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SHORT TERM STRATEGIES FOR SOLVING A VARIANT OF PERIOD VEHICLE ROUTING PROBLEM

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Key words: heuristics, integer programming, postponable delivery.

ABSTRACT

A company faces a variant of Period Vehicle Routing Problem (PVRP). Because of seasonal fluctuation in demand, the company outsources delivery to avoid maintaining excess vehicles or facing vehicle shortages. Customer orders can be classified into two groups, those that must be satisfied within two days, and those that must be fulfilled within three days. Currently, the company satisfies most orders the next day based on experience rather than any formal system. The objective of the studied company is to satisfy all received orders and minimize monthly transportation costs. This study proposes two short term strategies for the studied company. Both the mathematical programming model and heuristic algorithms are developed to compare the performance of the two strategies. Extensive computational results are provided based on real world data. The proposed strategies achieve promising savings in transportation costs.

I. INTRODUCTION

With increased competitiveness among supply chains and rising fuel costs, periodic delivery operations are attracting increasing scrutiny. Periodic deliveries have numerous real world applications, including vending machine replenishment, waste collection, courier services, and automobile parts distribution. These problems are modeled as Period Vehicle Routing Problems (PVRP). The objective of PVRP is to minimize the total travel cost for several vehicles that must visit a number of customers during a planning period of several days.

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Our motivation for this study stems from observations of a leading food manufacturer in Taiwan. The ice cream department of this studied company is actively introducing new technology from overseas and developing new products to enhance its competitiveness. Delivering ice cream products to its customers is a key operation of this company. Because of seasonal fluctuation in demand, the company outsources delivery to avoid maintaining excess vehicles or facing of vehicle shortages. The outsourcing logistics company takes responsibility for delivery according to the instructions of the studied company. Customer orders can be classified into two groups, namely those to be delivered within two days, and those to be delivered within three days.

Currently, the studied company satisfies most orders on a next day timeframe based on experience rather than any formal system. The goal of the studied company is to satisfy all received orders and minimize monthly transportation costs. The purpose of this paper is to propose two short term strategies for the studied company. In order to compare the performance of two strategies, both the mathematical programming model and heuristic algorithms are developed.

The first PVRP was introduced by Beltrami and Bodin (1974) for assigning trucks in municipal waste collection. Russell and Igo (1979) provided a formal definition of the PVRP. Other heuristic algorithms were developed by Christofides and Beasley (1984), and Tan and Beasley (1984) in 1980s. Russell and Gribbin (1991) proposed a solution method that consists of an initial route design using a network approximation. Chao et al. (1995) developed a metaheuristic method to solve the PVRP. The initial solution was obtained using the formulation by Christofides and Beasley (1984) and the optimal solution was approached through iterative improvement steps. Cordeau et al. (1997) presented a tabu search heuristic for solving several routing problems, including the PVRP. Moreover, Drummond et al. (2001) introduced a parallel genetic algorithm for the PVRP. Alegre et al. (2007) addressed a scatter search algorithm for instances with long time periods. Additonally, Hemmelmayr et al. (2009) developed a metaheuristic solution for the PVRP based on variable neighborhood search. Recently, Coene et al. (2010) proposed a two phase algorithm for the PVRP. Gulczynski et al. (2010) presented a heuristic for the PVRP using an integer programming model and the enhanced record-to-record travel algorithm developed by Groër et al. (2010). The heuristic algorithm produced high quality results on standard benchmark instances. Hamzadayi et al. (2013) developed a nested simulated annealing procedure for solving a periodic traveling salesman problem for a retail distribution system.

Some variants of PVRP have been discussed in the literature. First, Cordeau et al. (1997) proposed a tabu search heuristic algorithm for the period and multi-depot vehicle routing problem. Hadjiconstantinou and Baldaccci (1998) then extended the PVRP to Multi-Depot Vehicle Routing Problem (MDVRP). Subsequently, Angelelli and Speranza (2002) considered a PVRP with intermediate facilities that is similar to the MDVRP. Ngueveu et al. (2013) presented new lower bounding procedures and an exact method for the m-peripatetic vehicle routing problem. Second, Cordeau et al. (2001) extended the earlier work by Cordeau et al. (1997) to include time windows (PVRPTW). Finally, Baptista et al. (2002) presented a PVRP case study in which customer demand is a random variable and the objective function is modified to treat demand as profit. Making visit frequency a decision of PVRP, Francis et al. (2006) suggested a PVRP with service choice (PVRPSC). The latest work is the Dynamic Multi-Period Routing Problem (DMPRP) of Angelelli et al. (2009). They considered a distribution system in which customer orders arrive dynamically, and orders arising on a certain day must be satisfied on either the same day or the following day.

Given a three day planning period, the studied company faces a variant of PVRP. This variant of PVRP we study in this paper is characterized by some distinctive features. First of all, orders arising on a certain day (d = 0) must be satisfied within two days (d ≤ 2) or three days (d ≤ 3). Thus, these orders can be classified as unpostponable (d = 1) or postponable (d = 2 or d = 3). Secondly, at the beginning of a three-day planning period some orders are already known, while others may arrive during the three-day planning period (orders are unknown). Thirdly, the studied company is outsourcing the delivery of products by truckload contract. Within the same region, the daily operating cost of a certain type of truck is identical, so the travel distance is not a major concern. In order to minimize the transportation cost, we attempt to find the optimal combination number of trucks rather than minimize the travel distance. This characteristic differs from traditional VRP. Fourthly, the temperature required to preserve ice cream products is -18°C. Frequent opening of a truck door will cause sudden temperature increases, so the number of delivery locations for each truck cannot exceed three to prevent the products from melting. Lastly, rolling the planning period of three days forward every day, we repeatedly solve the variant of PVRP to minimize monthly transportation cost.

The study has two main contributions. First, to the best of our knowledge, it presents the first application of a variant of PVRP to ice cream distribution. Second, from a problem oriented perspective, the computational results of the strategies show that heuristic algorithms yield significant cost savings.

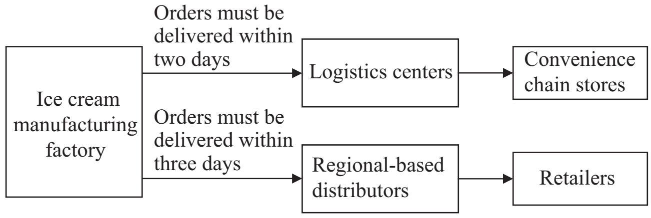

Fig. 1. Ice cream distribution channels of the studied company.

The remainder of this paper is organized as follows. Next section is the description of the problem, and heuristic algorithms are introduced in Section III. Section IV presents some computational results. Finally some concluding remarks and suggestions for future research are provided in Section V.

II. PROBLEM DESCRIPTION

1. Characteristics of the Ice Cream Industry

Numerous products compete in the domestic ice cream market in Taiwan. In terms of market share, popsicles and ice cream bars account for 55-60%, ice cream represents approximately 35%, while ice cream soft-drinks and sandwiches account for about 5%, each with different price positioning and consumer groups. Although the peak season for ice cream consumption lasts approximately six months (from April to September), ice cream products suffer from a short life cycle. To remain competitive, ice cream manufacturers must constantly introduce new products. Recently, rising living standards and purchasing power in Taiwan have led to many imported ice cream products entering the Taiwan market. According to a survey commissioned by the studied company, the total retail market value of packaged ice cream in 2009 was about $3.2 billion NTD, growing by about 7% compared with $3 billion NTD in 2008.

2. Ice Cream Distribution Channels of the Studied Company

The studied company is a leading food manufacturing company in Taiwan. The company is a listed company with a business scope that covers food manufacturing, retailing, and logistics. The ice cream department of the company is actively introducing overseas technology and developing differentiated new products to enhance its competitiveness.

The studied company distributes ice cream through two distribution channels, as shown in Fig. 1. One channel delivers products from the factory to convenience chain stores via logistics centers. The other channel delivers products from the factory to retailers via the warehouses of regional distributors. Because of the high storage costs of ice cream, both the logistics centers and regional distributors employ “small amount, multiple trips” delivery strategies to reduce storage costs. However, the two channels differ in delivery time requirements. Orders from logistics centers must be satisfied within two days after order receipt. Nevertheless, regional distributors are more flexible and can accept delivery within three days of order receipt.

