



A SEQUENTIAL MULTIPLE PACKET TRANSMISSION POLICY FOR MODEL-BASED NETWORKED CONTROL SYSTEMS

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A SEQUENTIAL MULTIPLE PACKET TRANSMISSION POLICY FOR MODEL-BASED NETWORKED CONTROL SYSTEMS

Sheng-Hsiung Yang and Jenq-Lang Wu

Key words: networked control systems, model-based control, multiple packet transmission, linear matrix inequality.

ABSTRACT

This paper considers the stabilization of discrete-time model-based networked control systems under a sequential multiple packet transmission policy. The dynamics of considered model-based sequential multiple packet transmission NCSs are modeled as switched control systems. A sufficient condition for the existence of stabilizing controllers is derived by the switched Lyapunov function approach. Stabilizing networked feedback controllers can be obtained by solving linear matrix inequalities and equalities. A numerical example is proposed for verification.

I. INTRODUCTION

The stability analysis and control synthesis problems of networked control systems (NCSs) have attracted much attention in the last decades. The information (measured data or control signals) in an NCS is exchanged among distributed control system components (sensors, controllers, and actuators) via a real-time network channel. Comparing to the traditional point-to-point wiring feedback control systems, NCSs have the advantages of reduced wiring, low cost, simple installation, easy maintenance, and high reliability.

In an NCS, since data are exchanged via a network channel, some network-induced problems, such as uncertain transmission delay, packet dropout, finite data transmission rate, quantization error, and multiple packet transmission, etc., can degrade control performances. They must be taken into consideration in designing networked feedback control laws. In the literature, the research on NCSs focuses on dealing with finite communication bandwidth (see e.g., Hristu and Mor-

gansen, 1999; Wong and Brockett, 1999; Tatikonda and Mitter, 2004; Zhang et al., 2011; Al-Areqi et al., 2015), network scheduling (see e.g., Orihuela, 2014), network-induced delay (see e.g., Zhang et al., 2001; Walsh et al., 2002; Carnevale et al., 2007; Gao and Chen, 2007; Ma et al., 2007; Li et al., 2014), and packets dropout (see e.g., Wu and Chen, 2007; Zhang and Yu, 2007; Chiuso and Schenato, 2011; Li et al., 2014; Wang and Han, 2015), etc. For a large NCS, the sensors and actuators may distribute at different places and therefore, the measured data (or the control signal) cannot be transmitted (via network channel) in a single packet. In this case, the effect of multiple packet transmission must be considered in synthesizing controllers. However, in the literature only a few results have been proposed on the design of feedback laws for NCSs under multiple packet transmission policies (see e.g., Hu and Yan, 2008; Wu et al., 2010, 2011; Wu and Yang, 2013; Li et al., 2014). In (Hu and Yan, 2008), an NCS under multiple packet transmission policy and possible packet dropout is modeled as a jumped system. Stability analysis and controller synthesis problems are investigated. In (Li et al., 2014), a sliding mode predictive control approach was proposed for NCSs under multiple packet transmission. In (Wu et al., 2011), a separation principle for NCSs with multiple packet transmission is proposed. In (Wu et al., 2010), robust H_∞ control for uncertain NCSs under multiple packet transmission was considered. In (Wu and Yang, 2013), for NCSs under a new multiple packet *sequential* transmission policy, both state feedback and output feedback stabilization problems were discussed. Based on the multiple Lyapunov function approach, stabilizing networked feedback laws were obtained by solving linear matrix inequalities (LMIs). In this study, system uncertainties have not been considered. By the setting in (Wu and Yang, 2013), extending the approach of (Wu and Yang, 2013) to overcome parameter uncertainties will in general lead to conservative results.

