Journal of Marine Science and Technology

Volume 23 | Issue 1 Article 12

ESTIMATED STATE FEEDBACK FUZZY CONTROL FOR PASSIVE DISCRETE TIME-DELAY MULTIPLICATIVE NOISED PENDULUM SYSTEMS

Wen-Jer Chang
Department of Marine Engineering, National Taiwan Ocean University, Keelung, Taiwan, R.O.C.,
wjchang@mail.ntou.edu.tw

Chun-Hung Lin
Department of Marine Engineering, National Taiwan Ocean University, Keelung, Taiwan, R.O.C.

Cheung-Chieh Ku
Department of Marine Engineering, National Taiwan Ocean University, Keelung, Taiwan, R.O.C.

Recommended Citation

Chang, Wen-Jer; Lin, Chun-Hung; and Ku, Cheung-Chieh (2015) "ESTIMATED STATE FEEDBACK FUZZY CONTROL FOR PASSIVE DISCRETE TIME-DELAY MULTIPLICATIVE NOISED PENDULUM SYSTEMS," *Journal of Marine Science and Technology*: Vol. 23: Iss. 1, Article 12.
DOI: 10.6119/JMST-014-0515-1
Available at: https://jmstt.ntou.edu.tw/journal/vol23/iss1/12

ESTIMATED STATE FEEDBACK FUZZY CONTROL FOR PASSIVE DISCRETE TIME-DELAY MULTIPLICATIVE NOISED PENDULUM SYSTEMS

Acknowledgements

The authors would like to express their sincere gratitude to anonymous reviewers who gave us some constructive comments, criticisms and suggestions.

Journal of Marine Science and Technology, Vol. 23, No. 1, pp. 98-108 (2015)
DOI: 10.6119/JMST-014-0515-1

ESTIMATED STATE FEEDBACK FUZZY CONTROL FOR PASSIVE DISCRETE TIME-DELAY MULTIPLICATIVE NOISED PENDULUM SYSTEMS

Wen-Jer Chang, Chun-Hung Lin, and Cheung-Chieh Ku

Key words: T-S fuzzy model, multiplicative noise, parallel distributed compensation, passivity theory, discrete Jensen inequality.

ABSTRACT

This paper develops a fuzzy controller design method via estimated state feedback scheme for discrete-time nonlinear stochastic time-delay systems which are presented by Takagi–Sugeno fuzzy model with multiplicative noises. The proposed design method is accomplished by the concept of parallel distributed compensation. Using the Lyapunov-Krasovskii function and passivity theory, the stability conditions are derived to guarantee the stability and attenuation performance. An algorithm is proposed in this paper to solve the derived stability conditions that belong to non-strict linear matrix inequality problems. Moreover, the discrete Jensen inequality and free-weighting matrix technique are employed to decrease the conservatism of the proposed design method. Finally, the control of a nonlinear time-delay pendulum system is provided to illustrate the effectiveness and usefulness of the proposed fuzzy control method.

I. INTRODUCTION

Takagi–Sugeno (T-S) fuzzy model (Tseng and Chen, 2001) is a well-known popular tool to describe nonlinear systems in terms of fuzzy sets. In the T-S fuzzy model, the linear subsystems in the consequence part of IF-THEN rules are proposed to describe the local dynamics of nonlinear systems. Based on the T-S fuzzy model, stability and stabilization problems of the nonlinear systems have been investigated through quadratic Lyapunov function. According to the Lyapunov stability theory, sufficient conditions are derived to guarantee the stability of all local subsystems. These sufficient stability conditions depend on the existence of a common positive definite matrix. Moreover, the concept of Parallel Distributed Compensation (PDC) (Tanaka and Wang, 2001) is applied to design the fuzzy controller for T-S fuzzy model by blending several linear subcontrollers with the same premise variable of the T-S fuzzy model. Many literatures (Tanaka and Wang, 2001; Tseng and Chen, 2001; Hwang and Lin, 2004; Hsiao et al., 2005a; Hsiao et al., 2005b; Wang and Lin, 2005; Chang and Chang, 2006; Lin et al., 2007; Dong and Yang, 2009; Ma and Boukas, 2009; Chang et al., 2012; Chang et al., 2013; Chang and Huang, 2014) have discussed the stability and stabilization problems of nonlinear systems via T-S fuzzy model and PDC technique. Most of the above literatures are state feedback case or output feedback case. However, both of state feedback and output feedback schemes can no longer be applied under the conditions of immeasurable states. Hence, several authors have proposed fuzzy observer design method (Choi, 2007; Tseng, 2008; Chen and Chang, 2009; Kim and Park, 2009; Tseng and Chen, 2009; Gassara et al., 2010; Wen and Ren, 2010) for the nonlinear systems represented by T-S fuzzy modeling technique. According to the above-mentioned motivations, the fuzzy control via estimated state feedback scheme for the discrete-time nonlinear stochastic time-delay systems is thus investigated in this paper.

