Journal of Marine Science and Technology

Volume 23 | Issue 3 Article 16

REGRESSION MODELING OF RESILIENT MODULUS OF UNBOUND AGGREGATES

Zahid Hossain
Civil Engineering Arkansas State University, USA., mhossain@astate.edu

Musharraf Zaman
School of Civil Engineering and Environmental Science, Professor of Petroleum Engineering, and Associate Dean for Research and Graduate Education, College of Engineering, USA.

Curtis Doiron
Former Undergraduate Research Assistant, College of Engineering, USA.

Recommended Citation
Hossain, Zahid; Zaman, Musharraf; and Doiron, Curtis (2015) "REGRESSION MODELING OF RESILIENT MODULUS OF UNBOUND AGGREGATES," *Journal of Marine Science and Technology*: Vol. 23: Iss. 3, Article 16.
DOI: 10.6119/JMST-014-0416-6
Available at: https://jmstt.ntou.edu.tw/journal/vol23/iss3/16

REGRESSION MODELING OF RESILIENT MODULUS OF UNBOUND AGGREGATES

Acknowledgements

The authors would like to express their sincere appreciation to the Oklahoma Department of Transportation for providing financial support for this project. Assistance given by the Oklahoma Transportation Center is greatly appreciated. Thanks are due to Ms. Aissata Cissie, from the University of Oklahoma, for her assistance with the initial data entry operation in this study.

Journal of Marine Science and Technology, Vol. 23, No. 3, pp. 388-398 (2015)
DOI: 10.6119/JMST-014-0416-6

REGRESSION MODELING OF RESILIENT MODULUS OF UNBOUND AGGREGATES

Zahid Hossain[1], Musharraf Zaman[2], and Curtis Doiron[3]

Key words: resilient modulus, aggregates, regression modeling, MEPDG, correlations.

ABSTRACT

Resilient response of granular aggregate is a key input parameter in all three hierarchical levels of the new Mechanistic-Empirical pavement Design Guide (MEPDG), which requires a comprehensive evaluation of resilient modulus (M_r) database(s) for local conditions. To this end, M_r data and other routine properties (gradation, LA Abrasion loss, standard Proctor, and unconfined compressive strength (UCS)) of 105 samples of two different types of aggregate (limestone and sandstone) were analyzed in this study. A total of four stress-based regression models were evaluated using a statistical software package ("SPSS", Version 17), and material constants (k_1, k_2, and k_3) for these aggregates were determined. The octahedral model was found to outperform the other models and is recommended for use in Level 1 analysis. Correlation equations for material constants, required for Level 2, were developed using routine aggregate properties, and the universal model was found to be the "best fit" model with a R^2 value of 0.57. Default (Level 3) M_r values for these aggregates were also estimated using the average material constants. The estimated M_r values obtained from different models were in agreement with each other, and the variations of M_r values were within 4%. However, all of these models would result in conservative designs compared to the MEPDG recommended typical values. The findings of this study are expected to be helpful in the implementation of the MEPDG in Oklahoma and elsewhere.

I. INTRODUCTION

The new Mechanistic-Empirical Pavement Design Guide (MEPDG), developed under the National Cooperative Highway Research Program (NCHRP) project 1-37A, is less empirical than the widely used 1993 American Association of State Highway and Transportation Officials (AASHTO) Design Guide (Kim et al., 2009). The MEPDG uses a hierarchical approach to achieve a target design reliability of material properties for the analysis and design of pavement structures (NCHRP, 2004; Papagiannakis and Masad, 2008). Among others, resilient modulus (M_r) of granular base is an important input parameter in all three hierarchical levels of the MEPDG (NCHRP 2004). Level 1 requires material constants (k_1, k_2, and k_3) from actual M_r test data, and it provides the highest level of design reliability. Level 2 uses correlations to determine M_r from other aggregate properties, and gives an intermediate level of reliability. Level 3, the lowest reliability level, uses default values based on soil classifications (NCHRP, 2004).

In the past, researchers around the globe conducted significant amounts of studies to evaluate resilient properties of subgrade soils, but there had been a limited number of studies that focused on evaluating aggregates for the MEPDG applications. A previous study by Zaman et al. (1998) examined M_r of limestone and sandstone aggregates in Oklahoma. These researchers used the *bulk stress* ($k\sim\theta$) model to estimate M_r of aggregates. These researchers also established correlation equations of M_r with unconfined compressive strength (UCS) and elastic modulus (EM) for selective stress levels. Since the M_r value is stress-dependant, it was recommended that stress values be included in the correlations (Zaman et al., 1998). Also, the establishment of correlations of material constants (k_1, k_2, and k_3) with routine aggregate test data was out of scope of these studies.

Richardson et al. (2009) tested five unbound granular base materials in Missouri. The base materials were tested at two different gradations (as-delivered and with an increased amount of percent passing No. 200 Sieve (P200) than the former). These researchers used the MEPDG-recommended octahedral model to determine the material constants. It was reported that all of the individual samples' coefficient of correlation (R^2) values were greater than 0.90, thus satisfying the recommendations of the MEPDG. These researchers also observed very good repeatability of test results among replicate samples. It was recommended that the reported material

Paper submitted 11/06/09; revised 02/26/10; accepted 04/16/10. Author for correspondence: Zahid Hossain (e-mail: mhossain@astate.edu).
[1] *Civil Engineering Arkansas State University, USA.*
[2] *School of Civil Engineering and Environmental Science, Professor of Petroleum Engineering, and Associate Dean for Research and Graduate Education, College of Engineering, USA.*
[3] *Former Undergraduate Research Assistant, College of Engineering, USA.*

Table 1. Grain size distributions of tested aggregates.

US Standard Sieve Size or No	Sieve Opening (mm)	% Passing of Limestone at Meridian	% Passing of Limestone at Richard Spurs	% Passing of Sandstone at Sawyer	ODOT limit % Passing for Coarser	ODOT limit % Passing for Finer	ODOT limit % Passing for Median
1-1/2 in	38.1	98.2	100	100	100	100	100
1-1/4 in	31.75	91.8	98.1	95.0	85	100	90
1.0 in	25.4	81.5	91.2	84.0	60	100	80
0.75 in	19.0	71.4	79.5	70.0	40	100	70
0.5 in	12.7	58.8	63.8	54.8	35	85	60
0.375 in	9.5	51.7	59.3	47.8	30	75	52.5
No. 4	4.75	38.6	48.6	34.5	25	60	42.5
No. 40	0.425	14.2	14.8	20.3	8	26	17
No. 200	0.075	6.3	5.6	4.8	4	12	8

constants be used as inputs for Level 1 analysis and design for tested aggregates.

Yohannes et al. (2009) characterized several unbound granular materials from Minnesota for pavement applications, including the MEPDG, by conducting M_r tests. These researchers also used a 3-D discrete element method (DEM)-based model, capable of accounting for aggregate shape, coefficient of friction, gradation, stiffness, and other properties, to estimate M_r. The simulation results were in a good agreement with the experimental observations.

