Journal of Marine Science and Technology

Volume 23 | Issue 4 Article 15

ELASTOHYDRODYNAMIC LUBRICATION OF CIRCULAR CONTACTS AT PURE SQUEEZE MOTION INVOLVING THE MIXTURE OF TWO LUBRICANTS

Li-Ming Chu
Department of Mechanical Engineering, Southern Taiwan University of Science and Technology, Tainan City 71005, Taiwan, lmchu@mail.stust.edu.tw

Yuh-Ping Chang
Department of Mechanical Engineering, Kun Shan University, Tainan City 71070, Taiwan

Jer-Jia Sheu
Department of Mechanical Engineering, Southern Taiwan University of Science and Technology, Tainan City 71005, Taiwan

Recommended Citation
Chu, Li-Ming; Chang, Yuh-Ping; and Sheu, Jer-Jia (2015) "ELASTOHYDRODYNAMIC LUBRICATION OF CIRCULAR CONTACTS AT PURE SQUEEZE MOTION INVOLVING THE MIXTURE OF TWO LUBRICANTS," *Journal of Marine Science and Technology*: Vol. 23: Iss. 4, Article 15.
DOI: 10.6119/JMST-015-0325-1
Available at: https://jmstt.ntou.edu.tw/journal/vol23/iss4/15

ELASTOHYDRODYNAMIC LUBRICATION OF CIRCULAR CONTACTS AT PURE SQUEEZE MOTION INVOLVING THE MIXTURE OF TWO LUBRICANTS

Acknowledgements

The authors would like to express their appreciation to the Ministry of Science and Technology (MOST 103-2221-E-218- 036) and the National Science Council (NSC 102-2221-E218-042) in Taiwan for financial support.

Journal of Marine Science and Technology, Vol. 23, No. 4, pp. 516-522 (2015)
DOI: 10.6119/JMST-015-0325-1

ELASTOHYDRODYNAMIC LUBRICATION OF CIRCULAR CONTACTS AT PURE SQUEEZE MOTION INVOLVING THE MIXTURE OF TWO LUBRICANTS

Li-Ming Chu[1], Yuh-Ping Chang[2], and Jer-Jia Sheu[1]

Key words: EHL, squeeze film, volume fraction.

ABSTRACT

This study investigated the pure squeeze elastohydrodynamic lubrication (EHL) motion of circular contacts by using mixed rheology fluid models under constant load conditions. The lubricant was a homogeneous mixture of a Newtonian base oil and couple stress fluid at various volume fractions. The simulation results revealed that the effect of the couple stress fluid was equivalent to enhancing the viscosity of the lubricant. Therefore, the central film thickness and minimum film thickness increased with the volume fraction of the couple stress fluid (v). The central pressure decreased as v increased. In addition, the normal squeeze velocity followed the ensuing order: Newtonian lubricant > mixed lubricants > couple stress lubricant.

I. INTRODUCTION

Numerous mechanical elements with contact pairs, such as gear teeth, cams/followers, piston rings/cylinders, and rolling element bearings, as well as the stretching process of metal sheets, have an elastic dimple at the center of the contact region because of the squeeze effect. The working conditions are not only severe but hazardous. Therefore, lubricants are crucial substances. However, almost all practical lubricants are a mixture comprising a base fluid and several additives. Oliver (1988) and Spike (1994) have proven that additives lead to load enhancement and friction reduction. Therefore, the effects of additives on the fluid rheology of a lubricant have received considerable research attention.

Paper submitted 03/11/14; revised 08/19/14; accepted 03/25/15. Author for correspondence: Li-Ming Chu (e-mail: lmchu@mail.stust.edu.tw).
[1] Department of Mechanical Engineering, Southern Taiwan University of Science and Technology, Tainan City 71005, Taiwan, R.O.C.
[2] Department of Mechanical Engineering, Kun Shan University, Tainan City 71070, Taiwan, R.O.C.

