Journal of Marine Science and Technology

Volume 23 | Issue 5 | Article 2

THE DESIGN OF A HULL FORM WITH THE MINIMUM TOTAL RESISTANCE

Bao-Ji Zhang
College of Ocean Science and Engineering, Shanghai Maritime University, Shanghai, 201306, China., zbj1979@163.com

Ai-qin Miao
College of Ocean Science and Engineering, Shanghai Maritime University, Shanghai, 201306, China.

Recommended Citation

Zhang, Bao-Ji and Miao, Ai-qin (2015) "THE DESIGN OF A HULL FORM WITH THE MINIMUM TOTAL RESISTANCE," *Journal of Marine Science and Technology*: Vol. 23: Iss. 5, Article 2.
DOI: 10.6119/JMST-014-0605-1
Available at: https://jmstt.ntou.edu.tw/journal/vol23/iss5/2

THE DESIGN OF A HULL FORM WITH THE MINIMUM TOTAL RESISTANCE

Acknowledgements

Foundation item: the National Natural Science Foundation of China (No. 51009087); Shanghai Natural Science Fund Project (No: 14ZR1419500); the Research Innovation Projects of 2013 Shanghai Postgraduate (20131129); the Top Discipline Projects of the Shanghai Municipal Education Commission.

Journal of Marine Science and Technology, Vol. 23, No. 5, pp. 591-597 (2015)
DOI: 10.6119/JMST-014-0605-1

THE DESIGN OF A HULL FORM WITH THE MINIMUM TOTAL RESISTANCE

Bao-Ji Zhang and Ai-qin Miao

Key words: boundary layer, viscous separation, Rankine source method, NLP.

ABSTRACT

In order to obtain a hull form which exhibits low resistance and highly-efficient energy-saving performance, the overall resistance should be calculated as the sum of wave-making and viscous resistance, in which the total resistance corresponds to the objective function whereas the hull geometry parameters correspond to design variables. Apart from considering the limited conditions due to appropriate displacement, we further ponder over the boundary-layer viscous separation caused by additional constraints. We then proceed to apply the Nonlinear Programming Method (NLP) to determine the hull form with the minimum total resistance. This paper aims to optimize the streamlined design of the S60 so as to get an improved hull form in which lower resistance and smoother hull lines are evident. This suggests that there is no significant increase in viscous resistance during the process of hull form optimization with the wave-making resistance as the objective function. Therefore, this confirms the feasibility of optimizing the hull form by the NLP method.

I. INTRODUCTION

A hull form with the minimum resistance is a common pursuit in the field of ship design. Nonetheless, previous hull form optimization schemes were mainly dependent on a series of ship model tests carried out at the initial stage, through which a new hull form was obtained via the transformation of the original hull form. However, since designers generally conduct hull form modifications according to principles derived from the personal experience of determining the preferable resistance performance, the optimal process of the practical hull form becomes a system which lacks scientific evaluation in terms of engineering (Zhao et al., 2010). During the 1980s, a series of research papers which mathematically combined hydromechanics with optimization techniques for hull form generation were published (Hsiung, 1981, 1984; Suzuki and Iokamori, 1999). However, this research determines a hull form with minimum wave-making resistance by using the bow-body (or the first half body) as the only objective of design and by keeping the stern lines (or the rear half body) fixed. Nevertheless, the design of the stern lines is a complex decision-making process, during which the effects of waves, viscosity, sea-keeping and propulsion performance must be considered comprehensively (Ma and Tanaka, 1994, 1995; Zhang and Xu, 2010) since viscous resistance may increase or decrease due to slight changes of the stern lines. Should boundary-layer viscous separation appear at the stern, the viscous resistance will obviously increase. Optimizing the lines of the stern or even the whole hull proves to be so difficult that scares off researches in this field. Based on the Michell integral method, Ma et al. (1994) proposed a simple separation judgment for two-dimensional turbulence which, with satisfactory results, was applied to design the hull form with minimum total resistance. In recent years, with the rapid development of computer technology and the continuous progress of numerical theory, the practicability of the CFD assessment has been certainly enhanced. Also, a comprehensive design process (Zhang, 2012) and a design of a hull form based on CFD have gradually become feasible (Mahmood and Huang, 2012).