A recent study by Xiao et al. (2011) established correlations between M_r and aggregate physical properties by utilizing Minnesota aggregate property database containing M_r data of 376 aggregate specimens (four types of aggregates). It was reported that some basic aggregate parameters, namely P200, moisture content (MC) and dry density (MDD), were somewhat correlated with M_r. However, aggregate particle surface properties (flat and elongated ratio, angularity index, and surface texture index) were also found to be highly correlated predictor variables. Without surface properties of the aggregates, the R^2 values of correlation equations of k_1, k_2, and k_3 were found to be only 0.14, 0.32, and 0.39, respectively. It was reported that the R^2 values of the correlations increased significantly when aggregate surface properties were used with aggregate physical properties. Thus, the R^2 values of correlation equations of k_1, k_2, and k_3 were reported as 0.58, 0.50, and 0.53, respectively.

The default values and correlations for M_r provided by the MEPDG are based on a limited number of tests and climatic conditions. Previous studies (e.g., Tutumluer and Pan, 2008) reported that aggregate morphology, mineralogy, and textual characteristics play a significant role on the M_r value of unbound aggregate materials. Since the morphological, mineralogical, and textural characteristics of Oklahoma aggregates are different from those in the literature, those default values and correlations may not be applicable for Oklahoma.

As noted earlier, several state departments of transportation (DOTs) have already created or are in the process of creating M_r databases for local aggregates. These agencies have found their M_r databases to be useful tools for improving pavement designs and analyses using the MEPDG (Titi et al., 2006; Wang, 2009). The Oklahoma Department of Transportation (ODOT) is actively working toward implementing the MEPDG for flexible pavements (Hossain et al., 2011). A successful implementation of the MEPDG will require a comprehensive database and its assessment through local calibrations. The present study is expected to provide useful data and correlations that can be used to calibrate the MEPDG according to Oklahoma's conditions and materials. The findings of this study are also expected to be useful for other researchers to gain a better understanding of how to evaluate and incorporate materials from other regions into the MEPDG.

II. TEST MATERIALS AND METHODOLOGY

M_r test (AASHTO T 307) data for 105 samples of two commonly used aggregates (limestone and sandstone) in Oklahoma were analyzed in this study. The limestone aggregates were obtained from quarries at Meridian in Marshal County, and Richard Spurs (RS) in Comanche County. Traditionally, RS aggregate is a good quality aggregate, whose M_r values are significantly higher (at least two times) than those of Meridian aggregates (Zaman et al., 1998). For example, at a bulk stress of 100 psi (690 kPa), the M_r value of the RS aggregate is 53.4 psi (368.2 kPa), while that of the Meridian aggregate is 19.16 psi (132.1 kPa). Sandstone aggregate in this study was obtained from a quarry at Sawyer in Choctaw County. Other test data included in the database were sieve analysis (AASHTO T 11 and AASHTO T 27), LA Abrasion loss (AASHTO T 96), standard Proctor (AASHTO T 180), and unconfined compressive strength (AASHTO T 208). The gradation (average of three replicates) data for these aggregates, along with the ODOT requirements, are shown in Table 1. The index properties (AASHTO T 89 and AASHTO T 90) were determined from particles finer than 425 μm (passing Sieve No. 40). The specimens for M_r tests were compacted at optimum moisture content (OMC) and at or above 95% MDD. The MDD values were obtained from the moisture-density tests (AASHTO T 180). According to the AASHTO M 145 specifications, these aggregates were classified as A-1-a.

Table 2. Basic statistical parameters of resilient modulus of tested aggregates.

Development Dataset								
Seq. No.	σ_3 (kPa)	σ_d (kPa)	Mean (MPa)	SD (MPa)	Min (MPa)	Max (MPa)	Skew (MPa)	Kurt (MPa)
1	41.4	13.8	74.50	35.43	15.59	176.09	1.03	0.66
2	41.4	27.6	93.88	33.03	39.74	179.97	0.63	-0.45
3	41.4	41.4	104.30	40.86	33.95	275.6	1.32	2.60
4	41.4	55.2	106.40	49.74	27.67	305.09	1.33	2.73
5	41.4	68.9	118.78	44.48	58.77	257.55	1.08	0.82
6	27.6	13.8	139.57	47.64	69.38	277.05	0.80	0.13
7	27.6	27.6	153.35	63.32	39.68	356.21	0.86	0.37
8	27.6	41.4	170.53	64.40	76.55	373.71	1.09	1.07
9	27.6	55.2	177.76	62.28	88.67	359.45	0.73	-0.05
10	27.6	68.9	163.11	66.97	43.06	397.48	0.88	0.85
11	13.8	13.8	187.29	71.68	56.17	400.58	0.89	0.51
12	13.8	27.6	209.20	74.25	91.71	418.02	0.78	0.13
13	13.8	41.4	202.95	77.92	59.25	462.53	1.06	0.95
14	13.8	55.2	219.39	80.87	72.55	462.87	0.93	0.56
15	13.8	68.9	249.63	83.92	80.11	478.37	0.59	-0.14
Evaluation Dataset								
Seq. No.	σ_3 (kPa)	σ_d (kPa)	Mean (MPa)	SD (MPa)	Min (MPa)	Max (MPa)	Skew (MPa)	Kurt (MPa)
1	41.4	13.8	53.12	7.31	40.72	62.91	-0.43	-0.48
2	41.4	27.6	102.08	54.52	51.47	189.47	0.96	-0.54
3	41.4	41.4	102.10	38.01	61.32	161.18	0.75	-0.74
4	41.4	55.2	109.36	52.73	42.71	174.16	0.27	-1.84
5	41.4	68.9	121.41	47.72	62.17	184.58	0.28	-1.46
6	27.6	13.8	72.76	6.19	60.36	81.3	-0.76	1.01
7	27.6	27.6	157.69	81.21	57.06	277.04	0.62	-1.14
8	27.6	41.4	169.07	58.45	78.59	250.13	0.03	-0.59
9	27.6	55.2	177.18	55.47	77.56	241.93	-0.68	-0.43
10	27.6	68.9	160.12	62.70	41.12	226.39	-0.62	-0.02
11	13.8	13.8	99.92	23.52	64.97	145.03	0.47	0.79
12	13.8	27.6	207.31	68.55	74.8	275.59	-0.81	0.13
13	13.8	41.4	206.76	62.39	97.15	289.87	-0.14	-0.19
14	13.8	55.2	218.14	88.88	67.97	348.31	0.15	0.16
15	13.8	68.9	274.77	66.16	184.17	353.85	0.08	-1.84

Note: σ_3 = confining pressure, σ_d = axial stress, Min = Minimum, Max = Maximum, SD = Standard deviation, Skew = skewness, and Kurt = kurtosis.

Since the estimated coefficients of a regression analysis can be profoundly influenced by outliers (Norusis, 2002), these observations were identified and discarded from further analyses. To this end, M_r data located outside the range of ±1.5 standard deviations from the average M_r value of a given sequence for each aggregate type were treated as outliers. A value of 1.5 standard deviation was selected for locating outliers after plotting and careful examining histograms of M_r values. In the case of one standard deviation, a significant portion (about 15%) of M_r values would get discarded. On the other hand, a value of two standard deviations did not improve the normal probability plots M_r data. Thus, a value of 1.5 was chosen as the standard deviation and about 5% of the M_r data was found to be outliers. The M_r data were then randomly divided into two sets. The development dataset was used to develop the models, while the evaluation dataset was used to validate the models. Thus, the evaluation dataset was independent of those used in the development dataset. Using the complete dataset to evaluate the models would be an "autocorrelation," which may improve the fit of the equation to the data set. However, a good fit of a model to the data is not the only goal of regression analysis. Rather, it is important to know how the model validates to a new dataset. Having independent datasets will clarify how the model validates to a new dataset without being influenced by the "autocorrelation" factors. In the current study, the development dataset consisted of 96 samples, whereas the evaluation dataset included 9 samples.