The flow behavior of a Newtonian lubricant blended with various additives cannot be described accurately using classical continuum theory. Classical Newtonian and non-Newtonian theories cannot predict the flow behavior of the lubricants correctly as the additives are added to the base oil (Kumar et al., 2008a). Dai and Khonsari (1994) developed a general lubrication theory for a mixture of two incompressible fluids. The base oil and additive oils are Newtonian and non-Newtonian fluids, respectively. The effects of a mixture on the performance of elastohydrodynamic lubrication (EHL) rolling/sliding line contacts were investigated numerically (Kumar et al., 2008a). The EHL characteristics computed for polymer-modified oils are found to depend upon the effective viscosity of the lubricant mixture. Therefore, mixture theory (Dai and Khonsari 1994) has been applied to determine the correct flow behavior of lubricants. In addition, a previous study investigated the effects of a mixed rheological fluid model on the thermal EHL behavior of rolling/sliding line contacts (Kumar et al., 2008b). The combined effects of the flow rheology and surface roughness should be considered when the operating condition falls within the EHL region. Under the assumption of a homogeneous mixture, Li (1998) derived the average type Reynolds equation that includes the coupling effects of surface roughness and flow rheology for a mixture of two fluids.

When two bodies approach each other along the normal direction, an extremely high pressure is generated in the lubricating film because of the squeeze effects. Therefore, an elastic dimple occurs at the center of the contact region. The related problems are called transient EHL problems, and they occur in numerous mechanical elements. Several numerical solutions have been proposed for solving the problem associated with the formation of dimples in pure squeeze motion; these solutions have been proposed by several studies including Christensen (1970), Lee and Cheng (1973), Yang and Wen (1991), and Chang (1996). These studies have often used the ball-dropping case because it includes all the effects of pure squeeze motion that are of interest. Chu et al. (2004) and Wang et al. (1992) have calculated the pressure distribution by using elastic deformation theory and modifying the apparent viscosity in the Reynolds equation at pure squeeze motion. Safa and Gohar (1986) investigated pure impact problems by using thin film transducers to measure the pressure in the contact region during impact. They discovered two pressure peaks during the impact period. The first peak corresponded to the stage of impact where the impact force reached its maximum. At the very end of the rebound process, immediately before the ball left the lubricated surface, a sharp contact center pressure peak (the second peak) was also found. Dowson and Wang (1994) as well as Larsson and Högund (1995) analyzed the bouncing of an elastic sphere on an oily plate. These analyses were restricted to normal motion in the first instance in order to develop the numerical technique and to relate the overall findings to the results presented by Safa and Gohar (1986). However, few studies have investigated the EHL circular contact region containing mixed lubricants.

This paper explored the pure squeeze EHL motion of circular contacts by using a mixture of Newtonian and couple stress fluids under constant load conditions. The transient modified Reynolds equation was derived in polar coordinates according to the Stokes microcontinuum theory (1966). The finite difference method and the Gauss-Seidel iteration method were used to solve the transient modified Reynolds equation, the elasticity deformation equation, the load balance equation, and the lubricant rheology equations simultaneously. The transient pressure profiles, film shapes, and normal squeeze velocities during the pure squeeze process under various operating conditions in the EHL regime are discussed in this paper.

II. THEORETICAL ANALYSIS

1. Modified Reynolds Equation

Two spheres approach one another in terms of an equivalent sphere approaching a plane. Consider the squeeze film mechanism as shown in Fig. 1, an elastic sphere of radius R is approaching an infinite plate with a velocity under constant load condition. The lubricant in the system is taken to be a compressible mixture fluid.

According to the Stokes microcontinuum theory (1966), the field equations of an incompressible coupled stress fluid in the absence of body forces and body couples are expressed as follows:

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

$$\rho \frac{D\mathbf{V}}{Dt} = -\nabla p + (\mu - \eta \nabla^2)\nabla^2 \mathbf{V} \tag{2}$$

where $\mathbf{V}$ is the velocity vector, ρ is the density, p is the pressure, μ is the classical viscosity coefficient, and η is a new material constant with the dimension of momentum responsible for the couple stress fluid property. Since the ratio η/μ_0 has the dimensions of length squared, the dimension of $l = (\eta/\mu_0)^{1/2}$ characterizes the material length of the couple stress fluids, and l is assumed to be a material constant in the present analysis.

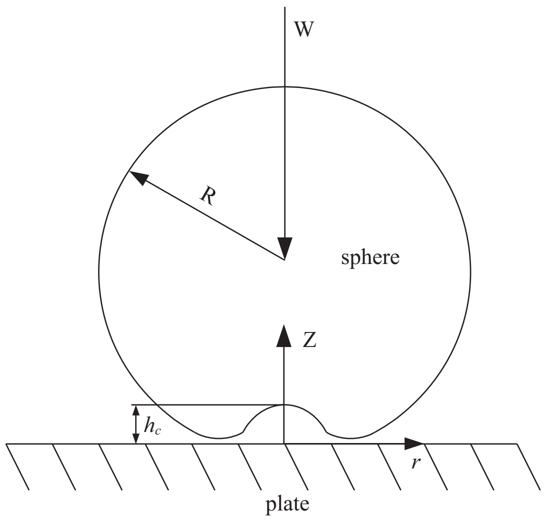

Fig. 1. Geometry of EHL of circular contacts under pure squeeze motion.

