	β_k	γ_k	α_k	β_k	γ_k
The incidence of three different conditions this season ($\alpha_k/\beta_k/\gamma$)	0.05/0.94/0.01	0.01/0.97/0.02	0.01/0.95/0.04	0.04/0.94/0.02	0.02/0.96/0.02	0.02/0.95/0.03
	Season 3			Season 4		
The condition of the last season	α_k	β_k	γ_k	α_k	β_k	γ_k
The incidence of three different conditions this season ($\alpha_k/\beta_k/\gamma$)	0.05/0.93/0.02	0.01/0.98/0.01	0.01/0.94/0.05	0.03/0.95/0.02	0.02/0.97/0.01	0.02/0.93/0.05

α_k: Possibility of the actual demand in season k increasing by.ρ
β_k: Possibility of the actual demand in season k is exactly same as the expected demand.
γ_k: Possibility of the actual demand in season k decreasing by.ρ

Table 2. The constant incidence of three different conditions in season k.

	α_k	β_k	γ_k
Season 1	0.0104	0.9693	0.0203
Season 2	0.0204	0.9594	0.0202
Season 3	0.0104	0.9791	0.0105
Season 4	0.0202	0.9692	0.0106

2.3.3. The total profit

To determine the function of the total profit of the system, it is quite essential to calculate total cost accurately. In this model, the total cost includes vendor's total cost, buyers' total cost and production cost. The vendor's total cost and buyers' total cost have already been shown in previous sections.

$$\text{Production cost} = P_c \sum_{k=1}^{4} D_k \tag{20}$$

Thus, total cost = vendor's total cost in one year + buyers' total cost in one year + production cost

$$= \text{Eq.}(12) + \text{Eq.}(19) + \text{Eq.}(20)$$

$$= \sum_{k=1}^{4} \left(S\frac{D_k}{Q_k} + \frac{H_v}{2P}\left(D_k{}^2 + 2D_k{}^2 I_A - I_B\right) + \rho C_s \alpha_k D_k + \rho C_d \gamma_k D_k + \sum_{j=1}^{m}\left(H_{bj}\left(\frac{q_j}{2} + F\sigma\sqrt{L}\right)\right) + \frac{C_o m D_k}{Q_k} + C_t \sum_{j=1}^{m} n_{jk} + P_c D_k \right) \tag{21}$$

$$\text{Total revenue} = S_p \sum_{k=1}^{4} D_k \tag{22}$$

Since profit is equal to revenue subtract cost, the total profit of the whole system is given by

$$= \text{Eq.}(22) - \text{Eq.}(21)$$

Table 3. Values off all parameters.

Season	1^{st}	2^{nd}	3^{rd}	4^{th}
Demand D_k	mx900 units/buyer	mx1000 units/buyer	mx1200 units/buyer	mx600 units/buyer
Setup cost S	300 \$			
Production P	5000 units/season			
Vendor's holding cost H_v	5 \$/unit/season			
Buyers' holding cost H_{bj}	2 \$/unit/season			
Ordering cost C_o	30 \$/period			
Subcontracting cost C_s	3 \$/unit			
Disposing cost C_d	4 \$/unit			
Transportation cost C_t	40 \$/shipment			
F	20 units			
P	5%			
α_k	0.0104	0.0204	0.0104	0.0202
β_k	0.9693	0.9594	0.9791	0.9692
γ_k	0.0203	0.0202	0.0105	0.0106
σ	1.645 (for 95% confidence)			
L	0.05 season			
Sale price S_p	35 \$/unit			
Production cost P_c	5 \$/unit			

*According to practical experience, this paper sets the confidence interval in the case as 95%. Thus the confidence level σ is 1.645.

Table 4. The optimum solution of the case.

	Season 1	Season 2	Season 3	Season 4
Q_k (unit)	1800	1333.33	1600	1200
n_k (time)	1	1	1	1
Period (time)	2	3	3	2
M	4			
Total profit ($)	273112.7			

$$= S_p \sum_{k=1}^{4} D_k - \sum_{k=1}^{4} \left(S\frac{D_k}{Q_k} + \frac{H_v}{2P}\left(D_k{}^2 + 2D_k{}^2 I_A - I_B\right) + \rho C_s \alpha_k D_k + \rho C_d \gamma_k D_k + \sum_{j=1}^{m} \left(H_{bj} \left(\frac{q_j}{2} + F\sigma\sqrt{L} \right) \right) + \frac{C_o m D_k}{Q_k} + C_t \frac{D_k}{Q_k} \sum_{j=1}^{m} n_{jk} + P_c D_k \right) \tag{23}$$

Where m, n_{jk} and Q_k are decision variables.

