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Yu-Xiang Ma
State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian.

Xiao-Zhou Ma
State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian.

Guo-Hai Dong
State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian., ghdong@dlut.edu.cn

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VARIATIONS OF STATISTICS FOR RANDOM WAVES PROPAGATING OVER A BAR

Yu-Xiang Ma, Xiao-Zhou Ma, and Guo-Hai Dong

Key words: random waves, wave statistical parameters, freak waves, wavelet-based bicoherence.

ABSTRACT

A series of physical experiments were conducted on the variations of statistics (skewness, kurtosis, groupiness) in random waves propagating over a submerged symmetrical bar. Random waves were generated using JONSWAP spectra while varying initial spectral width, wave height and peak frequency. It was found that the initial spectral width has a negligible effect on the variations of these statistical parameters. An abrupt change in wave groupiness is caused by wave breaking. Variations in the skewness and kurtosis mainly depend on the local water depth and wave height and period. Furthermore, the relationship between the skewness and kurtosis in the shoaling region is well predicted by the formula of Mori and Kobayashi (1998), but on the crest of the bar, the formula should be adjusted. Additionally, extreme waves that satisfy the definition of freak waves can be formed in the shoaling region close to the top of the bar. The probability occurrence of the freak waves has a negligible relationship with the initial spectral width, but the appearance of the extreme waves encounters with the increase of groupiness.

I. INTRODUCTION

Wave behavior plays a critical role in determining the design of coastal structures and the description of many coastal processes (Goda, 2010). Hence, a clear understanding of wave characteristics in the nearshore zone, particularly the probability of occurrence of huge waves, is urgent. The sudden appearance of extreme large waves can cause severe damage of structures and human casualties (Dysthe et al., 2008). In deep water, the occurrence of freak waves is closely related to wave statistics, such as kurtosis and skewness, such as kurtosis and skewness (Mori and Janssen, 2006; Tao et al., 2012). However, recent studies (Sergeeva et al., 2011; Trulsen et al., 2012; Zeng and Trulsen, 2012) found that when waves propagate over a slope bottom, the skewness and kurtosis can reach to a maximum value near the shallower side of the slope and extreme waves can be formed. Kashima et al. (2013) pointed out freak waves can also be generated by the shoaling effect. Katsardi et al. (2013) experimentally determined that in shallow water large waves can be formed that cannot be described by any existing wave height distributions. Freak waves have also been identified in coastal regions based on field observations (Chien et al., 2002; Nikolkina and Didenkulova, 2011; Doong et al., 2012; Wang et al., 2014). Hence, research on the predication of the appearance of these extreme waves and the behind mechanisms behind this process is crucial in coastal engineering.

Investigation of waves propagating over a submerged bar is an effective means of assessing the wave behaviors in coastal region (Beji and Battjes, 1993). The purpose of this work is to study changes in statistical parameters (groupiness, kurtosis and skewness) of random waves passing over a submerged bar. The main factors controlling the variations of these parameters will be investigated. Investigation of the relationship between these statistics and the occurrence of extreme waves is the primary object.

The paper is organized as follows. The experimental configuration and measurement techniques are discussed in Section 2. The data analysis methods are introduced briefly in Section 3, and a detailed discussion of the experimental data is presented in Section 4. Lastly, the conclusions of this research are given in Section 5.

II. EXPERIMENTAL SET-UP

The experiment was conducted in a wave flume located in the State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology. This wave flume is 50.0 m long, 3.0 m wide and 1.0 m deep. In this study, the water depth was h = 0.45 m. A submerged isosceles trapezoidal bar with slopes of 1:20 and a 3 m horizontal crest was constructed. Details of the experimental configuration are shown in Fig. 1.

Random wave simulations based on JONSWAP spectra with various significant wave heights H_s, peak frequencies f_p and different peak enhancement factor γ. Details of the wave

Paper submitted 12/05/14; revised 01/24/15; accepted 06/10/15. Author for correspondence: Guo-Hai Dong (e-mail: ghdong@dlut.edu.cn).
State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian.

Table 1. Wave parameters.

Case	f_p(Hz)	H_s(m)	γ
1	1.0	0.049	1.5
2	1.0	0.049	3.3
3	1.0	0.049	5
4	1.0	0.049	7
5	1.0	0.025	3.3
6	1.0	0.068	3.3
7	0.8	0.049	3.3
8	1.2	0.049	3.3
9	1.5	0.049	3.3

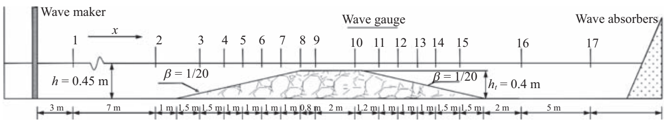

Fig. 1. Schematic drawing of the experimental setup.