3. Ice Cream Delivery Practices of the Studied Company

The ice cream market is characterized by marked peak and trough seasons. Therefore, product delivery using self-owned vehicles will result in insufficient vehicles during the peak season and excess vehicles during the trough season. Presently, the studied company is outsourcing the delivery of ice cream products by truckload contract to avoid maintaining excess vehicles or facing vehicle shortage. The truck door must be opened during ice cream delivery, increasing the temperature inside the truck and affecting the product quality. Hence, each truck cannot service more than three delivery locations per trip. The studied company uses two types of trucks, a 15-ton truck and a 20-ton truck, to delivery ice cream. The loading capacity of a 15-ton truck and a 20-ton truck are 900 cubic feet and 1200 cubic feet, respectively. Because all shipments of the studied company are located within the same geographical region, the studied company signed the contract with truckload carriers based on a daily freight allowing each truck to delivery three shipments per day. Currently, the daily freight of a 15-ton truck is NT$16,500; meanwhile, the daily freight of a 20-ton truck is NT$17,500.

The order processing of the studied company involves collecting orders from various logistics centers and regional distributors. Furthermore, the studied company passes the order information to the outsourcing logistics company. The outsourcing logistics company then takes responsibility for delivery based on the instructions of the studied company. Currently, the studied company creates the daily delivery plan based on experience rather than any formal system. By carefully examining the daily delivery plan, this study found that the studied company satisfies most orders on the next day and that current operation frequently results in assigning excess trucks or trucks with high wasted loading capacity.

4. Proposed Short Term Strategies

Discussion with the logistics manager of the studied company clarified that the objective is to satisfy all received orders and minimize monthly transportation costs. Based on a three day planning period, the company faces a variant of PVRP. With a planning period of one month, a common approach can be used to solve the problem based on the repeated solution of a planning period of three days (namely re-optimization). Although this approach is widely accepted, no guarantee exists that a good solution can be obtained without the right objective to optimize. Hence, this study proposes two short term strategies.

Strategy I satisfies all orders on the next day. Since the studied company satisfies most orders the next day based on personal experience, this study follows similar reasoning by solving the problem using a formal method, an integer programming model. Clearly, a formal method outperforms personal experience and saves transportation costs.

Strategy II satisfies all orders within the due dates and minimizes the number of assigned trucks. As mentioned above, the current operation of the studied company frequently results in assigning excess trucks or trucks with high wasted loading capacity. It is reasonable to fully load each truck and to minimize wastage of loading capacity. Transportation costs are reduced when the number of assigned trucks is decreased.

Two mathematical models were formulated to solve Strategies I and II. Due to limited spaces, we will not present mathematical models in the paper. However, they are available upon request.

III. HEURISTIC ALGORITHMS

This section describes two algorithms, called HS1 and HS2, for solving the proposed Strategy I and Strategy II, respectively. To see how Strategy I and Strategy II work out in actual implementation, this study shows the pseudo codes of HS1 and HS2 as follows.

Both heuristic algorithms can be divided into three main steps. The following describes HS1 and HS2 by separately examining their main steps.

Algorithm HS1

```
1. for i ← 1 to n
2.   Total loading capacity ← Total loading capacity + Demand [i]
3. do Min_Car_No ← ⌈n/3⌉
4. Combination1 (Min_Car_No, Total Capacity, C20T_No, C15T_No)
5. Initial (n, Demand, Due date, C20T_No, C15T_No, Assigned, Unassign)
6. for i←1 to n
7.   if due date [i]=1
8.     then color [i]=Red
9.     else if due date [i]=2
10.       then color [i]=yellow
11.       else color [i]=green
12. Unassign_empty (n, Unassign, Empty)
13. if Empty = TRUE
14.   then STOP
15.   else Insertion (n, Demand, Due date, C20T_No, C15T_No, Assigned, Unassign)
16. if Empty= TRUE
17.   then STOP
18.   else if C15T_No >0
19.   then C20T_No ← C20T_No+1
20.     C15T_No← C15T_No-1
21.   Insertion (n, Demand, Due date, C20T_No, C15T_No, Assigned, Unassign)
22.     else C15T_No ← C15T_No+1
23.   Insertion ( n, Demand, Due date, C20T_No, C15T_No, Assigned, Unassign)
24. STOP
```

Algorithm HS2

```
1.  for i ←1 to n
2.      Total loading capacity ← Total loading capacity +
        Demand [i]
3.  Combination2 (Total capacity, C20T_No, C15T_No)
4.  Initial (n, Demand, Due date, C20T_No, C15T_No,
    Assigned, Unassign)
5.  for i←1 to n
6.  if due date[i]=1
7.  then color[i]=Red
8.  else if due date[i]=2
9.      then color[i]=yellow
10.     else color[i]=green
11. Unassign_empty (n, Unassign, Empty)
12. if Empty = TRUE
13.     then STOP
14. else Insertion (n, Demand, Due date, C20T_No, C15T_No,
    Assigned, Unassign)
15.     Reduction (n, Demand, Due date, C20T_No, C15T_No,
        Assigned, Unassign)
16. STOP
```

1. Determining the Vehicle Combination

The first step of HS1 and HS2 is determining the number and type of trucks assigned to serve customers. At the beginning of HS1 and HS2, the total loading capacity is obtained by summing the required loading capacity of all customers (lines 1-2). Besides the total loading capacity, the minimum number of trucks is also calculated (line 3). Procedures Combination1 and Combination2 are used to determine the vehicle combination for Strategy I and Strategy II, respectively. Based on the total loading capacity and minimum number of trucks, procedure Combination1 looks up the vehicle combination based on Table 1. For example, the number of customers waiting for service on the first day is 9 and the total loading capacity is 4255 cubic feet (see Table 2). The 13th combination in Table 1 is chosen because it satisfies both the number of served customers and total loading capacity as well as minimizing total cost. One 15-ton truck and three 20-ton trucks are suggested for delivery by procedure Combination1.

Procedure Combination2 also looks up the vehicle combination in Table 1. The difference between both strategies (procedures) is that Strategy II satisfies every order within its due date rather than pursing next day order satisfaction. Consequently, procedure Combination2 does not consider the minimum number of trucks. Using the data mentioned above as an input, and searching down the rightmost column of Table 1, Combination2 finds the 13th vehicle combination that can satisfy the total capacity 4255 cubic feet. Next, Combination2 chooses the vehicle combination by moving one row up and suggesting the 12th vehicle combination. The intuitive reason for moving one row up is to decrease unused loading capacity and minimize the transportation cost, because with Strategy II there is no need to satisfy all orders the next day. Finally, two 15-ton and two 20-ton trucks are be assigned for delivery.

Table 1. Combination of various types of vehicles configuration.

Combination	15-ton truck	20-ton truck	Total number of trucks	Maximum number of served customers	Total cost	Load capacity (cubic feet)
1	1	0	1	3	16500	900
2	0	1	1	3	17500	1200
3	2	0	2	6	33000	1800
4	1	1	2	6	34000	2100
5	0	2	2	6	35000	2400
6	3	0	3	9	49500	2700
7	2	1	3	9	50500	3000
8	1	2	3	9	51500	3300
9	0	3	3	9	52500	3600
10	4	0	4	12	66000	3600
11	3	1	4	12	67000	3900
12	2	2	4	12	68000	4200
13	1	3	4	12	69000	4500
14	0	4	4	12	70000	4800
15	5	0	5	15	82500	4500
16	4	1	5	15	83500	4800
17	3	2	5	15	84500	5100
18	2	3	5	15	85500	5400
19	1	4	5	15	86500	5700
20	0	5	5	15	87500	6000
21	6	0	6	18	99000	5400
22	5	1	6	18	100000	5700
23	4	2	6	18	101000	6000
24	3	3	6	18	102000	6300
25	2	4	6	18	103000	6600
26	1	5	6	18	104000	6900
27	0	6	6	18	105000	7200

2. Initial Solution Construction

Fig. 2 illustrates an example of constructing an initial solution based on the first day order. The detail for constructing the initial solution is described as follows.

(1) Set the unused truck capacity of all trucks.
(2) Arrange all trucks according to descending order of capacity.
(3) Sort all customers according to descending order of required loading capacity. Starting at the top of the list, do the following.
(4) If the unused truck capacity is greater than or equal to the required loading capacity of the customer, proceed to step (5) otherwise execute step (6).
(5) Assign the customer to this truck, update the unused capacity of the truck and set the truck index to another truck.
(6) Set the truck index to another truck. Go to step (4).