2.3. The algorithm

Hans Siajadi [8] and Jhih Ping Lin [16] adopted the integrated inventory model to obtain optimal benefits of the whole system. Based on that, this paper uses Markov Chain to optimize the model.

The algorithm is shown as follows.

Step 1. Use the spirit of Markov Chain to forecast future demand. The derivation is given in appendix 3.

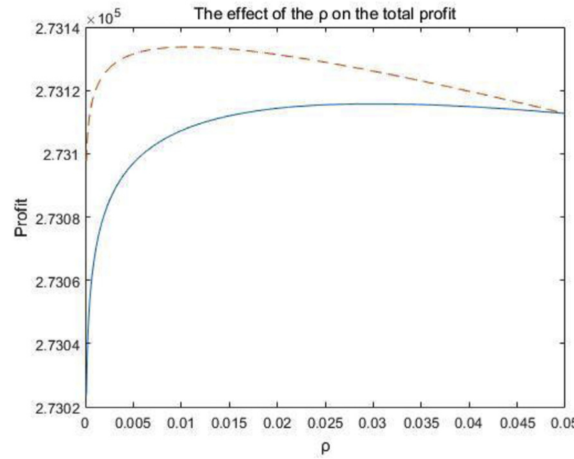
Step 2. Substitute all data into the objective function.

Step 3. Determine the second partial derivatives of the objective function.

Step 4. Determine the optimum solution of the case so that the highest value of the objective functions can be obtained.

According to the actual situation, decision makers can determine the confidence interval to the safety stock by themselves to make the most efficient strategies for their companies. The confidence level σ will change with the confidence internal, a result in the change of the optimal solution of the whole system.

To explain the new model more effectively and efficiently, an example of the solution is given in the next section.



ρ: The parameter representing the percentage by which the demand in season k might increase or decrease.
---: The percentage by which the demand in season k might decrease
__: The percentage by which the demand in season k might increase

Fig. 5. The effect of the ρ on the total profit.

3. Case study

In order to prove the feasibility and practicability of the new model, as well as explaining the usage of the new model much more distinctly, a practical case is developed as follows.

3.1. Numerical illustration

Table 3 shows the values of all parameters in this case.

From Table 3, we substitute the values into Eq. (5), Eq. (6) and Eq. (20):

The total cost

Table 5. The effect of parameters changes on the increase of the total profit.

Parameter	S	H_v	H_b	C_o	C_t
The total profit + 1%	− 91.3%	− 1.8%	− 44.5%	− 227.6%	− 170.7%

$$= \sum_{k=1}^{4}\left(300\frac{D_k}{Q_k}+\frac{1}{2000}\left(D_k{}^2+2D_k{}^2\sum_{j=1}^{m}\frac{5000n_k-d_{jk}n_k-5000+d_{jk}}{d_{jk}n_k}-\sum_{j=1}^{m}d_{jk}\frac{5000(n_k-1)+2d_{jk}-2n_k\sum_{i=1}^{j}\frac{d_{jk}}{n_k}}{n_k}\right)\right.$$
$$\left.+0.15\alpha_kD_k+0.2\gamma_kD_k+\frac{Q_k}{n_k}+2m\,F\sigma\sqrt{L}\;+\frac{30mD_k}{Q_k}\;+40\frac{D_k}{Q_k}\mathrm{m}n_k\right) \tag{24}$$

3.2. The optimum solution

From Eq. (24), the total profit is given by:

$$= \sum_{k=1}^{4}\left(30D_k-\left(300\frac{D_k}{Q_k}+\frac{1}{2000}\left(D_k{}^2+2D_k{}^2\sum_{j=1}^{m}\frac{5000n_k-d_{jk}n_k-5000+d_{jk}}{d_{jk}n_k}\right.\right.\right.$$
$$\left.\left.\left.-\sum_{j=1}^{m}d_{jk}\frac{5000(n_k-1)+2d_{jk}-2n_k\sum_{i=1}^{j}\frac{d_{jk}}{n_k}}{n_k}\right)+0.15\alpha_kD_k+0.2\gamma_kD_k+\frac{Q_k}{n_k}+2m\,F\sigma\sqrt{L}\;+\frac{30mD_k}{Q_k}\;+40\frac{D_k}{Q_k}\mathrm{m}n_k\right)\right) \tag{25}$$

The schematic diagram of the function is given in appendix 4.