Countries that are technologically advanced in shipbuilding, such as the United Kingdom, the United States, Japan and South Korea, have set up a design framework for the hull form optimization based on CFD and achieved remarkable results. However, the CFD calculation requires several hours and even longer time in order to calculate a hull form plan. If the optimization method is combined with the CFD to optimize a hull form, it would cost a long time, and would not meet the demands of the shipbuilding market to develop new hull forms (Zhang, 2009). Therefore, a feasible method of the hull form optimization design requires quick generation of hull forms with optimal output of the total resistance.

According to the judgment criteria for boundary layer separation suggested by Tanka (1997), hull design method of minimum total resistance based on Rankine source method is researched with consideration to the boundary-layer viscous separation. Its objective function is total resistance, which can

Paper submitted 02/22/13; revised 04/12/14; accepted 06/05/14. Author for correspondence: Bao-Ji Zhang (e-mail: zbj1979@163.com).
College of Ocean Science and Engineering, Shanghai Maritime University, Shanghai, 201306, China.

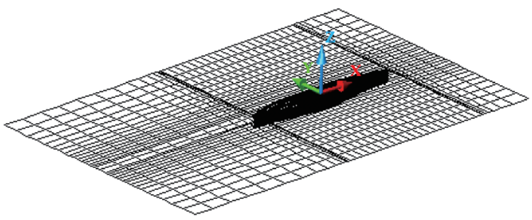

Fig. 1. Coordinate system of flow.

be calculated by the sum of wave-making resistance and viscous resistance. Also its design variables are the parameters of hull geometry shape. Then, considering the added constraint conditions of boundary-layer viscous separation and combining with nonlinear programming, the optimized model can be built. Optimized calculation of S60 was made as an example. The resistance of the obtained improved hull decreased apparently and the hull form became more smoothly. The result shows that viscous resistance do not increase significantly adopting hull optimized method with wave-making resistance as mainly part of the objective function. Thus, the hull optimized method is feasible.

II. RANKINE SOURCE METHOD

The Rankine source method (Tarafdera and Suzuki, 2008) is a numerical calculation method used to analyze wave-making resistance. It involves a double model flow rather than the uniform flow found in thin-ship theory. In a simulator, the Cartesian coordinate system is placed on the ship hull, and the x-axis and y-axis are fixed on the undisturbed water surface. The x-axis runs along the uniform flow toward the stern, where the z-axis points vertically upwards as shown in Fig. 1.

The velocity potential ϕ around a ship hull is expressed as the sum of basic velocity potential ϕ_0 of double model flow and the perturbed wave potential ϕ_1 with free- surface effect, namely

$$\phi = \phi_0 + \phi_1 \tag{1}$$

where

$$\phi_0(x, y, z) = Ux - \iint_{S_0} \sigma_0(x', y', z')\frac{1}{r_0}dS \tag{2}$$

$$\phi_1(x, y, z) = -\iint_{S_1} \sigma_1(x', y')\frac{1}{r_1}dx'dy' - \iint_{S_0} \Delta\sigma_0(x', y', z')\frac{1}{r_0}dS \tag{3}$$

$$r_0 = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$r_1 = \sqrt{(x-x')^2 + (y-y')^2 + z^2}$$

r_0 and r_1 denote the distance to the field point (x, y, z) from source points (x', y', z') and $(x', y', 0)$ respectively. S_0 is the hull surface of the double model and S_1 is the undisturbed free-surface ϕ_0 which should be calculated by the Hess-Smith method.

The equation of the free-surface can be expressed as:

$$z = \zeta(x, y) \tag{4}$$

The free-surface must satisfy the following boundary conditions:

a) The dynamic conditions

$$g\zeta + \frac{1}{2}(\phi_x^2 + \phi_y^2 + \phi_z^2 - U^2) = 0, (z = \zeta) \tag{5}$$

b) The kinematic conditions

After eliminating ζ from Eqs. (4) and (5), we get Eq. (6)

$$\phi_x\zeta_x + \phi_y\zeta_y - \phi_z = 0 \quad (z = \zeta) \tag{6}$$

$$\frac{1}{2}\left\{\phi_x(\nabla\phi)_x^2 + \phi_y(\nabla\phi)_y^2 + \phi_z(\nabla\phi)_z^2\right\} + g\phi_z = 0 \tag{7}$$

c) The free-surface linearizing conditions

$$\phi_{0l}{}^2\phi_{1ll} + 2\phi_{0l}\phi_{0ll}\phi_{1l} + g\phi_{1z} = -\phi_{0l}{}^2\phi_{0ll} \quad \text{on } (z = 0) \tag{8}$$

where the subscript l denotes the differentiation of the velocity potential along a streamline, ϕ_{0l} denotes the differentiation of the double-body velocity potential along a streamline on $z = 0$ and ϕ_{1l} denotes the differentiation of the perturbed velocity potential along a streamline on $z = 0$.