Fig. 2. Photo of the experimental setup.

parameters are presented in Table 1. As the width of the flume is 3.0 m, which was longer than the primary wave length used in this study, the cross-tank modulation may be occurred. Hence, to insure two-dimensionality of the wave field, a surface-ground steel wall was installed to longitudinally divide the flume longitudinally into two sections with widths of 0.8 m and 2.2 m (Fig. 2). The narrower section was chosen to be the working section (Fig. 2).

The water surface elevations were recorded using capacitance-type wave gauges at 17 different locations along the flume. The absolute accuracy of each gauge is on the of order ± 1 mm. Prior to use, each wave gauge was examined for soundness and then calibrated. The first gauge was located at $x = 3.0$ m and was used to measure the initial wave parameters and to insure the integrity of the shore-directed waves. The wave heights shown in Table 1 are the measured significant wave heights at the first gauge location ($x = 3.0$ m); the calculations were completed using the zero up-crossing method. To obtain a sufficient number of samples for a statistics analysis, 20 realizations of the random waves with the same imposed spectrum were generated with different sets of random phases. For each experimental cases, 200 s of data were collected; hence, approximately 4200 individual waves were measured at each location.

III. ANALYTICAL METHODS

1. Continuous Wavelet Transform

According to Torrence and Compo (1998), the continuous wavelet transform $WT(a, \tau)$ of a time series $\eta(t)$ is defined as:

$$WT(a,\tau) = \int_{-\infty}^{\infty} \eta(t)\psi_{a,\tau}^{*}(t)\mathrm{d}t, \tag{1}$$

where the asterisk denotes the complex conjugate, and $\psi_{a,\tau}(t)$ represents a family of functions called wavelets that are constructed by translating in time by τ and dilating with scale, a, a mother wavelet function $\psi(t)$. The scale a can be interpreted as the reciprocal of the frequency, $f = 1/a$. The $\psi_{a,\tau}(t)$ expression is defined as:

$$\psi_{a,\tau}(t) = |a|^{-0.5}\psi(\frac{t-\tau}{a}). \tag{2}$$

One of the most extensively used mother wavelets in ocean engineering is the Morlet wavelet; it is a plane wave modulated by a Gaussian envelope and is defined as:

$$\psi(t) = \pi^{-1/4}\exp(-\frac{t^2}{2})\exp(i\omega_0 t), \tag{3}$$

where ω_0 is the peak frequency of the wavelet, which is usually chosen to be 6.0 (Farge, 1993).

Using the wavelet coefficients, $WT(a, \tau)$, one can defined the local wavelet energy density or the scale-averaged wavelet power, $W(\tau)$:

$$W(\tau) = \int_{0}^{\infty} \frac{|WT(a,\tau)|^2}{a}\mathrm{d}a, \tag{4}$$

2. Wave Groupiness Factor

Dong et al. (2008a) proposed a groupiness factor (GF) by $W(\tau)$ to quantify the groupiness of wave trains:

$$GF = \sqrt{\frac{1}{T}\int_0^T \left[W(t) - \overline{W(t)}\right]^2 \mathrm{d}t} \,/\, \overline{W(t)}, \tag{5}$$

where $\overline{W(t)}$ is the mean of $W(t)$ over time, and T is the length in time of the time series.

3. Wavelet Based Bispectrum

The wavelet based bispectrum is defined as (Dong et al., 2008b):

$$B(f_1, f_2) = \int_T \{WT(f_1,\tau)WT(f_2,\tau)WT^*(f,\tau)\}\,d\tau, \quad (6)$$

where $x(t)$ and $y(t)$ are the time series embedded in the respective WT_s, and f_1, f_2 and f must satisfy the frequency sum rule:

$$f = f_1 \pm f_2. \quad (7)$$

The bispectrum measures the amount of phase coupling in the interval T that occurs between wavelet components of frequency f_1 and f_2 of $x(t)$ and wavelet component f of $y(t)$ in a manner such that the sum rule is satisfied. If the imaginary part of B is positive, then energy is transferred from the f_1 and f_2 components to the f component. For negative imaginary part of B, the energy transfer reverses and the f_1 and f_2 components grow at the expense of the f component (Herbers et al., 2000).