It is well-known that a normally distributed dataset is desired for developing good statistical models and correlations. Hence, basic statistical parameters (minimum, maximum, mean, and standard deviation) along with two other factors

Table 3. Basic statistical parameters for tested aggregate samples.

Parameter	Minimum	Maximum	Mean	Standard Deviation	Skewness	Kurtosis
SPGR	2.472	2.701	2.635	0.008	-0.405	-1.418
LA (%)	24	38	27	3.76	1.48	1.80
P_4	34.5	48.6	42.0	6.7	-0.08	-1.94
P_{40}	14.2	20.3	16.8	2.7	0.48	-1.78
P_{200}	4.8	6.3	5.4	0.5	0.20	-0.98
LL	13	22	16	3.2	0.38	-1.27
PI	3	9	4	1.7	2.09	3.12
UCS, psi (kPa)	17.5 (120.6)	45.9 (316.6)	34.4 (237.3)	8.3 (57.6)	-0.44	-1.00
MDD, pcf (kN/m^3)	133.0 (20.9)	149.0 (23.4)	142.5 (22.4)	5.7 (0.9)	-0.16	-1.40
OMC (%)	4.6	7.5	5.5	0.9	0.67	-0.56

Note: SPGR = Specific gravity, P_4 = % passing #4 sieve, P_{40} = % passing #40 sieve, P_{200} = % passing #200 sieve, LL = liquid limit, PI = plasticity index, UCS = unconfined compressive strength, MDD = maximum dry density, and OMC= optimum moisture content.

(skewness and kurtosis) were selected to determine distributions of the routine aggregate test data. Skewness is a measure of the asymmetry of the probability distribution of a random dataset. A positive skew indicates that the tail on the right side of the probability density function is longer than the left side; a negative skew indicates that the tail on the left side is longer than the right side. A skew value of zero indicates that the values are relatively evenly distributed on both sides of the mean. Kurtosis is a measure of the "peakedness" of the probability distribution of a random dataset. A positive kurtosis distribution has a sharper peak and longer and fatter tails, while a negative kurtosis distribution has a more rounded peak and shorter, thinner tails.

The statistical parameters of the M_r test data of each of 15 sequences for the development and evaluation datasets are presented in Table 2. The average M_r values in the development and evaluation datasets range from 74.50 MPa (Sequence #1) to 149.63 MPa (Sequence #15), and from 53.12 MPa to 274.77 MPa, respectively. The skewness and kurtosis values of the M_r data in the development were found to be in the range from 0.59 to 1.33, and from -0.45 to 2.73, respectively. Thus, the distribution of the M_r data in the development dataset can be treated as normally distributed. In regard to the distribution, a similar trend is seen in the case of M_r data in the evaluation dataset.

The statistical parameters of routine test properties of aggregates of the entire dataset are presented in Table 3. The mean value of specific gravity (SPGR) was found to be 2.63, with a standard deviation of 0.08. The skewness and the kurtosis values were found to be -0.45 and -1.48, respectively. The mean value of the LA Abrasion loss was found to be 26.93, with a standard deviation of 3.76. The skewness and the kurtosis values for LA Abrasion loss in the dataset were found to be 1.48 and 1.80, respectively. So, the LA Abrasion loss values were considered normally distributed. Based on the statistical parameters, test data of other aggregate properties (P_4, P_{40}, P_{200}, LA Abrasion (LA), liquid limit (LL), plasticity index (PI), UCS, MDD, and OMC) were also treated as normally distributed.

III. REGRESSION MODELING

1. Selection of Models

Among the several stress-based models available in the literature for characterizing resilient response of unbound granular base materials, four commonly used and relatively simple models were considered in this study. The bulk stress ($k\sim\theta$) model (Model 1 as shown in Eq. (1)), referenced by Hicks and Monismith (Hicks and Monismith, 1971), was used in related previous studies (e.g., Zaman et al., 1998) to estimate M_r of limestone and sandstone aggregates. The $k\sim\theta$ model was recommended in the 1986 AASHTO Guide (Thompson et al., 1998). The major limitations of the $k\sim\theta$ model are that it neglects the important effects of shear stress on M_r (Thompson et al., 1998). Also, the $k\sim\theta$ model can only represent a very limited range of stress paths, and is thus expected to give erroneous results (Brown and Pappin, 1991). The basic two-parameter [$k\sim f(\sigma_d, \sigma_3)$] model, also known as UT-Austin model (Model 2 as shown in Eq. (2)), introduced by Pezo (1993) and referenced by Andrei et al. (2004), is based on confining and deviatoric stresses and is being used by several state agencies. The $k\sim f(\sigma_d, \sigma_3)$ model was an outcome of regression analyses of expressing the axial strain in terms of applied confining and deviatoric stresses from laboratory tests. Von Quintus and Killingsworth (1997) recommended the universal (bulk stress and deviatoric stress-based [$k\sim f(\theta, \sigma_3)$] model (Model 3, as shown in Eq. (3)) for estimating M_r values required in the 1993 AASHTO Guide. It was introduced by Uzan (1985). The $k\sim f(\theta, \sigma_3)$ model, which includes the shear stress effects, is an improvement of the well-known $k\sim\theta$ model. An extended stress-based model, the octahedral ($k\sim\tau_{\text{oct}}$) model (Model 4 as presented in Eq. (4)), has been recommended by the MEPDG (NCHRP, 2004). The $k\sim\tau_{\text{oct}}$ model, proposed by Witczak and Uzan (1998), is a modification of the universal model by replacing the deviatoric stress with the octahedral shear stress term along with a numerical factor.

$$M_r = k_1 P_a \left(\frac{\theta}{P_a}\right)^{k_2} \tag{1}$$

Table 4. Material constants for selected regression models ($R^2 > 0.9$).