d) Hull boundary conditions

$$\frac{\partial\phi_0}{\partial n} = 0, \frac{\partial\phi_1}{\partial n} = 0 \text{ on } S_0 \tag{9}$$

The surface of the double model hull is divided into M_0 panels and the undisturbed free surface is divided into M_1 panels combined with discretization of boundary conditions (8) and (9) as

on the hull surface

$$\sum_{j=1}^{M_1}\sigma_1(j)N_1(ij) + \sum_{j=1}^{M_0}\Delta\sigma_0 N_0(ij) = 0$$

$$(i = 1, 2, \ldots, M_0 \text{ on } S_0) \tag{10}$$

$$N_k(ij) = -\iint_{\Delta S_K}\frac{\partial}{\partial n}(\frac{1}{r_k})dS, \quad k = 0, 1$$

on the undisturbed free-surface

$$\sum_{j=1}^{M_1}\sigma_1(j)A_1(ij)+\sum_{j=1}^{M_0}\Delta\sigma_0(j)A_0(ij)-2\pi g\sigma_1(i)=B(i)$$

$(i = 1, 2, \ldots, M_1$ on $S_1)$ (11)

$$A_k(ij)=\phi_{0l}^2(i)CL_k(ij)+2\phi_{0l}(i)\phi_{0ll}(i)L_k(ij)\ (k=0,1)$$

$$B(i)=-\phi_{0l}^2(i)\phi_{0ll}(i)$$

The calculation of the differentiation along a streamline in Eqs. (8) and (11) applies a four point upwind difference scheme along the streamline proposed by Dawson (1972) to meet non-wave radiation conditions at the front of the hull. This reflects the physical truth that the upstream disturbance only transmits itself to the downstream whereas the downstream disturbance indirectly affects the upstream. Actually, the calculation applies a low-order unilateral difference scheme both near the stern and in the calculation area in order to increase the numerical viscosity reflecting the actual fluid effects.

Based on Eqs. (10) and (11) and by combining the upstream conditions, the final set of equations is a full matrix, asymmetric and without preferential diagonal. This set of equations is solved via the Gauss elimination method. Then σ_1 and $\Delta\sigma_0$ of the discrete sources on hull surface and the still water surface are obtained, which can be used to express the perturbed wave potential ϕ_1 later ignoring high order terms.

$$p-p_0=\frac{1}{2}\rho(U^2-\phi_{0x}^2-\phi_{0y}^2-\phi_{0z}^2-2\phi_{0x}\phi_{1x}-2\phi_{0y}\phi_{1y}-2\phi_{0z}\phi_{1z}) \tag{12}$$

Also, the wave height of the free surface can be expressed as:

$$\zeta(x,y)=\frac{1}{2g}(U^2-\phi_{0x}^2-\phi_{0y}^2-2\phi_{0x}\phi_{1x}-2\phi_{0y}\phi_{1y}) \tag{13}$$

The wave-making resistance obtained by the integration of pressure on the hull surface is:

$$R_w=\sum_{i=1}^{M_0}\{p(i)-p_0\}n_{xl}\Delta S_{0l} \tag{14}$$

where ΔS_{0l} denotes the area of a panel on the hull surface, and n_{xl} is the direction cosine of the normal to the panels.

III. THE OPTIMAL MODEL

1. Objective Function

In the present study, the total resistance R_T is selected as an objective function in the optimization process, where the R_T is expressed as the sum of wave-making resistance R_w and viscous resistance $(1 + k)\ R_F$, namely,

$$R_T=P\cdot R_w+(k+1)\cdot R_F\to\min \tag{15}$$

where the correction coefficient of wave-making resistance P can be expressed as a ratio which is calculated by dividing the experiment value into the theoretical computational value at the design speed of original hull.