In this paper, motivated by the framework of model-based NCSs (Montestruque and Antsaklis, 2003, 2004, 2007; Garcia and Antsaklis, 2013a, 2013b; Mehta et al., 2013; Song et al., 2013), we consider the controller synthesis problem for uncertain NCSs under sequential multiple packet transmission. The concept of model-based NCSs was firstly introduced in (Montestruque and Antsaklis, 2003). It has been shown that

model-based NCSs can achieve stability under extremely low network usage, and are robust with respect to system uncertainties. In this paper, we adopt the model-based framework for designing controllers to robustly stabilize discrete-time uncertain NCSs under a multiple packet sequential transmission policy. This framework is very different from that discussed in (Wu and Yang, 2013). Moreover, unlike classical model-based NCSs (all states of the model are updated simultaneously), since the measured data (from sensors) are transmitted sequentially, the states of the approximation model are updated sequentially but not simultaneously. In this paper, sufficient conditions for the existence of stabilizing controllers are derived. And, a stabilizing feedback law can be obtained by solving linear matrix inequalities and equalities. We do not use the state augmentation technique as (Wu and Yang, 2013) and therefore the dimensions of matrix inequalities and equalities to be solved are much less than those in (Wu and Yang, 2013). For simplification, here we consider the case that only the measured data from sensors to controller are transmitted via network. The control signal generated by the controller is directly fed to the actuators without delay. Moreover, we focus on dealing with the effect of multiple-packet transmission and therefore we assume that the data is transmitted ideally (no delay, no packet dropout, and no quantization error, et al.) as assumed in (Wu and Yang, 2013). For NCSs under some particular protocols (e.g., CAN bus), this assumption can be reasonable for some applications.

II. MAIN RESULTS

Consider a model-based NCS equivalently shown as Fig. 1. The dynamic of the plant is described by

$$x(k+1) = Ax(k) + Bu(k) \quad (1)$$

where $x(k) \in \mathfrak{R}^n$ is the system state, $u(k) \in \mathfrak{R}^m$ is the control input, and A and B are constant matrices with appropriate dimensions. Suppose that the parameters in matrices A and B are not known exactly. An approximation model of the plant is established at the controller node:

$$\hat{x}(k+1) = \hat{A}\hat{x}(k) + \hat{B}u(k) \quad (2)$$

where $\hat{x}(k) \in \mathfrak{R}^n$ is the model state, and \hat{A} and \hat{B} are known constant matrices with appropriate dimensions. Suppose that $A = \hat{A} + \Delta A$ with $\Delta A = \rho E$ and $B = \hat{B} + \Delta B$ with $\Delta B = \delta H$, where E and H are known constant matrices and uncertain parameters $|\rho| \leq 1$ and $|\delta| \leq 1$.

In this paper we consider the case that the sensors distribute at different places and therefore the measured states cannot be transmitted to the controller in a single packet. They must be transmitted sequentially at different times via the network channel. In Fig. 1, this mechanism is equivalently described

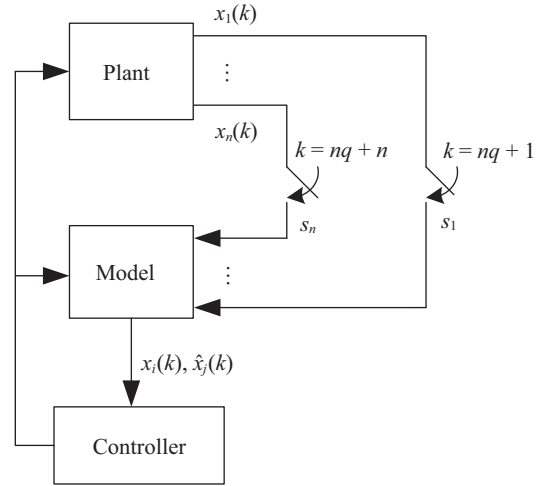


Fig. 1. A configuration of model-based NCSs.

as switches s_1, s_2, \dots, s_n , which are sequentially closed (and opened). Under the scheduling network protocol, the measured states can be transmitted periodically and therefore the states of the approximation model are updated periodically.

Without loss of generality, at $k = nq + i, i \in \{1, 2, \dots, n\}$ and $q = 0, 1, 2, \dots$, let the measured i -th state $x_i(nq + i)$ be transmitted to the approximation model and update the model state $\hat{x}_i(nq + i)$. That is,

$$\hat{x}_i(nq + i) = x_i(nq + i). \quad (3)$$

The other states of the approximation model are generated by the dynamic equation (2). Therefore, at $k = nq + i$, the information that can be utilized for generating control signal are $\hat{x}_1(k), \hat{x}_2(k), \dots, \hat{x}_i(k) = x_i(k), \hat{x}_{i+1}(k), \dots$, and $\hat{x}_n(k)$.