Model No. and Name	Aggregate Type	Statistical Parameter	k_1	k_2	k_3
Model 1 (Bulk stress model)	Limestone N = 29	Maximum	3807.16	0.71	N/A
		Minimum	483.76	0.312	N/A
		Standard Deviation	591.03	0.084	N/A
		Average	1102.97	0.530	N/A
	Sandstone N = 31	Maximum	1065.32	0.702	N/A
		Minimum	429.74	0.407	N/A
		Standard Deviation	178.83	0.066	N/A
		Average	685.44	0.550	N/A
Model 2 (UT-Austin model)	Limestone N = 43	Maximum	5884.22	0.532	0.718
		Minimum	918.731	-0.043	0.029
		Standard Deviation	880.31	0.153	0.183
		Average	2123.81	0.292	0.232
	Sandstone N = 35	Maximum	2042.30	0.528	0.447
		Minimum	645.64	-0.100	0.056
		Standard Deviation	345.32	0.131	0.084
		Average	1468.19	0.357	0.174
Model 3 (Universal model)	Limestone N = 41	Maximum	3947.63	0.771	0.739
		Minimum	501.35	-0.062	-0.172
		Standard Deviation	559.43	0.229	0.249
		Average	1160.04	0.427	0.101
	Sandstone N = 35	Maximum	1348.6	0.774	0.500
		Minimum	404.93	-0.154	-0.155
		Standard Deviation	251.05	0.192	0.137
		Average	721.93	0.522	0.0107
Model 4 (Octahedral model)	Limestone N = 36	Maximum	3894.7	0.633	4.182
		Minimum	126.56	-1.654	-0.38
		Standard Deviation	612.11	0.487	0.919
		Average	860.56	-0.0217	0.939
	Sandstone N = 31	Maximum	1110.10	1.005	1.905
		Minimum	277.48	-0.513	-0.875
		Standard Deviation	255.23	0.386	0.702
		Average	637.49	0.298	0.433

Note: N = No. of samples.

$$M_r = k_1 P_a \left(\frac{\sigma_3}{P_a}\right)^{k_2} \left(\frac{\sigma_d}{P_a}\right)^{k_3} \tag{2}$$

$$M_r = k_1 P_a \left(\frac{\theta}{P_a}\right)^{k_2} \left(\frac{\sigma_d}{P_a}\right)^{k_3} \tag{3}$$

$$M_r = k_1 P_a \left(\frac{\theta}{P_a}\right)^{k_2} \left(\frac{\tau_{oct}}{P_a}+1\right)^{k_3} \tag{4}$$

where,

σ_1, σ_2, and σ_3 = principal stresses, where $\sigma_2 = \sigma_3$,

σ_d = cyclic (deviatoric) stress = σ_1-σ_3,

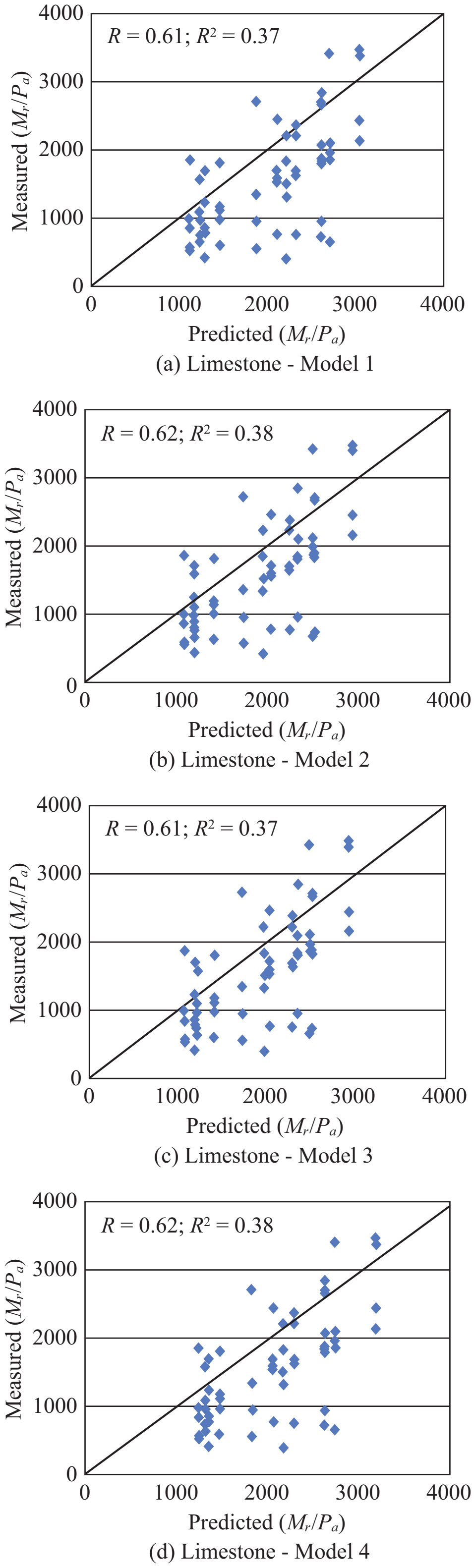
θ = bulk stress = $\sigma_1 + \sigma_2 + \sigma_3 = 3\sigma_3 + \sigma_d$,

τoct = octahedral shear stress

$$= \sqrt{\frac{1}{3}(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2} = \frac{\sqrt{2}}{3}\sigma_d, \text{ and}$$

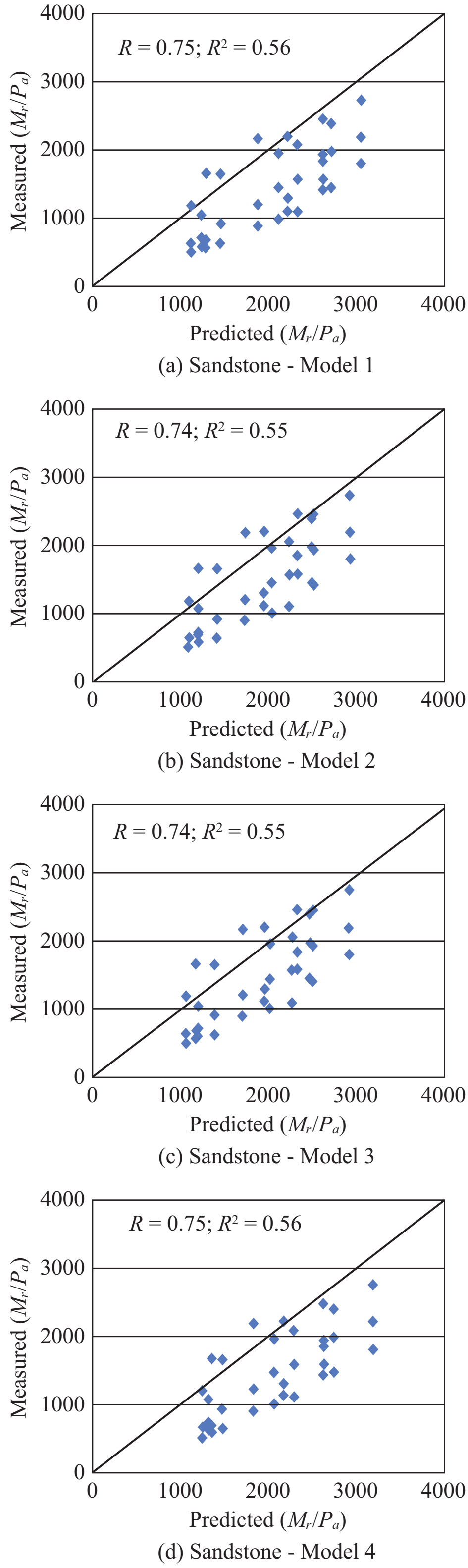
P_a = atmospheric pressure (14.7 psi [101 kPa]).