The form factor k is calculated by the following formula (Hirayama and Ando, 2008):

$$k=(\frac{V^{1/3}}{L})\cdot(0.5C_B+\frac{2\gamma^{1.3}}{C_B}) \tag{16}$$

where, $\gamma=(b/L)/\{1.3(1-C_B)-0.031\cdot l_{cb}\}$, V is the displacement volume, b the breath molded, C_B the block coefficient, γ the full degree of stern and l_{cb} the longitudinal position of the center of buoyancy.

The R_w is calculated by the Rankine source method illustrated below:

$$R_W=\frac{1}{2}\cdot\rho\cdot U^2\cdot L^2\cdot C_{W,L} \tag{17}$$

where C_w is the wave-resistance coefficient, U is the design speed, L is the ship length between the perpendiculars and ρ is the density of the fluid.

The frictional resistance is calculated by the following formula:

$$R_F=\frac{1}{2}\cdot\rho\cdot U^2\cdot S^2\cdot C_{f0}$$

$$C_{f0}=0.463(\log_{10}\mathrm{Re})^{(-2.6)} \tag{18}$$

where C_{f0} is the non-dimensional friction drag coefficient，Re is the Reynolds number based on the body length and S is the wet-surface area.

2. Design Variables

This paper takes the whole hull as the optimal design length, in which the waterlines, the bottom of the hull and the bow and the stern of the hull body are fixed as shown in Fig. 2.

An improved hull surface $y(x, z)$ can be achieved by multiplying modification function $w(x, z)$ (Hirayama and Ando, 2008) on the basis of the original hull surface $f_0(x, z)$, namely

$$y(x,z)=f_0(x,z)\cdot w(x,z) \tag{19}$$

$$w(x,z)=1-\sum_m\sum_n\alpha_{mn}$$

$$\sin\left[\pi(\frac{x-x_0}{x_{\min}-x_0})^{m+2}\right]\cdot\sin\left[\pi(\frac{\beta-z}{\beta+T})^{n+2}\right]$$

$m, n = 1, 2, 3,\ -L/2\le x\le 0$

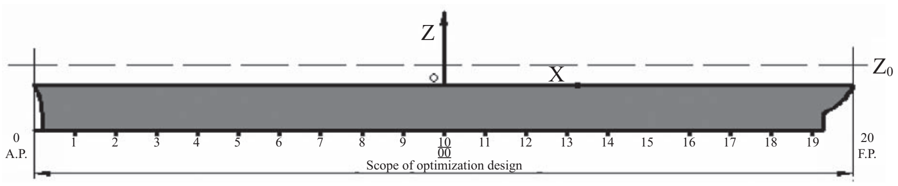

Fig. 2. The scope of optimization design of the S60 hull form.

$$w(x,z) = 1 - \sum_m \sum_n \alpha_{mn}$$

$$\sin\left[\pi(\frac{x-x_0}{x_{\max}-x_0})^{m+2}\right] \cdot \sin\left[\pi(\frac{\beta - z}{\beta + T})^{n+2}\right]$$

$$m, n = 1, 2, 3, \ 0 \le x \le L/2$$

$$w(x, z) > 0 \ \ (x > x_0, z < z_0)$$

In the above equation, L represents the longitudinal coordinate of the front-most part of the bow and T is generally the greatest depth which can be modified. If the baseline remains unchanged, then T is the draft. By fixing m, n = 1, 2, 3, 18 variables for A_{mn}, are obtained along with 18 design variables. B can also be a design variable, which ranges from T to z_0. In sum, the number of the design variables will not surpass 20.

3. Constraints

Two basic constraints were selected as follows:

(1) All offsets are non-negative, namely $y(i, j) \ge 0$, where $y(i, j)$ denotes the coordinate of the hull form;
(2) The displacement should meet the demand of

$$\nabla \ge \nabla_0;$$

where ∇ and ∇_0 denote the displacement of the improved hull form and the original hull respectively.