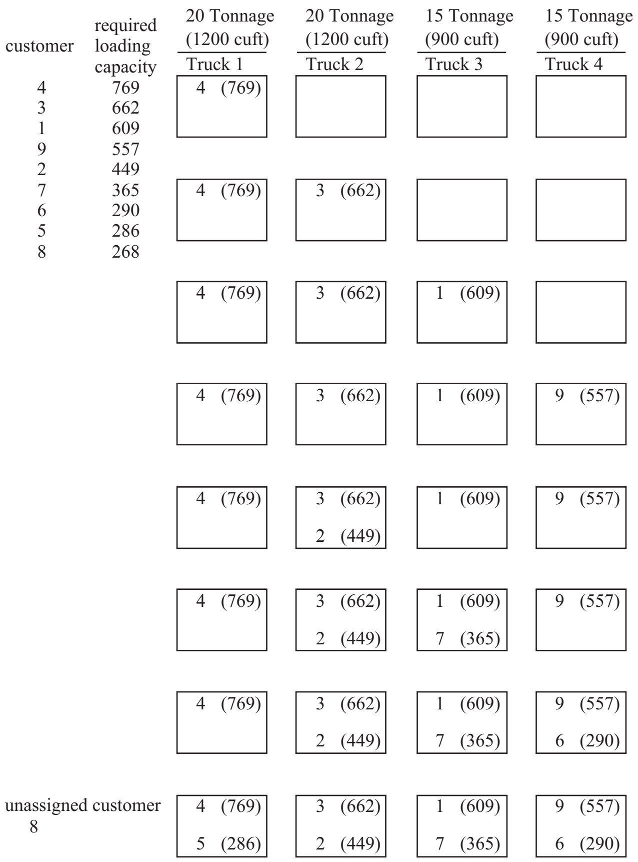

Fig. 2. An illustrative example of constructing an initial solution.

(7) Repeat Steps (4), (5), and (6) until no more customers can be assigned.
(8) Output all assigned trucks and unassigned customers.

Combining steps (1) and (2) can reduce wasted truck capacity and the unassigned loading capacity, because it allows customers to be assigned a high loading capacity from the beginning. As the process continues, the program keeps assigning customers high loading capacities until finally, the unassigned customers and unassigned loading capacities become small.

3. Refining Stage

Procedure Unassign-empty is used to check whether the unassigned customer is empty. If true, the program stops otherwise the program moves down to the refining stage. This stage comprises two main procedures, Insertion and Reduction.

1) Insertion Procedure

The main function of the Insertion procedure is allocating the unassigned customers to one of the current trucks. This procedure consists of three procedures, direct insertion, (1-1) exchange and (2-1) exchange. Figs. 3-5 show the main idea of each procedure.

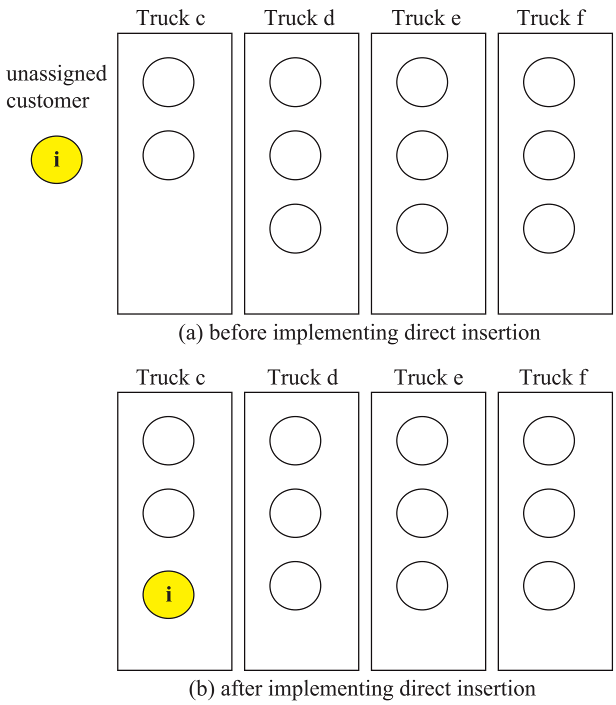

Fig. 3. An illustrative example of direct insertion procedure.

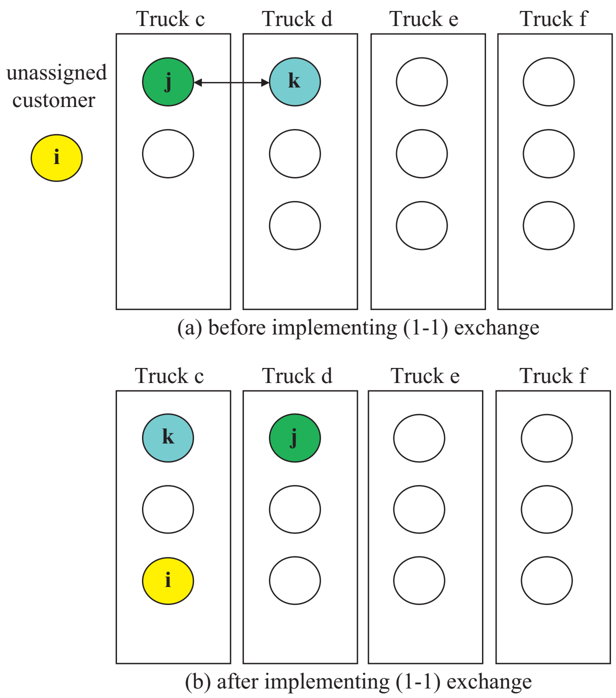

Fig. 4. An illustrative example of (1-1) exchange procedure.

Let demand[i] denote the required loading capacity of customer i, unused[c] represent the unused truck capacity of truck c and assigned_no[c] be the number of assigned customers of truck c, respectively. Given an initial solution, this study searches for all customers i and truck c. Fig. 3 shows an illustration of a direct insertion that can be obtained by

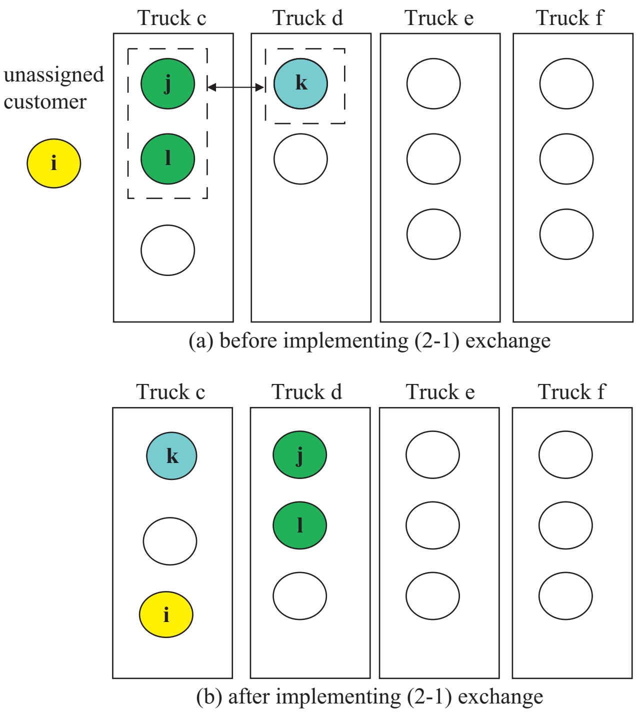

Fig. 5. An illustrative example of (2-1) exchange procedure.