The optimal solution of the case is illustrated in Table 4:

From Table 4, we substitute all the values in Eq. (25), the maximal value of the system's profit is \$273112.7.

3.3. Sensitivity analysis

In the previous section, the optimum solution of the practical case has already been found. In the reality, however, in order to improve customers' satisfaction and achieve the goal of sustainable development, decision makers still have to develop a feasible plan to optimize the supply chain system. In this section, a sensitivity analysis is perfomed to find out how the out put into the model could be influenced by changes in the given parameters ρ, S, H_v, H_b, C_o *and* C_t and the feasible way to cut the cost of the supply chain system.

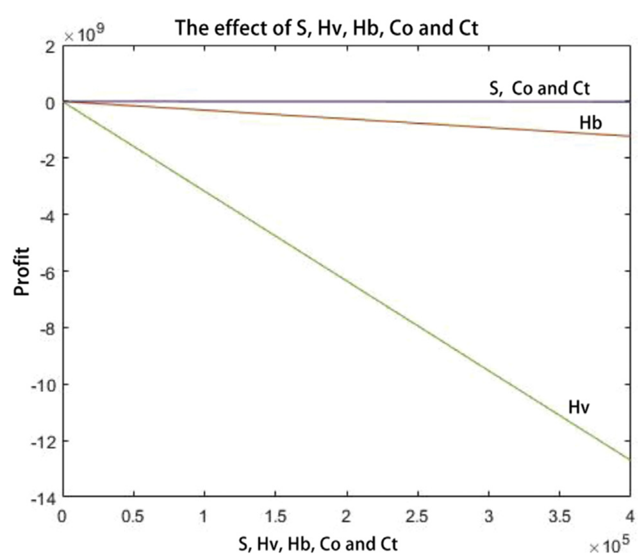


Fig. 6. The relationships between the total profits and parameters S, H_v, H_b, C_o *and* C_t.

1. Suppose that with every 10$ be invested into the system, the percentage by which the demand in season k might increase would decrease by 10%, and with every 5$ be invested into the system, the percentage by which the demand in season k might decrease would decrease 10%. As indicated in Fig. 5, if the percentage by which the demand in season k might increase is the only parameter to be considered, it is quite easy to find out that when ρ decreases to 0.03, the system would have the highest profit of 273115.7$. If the percentage by which the demand in season k might decrease is the only parameter to be considered, the system would have the highest profit of 273133.7$ when ρ decreases to 0.011.
2. Table 5 illustrates that in order to make the total profit increase by 1%, setup cost (S) is supposed to decrease by 91.3%, which means that only as the setup cost approaches 0, could the total profit increase obviously. In other words, it is quite hard for decision makers to gain higher profits by cutting down the setup cost.
3. Table 5 illustrates that in order to make the total profit increase by 1%, vendor's holding cost (H_v) is supposed to decrease by $-$ 1.8%. Since the ratio of the increasing rate of the total profit and the decreasing rate of the vendor's holding cost is close to -1, as the vendor's holding cost falls, the total profit would increase rapidly.
4. Table 5 illustrates that in order to make the total profit increase by 1%, buyers' holding cost (H_b) is supposed to decrease by 44.5%. Which means that as buyers' holding cost falls, the total profit would slowly increase.
5. Table 5 illustrates that in order to make the total profit increase by 1%, ordering cost (C_o) is supposed to decrease by 227.6%. Since the decreasing rate of the ordering cost is even higher than 100%, it is quite impossible for decision makers to gain higher profits by reducing the ordering cost.
6. Table 5 illustrates that in order to make the total profit increase by 1%, transportation cost (C_t) is supposed to decrease by $-$ 170.7%. Just like the decreasing rate of the ordering cost, the decreasing rate of the transportation cost is quite high-even higher than 100%. Thus, it is impossible for decision makers to gain higher profits by reducing the transportation cost.