The stochastic behaviors of systems are usually implied as unmeasured signals and considered as disturbance effects on systems. In order to study the stochastic behaviors, the fuzzy control of nonlinear stochastic systems has been discussed via T-S fuzzy model (Chang et al., 2010; Ku et al., 2010; Chang et al., 2011; Chang et al., 2012, Chang and Huang, 2014). In (Chang et al., 2010; Ku et al., 2010; Chang et al., 2011; Chang et al., 2012, Chang and Huang, 2014), the consequence part of T-S fuzzy model is described as stochastic differential equations (Eli et al., 2005). Via stochastic differential equations, the stochastic systems are structured as deterministic systems with multiplicative noise terms. The T-S fuzzy model with multiplicative noise has been widely employed to propose the

Paper submitted 05/10/12; revised 11/12/13; accepted 05/15/14. Author for correspondence: Wen-Jer Chang (e-mail: wjchang@mail.ntou.edu.tw).
Department of Marine Engineering, National Taiwan Ocean University, Keelung, Taiwan, R.O.C.

stability criteria for the nonlinear stochastic systems. Besides, the time delay and external disturbance are common and wicked phenomenon in many industry and engineering systems. They are usually the source of instability or poor performance of dynamic control systems (Chang et al., 2010; Ku et al., 2010; Liang et al., 2010; Chang et al., 2011; Chang et al., 2012, Chang and Huang, 2014). The control issues of nonlinear time-delay systems have attracted much interest via T-S fuzzy model (Chang and Chang, 2005; Hsiao et al., 2005b; Yang et al., 2005; Lin et al., 2008). Moreover, the disturbance effects are usually caused by external perturbations in the environments. For attenuating these effects, dissipative theory and its special case passivity theory have been used to discuss the energy change of external disturbance in the control systems (Wang et al., 2003; Chang et al., 2010; Ku et al., 2010; Liang et al., 2010; Chang and Huang, 2014). According to the results of (Lozano et al., 2001), one can find that the passivity theory includes several control schemes such as positive real theory, H_∞ control scheme, L_2 control scheme and so on.

In this paper, a fuzzy control via estimated state feedback scheme for the discrete-time nonlinear stochastic systems with time delay is discussed and investigated by using the T-S fuzzy model. Applying passivity theory and Lyapunov theory, the sufficient conditions are derived to guarantee the stability of the considered T-S fuzzy systems. Besides, the discrete Jensen inequality (Zhu and Yang., 2008) and free-weighting matrix technique (Souza et al., 2009) are employed to reduce the conservatism of the proposed stability conditions. Since the derived stability conditions do not belong to strict Linear Matrix Inequality (LMI) problems, cone complementarity (Ghaoui et al., 1997) is used to provide a sub-optimal algorithm to solve the non-strict LMI conditions. After solving the non-strict LMI stability conditions, an observer-based fuzzy controller can be constructed to stabilize the closed-loop system based on the PDC technique. Finally, a numerical example for the control of a nonlinear pendulum system is provided to demonstrate the application and effectiveness of proposed observer-based fuzzy controller design method.