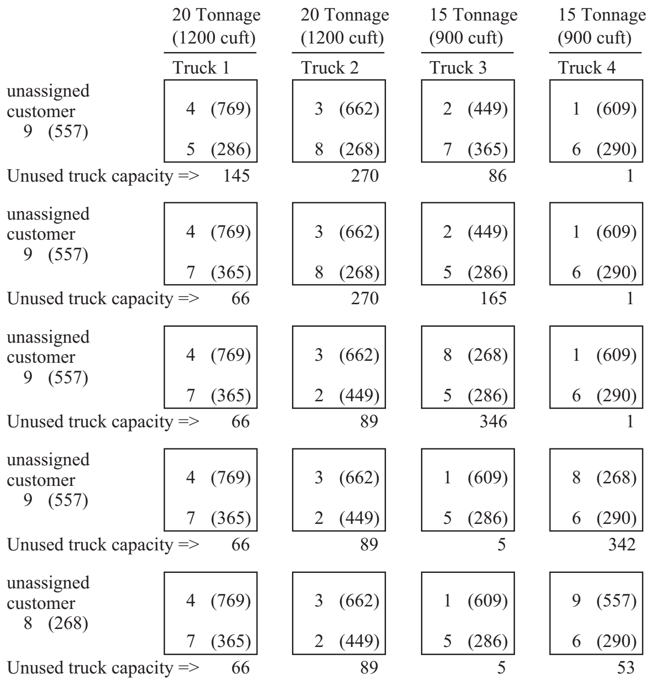

Fig. 6. An illustrative example of the Reduction procedure.

inserting the unassigned customer i into a truck c satisfying demand[i] ≤ unused[c] and assigned_no[c] ≤ 2.

As illustrated in Fig. 4, the (1-1) exchange is achieved by exchanging customer j with customer k and inserting the unassigned customer i into a truck c satisfying demand[k] < demand[j], demand[i] ≤ unused[c] + demand[j] - demand[k], demand[j] ≤ unused[d] and assigned_no[c] ≤ 2.

Similarly, the (2-1) exchange is obtained by exchanging customers j and l with customer k and inserting the unassigned customer i into a truck c satisfying demand[k] < demand[j] + demand[l], demand[j] + demand[l] ≤ unused[d], demand[i] ≤ unused[c] + demand[j] + demand[l] - demand[k], and assigned_no[c] ≤ 2 as shown in Fig. 5.

2) Reduction Procedure

The goal of the Reduction procedure is to reduce wasted truck capacity and minimize the required loading capacities of unassigned customers. To achieve this goal, a local search is performed.

For a given customer i allocated to truck c, this study searches for a customer k that satisfies demand[k] > demand[i], demand[k] - demand[i] ≤ unused[c], and maximize(demand[k] - demand[i]), ∀ k. Wastage of the loading capacity of truck c can be reduced by exchanging customer i with customer k. As the search continues, the program keeps reducing the wasted loading capacity of other trucks. Finally, the total wastage of truck loading capacity and the required loading capacities of unassigned customers are reduced.

Fig. 6 illustrates an example of the Reduction procedure. Starting with customer 4, no customer k can be found that satisfies demand[k] > demand[i] and other conditions. For customer 5, customers 7 and 6 can be found that satisfy demand[k] > demand[i] (365 > 286 and 290 > 286), demand[k] - demand[i] ≤ unused[c] (79 < 145 and 4 < 145). Based on the last condition, maximize(demand[k] - demand[i]), ∀ k (i.e., maximize (79, 4)), the procedure identifies customer 7 who meets all conditions. Consequently, the program exchanges customer 5 with customer 7. Moving to customer 3, no customer k can be found that satisfies all conditions. Using similar reasoning, the Reduction procedure searches all customers. The final result shows that the required loading capacity of unassigned customer 8 is just 208 cubic feet rather than 557 cubic feet. A reduction of 289 cubic feet is thus achieved. Furthermore, wastage of truck loading capacity is also decreased except for truck 4.

IV. NUMERICAL EXAMPLES

1. Application Data and Test Equipment

To compare the performance of two proposed strategies, the study gathered order data from the studied company. Fig. 7 shows detailed locations of logistics centers and regional retailers. In Fig. 7, D1 represents the ice cream factory of the studied company, a triangle denotes a logistics center and a circle stands for a regional retailer. Table 2 summarizes order data for a four week period during the peak season. For example, the order of the first customer on the first day is 609/A1, where 609 is the required loading capacity (measured in cubic feet) and A1 is customer location which can be found by looking up the table on the right of Fig. 7.

Table 2. Order data for four weeks.

Customer \ Order date	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 8	Day 9	Day 10	Day 11	Day 12	Day 13	Day 15	Day 16	Day 17	Day 18	Day 19	Day 20	Day 22	Day 23	Day 24	Day 25	Day 26	Day 27
1	609	422	783	440	563	215	230	411	432	147	132	187	817	314	663	245	465	481	432	605	623	213	344	211
	A1	**A2**	**A1**	**A3**	**A1**	**A3**	**A1**	**A2**	**A4**	**A1**	**A3**	**A1**	**A3**	**A1**	**A4**	**A3**	**A2**	**A1**	**A3**	**A2**	**A1**	**A6**	**A2**	**A1**
2	449	449	204	197	264	324	238	111	236	326	245	321	206	448	572	384	252	302	122	267	286	342	325	442
	A3	**A5**	**A3**	**A6**	**A2**	**A5**	**A2**	**A3**	**A5**	**A2**	**A5**	**A2**	**A4**	**A2**	**A6**	**A7**	**A4**	**A6**	**A5**	**A4**	**A3**	**A7**	**A5**	**A3**
3	662	108	273	274	453	336	346	234	512	784	502	432	804	410	234	306	534	267	85	213	397	555	231	386
	A4	**A6**	**A4**	**A7**	**A4**	**A6**	**A4**	**A5**	**A6**	**A3**	**A6**	**A4**	**A7**	**A5**	**B2**	**A8**	**B1**	**A8**	**A7**	**A6**	**A5**	**A8**	**A6**	**A4**
4	769	286	668	433	153	141	555	246	108	255	208	223	136	523	496	88		170	796	102	120	324	233	298
	A6	**A7**	**A5**	**B2**	**B1**	**A7**	**A7**	**A6**	**A8**	**A4**	**B2**	**B3**	**A8**	**A6**	**B4**	**B3**		**B2**	**A8**	**A7**	**B1**	**B3**	**B1**	A8
5	286	290	121	410	138	132	341	213	136	863	173	122	570	256	633	143		612	417	434	134	218	124	98
	A8	**A8**	**A6**	**B5**	**B3**	**A8**	**A8**	**A7**	**B3**	**A7**	**B8**	**B5**	**B4**	**B1**	**B7**	**B4**		**B6**	**B1**	**B6**	**B5**	**B4**	**B2**	**B3**
6	290	101	118		403	203	290	342		721			296	490	63	283			168	263	293	109	63	74
	B1	**B2**	**A8**		**B8**	**B2**	**B1**	**A8**		**A8**			**B7**	**B2**	**B8**	**B7**			**B3**	**B7**	**B8**	**B7**	**B5**	**B4**
7	365	162	454			148	345	552		90				218		448			398			89	132	
	B3	**B5**	**B3**			**B7**	**B3**	**B1**		**B4**				**B3**		**B8**			**B4**			**B8**	**B6**	
8	268	267	397				283	638		322				247					65					
	B4	**B6**	**B4**				**B4**	**B2**		**B5**				**B5**					**B5**					
9	557	456	244				116	432		236				162					256					
	B7	**B7**	**B8**				**B5**	**B7**		**B6**				**B6**					**B8**					
10		201					540	571																
		B8					**B6**	**B8**																
Total loading capacity	**4255**	**2742**	**3262**	**1754**	**1974**	**1499**	**3284**	**3750**	**1424**	**3744**	**1260**	**1285**	**2829**	**3068**	**2661**	**1897**	**1251**	**1832**	**2739**	**1884**	**1853**	**1850**	**1452**	**1509**

Data source: provided by the studied company.

Number	Location
A1	Nuan-nuan District, Keelung City
A2	Taipei City
A3	Yingge Township, Taipei County
A4	Yingge Township, Taipei County
A5	Dayuan Township, Taoyuan County
A6	Guishan Township, Taoyuan County
A7	Taoyuan City
A8	Luchu Township, Taoyuan County
B1	Ruifang Township, Taipei County
B2	Songshan District, Taipei City
B3	Tucheng City, Taipei County
B4	Luchou City, Taipei County
B5	Dasi Township, Taoyuan County
B6	Taoyuan City
B7	Hsinchu City
B8	Chunan Township, Miaoli County

Fig. 7. Locations of logistics centers and regional retailer warehouses.

All computations were performed using a Pentium 4 (3 GHz, 512 MB) machine with window XP operating system. The LINGO commercial software was used to implement the mathematical model and the heuristic algorithms were programmed using the FORTRAN language.

2. Analysis of Numerical Examples

This section tested the performance of two proposed strategies with real data. The proposed strategies are shown to provide cost benefits over the current implementation.