The relationships between the total profits and five cost parameters (S, H_v, H_b, C_o *and* C_t) is presented clearly in Fig. 6, where the rate of change in the total profits relatives to S, H_v, H_b, C_o *and* C_t is shown respectively.

The sensitivity analysis of the given parameters ρ, S, H_v, H_b, C_o *and* C_t is shown as above. The feasible way of adjusting these parameters to help the system achieve a higher profit will be introduced in the next section.

4. Conclusions

This study makes potential contributions to the literature. First, we discuss the issue of the one-vendor and multiple-buyers integrated inventory model with Markov chain demands to forecast the fluctuation of future demand. Then, we determined the optimal solution for decision makers to reduce the total cost of the whole supply chain system. Finally, we provide the systematic-model that consider the fluctuating prospective demand to support decision makers seeking to gain maximum profits in real-world businesses.

Through case analysis, we can find out that when the numbers of periods occurred in four seasons are 2, 3, 3 and 2 respectively, the quantities of demands per period for all buyers in four seasons are 1800 units, 1333.33 units, 1600 units and 1200 units correspondingly, and the number of buyers is 4, the whole system can reach its optimum profit of $273112.7. However, nowadays, shortages and overages of goods often occur in a supply chain, leading to much higher costs. Thus the significance of optimizing the supply chain system has to be thoroughly understood by decision makers, who should never be satisfied with the existing strategy.

Meanwhile, according to the real situation, it is quite difficult for companies to the total cost by increasing the price of their goods. However, compared to other sections, the cost of supply chain (vendor and buyer's holding cost, ordering cost, subcontracting cost, disposing cost, transportation cost and etc.) is easier to control. Thus, if companies want to enhance their profits, controlling some of those costs and reducing them efficiently is an effective approach for them. According to Fig. 6 and Table 5, parameters ρ, $S, H_v, H_b, C_o and C_t$ all have certain influence on the total profit in the case study. As indicated in Table 5, the obtained savings increased rapidly as the holding costs, especially the vendor's holding cost, fell. In other words, among the parameters, as the vendor's holding cost falls, the total profit has the highest increasing rate. Thus, in practice, decision makers can use the same way to find out the specific cost they must reduce in their

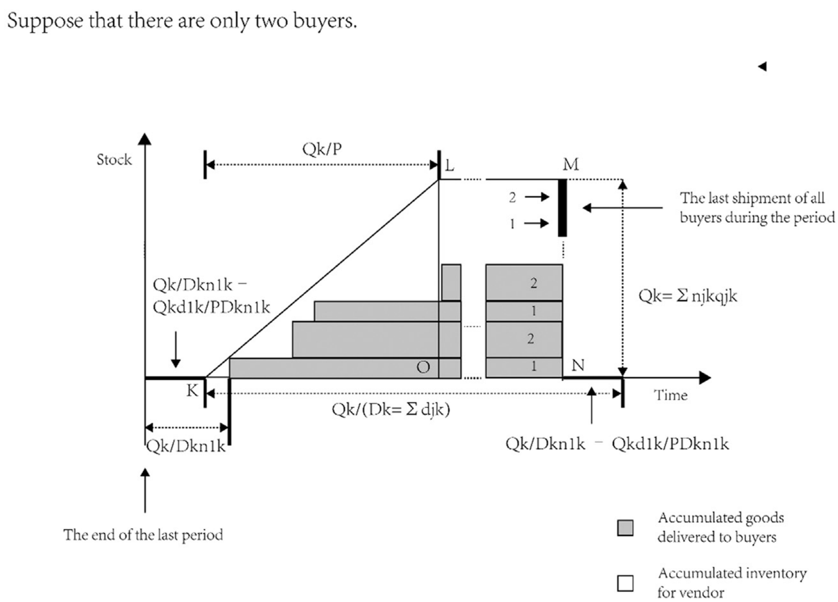


Fig. 7. This schematic diagram shows vendor's inventory level against time when there are only two buyers in the supply chain system. The darker part in the diagram represents the quantity of goods delivered to buyers.

supply chain systems, which can help companies improve profits in the most effective and efficient way.