This study developed the general governing equations for the EHL of two compressible fluids. The base fluid was assumed to be Newtonian and the other fluid was classified as a simple couple stress fluid. The mixture was homogeneous because this study assumed that no chemical reaction occurred and the constituent fluids retained their original mechanical properties after being mixed. The total shear stress is shared by the two fluids in proportion to their volume fractions:

$$\tau = (1-v)\mu\dot{\gamma} + v\mu_c\dot{\gamma} \tag{3}$$

where v and $(1\text{-}v)$ are the volume fractions of the couple stress fluid and Newtonian base oil, respectively. $\dot{\gamma} = \partial u / \partial z$ is the shear-strain rate and $\mu_c = \mu/[1-12(l/h)\bar{\mu}^{1.5}]$ is the viscosity of the couple stress fluid additive.

Under the general assumption that EHL is applicable to a thin film, the reduced momentum equations and continuity equation governing the motion of the lubricant given in polar coordinates can be derived. Integrating the reduced momentum equations by applying the boundary condition yields the velocity components. Substituting the velocity components into the continuity equation and integrating across the film thickness with the boundary conditions of $w(r, z)$, the transient modified Reynolds equation in polar coordinates for the mixture comprising a couple stress fluid and Newtonian fluid can be derived as follows:

$$\frac{\partial}{\partial r}\left\{\frac{\rho r h^3}{\mu} / \{(1-v\} \frac{\partial p}{\partial r}\right\} = 12r\frac{\partial \rho h}{\partial t} \tag{4}$$

or in dimensionless form as follows:

$$\frac{\partial}{\partial X}\left\{\frac{\bar{\rho} X H^3}{\bar{\mu}} / \{(1-\nu)+\} \frac{\partial P}{\partial X}\right\} = KX \frac{\partial \bar{\rho} H}{\partial T} \tag{5}$$

where

$$K = 8\pi / W$$

The origin of the radial coordinate, r, is at the center of the contact. The boundary conditions for Eq. (5) can be expressed as follows:

$$P(X \to \infty, T) = 0 \tag{6a}$$

$$\frac{\partial}{\partial X} P(0, T) = 0 \tag{6b}$$

$$P(X, T) \geq 0 \tag{6c}$$

2. Initial Stage

The squeeze motion has two stages: the initial stage and high-pressure stage. At the initial stage, the ball has a lubricant layer and starts squeezing the lubricant film away. Because the pressure is low, an isoviscous incompressible lubricant model is appropriate, and the elastic deformation can be disregarded. At this stage, the transient modified Reynolds equation in dimensionless form can be expressed as follows:

$$\frac{\partial}{\partial X}\left(H^3 X \frac{\partial P}{\partial X}\right) = KX \frac{\partial}{\partial T}(H) \tag{7}$$

The film thickness can be expressed as follows:

$$H = H_0 + \frac{X^2}{2} \tag{8}$$

and the central normal velocity is simply given as:

$$\frac{\partial H}{\partial T} = \frac{\partial H_0}{\partial T} = -V_0 \tag{9}$$

The pressure profile can be solved analytically from the Reynolds equation as follows:

$$P = \frac{KV_0}{4H^2} \tag{10}$$

For the constant load, the instantaneous load balance equation is:

$$\int_0^\infty PXdX = \frac{1}{3} \tag{11}$$

By using Eqs. (10) and (11), the central normal velocity can be expressed as follows:

$$V_0 = \frac{4H_0}{3K} \tag{12}$$

At the initial stage, the equation governing the transient hydrodynamic lubrication (HL) problem is solved analytically when a thin layer of oil initially separates the ball and plate at pure squeeze motion. This HL solution is applied as the initial condition.

3. High-Pressure Stage

When the pressure increases with time, the elastic deformation and effect of pressure on the viscosity cannot be neglected. This stage is denoted as the high-pressure stage, and presents a problem of pure squeeze motion in EHL. The coupled Reynolds, rheology, load equilibrium, and elasticity equations were solved numerically.