Table 3 shows the actual four-week delivery results of the studied company. For ease of understanding the contents of Table 3, this study explains the contents based on the delivery results of the second day. For example, the delivery result for the first customer was 609 (20-1) which consists of the required loading capacity 609 cubic feet and truck assignment (20-1). The first and second numbers in the parenthesis (20-1) stand for the truck tonnage and truck number, respectively. Hence, (20-1) indicates that the first 20-ton truck was assigned for delivery. Similarly, the delivery result of the third customer was 662 (20-2). This result denotes that the required loading capacity was 662 cubic feet and the second 20-ton truck was assigned for delivery. A total of four 20-ton trucks were deployed. These trucks provided a loading capacity of 4800 cubic feet for delivery. Summing the required loading capacities of customers, this study obtains the actual delivery

Table 3. Four week delivery results of the studied company.

Customer \ Delivery date	Day 2	Day 3	Day 4	Day 5	Day 6	Day 8	Day 9	Day 10	Day 11	Day 12	Day 13	Day 15	Day 16	Day 17	Day 18	Day 19	Day 20	Day 22	Day 23	Day 24	Day 25	Day 26	Day 27	Day 29	Total
1	609 (20-1)	422 (20-1)	783 (20-1)	440 (20-1)	563 (20-1)	215 (15-1)	230 (15-1)	411 (15-1)	432 (20-1)	147 (20-1)		187 (15-1)	817 (20-1)	314 (15-1)	663 (15-1)	245 (20-1)	465 (15-1)	481 (20-1)	432 (20-1)	605 (20-1)	623 (20-1)	213 (15-1)	344 (20-1)	211 (15-1)	
2	449 (20-1)	449 (20-1)	204 (20-1)	197 (20-1)		324 (15-1)	238 (15-1)	111 (15-1)	236 (15-1)	326 (20-2)	245 (20-1)	321 (15-1)	206 (20-1)	448 (20-1)	572 (20-1)	384 (20-1)	252 (15-1)	302 (20-1)	122 (20-1)	267 (20-1)	286 (15-1)	342 (15-1)	325 (20-1)	442 (15-2)	
3	662 (20-2)	108 (20-1)	273 (20-2)	274 (20-1)	453 (20-1)	336 (15-2)	346 (15-1)	234 (15-1)	512 (20-1)	784 (15-1)	502 (20-1)	432 (15-2)	804 (20-2)	410 (20-2)	234 (15-2)	306 (20-1)	534 (15-2)	267 (20-1)	85 (20-2)	213 (15-1)	397 (15-1)	555 (15-2)	231 (15-1)	386 (15-1)	
4	769 (20-3)	286 (15-1)	668 (20-2)	433 (15-1)	153 (15-1)	141 (15-2)	555 (20-1)	246 (15-2)	108 (15-1)	255 (20-2)	208 (20-1)	223 (15-3)	136 (20-2)	523 (20-2)	496 (20-1)		88 (15-2)	170 (15-1)	796 (20-2)	102 (15-1)	120 (15-1)	324 (15-3)	233 (20-1)	298 (15-3)	
5	286 (20-3)	290 (15-1)	121 (15-1)	410 (15-1)	138 (15-1)	132 (15-2)	341 (20-2)	213 (15-2)	136 (15-1)	863 (20-1)		122 (15-2)	570 (15-1)	256 (15-2)	633 (15-2)	143 (15-1)		612 (15-1)	417 (20-1)	434 (15-1)		218 (15-3)	124 (15-1)	98 (15-3)	
6	290 (20-2)		118 (15-1)		403 (15-1)	203 (15-3)	290 (20-2)	342 (15-2)		721 (20-3)		132 (15-3)	296 (15-1)	490 (20-1)	63 (15-1)	283 (15-1)			168 (15-1)	263 (20-1)	293 (20-1)	109 (15-3)		74 (15-3)	
7	365 (20-4)	162 (15-1)	454 (15-1)			148 (15-3)	345 (20-2)	552 (20-1)		90 (15-1)		173 (15-3)		218 (15-3)		448 (15-1)			398 (15-1)			89 (15-2)	132 (15-1)	63 (15-2)	
8	268 (20-4)	267 (20-2)	397 (15-2)			264 (15-3)	283 (20-1)	638 (20-2)		322 (20-2)				247 (15-3)					65 (20-2)			134 (15-1)			
9	557 (20-4)	456 (20-2)	244 (15-2)				116 (20-1)	432 (20-2)		236 (20-3)				162 (15-3)					256 (15-1)						
10		201 (20-2)	101 (15-2)				540 (15-2)	571 (20-1)																	
Actual delivery amount	4255	2641	3363	1754	1710	1763	3284	3750	1424	3744	955	1590	2829	3068	2661	1809	1339	1832	2739	1884	1719	1984	1389	1572	55058
15T vehicles	0	1	2	1	1	3	2	2	1	1	0	3	1	2	2	1	2	1	1	1	1	3	1	3	36
20T vehicles	4	2	2	1	1	0	2	2	1	3	1	0	2	2	1	1	0	1	2	1	1	0	1	0	31
Loading capacity	4800	3300	4200	2100	2100	2700	4200	4200	2100	4500	1200	2700	3300	4200	3000	2100	1800	2100	3300	2100	2100	2700	2100	2700	69600
Wasted loading capacity	545	659	837	346	390	937	916	450	676	756	245	1110	471	1132	339	291	461	268	561	216	381	716	711	1128	14542
Percentage of wasted loading capacity	11.4%	20.0%	19.9%	16.5%	18.6%	34.7%	21.8%	10.7%	32.2%	16.8%	20.4%	41.1%	14.3%	27.0%	11.3%	13.9%	25.6%	12.8%	17%	10.3%	18.1%	26.5%	33.9%	41.8%	20.9%
Cost	70000	51500	68000	34000	34000	49500	68000	68000	34000	69000	17500	49500	51500	68000	50500	34000	33000	34000	51500	34000	34000	49500	34000	49500	1136500

Table 4. Delivery results for Strategy I based on mathematical model and HS1.