The objective of this correspondence is to design a periodic state feedback control law

$$u(k) = \sum_{j \neq i} F_j \hat{x}_j(k) + F_i x_i(k), k = nq + i, \quad (4)$$

to exponentially stabilize the system (1).

Let

$$F_i = [F_{1,i} \quad \dots \quad F_{i,i} \quad \dots \quad F_{n,i}],$$

and define

$$T_i = \text{diag}\{1, \dots, 1, 0, 1, \dots, 1\},$$

where 0 is the (i,i) element. The feedback law (4) can be equivalently expressed as

$$u(k) = F_i(I - T_i)x(k) + F_i T_i \hat{x}(k), k = nq + i.$$

With the updating law (3), by (4) we can see that, at $k = nq +$

$i, i \in \{1, \dots, n\}$ and $q = 0, 1, \dots$, the dynamics of the closed-loop system can be described as

$$x(k+1) = Ax(k) + BF_i((I - T_i)x(k) + T_i\hat{x}(k)) \quad (5)$$

$$\hat{x}(k+1) = \hat{A}((I - T_i)x(k) + T_i\hat{x}(k)) + \hat{B}F_i((I - T_i)x(k) + T_i\hat{x}(k)). \quad (6)$$

That is,

$$x(k+1) = (A + BF_i(I - T_i))x(k) + BF_iT_i\hat{x}(k)$$

$$\hat{x}(k+1) = (\hat{A} + \hat{B}F_i)(I - T_i)x(k) + (\hat{A} + \hat{B}F_i)T_i\hat{x}(k),$$

$$k = nq + i, i \in \{1, \dots, n\}, \text{ and } q = 0, 1, \dots$$

Define the state error as:

$$e(k) = x(k) - \hat{x}(k).$$

The updating operation can be equivalently expressed as

$$e(qn + i) = T_i(x(qn + i) - \hat{x}(qn + i)). \quad (7)$$

That is,

$$e_i(nq + i) = 0.$$

By the definition of $e(\cdot)$, we have (at $k = nq + i$)

$$x(k+1) = (A + BF_i)x(k) - BF_iT_ie(k) \quad (8)$$

$$\begin{aligned} e(k+1) &= (A - \hat{A} + (B - \hat{B})F_i)x(k) - (\hat{A} + (B - \hat{B})F_i)T_ie(k) \\ &= (\Delta A + \Delta BF_i)x(k) - (\hat{A} + \Delta BF_i)T_ie(k). \end{aligned} \quad (9)$$

Let

$$\bar{x}(k) = \begin{bmatrix} x(k) \\ e(k) \end{bmatrix}.$$

The dynamics of the closed-loop system can be described as (at $k = nq + i$)

$$\begin{aligned} \bar{x}(k+1) &= \begin{bmatrix} A + BF_i & -BF_iT_i \\ \Delta A + \Delta BF_i & -(\hat{A} + \Delta BF_i)T_i \end{bmatrix} \bar{x}(k) \\ &= \begin{bmatrix} A + BF_i & -BF_i \\ \Delta A + \Delta BF_i & -\hat{A} - \Delta BF_i \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & T_i \end{bmatrix} \bar{x}(k) \end{aligned}$$

$$\begin{aligned} &= \left(\begin{bmatrix} \hat{A} & 0 \\ 0 & -\hat{A}T_i \end{bmatrix} + \begin{bmatrix} \Delta A & 0 \\ \Delta A & 0 \end{bmatrix} \right. \\ &+ \left. \left(\begin{bmatrix} \hat{B} \\ 0 \end{bmatrix} + \begin{bmatrix} \Delta B \\ \Delta B \end{bmatrix} \right) F_i \begin{bmatrix} I & -T_i \end{bmatrix} \right) \bar{x}(k) \\ &\equiv (\bar{A}_i + \Delta \bar{A} + (\bar{B} + \Delta \bar{B})F_iG_i) \bar{x}(k), \end{aligned} \quad (10)$$