The viscosity of the lubricant was assumed to be a function of pressure only. The relationship between viscosity and pressure used by Roelands et al. (1963) can be expressed as:

$$\bar{\mu} = \exp\{(9.67 + \ln \mu_0)[-1 + (1 + 5.1 \times 10^{-9} p)^z]\} \tag{13}$$

where μ_0 is the viscosity at ambient pressure and z is the pressure-viscosity index. According to Dowson and Higginson (1966), the relationship between density and pressure can be derived as follows:

$$\bar{\rho} = \frac{\rho}{\rho_0} = 1 + \frac{0.6 \times 10^{-9} p}{1 + 1.7 \times 10^{-9} p} \tag{14}$$

4. Elasticity Equation

The film thickness in a nominal point contact elastohydrodynamic conjunction can be expressed as follows:

$$h(r, t) = h_0(t) + \frac{r^2}{2R} + \delta(r, t) \tag{15}$$

The dimensionless film thickness between two elastic bodies in circular contact can be derived as follows:

$$H_i = H_0 + \frac{X_i^2}{2} + \bar{\delta}_i \tag{16}$$

To calculate the static deformation due to the pressure distribution, influence coefficients D_{ij} were introduced. The deformation can thus be computed at discrete points i as a sum of the deformation contributions from all pressure points j:

$$\bar{\delta}_i = \sum_{j=1}^{n} D_{ij} P_j \tag{17}$$

Table 1. Computational data.

G (Material parameter)	3500
Inlet viscosity of lubricant, Pa-s	0.04
Inlet density of lubricant, kg/m^3	846
Pressure viscosity coefficient, 1/GPa	15.91
Pressure-viscosity index (Roelands)	0.4836
Reduced radius, m	0.02
Density of balls, kg/m^3	7850
Elastic modulus of balls, GPa	200
Poisson's ratio of balls	0.3

where the influence coefficients, D_{ij}, are computed according to Yang and Wen (1991) and Larsson and Höglund (1995). For the constant load case, the rigid separation is an unknown variable at each time step. It can be determined by solving the transient Reynolds equation with the load balance equation. To reduce computation time, the analytical solution of the first stage discussed above can be used as the initial condition.

III. RESULTS AND DISCUSSION

The ball was assumed to accelerate continuously from the free surface of the lubricant layer until it achieves a quasi-static condition. To discuss the effects of the flow rheology of the mixture lubricants and elastic deformation on the squeezing motion, point contact EHL problems were discussed under non-isoviscous, compressible lubricant, and constant load conditions. Numerical solutions of film profiles and pressure distributions in pure squeeze motion were calculated using the parameters listed in Table 1. The initial falling height of the sphere was 20 μm.

The load equilibrium equation has to be included in the coupled transient modified Reynolds equation, the rheology equation, and the elastic deformation equation for the constant load conditions. The rigid separation thus becomes one of the unknown variables, and is solved simultaneously with the nodal pressures. In this study, at the initial stage, the elastic deformation and increase in fluid viscosity and density with pressure were neglected. This solution was derived in Eqs. (10) and (12), and it was applied as the initial condition. At the beginning, the upper limit of the computational region was set to $X_{\max}$ = 16.0. The modified Reynolds equation was discretized by using the central difference technique in the space domain and explicit technique in the time domain. When more than half of the region was cavitated, the maximum analyzed region $X_{\max}$ reduced to half of its initial region, and so on, until $X_{\max}$ = 2.0. The grid comprised 401 evenly distributed nodes in every calculation domain. The Gauss-Seidel iteration method was employed to calculate the film thickness and pressure distribution at each time step.

Under the conditions of Newtonian fluid and constant load, the operation and initial conditions derived by Yang and Wen (1991) were employed to solve the pure squeeze EHL motion of circular contacts problem by using the proposed algorithm. The numerical results of the central pressure and film thickness and those obtained by Yang and Wen (1991) are compared as shown in Fig. 2 and favorable consistency was observed. The discrepancies in the present analysis were caused by the finer grids and variation of the calculation region with time.