Obviously, the issue of controlling the demand fluctuation can facilitate companies reduce the total cost. The data analysis revealed that when the percentage by which the demand in season k might increase, decreases to 0.03, the system would have the highest profit of $273133.7. Then, when the percentage, by which the demand in season k might decrease, decreases to 0.011, the system would have the highest profit of $273133.7. Nevertheless, compared to reducing vendor's holding cost, controlling the demand fluctuation is quite inefficient in enhancing companies' profits.

In the preceding discussion suggests increasing the total profit to a specific level in the case mentioned above, the system have to invest much more money to cut down setup cost (S), buyers' holding cost (H_b), ordering cost (C_o) or

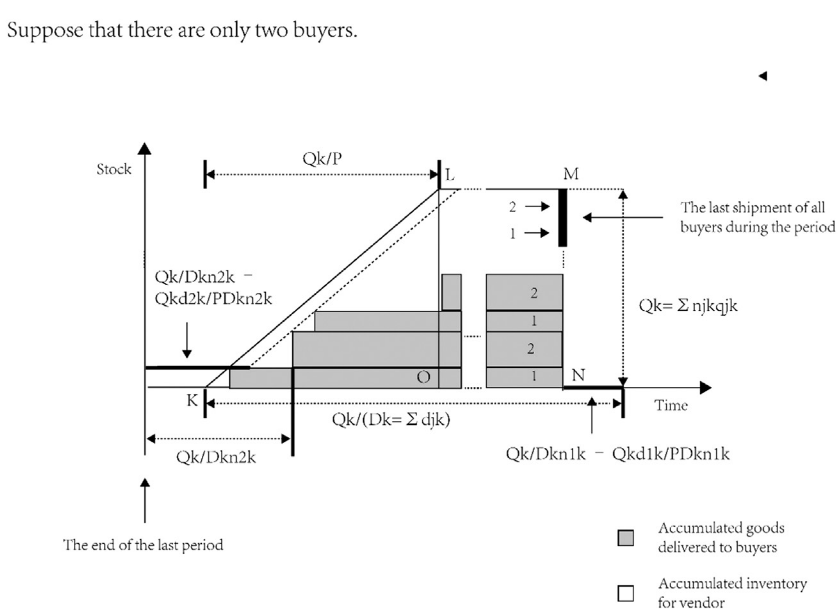


Fig. 8. Suppose the interval between two lots is constant, vendor's inventory level against time could be illustrated as above.

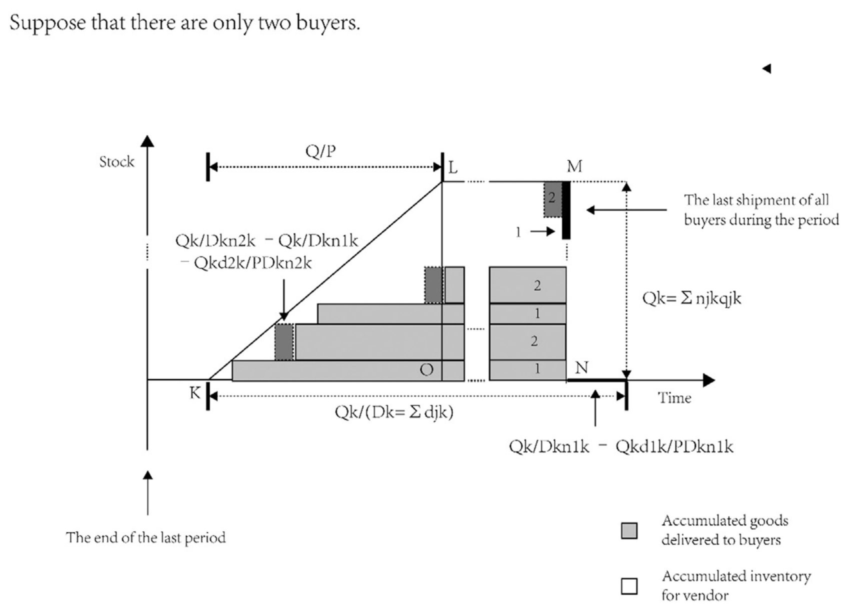


Fig. 9. In practice, the first lot of goods would always be sent to buyers as soon as they are produced, ignoring the interval between two lots.