IV. A METHOD OF JUDGING THE STERN BOUNDARY-LAYER VISCOSITY SEPARATION

The hull form optimization aims to minimize the total resistance under the non-separated condition near the stern. Since the problem of three-dimensional turbulence separation is too complex, this paper adopts a simple judgment of two-dimensional turbulence separation proposed by Ma and Tanaka (1997) to determine the separated domain of the original ship and improved hull form. The judgment of the boundary-layer viscosity separation is defined as:

$$C(s) \equiv \frac{1}{U^5}\frac{dU}{dS}\int_0^s U^4 ds \approx -2 \tag{20}$$

where U is the flow velocity on the outer edge of the boundary-layer. This paper takes Eq. (20) as the condition to determine whether the streamlines on the hull surface will eventually separate, which makes it effortless to obtain the separation point. U can be approximately substituted by the potential velocity on the hull surface by applying the Hess-Smith method to calculate potential velocity distribution on the hull surface and then using streamline tracking method to seek each of streamlines on the hull surface and the velocity distribution on these. After that, the $C(S)$ distribution of each streamline is calculated, paving the foundation for judging whether there is separation or not according to Eq. (20) to determine the location of the separation point. Finally, all the separation points are connected to each streamline to achieve the separated domain.

This separated domain will be used as the additional constraint

$$\left| y(i, j) - y_0(i, j) \right| \le b$$

where $y_0(i, j)$ and $y(i, j)$ respectively represent the coordinates of the surface of the original hull and those of the improved hull.

The optimization of the hull form is conducted under basic constraints to judge whether the shape of the improved hull meets the demands of the design, and then consideration on the additional constraints is taken, optimizing and re-calculating them. The controlling value b is judged from the resultant of the stern separation or through the experience of the author.

V. THE OPTIMAL METHOD

Generally speaking, in engineering optimization problems, the objective functions are nonlinear with respect to the design variables, and complex design constraints are imposed. To solve these optimization problems, a nonlinear programming (NLP) technique could be employed. In this study, SUMT was selected as the NLP to minimize the objective function under the design constraints.

A flow chart of the optimization calculation is shown in Fig. 3. Firstly, the offset file of the original hull is inputted, which includes the main elements of the original hull, the offsets and the design range, as well as the number of the design variables, the design speed and the initial parameters of the optimal calculation. Secondly, according to the Hess-Smith and the streamlines-tracking method, the velocity

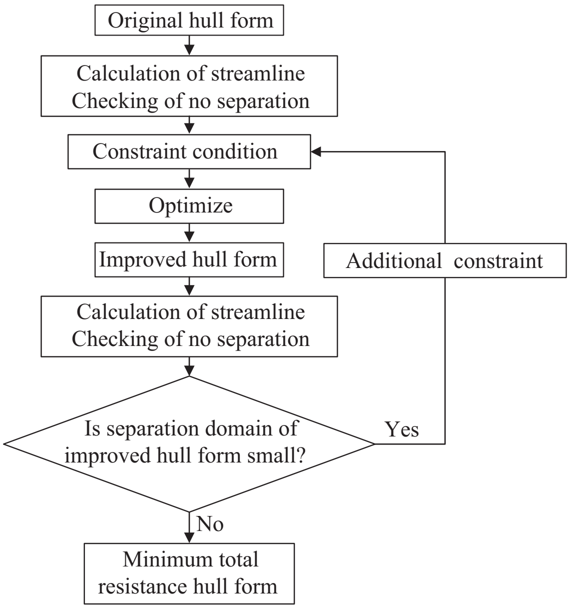

Fig. 3. Flow chart of the optimization calculation.

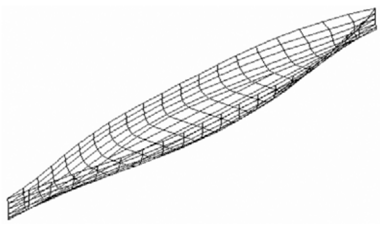

Fig. 4. Hull gird division of S60 hull form.

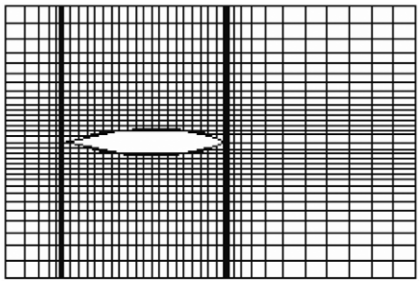

Fig. 5. Grid division of free surface.