II. CONTROL PROBLEM FORMULATIONS

The discrete-time nonlinear stochastic time-delay systems considered in this paper could be approximated by the following T-S fuzzy model in the IF-THEN form.

Plant part

Rule i: IF $z_1(k)$ is M_{i1} and $z_2(k)$ is M_{i2} and … and $z_n(k)$ is M_{in} THEN

$$x(k+1)=\mathbf{A}_i x(k)+\mathbf{B}_i u(k)+\mathbf{E}_i w(k)+\mathbf{A}_{di}x(k-\tau)+\left(\bar{\mathbf{A}}_i x(k)+\bar{\mathbf{B}}_i u(k)+\bar{\mathbf{A}}_{di}x(k-\tau)\right)\beta(k) \quad (1a)$$

$$y_m(k)=\mathbf{C}_{mi}x(k)+\mathbf{C}_{mdi}x(k-\tau)+\mathbf{D}_{mi}w(k) \quad (1b)$$

$$y_c(k)=\mathbf{C}_{ci}x(k)+\mathbf{C}_{cdi}x(k-\tau)+\mathbf{D}_{ci}w(k) \quad (1c)$$

$$x(k)=\varphi(k),\ k\in[-\tau\ \ 0] \text{ for i = 1, 2, …, r} \quad (1d)$$

where M_{ij} is the fuzzy set, z_1, z_2, …, z_n are the premise variables and r is the number of fuzzy rules. And $x(k)\in\Re^{n_x}$ is the state vector, $x(k-\tau)\in\Re^{n_x}$ is the time delay vector which satisfies $\tau>0$, $\varphi(k)$ is the initial state of the system for $-\tau\le k\le 0$, $u(k)\in\Re^{n_u}$ is the control input vector, $w(k)\in\Re^{n_y}$ is the disturbance vector, $y_c(k)\in\Re^{n_y}$ is the controlled output vector, $y_m(k)\in\Re^{n_y}$ is the measured output vector, τ denotes the integer representing the time delay of the fuzzy model and $\beta(k)$ is a scalar discrete type Brownian motion. Besides, $\mathbf{A}_i\in\Re^{n_x\times n_x}$, $\mathbf{B}_i\in\Re^{n_x\times n_u}$, $\mathbf{E}_i\in\Re^{n_x\times n_y}$, $\mathbf{A}_{di}\in\Re^{n_x\times n_x}$, $\bar{\mathbf{A}}_i\in\Re^{n_x\times n_x}$, $\bar{\mathbf{B}}_i\in\Re^{n_x\times n_u}$, $\bar{\mathbf{A}}_{di}\in\Re^{n_x\times n_x}$, $\mathbf{C}_{mi}\in\Re^{n_y\times n_x}$, $\mathbf{C}_{mdi}\in\Re^{n_y\times n_x}$, $\mathbf{D}_{mi}\in\Re^{n_y\times n_y}$, $\mathbf{C}_{ci}\in\Re^{n_y\times n_x}$, $\mathbf{C}_{cdi}\in\Re^{n_y\times n_x}$ and $\mathbf{D}_{ci}\in\Re^{n_y\times n_y}$ are known constant matrices. Here, it is assumed that $\beta(k)$ satisfies the properties: $E\{\beta(k)\}=0$, $E\{x(k)\beta(k)\}=0$ and $E\{\beta(k)\beta(k)\}=1$. Note that $E\{\bullet\}$ stands the mathematical expectation operator.

The final output of the fuzzy system can be inferred as follows.

$$x(k+1)=\sum_{i=1}^{r}h_i(z(k))\times\left(\mathbf{A}_i x(k)+\mathbf{B}_i u(k)+\mathbf{E}_i w(k)+\mathbf{A}_{di}x(k-\tau)+\left(\bar{\mathbf{A}}_i x(k)+\bar{\mathbf{B}}_i u(k)+\bar{\mathbf{A}}_{di}x(k-\tau)\right)\beta(k)\right) \quad (2a)$$

$$y_m(k)=\sum_{i=1}^{r}h_i(z(k))\left(\mathbf{C}_{mi}x(k)+\mathbf{C}_{mdi}x(k-\tau)+\mathbf{D}_{mi}w(k)\right) \quad (2b)$$