Using the aforementioned four models nonlinear regressions were performed, thus giving four sets of regression constants (k_1, k_2, and k_3) for each sample. Average regression constants (material constants) along with other important statistical parameters (maximum, minimum, and standard deviation) for the aggregate samples, satisfying the MEPDG recommended R^2 criterion of greater than 0.9, are presented in Table 4. As shown in Table 4, the lowest number of samples (57%; 60 out of 105) satisfied the MEPDG specified R^2 (≥ 0.9) requirement in the case of Model 1. The largest number of samples (about 74%; 78 out of 105) fulfilled the aforementioned R^2 criterion in the case of Model 2, followed by Model 3 (72%; 76 out of



Note: F-values with corresponding probabilities: Model 1: 1.75 (p = 0.01); Model 2: 2.04 (p = 0.002); Model 3: 2.05 (p = 0.002); Model 4: 1.82 (p = 0.006)

Fig. 1. **Predicted versus measured M_r/P_a values from regression analysis for limestone aggregate: (a) Model 1, (b) Model 2, (c) Model 3, and (d) Model 4.**



Note: F-values with corresponding probabilities: Model 1: 1.00 (p = 0.499); Model 2: 1.16 (p = 0.335); Model 3: 1.16 (p = 0.330); and Model 4: 1.03 (p = 0.467).

Fig. 2. **Predicted versus measured M_r/P_a values from regression analysis for sandstone aggregate: (a) Model 1, (b) Model 2, (c) Model 3, and (d) Model 4.**

105), which was followed by Model 4 (64%; 67 out of 105). The reason for relatively low percentages (up to 74%) of samples satisfying these models could be related to "poor" quality laboratory test data, which can be seen in Table 2. As seen in Table 2, the SD value of M_r data in the development set ranges from 33.03 MPa (Sequence 2) to 83.92 MPa (Sequence 15). The variation of M_r data in all 15 sequences are also significantly high. In general, quality M_r test data is expected to provide higher R^2 value.

2. Validation of Models

Validations of these models were done with average material constants obtained from the evaluation dataset. The measured and predicted M_r values for limestone and sandstone aggregates were then plotted, as shown in Figs. 1 and 2. Based on the R^2 values, all of the selected models showed similar performances for both aggregates. However, the F-values and corresponding probabilities (p-values) showed some variations in the "overall significance" of the models. It can be noted that the F-test is designed to test if two population variances are equal. It does this by comparing the ratio of two variances. So, if the variances are equal, the ratio of the variances will be 1. The p-value, on the other hand, is a statistical measure that is directly related to the significance level, which is an important component in determining whether the data obtained from a scientific research is statistically significant. The magnitude of p-value is interpreted as follows: (a) a small p-value (typically ≤ 0.05) indicates strong evidence against the null hypothesis, (b) a large p-value (> 0.05) indicates weak evidence against the null hypothesis, and (c) p-values very close to the cutoff (0.05) are considered to be marginal.

In general, limestone data were found to be better "fit" and more "explainable" than sandstone, irrespective of the type of model. Even though there were insignificant variations in performance among these models, based on the R^2 value, and the proximity of data points to the equality lines for limestone aggregate, Model 4 (*octahedral model*) outperformed Model 2, followed by Model 3, and then Model 1. For sandstone aggregate, Model 1 (bulk-stress model) showed the best performance, followed by Model 4, and then Model 2 and Model 3. However, based on the individual sample count that satisfied the MEPDG requirement for R^2 value, as mentioned earlier, Model 1 could not be recommended for sandstone aggregate. Therefore, Model 4 (*octahedral model*) was ranked as the "*best fit*" model, irrespective of aggregate type. Model 2 and Model 3 were ranked as the "second best fit" and "third best fit" models, respectively.

IV. CORRELATIONS

1. Stress-Based Model Correlations

To correlate M_r values of an aggregate with index properties, various multiple linear regression models have been used in the past. Tian et al. (1998) suggested a correlation equation (Eq. (5)) where bulk stress (θ), moisture content (MC), and percent passing from Sieve No. 200 (P_{200}) were used to predict M_r values of both limestone and sandstone aggregates. Even though a "good fit" was reported, that study was limited to the bulk-stress model (Model 1) only, and high standard deviations in the model parameters were reported. In another study by Pandey et al.(1998), M_r was correlated with unconfined compressive strength (UCS) and elastic modulus (EM), as shown in Eq. (6). However, it reported very low R^2 values (0.28 to 0.41), which suggests a "poor fit." The EM also called Young's modulus, which is the ratio of stress and strain (stiffness) due to the application of slow load within the elastic limit of the material. On the other hand, M_r is the elastic modulus based on the recoverable strain under repeated load simulating traffic conditions (fast loading rate).

$$\frac{M_r}{P_a} = A_0 + A_1 * \frac{\theta}{P_a} + A_2 * MC + A_3 * P_{200} \tag{5}$$

$$M_r = A_0 + A_1 * UCS + A_3 * EM \tag{6}$$

Some researchers also attempted to correlate M_r with relatively expensive field measurements. For instance, Ping et al. (2001) established a correlation with falling weight deflectometer (FWD) test results, but the backcalculated modulus was found to be 80% higher than the laboratory M_r data. California Bearing Ratio (CBR) has also been used as an indicator of the strength of aggregate bases. However, CBR values do not correlate well with M_r values due to differences in laboratory testing conditions (Pandey et al., 1998; Zaman et al., 1998; Tao et al., 2008). Results of the stress-based correlations pursued in this study are discussed below. Another major limitation of these correlations is that they do not consider stress as a variable. Consistent with the goal of the current study, correlation equations for k_1, k_2, and k_3 were developed based on aggregate parameters. The general form of these equations is shown in Eq. (7). The major limitation of these studies is that they established direct correlations of M_r instead of stress-based correlations, which are recommended by the MEPDG. Specific correlation equations for k_1, k_2, and k_3 for selected stress-based models for limestone and sandstone aggregates are given in Eqs. (8) through (18). Aggregate parameters used in these equations were selected based on their correlation strengths with material constants. The correlation strengths of these parameters were evaluated by calculating their Pearson coefficients. Pearson's correlation coefficient, which varies from -1 to +1, gives information about the degree of correlation as well as the direction of the correlation. If Pearson's correlation coefficient value is near ± 1, then it said to be a perfect correlation. When Pearson's correlation coefficient value lies around zero, there is no correlation.

$$\begin{aligned} K_i &= f(\mathrm{LA}, P_{40}, \mathrm{LL}, ..) \\ &= a_1 + a_2 * \mathrm{LA} + a_3 * P_{40} + a_4 * \mathrm{LL} + \ldots + a_n * \mathrm{OMC} \end{aligned} \tag{7}$$

where, a_1, a_2, …, and a_n are regression constants, and $k_{i = 1 \text{ to } 3}$ are material constants.