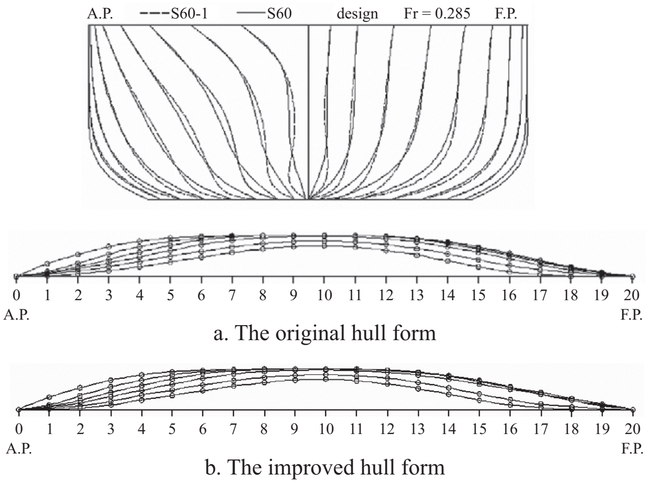

Fig. 6. The comparison of the body lines and waterlines between the improved hull form (S60-1) and the original hull form.

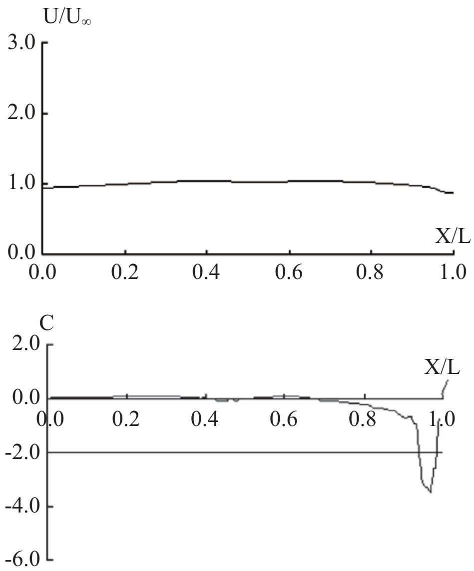

Fig. 7. The velocity and separation distribution along the streamlines on the loaded draft.

distribution of each streamline can be obtained on the hull surface, and the separated domain of the original hull can also be obtained. The improved hull form will be acquired by optimizing and calculating it according to the basic constraints (1), (2) and NLP. Subsequently, through further application of the basic two-dimensional separation judgment to get the separation point of each streamline of the improved hull form, the separation domain will be found. By the analysis of this domain and checking whether the domain is greater than the original domain, an additional constraint (3) can then be added to the domain that will eventually be returned to the initial condition and result in the repeated operation above. On the other hand, if an optimal calculation is completed, a hull form with a minimum total resistance will have been acquired.

VI. EXAMPLES

In this paper, the S60 hull form has been selected as the original hull to be optimized. The hull surface is divided into 100 panes, and the free surface is divided into 704 panes, as shown in Figs. 4 and 5. Fig. 6 shows the bodylines and waterlines of the original and improved hull form after optimization. These lines of the improved hull form change greatly in the astern area. Generally, the changes of the stern hull lines supposedly lead to the viscosity separation, thus resulting in the increase of the viscous resistance. So a determination that

Table 1. The result of the optimal calculation.

Hull form	Constraints	Design Fr	R_W/R_{W0}	R_T/R_{T0}	V/V_0	S/S_0
S60-1	(1), (2)	0.285	83.8%	92.8%	1.007	1.004
S60-2	(1), (2), (3)	0.285	86.8%	95.5%	1.003	1.000

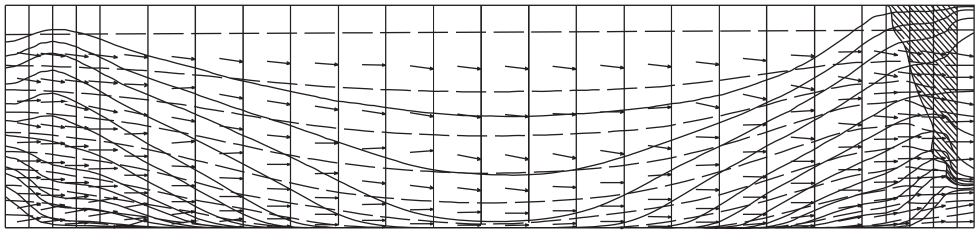

Fig. 8. The turbulence separation area of S60 (the original hull).