transportation cost (C_t) than just cutting down vendor's holding cost (H_v). In C_o's case, even if decision makers make a great effort turning C_o into 0, the total profits would not increase by more than 1%. Meanwhile, by reducing H_v by only 2%, decision makers can increase the system's total profit by more than 1%.Thus, it is quite obvious that under a limited budget, cutting down the vendor's holding cost is the most feasible and efficient way to optimize the supply chain system and gain a higher profit for the case. In real cases, the very cost of the supply chain system that should be cut down to help the company earns a greater profit may not be the vendor's holding cost (H_v), but the new proposed model discussed by this paper can help decision makers find out the most effective and efficient approach to cut down the supply chain system's total cost and enhance their companies' total profits.

Finally; in future research, we can add more assumptions, considerable elements and equations to forecast the fluctuation of the future demand. In addition, since some of parameters in this paper are fixed while they are actually quite fluctuant in real cases, we will keep improving our further research in more actual word complicities, such as adding the parameter of incidence of freight damage that may happen in transit to destinations. Furthermore, we hope that our research can be extended more realistic to help companies to reach advantages.

We consider the factor of our proposed model has more thoughtful consideration of real-world practical situations. In this study, based on our sensitivity analysis in all parameters, we provide some results which might be helpful for decision-makers. In addition, future research in this direction should also incorporate more real-world complexities and should attempt to develop more refined solution methodologies. Indeed, it is truly hoped that our experimental results are of great interest both for application and related research.

Appendix 1.

$$\mathrm{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, \ \mathrm{X}_t = \begin{bmatrix} x_t & y_t & z_t \end{bmatrix};$$

The schematic diagram of Markov Chain is shown in Fig. 1.

When

$$k=1, \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x_0 & y_0 & z_0 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & z_1 \end{bmatrix};$$

$$k=2, \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x_1 & y_1 & z_1 \end{bmatrix} = \begin{bmatrix} x_2 & y_2 & z_2 \end{bmatrix};$$