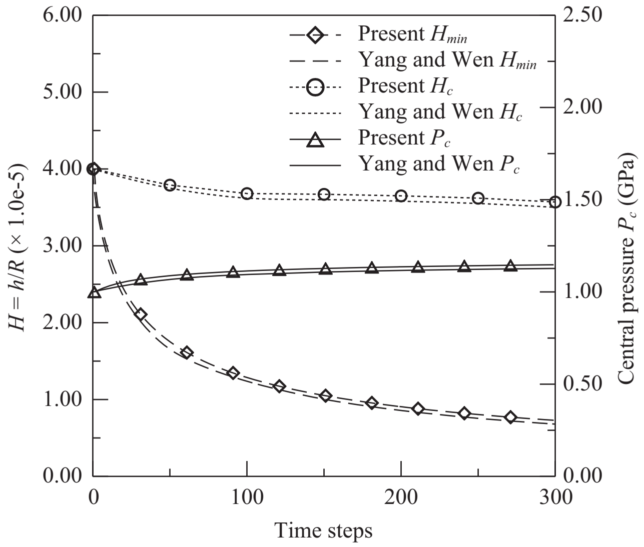

Fig. 2. Comparison of results obtained by Yang and Wen (1991) and those using the present method.

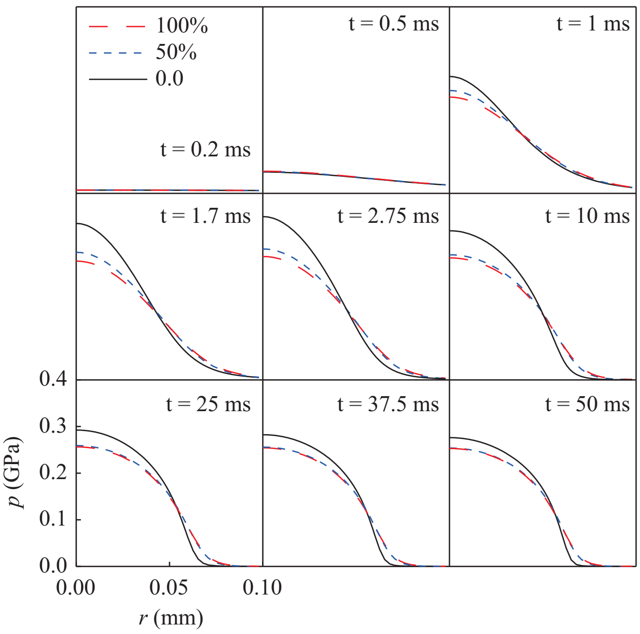

Fig. 3. Pressure distribution versus time using different volume fraction.

For constant load conditions, Figs. 3 and 4 respectively illustrate the relative change in the pressure distribution and film thickness for a flexible sphere approaching a lubricated flat surface with mixed fluids under the condition of $\bar{W}$ = 2.62 × 10^{-8} and G = 3500. As shown in Fig. 3, the pressure profile is

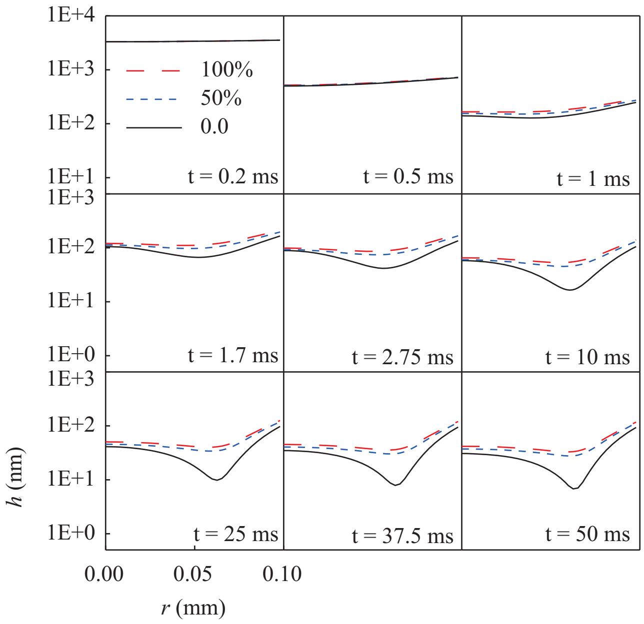

Fig. 4. Film thickness shape versus time using different volume fraction.