Customer \ Delivery date	Day 2	Day 3	Day 4	Day 5	Day 6	Day 8	Day 9	Day 10	Day 11	Day 12	Day 13	Day 15	Day 16	Day 17	Day 18	Day 19	Day 20	Day 22	Day 23	Day 24	Day 25	Day 26	Day 27	Day 29	Total
1	609 (20-1)	422 (15-1)	783 (20-1)	440 (15-1)	563 (15-1)	215 (15-1)	230 (15-1)	411 (15-1)	432 (15-1)	147 (15-1)	132 (15-1)	187 (15-1)	817 (15-1)	314 (15-1)	663 (15-1)	245 (15-1)	465 (15-1)	481 (20-1)	432 (15-1)	605 (20-1)	623 (20-1)	213 (15-1)	344 (15-1)	211 (15-1)	
2	449 (20-2)	449 (15-3)	204 (20-1)	197 (15-2)	264 (20-1)	324 (15-1)	238 (15-2)	111 (15-2)	236 (15-2)	326 (15-2)	245 (15-1)	321 (15-2)	206 (20-1)	448 (20-1)	572 (15-2)	384 (15-1)	252 (15-1)	302 (15-1)	122 (15-2)	267 (20-1)	286 (15-1)	342 (15-2)	325 (15-2)	442 (15-1)	
3	662 (20-2)	108 (15-1)	273 (20-2)	274 (15-2)	453 (20-1)	336 (15-2)	346 (15-1)	234 (15-3)	512 (15-2)	784 (15-3)	502 (15-2)	432 (15-2)	804 (15-2)	410 (20-2)	234 (15-2)	306 (15-2)	534 (15-2)	267 (15-1)	85 (20-1)	213 (20-1)	397 (20-1)	555 (15-1)	231 (15-1)	386 (15-2)	
4	769 (15-1)	286 (15-4)	668 (20-2)	433 (15-1)	153 (15-1)	141 (15-2)	555 (15-4)	246 (15-3)	108 (15-1)	255 (20-1)	208 (15-1)	223 (15-1)	136 (20-1)	523 (20-1)	496 (20-1)	88 (15-1)		170 (15-1)	796 (20-1)	102 (15-1)	120 (20-1)	324 (15-2)	233 (15-1)	298 (15-2)	
5	286 (20-1)	290 (15-3)	121 (20-1)	410 (15-2)	138 (15-1)	132 (15-2)	341 (15-2)	213 (15-2)	136 (15-1)	863 (20-1)	173 (15-2)	122 (15-1)	570 (20-1)	256 (20-2)	633 (20-2)	143 (15-3)		612 (20-1)	417 (15-2)	434 (15-1)	134 (15-1)	218 (15-2)	124 (15-3)	98 (15-2)	
6	290 (20-1)	101 (15-4)	118 (20-3)		403 (20-1)	203 (15-3)	290 (15-2)	342 (15-3)		721 (15-1)			296 (20-1)	490 (20-2)	63 (15-2)	283 (15-3)			168 (15-2)	263 (15-1)	293 (15-1)	109 (15-3)	63 (15-3)	74 (15-1)	
7	365 (20-3)	162 (15-1)	454 (20-3)			148 (15-3)	345 (15-3)	552 (20-1)		90 (15-3)				218 (20-1)		448 (15-2)			398 (15-1)			89 (15-1)	132 (15-3)		
8	268 (20-3)	267 (15-2)	397 (20-3)				283 (15-4)	638 (20-1)		322 (15-2)				247 (15-1)					65 (15-1)						
9	557 (20-3)	456 (15-4)	244 (20-2)				116 (15-1)	432 (15-1)		236 (15-2)				162 (15-1)					256 (20-1)						
10		201 (15-2)					540 (15-3)	571 (15-2)																	
Actual delivery amount	4255	2742	3262	1754	1974	1499	3284	3750	1424	3744	1260	1285	2829	3068	2661	1897	1251	1832	2739	1884	1853	1850	1452	1509	55058
15T vehicles	1	4	0	2	1	3	4	3	2	3	2	2	2	1	2	3	2	1	2	1	1	3	3	2	50
20T vehicles	3	0	3	0	1	0	0	1	0	1	0	0	1	2	1	0	0	1	1	1	1	0	0	0	17
Loading capacity	4500	3600	3600	1800	2100	2700	3600	3900	1800	3900	1800	1800	3000	3300	3000	2700	1800	2100	3000	2100	2100	2700	2700	1800	65400
Wasted loading capacity	245	858	338	46	126	1201	316	250	576	56	540	515	171	232	339	803	549	268	261	216	247	850	1248	291	10342
Percentage of wasted loading capacity	5.4%	23.8%	9.4%	2.6%	6.0%	44.5%	8.8%	3.9%	20.9%	4.0%	3.0%	28.6%	5.7%	7.0%	11.3%	29.7%	30.5%	12.8%	8.7%	10.3%	11.8%	31.5%	46.2%	16.2%	15.9%
Cost	69000	66000	52500	33000	34000	49500	66000	67000	33000	67000	33000	33000	50500	51500	50500	49500	33000	34000	50500	34000	34000	49500	49500	33000	1122500

Table 5. Delivery results for Strategy II based on mathematical model.

Customer \ Delivery date	Day 2	Day 3	Day 4	Day 5	Day 6	Day 8	Day 9	Day 10	Day 11	Day 12	Day 13	Day 15	Day 16	Day 17	Day 18	Day 19	Day 20	Day 22	Day 23	Day 24	Day 25	Day 26	Day 27	Day 29	Total
1	609 (15-1)	422 (15-1)	783 (20-1)	440 (15-1)	563 (20-1)	215 (15-1)		411 (20-1)	432 (20-1)	147 (15-1)	132 (15-2)	187 (15-1)	817 (15-1)	314 (20-1)	663 (20-1)		465 (15-2)	481 (15-1)	432 (15-2)	605 (20-1)	623 (20-1)	213 (20-1)	344 (15-1)	211 (15-4)	
2	449 (20-1)	449 (20-1)		197 (15-1)	264 (15-1)	324 (15-1)		111 (15-1)		326 (15-2)	245 (15-2)	321 (15-1)		448 (20-2)	572 (15-1)	384 (15-1)	252 (15-2)	302 (15-1)		267 (20-1)	286 (15-1)	342 (15-1)	325 (15-2)	442 (15-1)	
3	662 (20-1)		273 (20-1)	274 (15-2)	453 (15-1)	336 (15-2)	346 (15-1)	234 (20-1)	512 (20-1)	784 (15-3)	502 (15-1)	432 (15-2)	804 (15-2)	410 (15-1)	234 (15-2)	306 (15-1)	534 (15-1)	267 (15-2)	85 (15-3)	213 (20-1)	397 (15-1)	555 (20-1)	231 (15-1)	386 (15-4)	
4	769 (20-2)	286 (15-2)	668 (20-2)	433 (15-2)	153 (15-1)	141 (15-1)	555 (20-1)	246 (20-2)			208 (15-2)	223 (15-2)		523 (20-1)	496 (20-1)		88 (15-1)		796 (15-1)			324 (15-1)		298 (15-2)	
5	286 (15-1)	290 (15-2)	121 (20-1)		138 (20-1)	132 (15-2)	341 (20-1)			863 (20-1)		122 (15-2)	570 (15-3)	256 (20-1)	633 (15-2)	143 (15-2)	245 (15-1)	612 (15-2)	417 (15-3)	434 (15-1)		218 (15-1)		98 (15-3)	
6	290 (15-2)			204 (15-1)			290 (15-1)	342 (20-2)	213 (20-1)	721 (15-1)	255 (15-1)	173 (15-1)	296 (15-3)	490 (20-2)	63 (15-1)	283 (15-2)				263 (15-1)	293 (20-1)			74 (15-1)	
7	365 (20-2)	162 (20-1)	454 (15-1)	118 (15-2)	410 (20-1)		345 (20-2)	552 (20-1)		90 (20-1)	136 (15-1)				218 (15-1)	448 (15-2)			398 (15-3)	122 (15-1)	102 (20-1)			233 (15-4)	
8		267 (15-1)	397 (20-2)			403 (15-2)	283 (20-1)	638 (15-1)		322 (15-2)				247 (15-1)		162 (15-1)			65 (15-1)		168 (15-1)	134 (20-1)		124 (15-3)	
9	557 (15-2)	456 (20-1)	244 (15-1)					432 (15-2)		236 (20-1)									256 (15-2)					63 (15-2)	
10		201 (15-1)	108 (20-2)				540 (20-2)	571 (20-2)		236 (15-2)				206 (15-1)					170 (15-2)				120 (15-2)	132 (15-3)	
11		268 (15-2)	101 (15-1)				203 (15-1)	230 (15-2)		108 (15-3)				136 (20-2)									109 (15-1)		
12							148 (20-2)	238 (15-2)															89 (15-2)		
13								116 (15-1)																	
Actual delivery amount	3987	2801	3149	1666	1981	1551	3051	4121	1157	3833	1478	1458	2487	3030	2879	1726	1584	1662	2619	1904	1869	1786	1218	2061	55058
15T vehicles	2	2	1	2	1	2	1	2	0	3	2	2	3	1	2	2	2	2	3	1	1	1	2	4	44
20T vehicles	2	1	2	0	1	0	2	2	1	1	0	0	0	2	1	0	0	0	0	1	1	1	0	0	18
Loading capacity	4200	3000	3300	1800	2100	1800	3300	4200	1200	3900	1800	1800	2700	3300	3000	1800	1800	1800	2700	2100	2100	2100	1800	3600	61200
Wasted loading capacity	213	199	151	134	119	249	249	79	43	67	322	342	213	270	121	74	216	138	81	196	231	314	582	1539	6142
Percentage of wasted loading capacity	5.1%	6.6%	4.6%	7.4%	5.7%	13.8%	7.6%	1.9%	3.6%	1.7%	17.9%	19.0%	7.9%	8.2%	4.1%	4.1%	12.0%	7.7%	3.0%	9.3%	11.0%	15.0%	32.3%	42.8%	10.0%
Cost	68000	50500	51500	33000	34000	33000	51500	68000	17500	67000	33000	33000	49500	51500	50500	33000	33000	33000	49500	34000	34000	34000	33000	82500	1041000

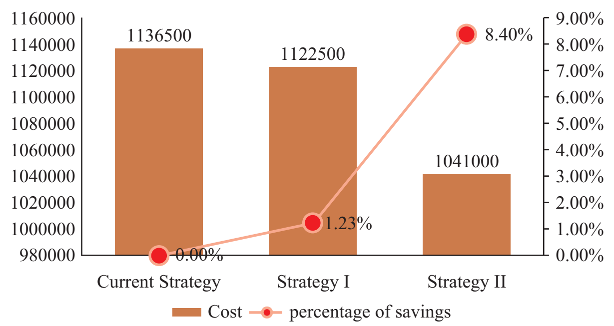

Fig. 8. Comparison of total costs and percentage of savings.