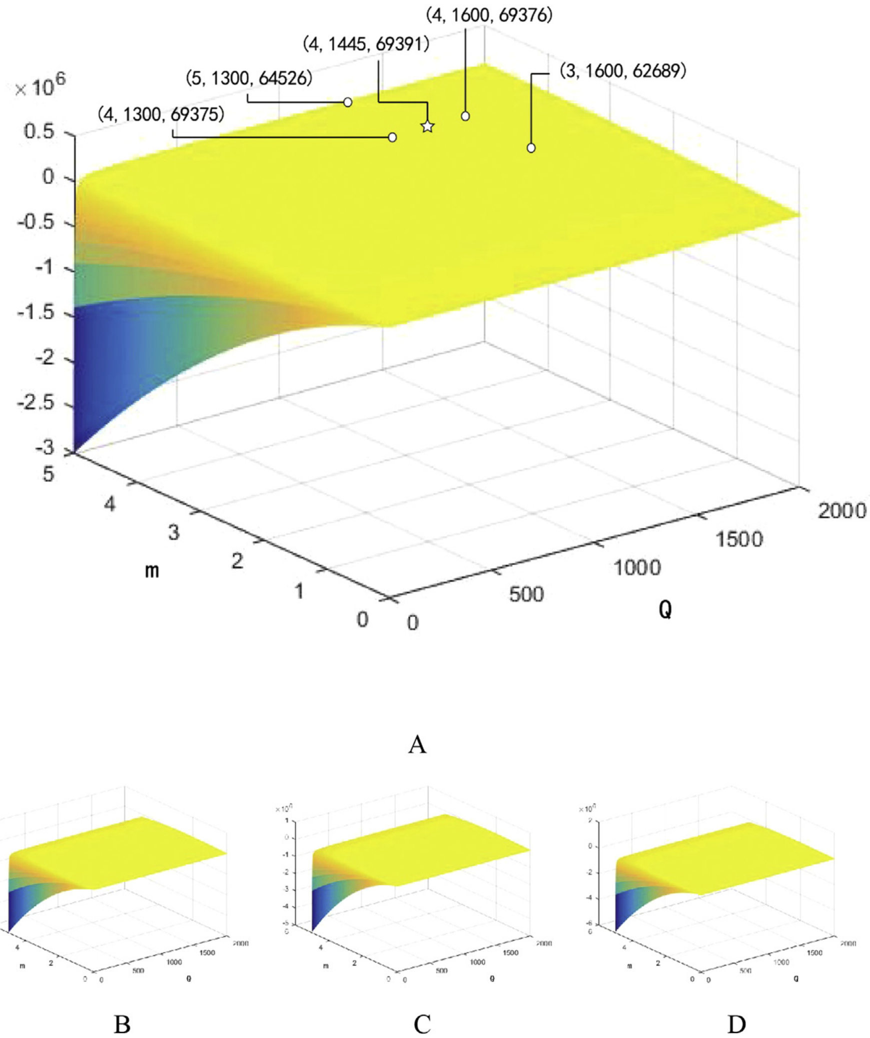


Fig. 10. Diagram A, B, C and D represent the schematic diagram of season 1's general function when n equals 1, 2, 3 and 4 respectively.

$k = \ldots;$

$$k=k,\ \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x_{t-1} & y_{t-1} & z_{t-1} \end{bmatrix} = \begin{bmatrix} x_t & y_t & z_t \end{bmatrix};$$

$$k=k+1,\ \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x_t & y_t & z_t \end{bmatrix} = \begin{bmatrix} x_t & y_t & z_t \end{bmatrix};$$

$k = \ldots$

Appendix 2.

Firstly, suppose that there are only two buyers in the system.

The area of the rectangle (the darker parts in Fig. 7) for each buyer in one period in season k is shown as follows.

For buyer 1, the area of the rectangle 1

$$= \frac{Q_k d_{1k}}{n_{1k} D_k}\left(\frac{Q_k}{n_{1k} D_k}(n_{1k}-1)+\frac{Q_k}{n_{1k} D_k}(n_{1k}-2) + \frac{Q_k}{n_{1k} D_k}(n_{1k}-3)+\ldots+\frac{Q_k}{n_{1k} D_k}\right) \tag{26}$$

For buyer 2, the calculating method of the area of the rectangle 2 is introduced as follows.

However, in practice, the first lot of goods would always be sent to buyers as soon as they are produced, ignoring the interval between two lots. The schematic diagram of this process is illustrated in Fig. 9.

According to Fig. 8 and Fig. 9, for buyer 2, the area of the rectangle 2

$$\begin{bmatrix} 0.05 & 0.94 & 0.01 \\ 0.01 & 0.97 & 0.02 \\ 0.01 & 0.95 & 0.04 \end{bmatrix} [\alpha_1 \quad \beta_1 \quad \gamma_1] = [\alpha_1 \quad \beta_1 \quad \gamma_1]$$

$$= \frac{Q_k d_{2k}}{n_{2k} D_k} \left(\frac{Q_k}{n_{2k} D_k}(n_{2k}-1) + \frac{Q_k}{n_{2k} D_k}(n_{2k}-2) + \frac{Q_k}{n_{2k} D_k}(n_{2k}-3) + \ldots + \frac{Q_k}{n_{2k} D_k} \right) + n_{2k} \left(\frac{Q_k d_{2k}}{n_{2k} D_k} \left(\frac{Q_k}{n_{1k} D_k}(n_{1k}-1) - \frac{Q_k}{n_{2k} D_k}(n_{2k}-1) - \frac{Q_k d_{2k}}{P n_{2k} D_k} \right) \right) \tag{27}$$