$$y_c(k)=\sum_{i=1}^{r}h_i(z(k))\left(\mathbf{C}_{ci}x(k)+\mathbf{C}_{cdi}x(k-\tau)+\mathbf{D}_{ci}w(k)\right) \quad (2c)$$

where

$$z(k)=\begin{bmatrix}z_1(k) & z_2(k) & \cdots & z_n(k)\end{bmatrix}^T,$$

$$\omega_i(z(k))=\prod_{j=1}^{n}\mu_{M_{ij}}(z_j(k)),\ h_i(z(k))=\frac{\omega_i(z(k))}{\sum_{i=1}^{r}\omega_i(z(k))},$$

$$\omega_i(z(k))\ge 0,\ h_i(z(k))\ge 0 \text{ and } \sum_{i=1}^{r}h_i(z(k))=1.$$

And, $\mu_{M_{ij}}\left(z_j(k)\right)$ is the grade of membership function corresponding to the fuzzy set M_{ij}.

In practice, not every state of the real systems is available to measure. Under the condition of immeasurable states, it is essential to design an observer for the systems. In this paper, the following fuzzy observer is proposed to deal with the state estimation for the nonlinear discrete-time stochastic time-delay systems (1).

Observer part

Rule i: IF $z_1(k)$ is M_{i1} and $z_2(k)$ is M_{i2} and … and $z_n(k)$ is M_{in} THEN

$$\hat{x}(k+1) = \mathbf{A}_i \hat{x}(k) + \mathbf{B}_i u(k) + \mathbf{A}_{di} \hat{x}(k-\tau) + \mathbf{L}_i \left(y_m(k) - \hat{y}(k)\right) \tag{3a}$$

$$\hat{y}(k) = \mathbf{C}_{mi} \hat{x}(k) + \mathbf{C}_{mdi} \hat{x}(k-\tau) \tag{3b}$$

$$\hat{x}(k) = \hat{\varphi}(k),\ k \in [-\tau\ \ 0] \text{ for i = 1, 2, …, r} \tag{3c}$$

where $\hat{x}(k) \in \Re^{n_x}$ is an estimate of $x(k)$, $\hat{x}(k-\tau) \in \Re^{n_x}$ is an estimate of $x(k-\tau)$, $\hat{\varphi}(k)$ is the initial state of the fuzzy observer for $-\tau \le k \le 0$, $\hat{y}(k) \in \Re^{n_y}$ is the output of the fuzzy observer and $\mathbf{L}_i \in \Re^{n_x \times n_y}$ is the observer gain. The overall fuzzy observer is thus inferred as follows.

$$\hat{x}(k+1) = \sum_{i=1}^{r} h_i\left(z(k)\right) \times \left(\mathbf{A}_i \hat{x}(k) + \mathbf{B}_i u(k) + \mathbf{A}_{di} \hat{x}(k-\tau) + \mathbf{L}_i\left(y_m(k) - \hat{y}(k)\right)\right) \tag{4a}$$

$$\hat{y}(k) = \sum_{i=1}^{r} h_i\left(z(k)\right)\left(\mathbf{C}_{mi} \hat{x}(k) + \mathbf{C}_{mdi} \hat{x}(k-\tau)\right) \tag{4b}$$

In this paper, the design of observer-based fuzzy controllers is performed through the concept of PDC. Hence, the observer-based fuzzy controller can be proposed as follows.

Controller part

Rule i: IF $z_1(k)$ is M_{i1} and $z_2(k)$ is M_{i2} and … and $z_n(k)$ is M_{in} THEN

$$u(k) = \mathbf{F}_i \hat{x}(k) \text{ for i = 1, …, r} \tag{5}$$

where $\mathbf{F}_i \in \Re^{n_u \times n_x}$ is the observer-based fuzzy controller gain.