where

$$\begin{aligned} \bar{A}_i &\equiv \begin{bmatrix} \hat{A} & 0 \\ 0 & -\hat{A}T_i \end{bmatrix}, \\ \Delta \bar{A} &\equiv \begin{bmatrix} \Delta A & 0 \\ \Delta A & 0 \end{bmatrix} = \rho \bar{E} \text{ with } \bar{E} = \begin{bmatrix} E & 0 \\ E & 0 \end{bmatrix}, \\ \bar{B} &\equiv \begin{bmatrix} \hat{B} \\ 0 \end{bmatrix}, \\ \Delta \bar{B} &\equiv \begin{bmatrix} \Delta B \\ \Delta B \end{bmatrix} = \delta \bar{H} \text{ with } \bar{H} = \begin{bmatrix} H \\ H \end{bmatrix}, \\ G_i &\equiv \begin{bmatrix} I & -T_i \end{bmatrix}. \end{aligned}$$

Let

$$A_{ci} \equiv \bar{A}_i + \Delta \bar{A} + (\bar{B} + \Delta \bar{B})F_iG_i.$$

The closed-loop system (10) can be expressed as

$$\bar{x}(k+1) = A_{ci}x(k), \quad k = nq + i. \quad (11)$$

In the sequel derivation, we will need the following lemma.

Lemma 1 (Wu et al., 2010)

Given matrices $R = R^T > 0$, $Q = Q^T$, H , and E of appropriate dimensions, then

$$Q + HFE + E^T F^T H^T < 0$$

for all F satisfying $F^T F \leq R$, if and only if there exists some $\varepsilon > 0$ such that

$$Q + \varepsilon^2 HH^T + \varepsilon^{-2} E^T RE < 0.$$

Now we are ready to present the main result.

Theorem 2

Consider system (1). There exists a feedback law as (4) such that the closed-loop system (11) is exponentially stable with

decay rate $0 < \lambda < 1$, if there exist positive definite matrices $S_i \in \mathfrak{R}^{2n \times 2n}$, $i = 1, \dots, n$, matrices $L_i \in \mathfrak{R}^{n \times n}$ and $N_i \in \mathfrak{R}^{1 \times n}$, $i = 1, \dots, n$, and scalars $\alpha_i > 0$ and $\beta_i > 0$, $i = 1, \dots, n$, satisfying

$$\begin{bmatrix} -S_i + \lambda S_i & * & * & * \\ \bar{A}_i S_i + \bar{B} N_i G_i & -S_{i+1} + \alpha_i I + \beta_i I & * & * \\ \bar{E} S_i & 0 & -\alpha_i I & * \\ \bar{H} N_i G_i & 0 & 0 & -\beta_i I \end{bmatrix} < 0, \quad (12)$$

$i = 1, 2, \dots, n-1$

$$\begin{bmatrix} -S_n + \lambda S_n & * & * & * \\ \bar{A}_n S_n + \bar{B} N_n G_n & -S_1 + \alpha_n I + \beta_n I & * & * \\ \bar{E} S_n & 0 & -\alpha_n I & * \\ \bar{H} N_n G_n & 0 & 0 & -\beta_n I \end{bmatrix} < 0, \quad (13)$$

and

$$L_i G_i = G_i S_i, \quad i = 1, 2, \dots, n. \quad (14)$$

In this case, the feedback law (4) with

$$F_i = N_i L_i^{-1}, \quad i = 1, 2, \dots, n, \quad (15)$$

is such that the closed-loop system (11) is exponentially stable with decay rate $\sqrt{1-\lambda}$.