4. Skewness and Kurtosis

Wave skewness is parameter that indicates the degree lack of symmetry of a wave profile relative to the horizontal axis. It can be defined through the real part of the bispectrum B (Elgar et al., 1985):

$$Ske = \frac{\sum\sum \mathrm{Re}\{B(f_1, f_2)\}}{E\left(\eta(t)^2\right)^{3/2}}, \quad (8)$$

where E denotes the expected value. Kurtosis is a parameter that quantifies the peakedness of the wave surface elevations and can be defined as (Mori and Janssen, 2006):

$$Kur = \frac{\langle \eta^4 \rangle}{\sigma^4}, \quad (9)$$

where < > brackets indicate an ensemble average, and σ is the standard deviation of the surface elevation.

IV. RESULTS AND DISCUSSION

Segments of the measured surfaces elevations of Case 1 at six different locations along the flume are shown in Fig. 3. The wave profiles are asymmetric about the mean water level with their crests sharper than their troughs as expected, on the upslope side due to shoaling and the effects of nonlinear wave-wave interactions.

Over the crest of the bar, the wave profiles become symmetrical with respect to both the horizontal and the vertical

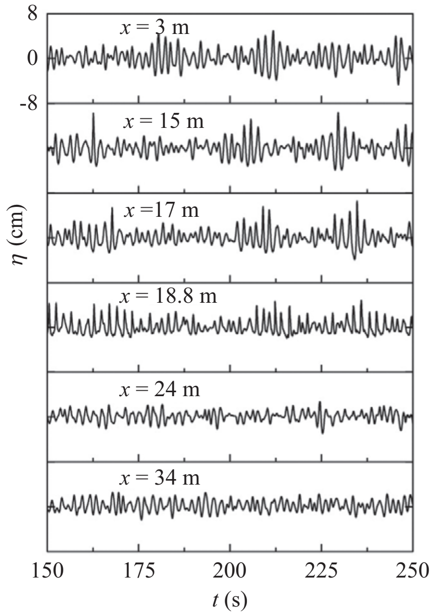

Fig. 3. Segments of the surface elevations for Case 1 measured at different locations.

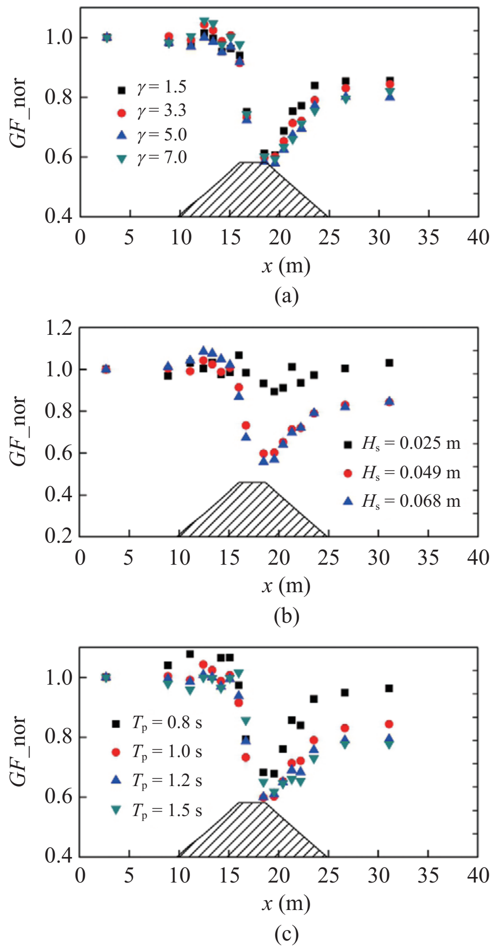

Fig. 4. Variations in the non-dimensional groupiness factor and their dependence on (a) the initial spectral bandwidth, (b) the initial wave height, and (c) the peak period.