The rest may be deduced by analogy.
Thus, for buyer j, the area of the rectangle j is

$$= \frac{Q_k d_{jk}}{n_{jk} D_k} \left(\frac{Q_k}{n_{jk} D_k}(n_{jk}-1) + \frac{Q_k}{n_{jk} D_k}(n_{jk}-2) + \frac{Q_k}{n_{jk} D_k}(n_{jk}-3) + \ldots + \frac{Q_k}{n_{jk} D_k} \right) + n_{jk} \left(\frac{Q_k d_{jk}}{n_{jk} D_k} \left(\frac{Q_k}{n_{1k} D_k}(n_{1k}-1) - \frac{Q_k}{n_{jk} D_k}(n_{jk}-1) + \frac{Q_k d_{1k}}{P n_{1k} D_k} - \sum_{i=1}^{j} \frac{Q_k d_{ik}}{P n_{ik} D_k} \right) \right)$$

$$= \frac{{Q_k}^2 d_{jk}}{{D_k}^2} \left(\frac{n_{jk}-1}{2n_{jk}} + \frac{n_{1k}-1}{n_{1k}} - \frac{n_{jk}-1}{n_{jk}} + \frac{d_{1k}}{P n_{1k}} - \sum_{i=1}^{j} \frac{d_{ik}}{P n_{ik}} \right)$$

$$= \frac{{Q_k}^2 d_{jk}}{2{D_k}^2 P} \left(\frac{2n_{1k}\left(P(n_{1k}-1) + d_{1k} - n_{1k} \sum_{i=1}^{j} \frac{d_{ik}}{n_i k} \right) - P n_{1k}(n_{jk}-1)}{n_{1k} n_{jk}} \right) \tag{28}$$

The sum of the area of rectangles for all buyers in one period in season k is

$$= \frac{{Q_k}^2}{2{D_k}^2 P} \left(\sum_{j=1}^{m} d_{jk} \frac{2n_{1k}\left(P(n_{1k}-1) + d_{1k} - n_{1k} \sum_{i=1}^{j} \frac{d_{ik}}{n_i} \right) - P n_{1k}(n_{jk}-1)}{n_{1k} n_{jk}} \right) \tag{29}$$

Appendix 3.

Step 1 Use the spirit of Markov Chain to find out the constant possibility of demand increases by ρ, remains unchanged and decreases by ρ in season k. All parameters are illustrated in Table 1:

According to Table 1, the constant possibility of demand increases by ρ, remains unchanged and decreases by ρ in season k can be shown as Table 2:

The deviation of Markov chain process is given as follows:

For season 1:

$$\alpha_1 = 0.0104, \beta_1 = 0.9693, \gamma_1 = 0.0203$$

For season 2:

$$\begin{bmatrix} 0.04 & 0.94 & 0.02 \\ 0.02 & 0.96 & 0.02 \\ 0.02 & 0.95 & 0.03 \end{bmatrix} [\alpha_2 \quad \beta_2 \quad \gamma_2] = [\alpha_2 \quad \beta_2 \quad \gamma_2]$$

$$\alpha_2 = 0.0204, \beta_2 = 0.9594, \gamma_2 = 0.0202$$

For season 3:

$$\begin{bmatrix} 0.05 & 0.93 & 0.02 \\ 0.01 & 0.98 & 0.01 \\ 0.01 & 0.94 & 0.05 \end{bmatrix} [\alpha_3 \quad \beta_3 \quad \gamma_3] = [\alpha_3 \quad \beta_3 \quad \gamma_3]$$

$\alpha_3 = 0.0104, \beta_3 = 0.9791, \gamma_3 = 0.0105$

For season 4:

$$\begin{bmatrix} 0.03 & 0.95 & 0.02 \\ 0.02 & 0.97 & 0.01 \\ 0.02 & 0.93 & 0.05 \end{bmatrix} [\alpha_4 \quad \beta_4 \quad \gamma_4] = [\alpha_4 \quad \beta_4 \quad \gamma_4]$$

$\alpha_4 = 0.0202, \beta_4 = 0.9692, \gamma_4 = 0.0106$

Appendix 4.

Since the general function of four seasons are almost the same, this article only uses the schematic diagram of season 1 to show that the function has a local maximum point.

According to Fig. 10, the general function of season 1 proved to have a local maximum value. Take the diagram A for example. Ignoring all constraints ($n_k \geq 1$; $Q_k \geq 0$; $Q_k \leq D_k$; m ≥ 1; n_k, m and $D_k/\ Q_k$ are integers), when n = 1, (4,1445,69,391) is the local maximum point of the function, and its local maximum value is shown as 69,391. All points around it have lower values, although the top of the surface looks like a plane.

Conflict of interest statement

Authors have no conflict of interest to declare.

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