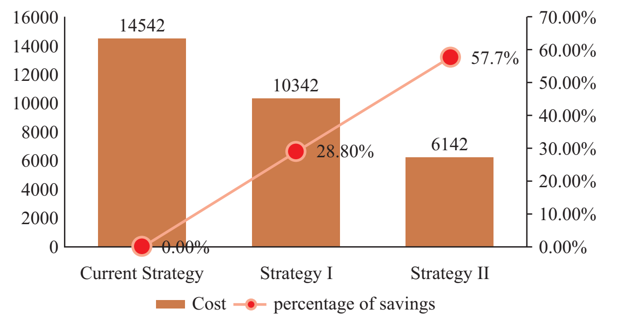

Fig. 9. Comparison of wasted loading capacities and percentage of savings.

amount, 4255 cubic feet. Subtracting actual delivery amount from loading capacity yields the wasted loading capacity which was 545 cubic feet, accounting for 11.4% of total available loading capacity. The daily cost per 20-ton truck is $17,500 and four trucks were assigned for a total cost of $70,000.

Examining Table 3 more closely reveals that the percentage of wasted loading capacity was greater than 10% every day and a total of 67 trucks were assigned. Overall and total transportation cost was $1,136,500, and the wasted loading capacity was 14542 cubic feet. In the long run, unnecessary waste of loading capacity significantly increased transportation costs.

The mathematical model is used to solve the variant of PVRP by using the same input data as in Table 2. Tables 4 and 5 summarize the delivery results for strategies I and II, respectively. To facilitate comparison, Figs. 8 and 9 compare total costs and wasted loading capacities, respectively. Clearly, both proposed strategies provide better results than current implementation. Strategy II is the best alternative, with the

Table 6. Delivery results for Strategy II based on heuristic algorithm HS2.

Customer \ Delivery date	Day 2	Day 3	Day 4	Day 5	Day 6	Day 8	Day 9	Day 10	Day 11	Day 12	Day 13	Day 15	Day 16	Day 17	Day 18	Day 19	Day 20	Day 22	Day 23	Day 24	Day 25	Day 26	Day 27	Day 29	Total
1	609 (15-1)	422 (15-2)	783 (20-2)	440 (20-1)	563 (20-1)	215 (15-1)	230 (15-1)	411 (20-1)		147 (20-1)	132 (15-1)	187 (15-2)	817 (15-3)	314 (20-1)	663 (20-1)	245 (15-1)	465 (15-2)	481 (15-2)	432 (20-2)	605 (15-2)	623 (20-1)	213 (15-1)	344 (15-1)	211 (15-3)	
2	449 (20-2)	449 (20-1)	204 (15-1)	197 (20-1)	264 (20-1)	324 (15-2)	238 (15-2)	111 (15-2)	236 (20-1)	326 (15-2)	245 (15-2)	321 (15-1)	206 (15-1)	448 (20-2)	572 (15-2)	384 (20-1)	252 (15-2)	302 (15-1)	122 (15-1)	267 (15-1)	286 (15-1)	342 (15-1)	325 (15-3)	442 (15-2)	
3	662 (20-2)		273 (20-1)		453 (15-1)	336 (15-1)	346 (20-1)	234 (15-3)	512 (20-1)	784 (15-1)	502 (15-2)	432 (15-2)		410 (20-3)	234 (15-2)	306 (15-1)	534 (15-1)	267 (15-2)		213 (15-2)	397 (20-1)	555 (20-1)	231 (15-2)	386 (15-2)	
4	769 (20-1)	286 (20-1)	668 (20-1)	433 (20-1)		141 (15-3)	555 (20-1)	246 (15-1)		255 (15-2)		223 (15-1)	136 (15-2)	523 (20-3)	496 (20-1)		143 (15-1)	170 (15-1)	796 (15-1)			324 (20-1)	233 (15-1)	298 (15-1)	
5	286 (15-1)	290 (15-1)	121 (20-2)	410 (15-1)		132 (15-2)		213 (15-2)		863 (20-2)			570 (15-2)	256 (20-2)	633 (15-1)		88 (15-1)		417 (20-1)	434 (15-1)		218 (20-1)	124 (15-3)	98 (15-1)	
6	290 (15-2)		118 (15-1)	101 (15-1)	403 (15-1)	203 (15-3)	290 (15-1)		342 (20-1)		721 (15-1)	208 (15-2)	296 (15-1)	490 (20-2)	63 (15-1)	283 (15-1)			168 (20-2)		293 (15-1)			74 (15-3)	
7	365 (20-1)	162 (15-2)	454 (15-1)	397 (15-1)	274 (20-1)	148 (15-3)	345 (15-1)	552 (15-2)		90 (15-1)		173 (15-1)	122 (15-1)		162 (15-1)	448 (20-1)			398 (20-2)	65 (15-2)	102 (20-1)		132 (15-2)	63 (15-1)	
8		267 (15-2)				153 (15-2)	283 (20-1)	638 (15-1)		322 (20-1)				247 (20-3)		218 (20-1)				85 (15-1)	263 (15-1)		109 (15-3)		
9	557 (15-2)	456 (20-1)	244 (20-2)			138 (15-1)	116 (15-2)	432 (20-1)		236 (15-2)									256 (20-1)			134 (15-1)	89 (15-2)		
10		201 (15-1)					540 (15-2)	571 (15-3)		432 (20-1)				804 (20-1)					612 (20-1)				120 (15-1)		
11		268 (15-1)	108 (20-1)					341 (20-1)		108 (20-2)															
12										136 (20-2)															
13																									
Actual delivery amount	3987	2801	2973	1978	1957	1790	2943	3749	1090	3699	1600	1544	2147	3492	2823	1884	1482	1220	3201	1669	1964	1786	1707	1572	55058
15T vehicles	2	2	1	1	1	3	2	3	0	2	2	2	3	0	2	1	2	2	1	2	1	1	3	3	42
20T vehicles	2	1	2	1	1	0	1	1	1	2	0	0	0	3	1	1	0	0	2	0	1	1	0	0	21
Loading capacity	4200	3000	3300	2100	2100	2700	3000	3900	1200	4200	1800	1800	2700	3600	3000	2100	1800	1800	3300	1800	2100	2100	2700	2700	63000
Wasted loading capacity	213	199	327	122	143	910	57	151	110	501	200	256	553	108	177	216	318	580	99	131	136	314	993	1128	7942
Percentage of wasted loading capacity	5.1%	6.6%	9.9%	5.8%	6.8%	33.7%	1.9%	3.9%	9.2%	11.9%	11.1%	14.2%	20.5%	3.0%	5.9%	10.3%	17.7%	32.2%	3.0%	7.3%	6.5%	15.0%	36.8%	41.8%	12.6%
Cost	68000	50500	51500	34000	34000	49500	50500	67000	17500	68000	33000	33000	49500	52500	50500	34000	33000	33000	51500	33000	34000	34000	49500	49500	1060500

lowest transportation cost of $1,041,000, and the lowest wasted loading capacity of 6142 cubic feet.

The advantage of strategy II is proven by a mathematical model. Based on Tables 4 and 5, numerical calculation demonstrates that strategy II outperforms strategy I. This study concludes that the strategy II is an alternative solution that should be considered by the studied company.

Because the complexity of the PVRP is NP-hard, the solution time of the mathematical model rapidly increases with problem size. From the computational results not shown here, the average run time is approximately 40 minutes and the worst case run time is about 6 hours for strategy II. Clearly, a heuristic algorithm is necessary for solving the variant of PVRP, although the mathematical model can provide the exact solution and is suitable for small scale problems.

Tables 4 and 6 present the delivery results for strategies I and II based on HS1 and HS2, respectively. The proposed heuristic algorithm, HS1 produces an identical delivery plan to the mathematical model. Table 7 lists a comparison of delivery results for strategy II based on the proposed heuristic algorithm, HS2 and the mathematical model. Table 7 indicates that for 13 out of 24 days, the heuristic algorithm, HS2 obtains the optimal solution of strategy II. The total cost is $1,060,500 and the wasted loading capacity is 7942 cubic feet.