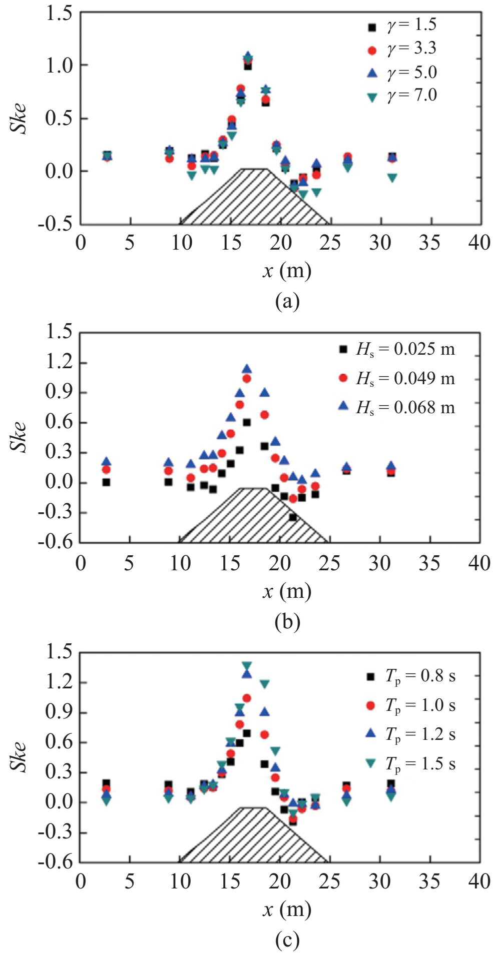

Fig. 5. Variations in skewness and their dependence on (a) the initial spectral bandwidth, (b) the initial wave height, and (c) the peak period.

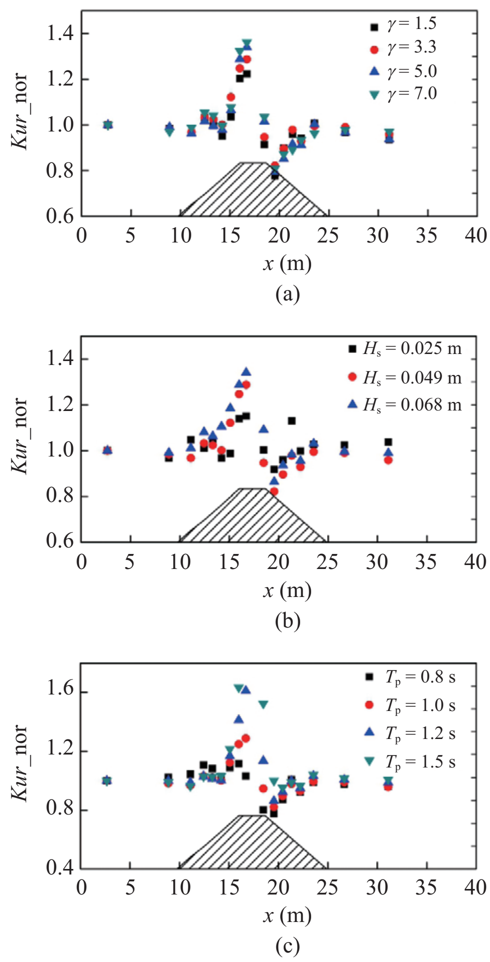

Fig. 6. Variations of non-dimensional kurtosis and their dependence on (a) the initial spectral bandwidth, (b) the initial wave height, and (c) the peak period.

axes. This is due to the deshoaling effect and release of some bound higher harmonics.

Fig. 4 illustrates the variations in the non-dimensional *GF* (based on the data measured at the first location) along the flume for all of the experimental cases. The results show that the groupiness of the wave trains is slightly increasing in the shoaling region, but decreases sharply on the crest of the bar, especially after wave breaking. In the deshoaling region, the groupiness increases with water depth. Ignoring the effects of breaking, it is found that the initial spectral width, initial wave height and peak period have negligible effects on the variations of wave groupiness. This indicats that breaking is the key factor, which is consistent with the previous studies (List, 1991; Dong et al., 2008a).

Variations in skewness for all the wave trains are shown in Fig. 5. The initial spectral width also has a ignorable effect on the evolution of skewness. However, the skewness increased with wave height and peak period, which was expected. Because increased of skewness is primary caused by triad wave-wave interactions (Elgar et al., 1985), the nonlinear interactions are stronger for wave trains with larger wave heights or periods.

It is noted that the variations in kurtosis for the measured surface elevations are dependent on the local water depth (See Fig. 6), and the influence of the spectral bandwidth is negligible, which is quite different from deep-water waves. Increasing wave height or period caused an increase in kurtosis, suggesting that the increases in kurtosis is related to the growth in skewness.