quite flat at a relatively large film thickness, but it becomes steeper with decreasing film thickness. When the sphere approaches the flat surface, the pressure profile almost converged to the Hertzian contact pressure. In this study, the peak pressure was always maintained at the center. It was found that the center pressure gradually increases with decreasing central film thickness from 0.2 to 2.75 ms, when the central film thickness decreases to 100 nm. After this stage, the pressure reversed its trend from 2.75 to 50.0 ms (i.e., the peak pressure decreased with the film thickness until it reached a stage at which the minimum film thickness and squeeze velocity were nearly zero). As shown in Fig. 3, the maximum pressure is maintained at the center, and the pressure decreases gradually to atmospheric pressure in the r-direction (X-direction) because of the effects of the elastic deformation and geometry. The pressure distribution at the central region followed the ensuing order: Newtonian lubricant > mixed lubricants > couple stress lubricant. As the loading is constant, the integration of the pressure distribution over the loading area is a constant. Therefore, the pressure distribution is found reverse outside the central region. As illustrated in Fig. 4, the film thickness reflects the following order: couple stress lubricant > mixed lubricants > Newtonian lubricant. This is because a higher equivalent viscosity results in greater film thickness. As shown in Fig. 4, the position of minimum film thickness moves farther away from the center ($r = 0$).

Fig. 5 shows the central pressure and film thickness versus time for different lubricants under constant load conditions. At the initial stage, the central pressure increased rapidly with time. The central pressure increased quickly to a maximum, and it subsequently decreased slowly with time to near the amplitude of the Hertzian pressure at the final stage. This

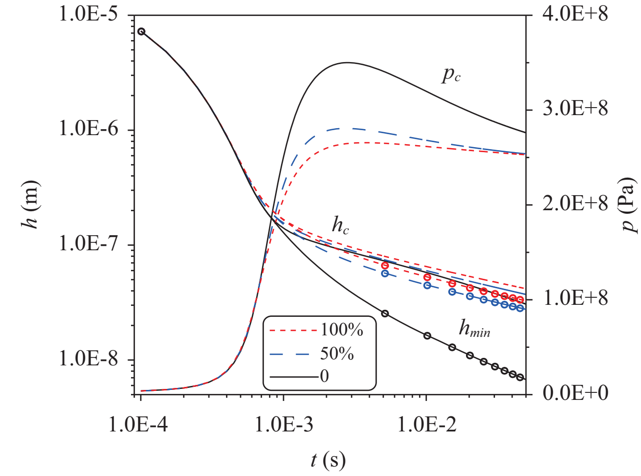

Fig. 5. Central Pressure and film thickness versus time using different volume fraction.

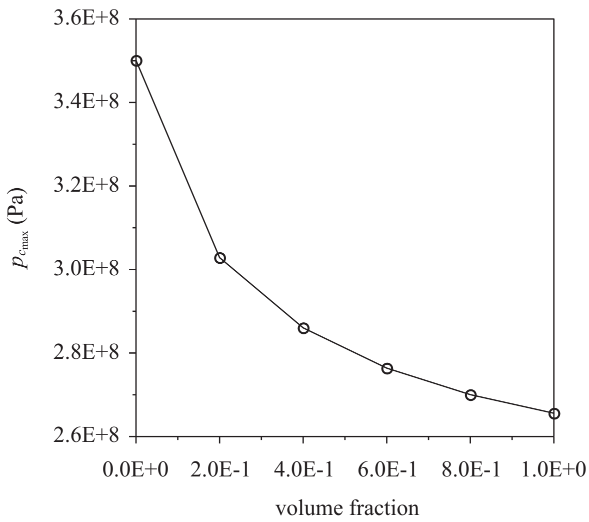

Fig. 6. Maximum central pressure versus volume fraction.

stage can be considered as the quasi-static condition. The central pressure followed the ensuing order: Newtonian lubricant > mixed lubricants > couple stress lubricant. The central and minimum film thicknesses decreased rapidly with time at the initial stage and subsequently decreased slowly with time. The film thickness followed the order couple stress lubricant > mixed lubricants > Newtonian lubricant.

Fig. 6 shows the maximum central pressure versus the volume fraction of the couple stress fluid using present model under constant load condition. As shown, the greater the volume fraction of the couple stress fluid is, the smaller the maximum central pressure is.

Fig. 7 shows the central pressure and film thickness versus the volume fraction of the couple stress fluid using present model under constant load condition. As shown, the greater the volume fraction of the couple stress fluid (v) is, the greater the central film thickness and the minimum film thickness are. The central pressure decreases as v increases.