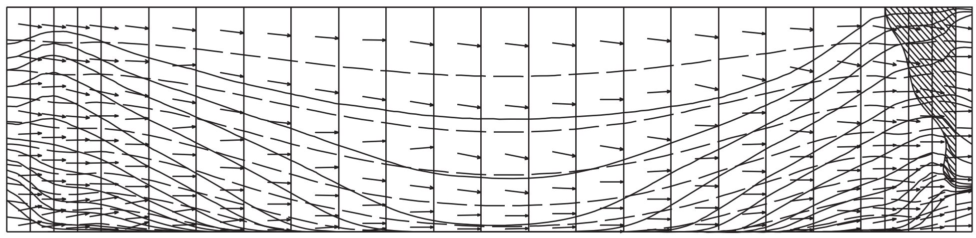

Fig. 9. The turbulence separation area of S60 (S60-1).

whether the viscous separation in this area is needed should be made. This paper applies the simple judgment of two-dimensional turbulence separation proposed by Ma and Tanaka (1997) to determine the separated domain. The result is as follows:

Fig. 7 shows the streamline velocity distribution curve and the C distribution curve of the hull on the design waterline. From the figure, the C distribution curve near the hull bow is not equivalent to -2, so the separation will not occur. However, when a separation and an attachment occur near the stern which can be judged from the separation of each streamline, the turbulent separated domains are accordingly procured as shown in Figs. 8 and 9.

In the figure, the shadow area is the range of separation, while Fig. 8 is the original hull and Fig. 9 the improved hull. Comparing the size of separated domains (side plan) in the two figures, the domain of the improved hull can be found being as large as the original hull. It turns out that the viscous resistance does not obviously increase in the optimal process considering the wave-making resistance as the main objective function. Therefore, this suggests the feasibility of optimizing the hull form by this method.

Table 1 shows the results of the optimal calculation. The wave-making resistance of the improved hull form (S60-1) was decreased by 16.2% and the total resistance was decreased by 7.2% under the design Fr. Meanwhile, the wave-making

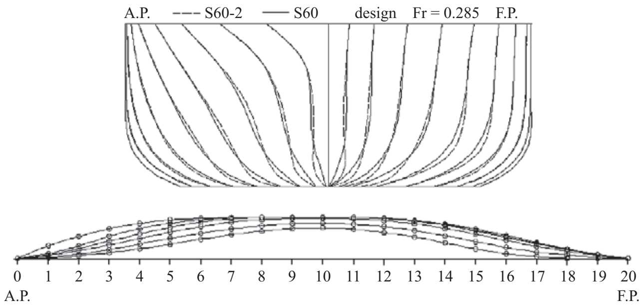

Fig. 10. The comparison of the body lines and waterlines between the improved hull form (S60-2) and the original hull form.

resistance of the improved hull form (S60-2) was decreased by 13.2% and the total resistance was decreased by 4.5% under the design Fr. Fig. 10 shows the comparison of the body lines and waterlines between the improved hull form (S60-2) and the original hull form. The bodylines and waterlines of S60-1 was slightly decreased in the outwardly bulged trend compared with S60-2. Fig. 11 indicates the comparison (S60-1) of the wave resistance coefficient curves between the original and the improved (S60-2) hull forms. Basically, the fluctuating trends of the wave resistance coefficient curves between the original and the improved hull forms are identical. At a certain range of Fr, the wave-making resistance of the

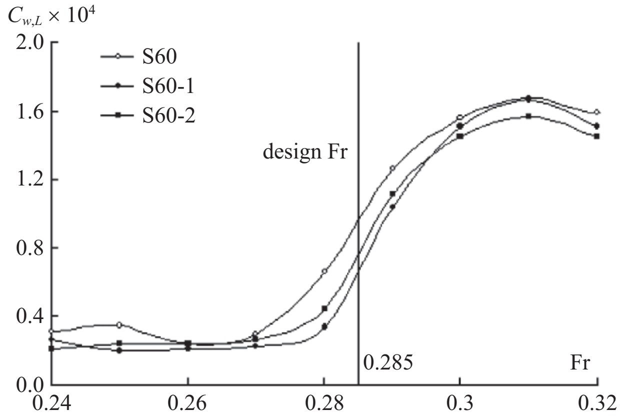

Fig. 11. Comparison of wave making resistance coefficient for two improved hulls.