Proof:

Let

$$P_i = S_i^{-1}, \quad i = 1, \dots, n,$$

and

$$V_i(\bar{x}) = \bar{x}^T P_i \bar{x}, \quad i = 1, \dots, n.$$

Consider the switched Lyapunov function

$$V(\bar{x}(k)) = V_{\sigma(k)}(\bar{x}(k)), \quad \sigma(k) = i \text{ if } k = nq + i.$$

If the matrix inequalities and equalities (12)-(14) are feasible, by noting (15), we have, for all $i = 1, \dots, n-1$,

$$\begin{bmatrix} -S_i + \lambda S_i & * \\ \bar{A}_i S_i + \Delta \bar{A} S_i + \bar{B} F_i G_i S_i + \Delta \bar{B} F_i G_i S_i & -S_{i+1} \end{bmatrix} \\ = \begin{bmatrix} -S_i + \lambda S_i & * \\ \bar{A}_i S_i + \bar{B} F_i G_i S_i & -S_{i+1} \end{bmatrix} + \begin{bmatrix} 0 & * \\ \Delta \bar{A} S_i & 0 \end{bmatrix} + \begin{bmatrix} 0 & * \\ \Delta \bar{B} F_i G_i S_i & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -S_i + \lambda S_i & (\bar{A}_i S_i + \bar{B} N_i G_i)^T \\ \bar{A}_i S_i + \bar{B} N_i G_i & -S_{i+1} \end{bmatrix} \\ + \begin{bmatrix} 0 \\ I \end{bmatrix} \rho \begin{bmatrix} \bar{E} S_i & 0 \end{bmatrix} + \begin{bmatrix} S_i \bar{E}^T \\ 0 \end{bmatrix} \rho \begin{bmatrix} 0 & I \end{bmatrix} \\ + \begin{bmatrix} 0 \\ I \end{bmatrix} \delta \begin{bmatrix} \bar{H} F_i G_i S_i & 0 \end{bmatrix} + \begin{bmatrix} S_i G_i^T F_i^T \bar{H}^T \\ 0 \end{bmatrix} \delta \begin{bmatrix} 0 & I \end{bmatrix} \\ \leq \begin{bmatrix} -S_i + \lambda S_i & (\bar{A}_i S_i + \bar{B} N_i G_i)^T \\ \bar{A}_i S_i + \bar{B} N_i G_i & -S_{i+1} \end{bmatrix} \\ + \alpha_i \begin{bmatrix} 0 \\ I \end{bmatrix} \begin{bmatrix} 0 & I \end{bmatrix} + \frac{1}{\alpha_i} \begin{bmatrix} S_i \bar{E}^T \\ 0 \end{bmatrix} \begin{bmatrix} \bar{E} S_i & 0 \end{bmatrix} \\ + \beta_i \begin{bmatrix} 0 \\ I \end{bmatrix} \begin{bmatrix} 0 & I \end{bmatrix} + \frac{1}{\beta_i} \begin{bmatrix} S_i G_i^T F_i^T \bar{H}^T \\ 0 \end{bmatrix} \begin{bmatrix} \bar{H} F_i G_i S_i & 0 \end{bmatrix}.$$

By Schur complement and noting (12), we have

$$\begin{bmatrix} -S_i + \lambda S_i & * \\ \bar{A}_i S_i + \Delta \bar{A} S_i + \bar{B} F_i G_i S_i + \Delta \bar{B} F_i G_i S_i & -S_{i+1} \end{bmatrix} < 0, \quad (16)$$

$i = 1, \dots, n-1$

Pre- and post-multiplying (16) by

$$\begin{bmatrix} P_i & 0 \\ 0 & P_{i+1} \end{bmatrix}$$

and by Schur complement, we can get

$$A_{ci}^T P_{i+1} A_{ci} - P_i + \lambda P_i < 0, \quad i = 1, \dots, n-1.$$

Similarly, it can be shown that

$$A_{cn}^T P_1 A_{cn} - P_n + \lambda P_n < 0.$$

That is, $\forall \bar{x}(k) \neq 0$,

$$\Delta V_{\sigma(k)}(\bar{x}(k)) = V_{\sigma(k+1)}(\bar{x}(k+1)) - V_{\sigma(k)}(\bar{x}(k)) \\ < -\lambda V_{\sigma(k)}(\bar{x}(k)).$$

This implies that the closed-loop system is exponentially stable with decay rate $\sqrt{1-\lambda}$ and completes the proof. ■