Model 1: For both aggregates, k_1 of Model 1 is expressed in terms of P_{200} and LL (Eq. (8)), and k_2 is expressed as a function of E and k_1 (Eq. (9)). The R^2 values of the equations of k_1 and k_2 are found to be 0.20 and 0.30, respectively, which implies no statistically significant correlations between these parameters.

$$k_1 = 243.479 * P_{200} - 1.811 * \text{LL} \tag{8}$$

$$k_2 = 0.618 - 8.775\text{E} - 5 * k_1 \tag{9}$$

Model 2: Material constants (k_1, k_2, and k_3) of Model 2 are expressed in terms of P_{200}, LL, and PI, as shown in Eqs. (10) through (12). The R^2 values of the equations of k_1, k_2, and k_3 are found to be 0.20, 0.56, and 0.49, respectively.

$$k_1 = 526.525 * P_{200} - 59.448 * \text{LL} \tag{10}$$

$$k_2 = 0.476 - 0.728 * k_1 \tag{11}$$

$$k_3 = 0.058 * \text{PI} \tag{12}$$

Model 3: In the case of Model 3, comparatively better R^2 values are observed for material constants, which are expressed in Eqs. (13) through (15). The R^2 values of the equations of k_1, k_2, and k_3 are found to be 0.38, 0.78, and 0.42, respectively.

$$k_1 = 259.440 * P_{200} - 1.951 * \text{UCS} \tag{13}$$

$$k_2 = 0.530 - 0.902 * k_3 \tag{14}$$

$$k_3 = -0.044 * \text{OMC} + 0.087 * \text{PI} \tag{15}$$

Model 4: Using five parameters (SPGR, LA, UCS, P_4, and OMC), expressions (Eqs. (16) through (18)) for k_1, k_2, and k_3 have been developed for both aggregates, and their R^2 values are found to be 0.23, 0.21, and 0.18, respectively.

$$k_1 = -425.926 + 1563.519 * \text{SPGR} + 41.445 * \text{LA} - 1.894 * \text{UCS} \tag{16}$$

$$k_2 = 3.196 - 0.040 * P_4 - 0.006 * \text{LA} - 0.002 * \text{UCS} - 0.151 * \text{OMC} \tag{17}$$

$$k_3 = -2.373 + 0.051 * P_4 - 0.039 * \text{LA} - 0.003 * \text{UCS} + 0.230 * \text{OMC} \tag{18}$$

The R^2 values of the established correlations are relatively low, which is not quite surprising to the authors. The authors are not aware of any literature in public domain that has reported strong correlations of material constants of stress-based models with aggregates' routine test properties. Such observations reiterate further research need in this area to incorporate other low expensive laboratory test parameters (e.g., surface characteristics) in establishing better correlations. Using the aforementioned correlation equations, material constants of aggregates for different regression models were estimated. The M_r/P_a values of aggregate samples were then estimated by using model Eqs. (1) through (4). The measured versus predicted M_r/P_a values are plotted in Figs. 3 and 4. The overall strengths of these models were evaluated by using their R^2 and F-values. Based on these statistical parameters, Model 3 showed the best performance for limestone aggregate, followed by Model 2, Model 1, and Model 4. For sandstone aggregate, Model 3 also showed the best performance, followed by Model 2, Model 1, and Model 4. Therefore, Model 3 (universal model) is recommended for both limestone and sandstone aggregates.

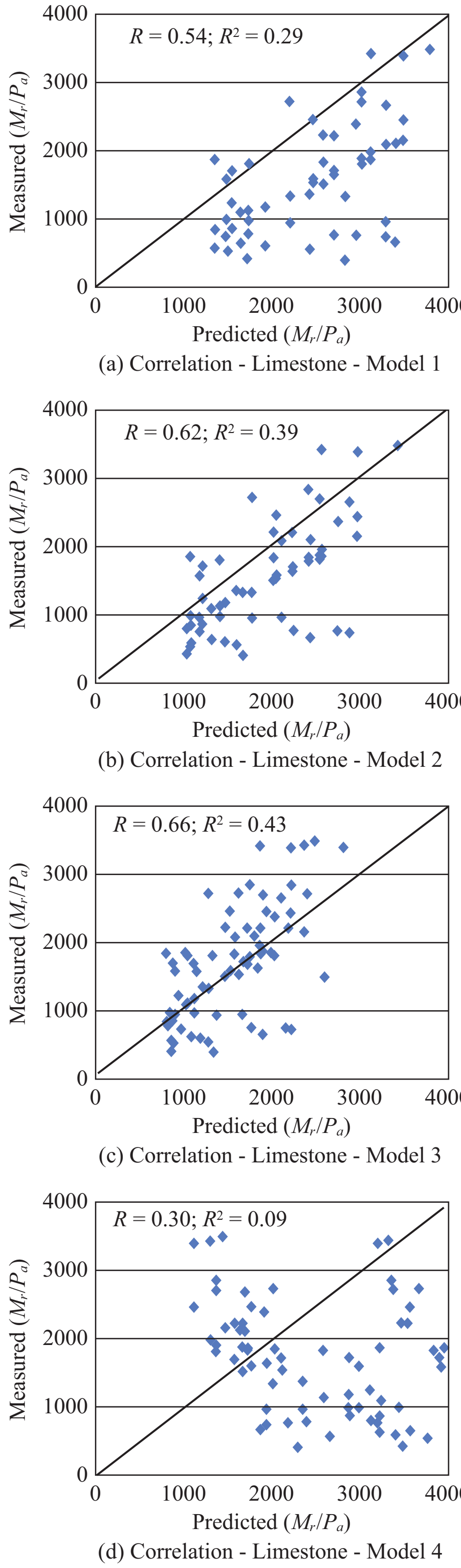
2. Direct Correlations of M_r

A multiple-linear regression (MLR) correlation for predicting M_r with eight engineering properties (SPGR, LA Abrasion value, P_{200}, LL, PI, UCS, MDD, and OMC) of aggregate was developed, as shown in Eq. (19). The calculated and predicted M_r values, based on the established MLR model, are plotted in Fig. 5. The R^2 and F values of the developed MLR correlation were found to be 0.39 and 21.7, respectively. It can be noted that a limited number of aggregate samples in the evaluation dataset had LA values in the pertinent data source (Pandey et al., 1998), which resulted in smaller number of data points in the chart. Furthermore, the predicted M_r values are for a particular bulk stress based on a selected state of stress, which is elaborated in the next section.

$$\frac{M_r}{P_a} = -122170.72 + 9945.29 * \text{SPGR} - 654.54 * LA + 13696.38 * P_{200} + 2299.25 * \text{LL} - 4061.14 * \text{PI} - 3.43 * UCS + 924.52 * \text{MDD} + 65.67 * \text{OMC} \tag{19}$$