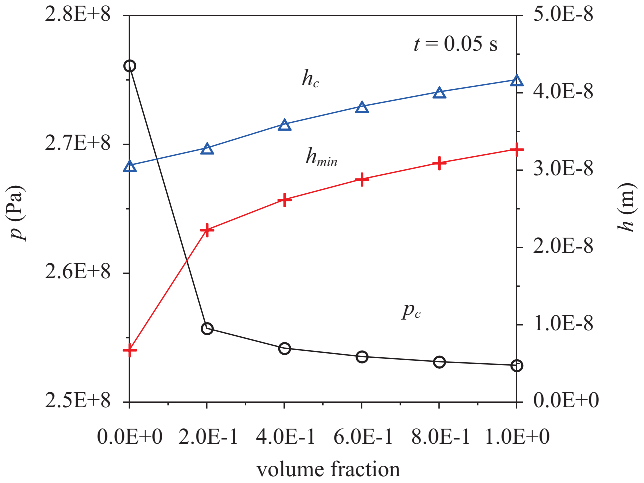

Fig. 7. Central Pressure and film thickness versus volume fraction.

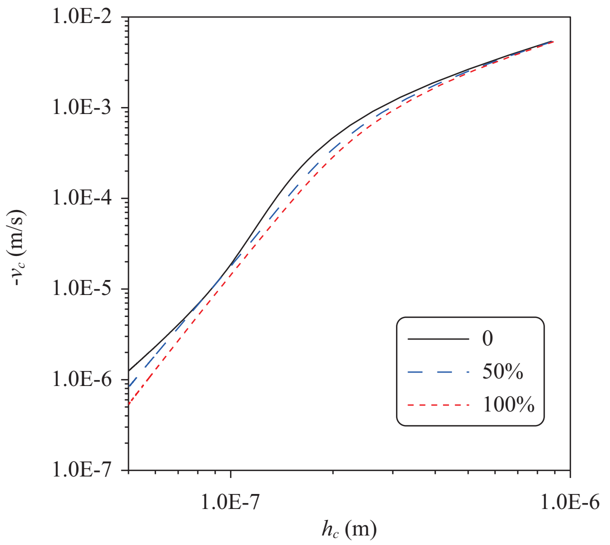

Fig. 8. Variation of central normal squeeze velocity with central film thickness.

Fig. 8 shows the relationship of the central normal squeeze velocity $-V_c$ and the central film thickness using present models under constant load condition. The central normal squeeze velocity decreased rapidly with decreasing central film thickness at the final stage. As shown, the normal squeeze velocity followed the ensuing order: Newtonian lubricant > mixed lubricants > couple stress lubricant.

IV. CONCLUSIONS

This study developed a numerical method for general applications with a mixture of Newtonian and couple stress fluids for investigating the pure squeeze motion in an isothermal EHL spherical conjunction under constant load condition. The conclusions based on the main results can be summarized as follows:

1. The effect of the couple stress is equivalent to enhancing the lubricant viscosity. Therefore, the greater the volume fraction of the couple stress fluid is, the greater the central film thickness and the minimum film thickness are.
2. The peak pressure is always kept at the center during the squeeze process. The greater the volume fraction of the couple stress fluid is, the smaller the central region pressure is. Due to the loading being constant, the pressure distribution is found reverse outside the central region.
3. The normal squeeze velocity follows the order Newtonian lubricant > mixture lubricants > couple stress lubricant.

ACKNOWLEDGMENTS

The authors would like to express their appreciation to the Ministry of Science and Technology (MOST 103-2221-E-218-036) and the National Science Council (NSC 102-2221-E-218-042) in Taiwan for financial support.

NOMENCLATURE

b	reference Hertzian radius at load w (m)
D_{ij}	influence coefficients for deformation calculation
E'	equivalent elastic modulus (Pa)
G	dimensionless material parameter, $\alpha E'$
h	film thickness
h_0	rigid separation
h_c	central film thickness
h_{min}	minimum film thickness
H	dimensionless film thickness, hR/b^2
H_0	dimensionless film thickness, h_0R/b^2
H_c	dimensionless film thickness, h_cR/b^2
H_{min}	dimensionless film thickness, $h_{min}R/b^2$
K	constant in Reynolds equation, $8\pi/W$
l	characteristic length of the couple stress fluids, $l = (\eta/\mu_0)^{1/2}$
L	dimensionless characteristic length of the couple stress fluids, lR/b^2
p	pressure (Pa)
p_c	central pressure (Pa)
p_h	reference Hertzian pressure at load w (Pa)
P	dimensionless pressure, p/p_h
r	radial coordinate (m)
R	ball radius (m)
t	time (sec)
T	dimensionless time, tE'/μ_0
v_0	normal velocity of the ball's center (m/s)
V_0	dimensionless normal velocity of the ball's center, $v_0\mu_0R/E'b^2$
w	load (N)
W	dimensionless load, $w/E'R^2$
X	dimensionless radial coordinate, r/b
z	pressure-viscosity index
α	pressure-viscosity coefficient
η	material constant responsible for couple stress pa-