III. A NUMERICAL EXAMPLE

Consider the control system

$$x(k+1) = Ax(k) + Bu(k) \tag{17}$$

where

$$A = \begin{bmatrix} 0.47 & 0.445 & 0.417 \\ 0.196 & -0.422 & 0.874 \\ 0.789 & 0.308 & 0.101 \end{bmatrix} + \rho \begin{bmatrix} -0.699 & -0.255 & 0.175 \\ -0.799 & 0.997 & -1.209 \\ -1.221 & -0.537 & -0.8 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.251 \\ -0.958 \\ -0.304 \end{bmatrix} + \delta \begin{bmatrix} -0.442 \\ 0.926 \\ -0.813 \end{bmatrix}$$

with $|\rho| \leq 1$ and $|\delta| \leq 1$. It can be seen that the considered uncertainties are large.

The approximation model of the plant established at the controller node is:

$$\hat{x}(k+1) = \hat{A}\hat{x}(k) + \hat{B}u(k) \tag{18}$$

where

$$\hat{A} = \begin{bmatrix} 0.47 & 0.445 & 0.417 \\ 0.196 & -0.422 & 0.874 \\ 0.789 & 0.308 & 0.101 \end{bmatrix}, \hat{B} = \begin{bmatrix} 0.251 \\ -0.958 \\ -0.304 \end{bmatrix}$$

It is easy to verify that the considered system (17) is open-loop unstable. The objective is to find a networked feedback law (4) with $n = 3$ such that the closed-loop system is exponentially stable. Given $\lambda = 0.2$, by solving the matrix inequalities and equalities (12)-(14), we have the following solutions (note that $P_i = S_i^{-1}$)

$$P_1 = \begin{bmatrix} 430.27 & 31.83 & 108.96 & 0 & 31.83 & 108.96 \\ 31.83 & 245.39 & -48.85 & 0 & -22.42 & -11.83 \\ 108.96 & -48.85 & 288.77 & 0 & -11.83 & -39.8 \\ 0 & 0 & 0 & 1.52 & 0 & 0 \\ 31.83 & -22.42 & -11.83 & 0 & 245.39 & -48.85 \\ 108.96 & -11.83 & -39.8 & 0 & -48.85 & 288.77 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 329.17 & 80.26 & 152.28 & -22.85 & 0 & -6.26 \\ 80.26 & 308.15 & -43.15 & 80.26 & 0 & -43.15 \\ 152.28 & -43.15 & 293.48 & -6.26 & 0 & -35.35 \\ -22.85 & 80.26 & -6.26 & 329.17 & 0 & 152.28 \\ 0 & 0 & 0 & 0 & 1.52 & 0 \\ -6.26 & -43.15 & -35.35 & 152.28 & 0 & 293.48 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 350.22 & 66.61 & 113.86 & -31.23 & 21.79 & 0 \\ 66.61 & 224.48 & -40.09 & 21.79 & -7.29 & 0 \\ 113.86 & -40.09 & 371.52 & 113.86 & -40.09 & 0 \\ -31.23 & 21.79 & 113.86 & 350.22 & 66.61 & 0 \\ 21.79 & -7.29 & -40.09 & 66.61 & 224.48 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.52 \end{bmatrix},$$

$$L_1 = 10^{-2} \times \begin{bmatrix} 0.33 & -0.09 & -0.17 \\ -0.19 & 0.53 & 0.21 \\ -0.33 & 0.21 & 0.6 \end{bmatrix},$$

$$L_2 = 10^{-2} \times \begin{bmatrix} 0.67 & -0.48 & -0.46 \\ -0.24 & 0.51 & 0.22 \\ -0.46 & 0.44 & 0.72 \end{bmatrix},$$

$$L_3 = 10^{-2} \times \begin{bmatrix} 0.53 & -0.29 & -0.39 \\ -0.29 & 0.64 & 0.31 \\ -0.19 & 0.16 & 0.42 \end{bmatrix}.$$