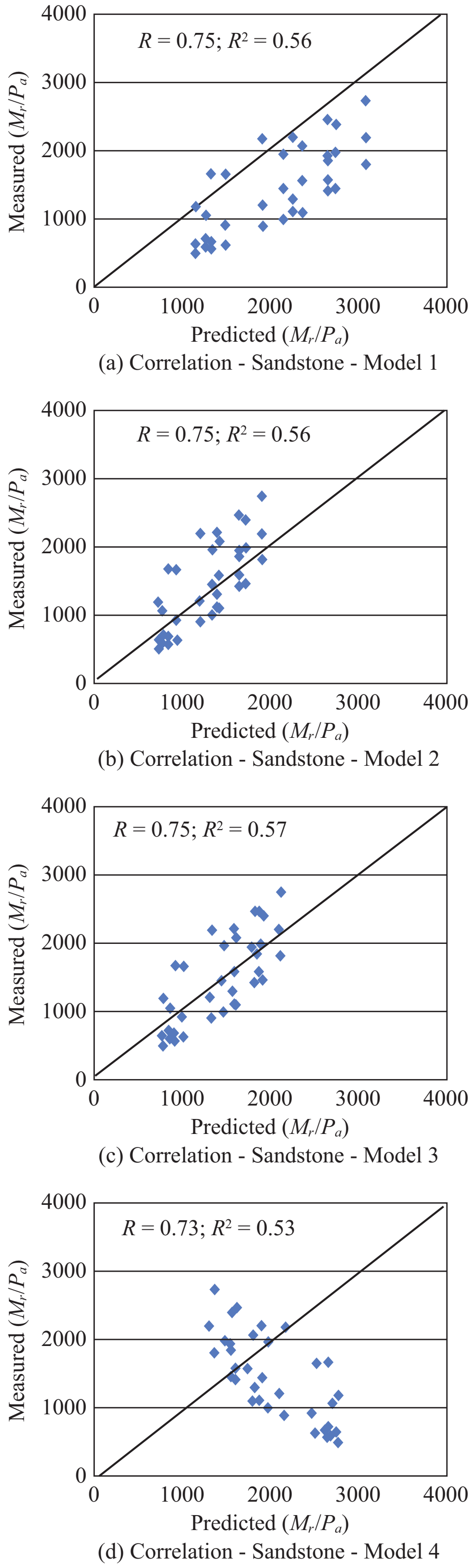
V. DEFAULT M_r VALUES

Previous studies (e.g., Zhu et al., 1998; Ping et al., 2001; Tao et al., 2008; Li et al., 2011) have reported that the M_r of a pavement material is very sensitive to the state of stress condition, which can be calculated from a layered elastic analysis or some other means (Khazanovich et al., 2006). The current study considered a state of stress corresponding to a confining pressure of 2 psi (13.78 kPa) and a deviatoric stress of 6 psi (41.34 kPa), as recommended by Ping et al. (2001). It is also reported that most aggregate bases are designed for bulk stress values ranging from 4.93 psi (34 kPa) to 29.73 psi (205 kPa) and regression models corresponding to a bulk stress value higher than 29.73 psi (205 kPa) are of very little practical



Note: F-values and corresponding probabilities: Model 1: 1.34 (p = 0.11); Model 2: 1.71 (p = 0.013); Model 3: 2.61 (p = 1.0E-04); and Model 4: 0.96 (p = 0.430).

Fig. 3. Predicted versus measured M_r/P_a values for limestone aggregate from stress-based correlations: (a) Model 1, (b) Model 2, (c) Model 3, and (d) Model 4.

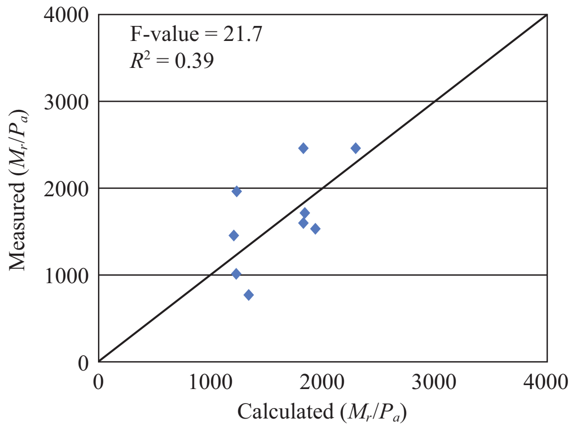


Note: F-values and corresponding probabilities: Model 1: 1.01 (p = 0.486); Model 2: 2.64 (p = 0.003); Model 3: 2.06 (p = 0.018); Model 4: 1.68 (p = 0.0.064).

Fig. 4. Predicted versus measured M_r/P_a values for sandstone aggregate from stress-based correlations: (a) Model 1, (b) Model 2, (c) Model 3, and (d) Model 4.

Table 5. Default M_r values for selected Oklahoma aggregates.

Aggregate Type	AASHTO Soil Type	MEPDG Suggested Typical Value, ksi (MPa)	Estimated from Model 2, ksi (MPa)	Estimated from Model 3, ksi (MPa)	Estimated from Model 4, ksi, (MPa)
Limestone	A-1-a	38 (262)	14.1 (97.42)	14.3 (98.40)	14.2 (98.03)
Sandstone	A-1-a	38 (262)	9.0 (62.35)	9.4 (65.12)	9.3 (64.03)



Note: F-value and the corresponding probability: F = 21.7 (p = 0.086).

Fig. 5. Calculated versus measured M_r values from multi-linear regression model correlation (direct).

significance (Zhu et al., 1998). Furthermore, it was also reported that the MLR correlation with a bulk stress of 29.73 psi (205 kPa) resulted in a "better fit" than that with a lower stress level (Khazanovich et al., 2006).

Default M_r values for limestone and sandstone aggregates were calculated using the average material constants obtained from regression modeling for the aforementioned state of stress (a bulk stress of 29.73 psi or 205 kPa) and are presented in Table 5. These M_r values can be used as Level 3 inputs in the MEPDG analysis and design. It was observed that the predicted typical M_r values obtained from different models were in agreement with each other, and the variations of M_r values among different models were within 4%. However, all of these models would result in conservative designs compared to the MEPDG recommended typical values. The predicted default M_r values corresponding to limestone and sandstone aggregates for Model 3 were found to be 38% and 25% of the MEPDG recommended typical values. The actual reasons for such variations in M_r values are unknown. However, it could be hypothesized that the differences in mineralogical and textural characteristics of tested aggregates from those considered in the nationally calibrated model contributed to the variations in M_r values. Another possible reason could be the state of stress considered in estimating the M_r value. An increase in confining pressure is expected to yield an increase in the M_r value.

In general, limestone aggregate showed higher (51%) M_r values than those of sandstone aggregate. While the actual reason is unknown, it is speculated that Meridian limestone aggregates consisted of relatively bigger size particles with higher interlocking potential than sandstone aggregates. However, this is not the case for Richard Spurs limestone aggregates. Furthermore, surface properties such as angularity, texture, or the cementing nature of limestone fines may have contributed to the increased M_r value (Chen et al., 2013). The cementing nature of fines in limestone aggregates is supported by their corresponding high UCS values compared to sandstone aggregates.

VI. CONCLUDING REMARKS

M_r data of 105 samples comprising two different types (limestone and sandstone) of aggregate in Oklahoma were analyzed in this study. According to the AASHTO M 145 specifications, these aggregates were classified as A-1-a. Four stress-based models were evaluated as possible models for M_r. Furthermore, correlation equations were developed to predict M_r values by using routine properties of aggregates. The findings of the current study are summarized below:

1. Among the four selected models, the octahedral model (Eq. (4)) was found to perform better than the others and is recommended for use in Level 1 analysis and design. Material constants for the selected aggregates, provided in Table 4, can be readily used by ODOT.
2. From the perspective of correlations, the universal model (Eq. (3)) was found to outperform the other models. The established correlation equations (Eqs. (13) through (15)) can be used to estimate material constants for Level 2 analysis and design.
3. Default M_r values for limestone and sandstone aggregates obtained from regression modeling were presented in Table 5. These M_r values can be used as Level 3 inputs in the MEPDG analysis and design.
4. The predicted typical (Level 3) M_r values obtained from the different models were in agreement with each other. Also, all of these models resulted in conservative designs compared to the MEPDG recommended typical values. Specifically, the predicted default M_r values corresponding to limestone and sandstone aggregates for the universal model (Eq. (3)) were found to be 38% and 25% of the MEPDG recommended typical values.
5. In general, limestone aggregate showed higher (51%) M_r values than sandstone aggregate. These observations may be justified by the fact that the limestone aggregate con-

tained bigger sized particles, which provides more interlocking potential than the sandstone aggregate.