To summarize the numerical examples explored in the study, Table 8 compares different strategies, including the results of assigned truck number, loading capacity, wasted loading capacity, total cost, % deviation from strategy II and average run time. Table 8 shows that HS1 always produces the optimal solution; HS2 solves the variant of PVRP in seconds, and the deviation from optimal solution is only 1.87%. In summary, strategy II is the best alternative for the studied company and the heuristic algorithms demonstrate good efficiency and accuracy in solving the variant of PVRP.

V. CONCLUSIONS

PVRP is an interesting problem with a variety of real world applications. This study presents a variant of PVRP with the objectives of satisfying all received orders and minimizing monthly transportation costs. Two short term strategies have been proposed for the studied company. Both the mathematical programming model and heuristic algorithms are developed to compare the performance of two strategies. Extensive computational results are also provided based on four weeks of real data. Due to the numerous variables and

Table 7. Comparison of delivery results for Strategy II based on heuristic algorithm HS2 and mathematical model.

Delivery date	Delivery amount		15T vehicles		20T vehicles		Wasted capacity		Percentage wasted capacity		Costs	
	H2	S2	H2	S2	H2	S2	H2	S2	H2	S2	H2	S2
Day2	3987	3987	2	2	2	2	213	213	5.1%	5.1%	68000	68000
Day3	2801	2801	2	2	1	1	199	199	6.6%	6.6%	50500	50500
Day4	2973	3149	1	1	2	2	327	151	9.9%	4.6%	51500	51500
Day5	1978	1666	1	2	1	0	122	134	5.8%	7.4%	34000	33000
Day6	1957	1981	1	1	1	1	143	119	6.8%	5.7%	34000	34000
Day8	1790	1551	3	2	0	0	910	249	33.7%	13.8%	49500	33000
Day9	2943	3051	2	1	1	2	57	249	1.9%	7.6%	50500	51500
Day10	3749	4121	3	2	1	2	151	79	3.9%	1.9%	67000	68000
Day11	1090	1157	0	0	1	1	110	43	9.2%	3.6%	17500	17500
Day12	3699	3833	2	3	2	1	501	67	11.9%	1.7%	68000	67000
Day13	1600	1478	2	2	0	0	200	322	11.1%	17.9%	33000	33000
Day15	1544	1458	2	2	0	0	256	342	14.2%	19.0%	33000	33000
Day16	2147	2487	3	3	0	0	553	213	20.5%	7.9%	49500	49500
Day17	3492	3030	0	1	3	2	108	270	3.0%	8.2%	52500	51500
Day18	2823	2879	2	2	1	1	177	121	5.9%	4.1%	50500	50500
Day19	1884	1726	1	2	1	0	216	74	10.3%	4.1%	34000	33000
Day20	1482	1584	2	2	0	0	318	216	17.7%	12.0%	33000	33000
Day22	1220	1662	2	2	0	0	580	138	32.2%	7.7%	33000	33000
Day23	3201	2379	1	3	2	0	99	81	3.0%	3.0%	51500	49500
Day24	1669	2619	2	1	0	1	131	196	7.3%	9.3%	33000	34000
Day25	1964	1904	1	1	1	1	136	231	6.5%	11.0%	34000	34000
Day26	1786	1786	1	1	1	1	314	314	15.0%	15.0%	34000	34000
Day27	1707	1218	3	2	0	0	993	582	36.8%	32.3%	49500	33000
Day29	1572	2061	3	4	0	0	1128	1539	41.8%	42.7%	49500	66000
Total	55058	55058	42	44	21	18	7942	6142	12.6%	10.0%	1060500	1041000

Table 8. Comparison of different strategies.

	Current strategy	Strategy I (mathematical model)	Strategy I (heuristic algorithm)	Strategy II (mathematical model)	Strategy II (heuristic algorithm)
15T vehicles	36	50	50	44	42
20T vehicles	31	17	17	18	21
Loading capacity	69600	65400	65400	61200	63000
Wasted loading capacity	14542	10342	10342	6142	7942
Total cost	1136500	1122500	1122500	1041000	1060500
% deviation from S2	9.17%	7.83%	7.83%	0.00%	1.87%
Average run time	-	About 10 minutes	Less than 1 second	About 40 minutes	Less than 3 seconds

constraints included in the model, the proposed mathematical programming model is only suitable for a small scale problem since the variant of PVRP is NP-hard. On the contrary, the proposed heuristic algorithms solve the variant of PVRP efficiently and accurately, and so can be applied in practice. All in all, the proposed strategy II achieves promising savings in transportation costs.

As for future research, monthly transportation costs may be improved using a new strategy, but developing such a strategy will require further study. Furthermore, with slight modification, the proposed heuristic algorithm can be applied to the delivery of refrigerated commodities, including frozen and chilled food and food stored at room temperature, such as meat, fish, vegetable, and milk, etc. In addition, the above mentioned applications, the proposed heuristic algorithm can be also applicable to freight forwarders and 3PL because they may involve the allocation of shipments at the warehouse to shipping routes with different destination ports. The shipments must be loaded into containers of varying sizes and costs, and the objective is to find an allocation that minimizes the total number of containers.

REFERENCES

Alegre, J., M. Laguna and J. Pacheco (2007). Optimizing the periodic pick-up of raw materials for a manufacturer of auto parts. European Journal of Operational Research 179, 736-746.

Angelelli, E., N. Bianchessi, R. Mansini and M. Speranza (2009). Short term strategies for a dynamic multi-period vehicle routing problem. Transportation Research Part C 17, 106-119.

Angelelli, E. and M. Speranza (2002). The period vehicle routing problem with intermediate facilities. European Journal of Operational Research 137, 233-247.

Baptista, S., R. Oliveira and E. Zuquete (2002). A period vehicle routing case study. European Journal of Operational Research 139, 220-229.

Beltrami, E. and L. Bodin (1974). Networks and vehicle routing for municipal waste collection. Networks 4, 65-94.

Chao, I. M., B. Golden and E. Wasil (1995). An improved heuristic for the period vehicle routing problem. Networks 26, 25-44.

Christofides, N. and J. Beasley (1984). The period routing problem. Networks 14, 237-256.

Coene, S., A. Arnout and F. Spieksma (2010). On a periodic vehicle routing problem. Journal of the Operational Research Society 61, 1719-1728.

Cordeau, J. F., M. Gendreau and G. Laporte (1997). A tabu search heuristic for periodic and multi-depot vehicle routing problems. Networks 30, 105-119.

Cordeau, J. F., M. Gendreau and G. Laporte (2001). A unified tabu search heuristic for vehicle routing problems with time windows. Journal of the Operational Research Society 52, 928-936.

Drummond, L., L. Ochi and D. Vianna (2001). An asynchronous parallel metaheuristic for the period vehicle routing problem. Future Generation Computer Systems 17, 379-386.

Francis, P., K. Smilowitz and M. Tzur (2006). The period vehicle routing problem with service choice. Transportation Science 40, 439-454.

Groër, C., B. Golden and E. Wasil (2010). A library of heuristics for solving the vehicle routing problem. Mathematical Programming Computation 2, 79-101.

Gulczynski, D., B. Golden and E. Wasil (2010). The split delivery vehicle routing problem with minimum delivery amounts. Transportation Research Part E 46, 612-626.

Hadjiconstantinou, E. and R. Baldacci (1998). A multi-depot period vehicle routing problem arising in the utilities sector. Journal of the Operational Research Society 49, 1239-1248.

Hamzadayi, A., S. Topaloglu and S. Yelkenci Kose (2013). Nested simulated annealing approach to periodic routing problem of a retail distribution system. Computers & Operations Research 40, 2893-2905.

Hemmelmayr, V., K. Doerner and R. Hartl (2009). A variable neighborhood search heuristic for periodic routing problems. European Journal of Operational Research 195, 791-802 .

Ngueveu, S. U., C. Prins and R. W. Calvo (2013). New lower bounds and exact method for the m-PVRP. Transportation Science 47, 38-52.

Russell, R. and D. Gribbin (1991). A multiphase approach to the period routing problem. Networks 21, 747-765.

Russell, R. and W. Igo (1979). An assignment routing problem. Networks 9, 1-17.

Tan, C. and J. Beasley (1984). A heuristic algorithm for the periodic vehicle routing problem. Omega 12, 497-504.