Based on the 3rd-order Stokes wave theory, Mori and Kobayashi (1998) derived a relationship between skewness (S) and kurtosis (K) for nearshore waves:

$$K = 3.0 + \left(\frac{4}{3}S\right)^2 \tag{10}$$

The relationship between the skewness and kurtosis for the measured waves at different regions is shown in Fig. 7. This shows that, the formula of Mori and Kobayashi (1998) fits

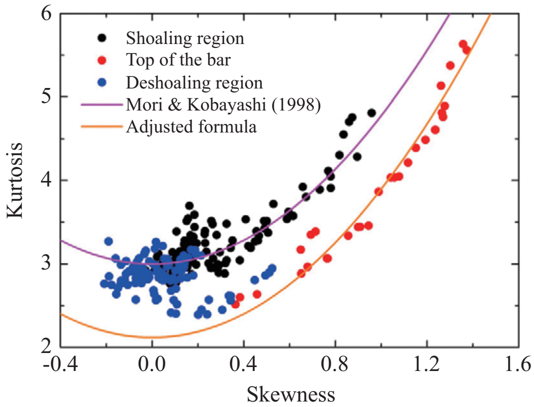

Fig. 7. Relationship between skewness and kurtosis.

Table 2. Probability of occurrence of freak waves.

Case	N_T	N_F(7#)	P_F(7#)
1	4514	5	0.11%
2	4345	21	0.48%
3	4230	9	0.21%
4	4177	10	0.24%
5	4228	7	0.17%
6	4308	5	0.12%
7	5123	12	0.23%
8	3725	10	0.26%
9	3101	21	0.68%

Note: N_T is the measured total wave number, N_F is the identified freak wave number and P_F is the probability of occurrence of freak waves ($\eta_c > 1.25H_s$).

well with the data measured on the upslope. In the deshoaling region, however, the relationship between skewness and kurtosis is near the Gaussian predication: i.e. the skewness is close to 0 and kurtosis is approaximately 3.0. On the crest of the bar, the formula of Eq. (10) clearly overestimates the kurtosis, although the measured data retain the shape of the function. Therefore, Eq. (10) has altered slightly to better reflect the experimental data:

$$K = 2.1 + \left(\frac{4}{3}S\right)^2 \tag{11}$$

The reason for the discrepancy between the theoretical formula and the measured data on the crest of the bar may primarily occur due to the partial release of the bound harmonics due to the sudden change in water depth.

As kurtosis can be used as an indicator to predict the occurrence of freak waves (Mori and Janssen, 2006), the significantly increase in kurtosis near the crest of the bar may suggest that freak waves can be formed in this area. Fig. 3 shows that some distinct large waves were formed near the crest of the bar. For example, for waves measured at $x = 15$ m, the maximum crest at approximately $t = 163$ s reach to 6.4 cm, which is more than the 1.25 times the background significant wave height. This satisfies the definition of freak waves (Dysthe, 2008).

The probability occurrence of freak waves is shown in Table 2. It is found that the maximum probability of occurrence of freak waves occurred at the region near the top of the bar (location No. 7). After wave breaking, the number of freak waves is dramatically reduced. The results shown in Table 2 also indicate that wave trains with the averaged γ values (γ = 3.3) are most likely to induce freak waves. Wave trains with the smaller γ values are the least likely to generate freak waves, indicating that the formation mechanism of freak waves in shallow water is subtly different from that in deep water. In deep-water condition, however, wave trains with a narrower bandwidth (i.e., larger γ) are the most likely to produce freak waves (Mori and Janssen, 2006). Increasing wave height decreases the probability of occurrence of freak waves due to intense wave breaking. However, wave trains with larger periods can increase the probability of occurrence of freak waves. As previously discussed, both skewness and kurtosis reach local maxima at the crest of the bar (See Figs. 5 and 6), although the maximum probability of occurrence of freak waves usually occur at the 7th measurement location. Meanwhile, the wave groupiness reaches its maximum immediately before the 7th location, suggesting that the appearance of extreme waves is related to the groupiness of wave trains in shallow water.

V. CONCLUSION

To investigate the factors influencing variations in the statistical parameters of random waves in shallow water, a series of physical experiments were conducted on waves propagating over a submerged bar. The wave groupiness, skewness and kurtosis are the statistical parameters as the main examined target. It is found that the initial spectral width has a negligible effect on the variations of these statistical parameters. Abrupt changes in wave groupiness were mainly induced by wave breaking. Variations in skewness and kurtosis are controlled by the local water depth, wave height and wave period, with the local water depth being the primary control. More than that, the relationship between the skewness and kurtosis in the shoaling region can be well predicted using the formula of Mori and Kobayashi (1998), but the formula should be adjusted slightly on the crest of the bar. Additionally, some freak waves can be formed in the shoaling region near the top of the bar. The probability of occurrence of the freak waves has a negligible correlation to the initial spectral width, but the appearance of the extreme waves increases with increasing groupiness.

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