ACKNOWLEDGMENTS

The authors would like to express their sincere appreciation to the Oklahoma Department of Transportation for providing financial support for this project. Assistance given by the Oklahoma Transportation Center is greatly appreciated. Thanks are due to Ms. Aissata Cissie, from the University of Oklahoma, for her assistance with the initial data entry operation in this study.

REFERENCES

Andrei, D., M. W. Witczak, C. W. Schwartz and J. Uzan (2004). Harmonized resilient modulus test method for unbound pavement materials, In Transportation Research Record, No. 1874, Transportation Research Board of the National Academies, Washington, D.C., 29-37.

Brown, S. F. and J. W. Pappin (1981). Analysis of pavement with granular bases. In Transportation Research Record, No. 810, Transportation Research Board of the National Academies, Washington D.C., 17-23.

Chen, J., W. Hsieh and M. Liao (2013). Effect of coarse aggregate shape on engineering properties of stone mastic asphalt applied to airport pavements. In the International Journal of Pavement Research and Technology, 6(5), 595-601.

Hicks, R. G. and C. L. Monismith (1971). Factors influencing the resilient properties of granular materials, Transportation Research Record 345, Transportation Research Board of the National Academies, Washington D.C., 15-31.

Hossain, Z., M. Zaman, C. Doiron and S. Cross (2011). Development of a flexible pavement database for local calibration of the MEPDG. Final Report, SPR No. 2209, Oklahoma Department of Transportation, Oklahoma City, OK.

Khazanovich, L., C. Celaro, B. Chadbourn, J. Zollars and S. Dai (2006). Evaluation of subgrade resilient modulus predictive model for use in mechanistic-empirical pavement design guide. In Transportation Research Record, No. 1947, Transportation Research Board of the National Academies, Washington D.C., 155-166.

Kim, S., K. Gopalakrishnan and H. Ceylan (2009). Iowa's rubbilized pavements: design properties of rubbilized PCC layers. In the 88th TRB Annual Meeting, Paper No. 09-3249 CD-ROM, Transportation Research Board of the National Academies, Washington D.C.

Li, L., J. Liu, X. Zhang and S. Saboundjian (2011). Resilient modulus characterization of Alaskan granular base materials, Paper No. 11-3155, In the 90th Annual Meeting, Poster Presentation, Transportation Research Board of the National Academies, Washington D.C.

NCHRP (2004). Guide for Mechanistic-Empirical Design of Pavement Structures: Part 2 – Design Inputs. ARA, Inc., ERES Consultants Division, Champaign, Illinois.

Norusis, M. J. (2002). SPSS 11.0 Guide to data analysis, Chapter 22 on Analyzing residuals, ISBN-10: 0130348309, Prentice Hall, New Jersey.

Pandey, K., M. Zaman and J. G. Laguros (1998). Resilient moduli of raw and stabilized aggregate bases and evaluation of layer coefficients for AASHTO flexible pavement design. Volume III, Final Report, Item 2199; ORA 125-4262, Oklahoma Department of Transportation, Oklahoma City, Oklahoma.

Papagiannakis, A. T. and E. A. Masad (2008). Pavement design and materials. Wiley and Inc. Sons, Hoboken, New Jersey.

Pezo, R. F. (1993). A General method of reporting resilient modulus tests of soils – a pavement engineer's point of view. Paper No. 93082, In TRB 72nd Annual Meeting, Transportation Research Board of the National Academies, Washington D.C.

Ping, W. V., Y. Zenghai, L. Chunshui and D. Bruce (2001). Measuring resilient modulus of granular materials in flexible pavements. In Transportation Research Record, No. 1778, Transportation Research Board of the National Academies, Washington, D.C., pp. 81-90.

Richardson, D. N., T. M. Petry, L. Ge, Y. Han and S. M. Lusher (2009). Resilient moduli of typical missouri soils and unbound granular base materials. Final Report, No. OR09-016, Missouri Department of Transportation, Jefferson City, Missouri.

Tao, M., M. Abu-Farsakh and Z. Zhang (2008). Characterization of unbound aggregates revealed through laboratory tests. In the International Journal of Pavement Research and Technology, 1(2), 72-75.

Thompson, M. R., E. Tutumluer and M. Bejarano (1998). Granular material and soil moduli review of the literature. Technical Research Report, Center for Excellence, Airport Pavement Research, COE Report No. 1, Under Grant No. DOT 95-C-001, Federal Aviation Administration, Washington, D.C.

Tian, P., M. Zaman and J. G. Laguros (1998). Resilient moduli of raw and stabilized aggregate bases and evaluation of layer coefficients for AASHTO flexible pavement design. Volume II, Final Report, Item 2199; ORA 125-4262, Oklahoma Department of Transportation, Oklahoma City, OK.

Titi, H., M. Elias and S. Helwany (2006). Determination of typical resilient modulus values for selected soils in Wisconsin. Final Report, Wisconsin Highway Research Program, Project ID 0092-03-11, Milwaukee, WI.

Tutumluer, E. and T. Pan (2008). Aggregate morphology affecting strength and permanent deformation behavior of unbound aggregate materials. In the ASCE Journal of Materials in Civil Engineering, 20(9), 1-11.

Uzan, J. (1985). Characterization of granular material. In Transportation Research Record, No. 1022, Transportation Research Board of the National Academies, Washington, D.C., 52-59.

Von Quintus H. L. and B. Killingsworth (1997). Analyses relating to pavement material characterizations and their effects on pavement performance. Publication Number FHWA-RD-97-085, Federal Highway Administration, Washington, D.C.

Wang, K. (2009). Database preparation for MEPDG PrepME. Presentation Slides, 2009 Southeastern Pavement Conference, New Orleans, Louisiana.

Witczak, M. K. and J. Uzan (1988). The universal airport pavement design system. Report I of V: Granular Material Characterization, Department of Civil Engineering, University of Maryland, College Park, Maryland.

Xiao, Y., E. Tutumluer and J. Siekmeier (2011). Resilient modulus behavior estimated from aggregate source properties. In Proceedings of Geo-Frontiers, Dallas, Texas.

Yohannes, B., K. Hill and L. Khazanovich (2009). Mechanistic modeling of unbound granular materials. Research Report MN/RC 2009-21, Minnesota Department of Transportation, St. Paul, Minnesota.

Zaman, M., J. Laguros, P. Tian, J. Zhu and K. Pandey (1998). Resilient moduli of raw and stabilized aggregate bases and evaluations of layer coefficients for AASHTO flexible pavement design. Final report, Submitted to ODOT, Item 2199, ORA 125-4262, Oklahoma.

Zhu, J., M. Zaman and J. G. Laguros (1998). Resilient moduli of raw and stabilized aggregate bases and evaluation of layer coefficients for AASHTO flexible pavement design, Volume IV, Final Report, Item 2199; ORA 125-4262, Oklahoma Department of Transportation, Oklahoma City, Oklahoma.