$$N_1 = 10^{-3} \times [0.09 \quad -0.34 \quad 0.88],$$

$$N_2 = 10^{-3} \times [-0.09 \quad -0.29 \quad 0.71],$$

$$N_3 = 10^{-3} \times [0.02 \quad -0.45 \quad 0.79].$$

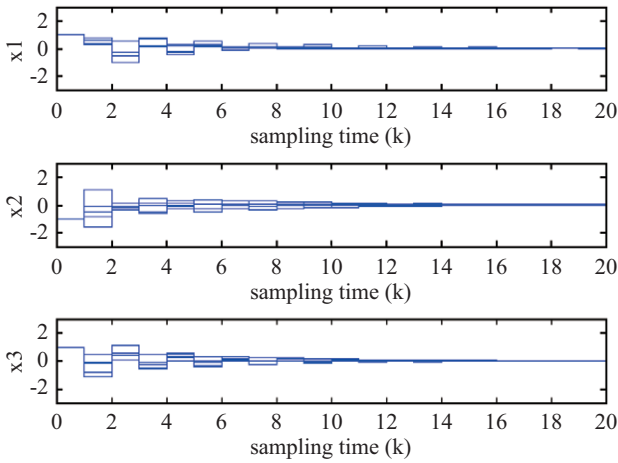
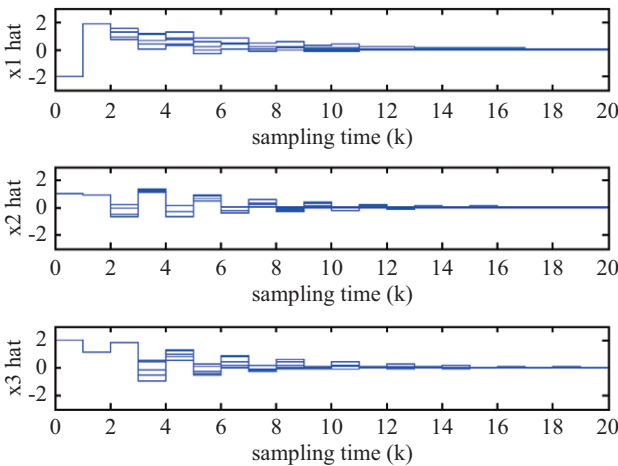
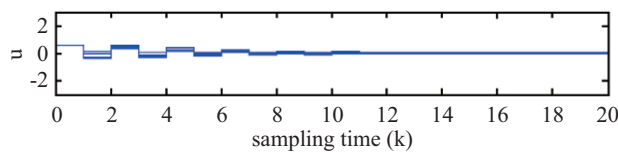
It can be verified that $P_1 > 0$, $P_2 > 0$ and $P_3 > 0$. By (15), the controller parameters are

$$F_1 = [0.212 \quad -0.125 \quad 0.249],$$

$$F_2 = [0.049 \quad -0.166 \quad 0.181],$$

$$F_3 = [0.056 \quad -0.128 \quad 0.332].$$

With these parameters, the feedback law (4) can exponentially stabilize the system (17). Under initial states $x(0) = [1 \quad -1 \quad 1]^T$ and $\hat{x}(0) = [-2 \quad 1 \quad 2]^T$, Fig. 2, Fig. 3, and Fig. 4, are the responses of the closed-loop system with five pairs of uncertain parameters $(\rho, \delta) = (0.6, 0.7)$, $(\rho, \delta) = (0.2, -0.8)$, $(\rho, \delta) = (0.9, 0.4)$, $(\rho, \delta) = (0.5, -0.5)$, and $(\rho, \delta) = (0.1, 0.87)$. It can be seen that in all cases the state trajectories converge to the origin.

Fig. 2. The responses of $x(k)$.Fig. 3. The responses of $\hat{x}(k)$.Fig. 4. The control inputs $u(k)$.

IV. CONCLUSIONS

In this paper, a model-based stabilizing feedback law is presented for uncertain NCSs under a sequential multiple-packet transmission policy. The dynamics of the resultant closed-loop NCSs can be modeled as periodic switched control systems. Stabilizing feedback laws can be obtained by solving linear matrix inequalities and equalities. The simulation verifies the theoretical results.

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