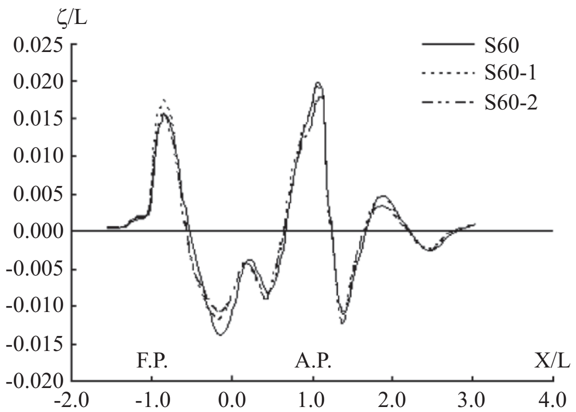

Fig. 12 . Comparison of the wave profiles along the S60 hull.

improved hull form has decreased to a certain degree. Fig. 12 shows the comparison of the waveform chart of one side of the ship between the original and the improved hull form. The wave height at one side of the improved hull form was decreased at different degrees in mid-ship and stern.

VII. CONCLUSION

This paper introduces a simple separation judgment for two-dimensional turbulence to evaluate the separation of the original and the improved hull forms respectively. It proposes a method of hull design with a minimum total resistance, which takes the whole hull lines as the design object, regards decreasing the wave-making resistance as the main goal, and simultaneously takes into consideration the effects of viscous separation at the stern. This paper also develops an optimal design procedure of the whole hull lines by taking S60 as the example to optimize the whole hull forms, obtaining the improved hull forms with the slight changes of the hull lines with obvious improvements of the performance of the total resistance. This idea can provide the theoretical basis and technical support for the lines optimization of the stern or even the whole hull, which will therefore accelerate the development of a new ship and the optimal technology of the hull form.

ACKNOWLEDGMENTS

Foundation item: the National Natural Science Foundation of China (No. 51009087); Shanghai Natural Science Fund Project (No: 14ZR1419500); the Research Innovation Projects of 2013 Shanghai Postgraduate (20131129); the Top Discipline Projects of the Shanghai Municipal Education Commission.

REFERENCES

Hirayama, A. and J. Ando (2008). Study on Multiobjective hull optimization for reducing resistance in still water and added resistance in waves-multi-objective optimization using real-coded genetic algorithm. The Japan Society of Naval Architects and Ocean Engineers 7, 213-222.

Hsiung, C. C. (1981). Optimal ship forms for minimum wave resistance. Journal of Ship Research 25(2), 95-116.

Hsiung, C. C. (1984). Optimal ship forms for minimum total resistance. Journal of Ship Research 28(3), 163-172.

Ma, K. and I. Tanaka (1994) A study of minimum resistance hull form with considerati-on of separation (2nd Report). Journal of the Kansai Society of Naval Architects of Japan 222, 41-47.

Ma, K. and I. Tanaka (1994). A study of minimum resistance hull form with consideration of separation (1st Report). Journal of the Kansai Society of Naval Arch- itects of Japan 221, 9-15.

Ma, K. and I. Tanaka (1997). A study of optimal hull from for minimum resistance. Journal of Hydrodynamics, Ser.A 12(1), 113-122.

Mahmood, S and D. B. Huang (2012). Computational Fluid Dynamics Based Bulbous Bow Optimization Using a Genetic Algorithm. J. Marine Sci. Appl 11, 286-294.

Suzuki, K. and N. Iokamori (1999). Studies on minimization of wave making resistance based on Rankine source method. Journal of the Kansai Society of Naval Architects of Japan 185, 9-19.

Tanaka, I. and K. Ma (1995). A method of determining hull form of low resistance and high propulsive efficiency. Journal of the Kansai Society of Naval Architects of Japan 223, 49-57.

Tarafdera, M. S. and K. Suzuki (2008). Numerical calculation of free-surface potential flow around a ship using the modified Rankine source panel method. Ocean Engineering 35(6), 536-544.

Zhang, B. J. (2009). Research on optimization design of hull Lines and minimum resistance hull form, Ph.D., Dissertation, Departrment of Ship Engineering, Dalian University of Technology, Dalian, China.

Zhang, B. J. (2012). Research on hull lines optimization for minimum resistance based on Rankine source method. Journal of Marine Science and Technology 20(1), 89-94.

Zhang, B. J. and L. P. Xu (2010). An optimization study of overall hull lines with consideration of viscous separation for stern. Journal of Shanghai Maritime University 31(4), 38-41.

Zhao, F., S. Z. Li and L. Yang (2010). An overview on the design optimization of ship hull based on CFD techniques. Journal of Ship Mechanics 14(7), 812- 821.