rameter
μ viscosity of lubricant (Pa-s)
μ_0 viscosity at ambient pressure (Pa-s)
$\bar{\mu}$ dimensionless viscosity, μ/μ_0
ρ density of lubricant (kg/m^3)
ρ_0 density of lubricant at ambient pressure (kg/m^3)
$\bar{\rho}$ dimensionless density of lubricant, ρ/ρ_0
δ elastic deformation (m)

REFERENCES

Chang, L. (1996). An efficient calculation of the load and coefficient of restitution of impact between two elastic bodies with a liquid lubricant. ASME Journal of Applied Mechanics 63(2), 347-352.

Christensen, H. (1970). Elastohydrodynamic theory of spherical bodies in normal approach. ASME J. Lubr. Technol. 92(1), 145-154.

Chu, H. M., R. T. Lee and Y. C. Chiou (2004). Study on pure squeeze elastohydrodynamic lubrication motion using optical interferometry and inverse approach. Proc. Instn Mech. Engrs, Part J: Journal of Engineer Tribology 218, 503-512.

Dai, F. and M. M. Khonsari (1994). A theory of hydrodynamic lubrication involving the mixture of two fluids. ASME Journal of Applied Mechanics 61, 634-641.

Dowson D. and G. R. Higginson (1966). Elastohydrodynamic Lubrication, Pergamon Press, 88-92.

Dowson, D. and D. Wang (1994). An analysis of the normal bouncing of a solid elastic ball on an oily plate. Wear 179, 29-37.

Kumar, P., S. C. Jain and S. Ray (2008a). Influence of polymeric fluid additives in EHL rolling/sliding line contacts. Tribology International 41, 482-492.

Kumar, P., S. C. Jain and S. Ray (2008b). Thermal elastohydrodynamic lubrication of rolling/sliding line contacts using a mixture of Newtonian and power law fluids. Proc. Instn Mech. Engrs, Part J: Journal of Engineer Tribology 222, 35-49.

Larsson, R. and E. Högund (1995). Numerical simulation of a ball impacting and rebounding a lubricated surface. ASME, J. of Tribology 117, 94-102.

Lee, K. M. and H. S. Cheng (1973). The pressure and deformation profiles between two normally approaching lubricated cylinders. ASME, J. Lubr. Technol. 95(3), 308-317.

Li, W. L. (1998). Surface roughness effects in hydrodynamic lubrication involving the mixture of two fluids. ASME Journal of Tribology 120, 772-780.

Oliver, D. R. (1988). Load enhancement effects due to polymer thickening in a short model journal bearings. J Non-Newtonian Fluid Mech. 30, 185-196.

Roelands, C. J. A., J. C. Vlugter and H. I. Watermann (1963). The viscosity temperature pressure relationship of lubricating oils and its correlation with chemical constitution. ASME Journal of Basic Engineering, 601-606.

Safa, M. M. A. and R. Gohar (1986). Pressure distribution under a ball impacting a thin lubricant layer. ASME, J. of Tribology 108, 372-376.

Spikes, H. A. (1994). The behavior of lubricants in contacts: current understanding and future possibilities. Proc. Instn Mech. Engrs, Part J: Journal of Engineer Tribology 28, 3-15.

Stokes, V. K. (1966). Couple stresses in fluids. Phys Fluids 9, 1709-1715.

Wong, P. L., S. Lingard and A. Cameron (1992). A simplified impact microviscometer. Tribology International 25 (6), 363-366.

Yang, P. R. and S. Z. Wen (1991). Pure squeeze action in an isothermal elastohydrodynamic lubricated spherical conjunction, Part 1: Theory and dynamic load results. Wear 142, 1-16.

Yang, P. R. and S. Z. Wen (1991). Pure squeeze action in an isothermal elastohydrodynamic lubricated spherical conjunction, Part 2: Constant speed and constant load results. Wear 142, 17-30.