The observer-based fuzzy controller (5) can be also represented as follows.

$$u(k) = \sum_{i=1}^{r} h_i\left(z(k)\right)\left(\mathbf{F}_i \hat{x}(k)\right) \tag{6}$$

Let us define the estimation error as follows.

$$e(x) = x(k) - \hat{x}(k) \tag{7a}$$

$$e(x+1) = x(k+1) - \hat{x}(k+1) \tag{7b}$$

$$e(x-\tau) = x(k-\tau) - \hat{x}(k-\tau) \tag{7c}$$

Substituting (6) into (2a), one can obtain the following closed-loop system from (7a).

$$x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(z(k)\right) h_j\left(z(k)\right) \times \left(\mathbf{A}_{ij} x(k) - \mathbf{B}_i \mathbf{F}_j e(k) + \mathbf{E}_i w(k) + \mathbf{A}_{di} x(k-\tau) + \left(\bar{\mathbf{A}}_{ij} x(k) - \bar{\mathbf{B}}_i \mathbf{F}_j e(k) + \bar{\mathbf{A}}_{di} x(k-\tau)\right)\beta(k)\right) \tag{8}$$

where $\mathbf{A}_{ij} = \mathbf{A}_i + \mathbf{B}_i \mathbf{F}_j$ and $\bar{\mathbf{A}}_{ij} = \bar{\mathbf{A}}_i + \bar{\mathbf{B}}_i \mathbf{F}_j$. From (7b) and (7c), one can infer the following estimation error equation.

$$e(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(z(k)\right) h_j\left(z(k)\right) \times \left(\mathbf{A}_{Lij} e(k) + \mathbf{E}_{Lij} w(k) + \mathbf{A}_{dij} e(k-\tau) + \left(\bar{\mathbf{A}}_{ij} x(k) - \bar{\mathbf{B}}_i \mathbf{F}_j e(k) + \bar{\mathbf{A}}_{di} x(k-\tau)\right)\beta(k)\right) \tag{9}$$

where $\mathbf{A}_{Lij} = \mathbf{A}_i - \mathbf{L}_i \mathbf{C}_{mj}$, $\mathbf{E}_{Lij} = \mathbf{E}_i - \mathbf{L}_i \mathbf{D}_{mj}$ and $\mathbf{A}_{dij} = \mathbf{A}_{di} - \mathbf{L}_i \mathbf{C}_{mdj}$. Then, the augmented closed-loop system can be expressed as the following form.

$$\bar{x}(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(z(k)\right) h_j\left(z(k)\right) \times \left(\mathbf{G}_{ij} \bar{x}(k) + \mathbf{G}_{di} \bar{x}(k-\tau) + \mathbf{G}_{Eij} w(k) + \left(\bar{\mathbf{G}}_{ij} \bar{x}(k) + \bar{\mathbf{G}}_{di} \bar{x}(k-\tau)\right)\beta(k)\right) \tag{10a}$$

$$y_c(k) = \sum_{i=1}^{r} h_i\left(z(k)\right) \times \left(\bar{\mathbf{C}}_{ci} x(k) + \bar{\mathbf{C}}_{cdi} x(k-\tau) + \mathbf{D}_{ci} w(k)\right) \tag{10b}$$

where

$$\bar{x}(k+1)=\begin{bmatrix} x(k+1) \\ e(k+1) \end{bmatrix},\ \bar{x}(k)=\begin{bmatrix} x(k) \\ e(k) \end{bmatrix},$$

$$\bar{x}(k-\tau)=\begin{bmatrix} x(k-\tau) \\ e(k-\tau) \end{bmatrix},\ \mathbf{G}_{ij}=\begin{bmatrix} \mathbf{A}_{ij} & -\mathbf{B}_i\mathbf{F}_j \\ \mathbf{0} & \mathbf{A}_{Lij} \end{bmatrix},$$

$$\mathbf{G}_{di}=\begin{bmatrix} \mathbf{A}_{di} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{dij} \end{bmatrix},\ \mathbf{G}_{Eij}=\begin{bmatrix} \mathbf{E}_i \\ \mathbf{E}_{Lij} \end{bmatrix},\ \bar{\mathbf{G}}_{ij}=\begin{bmatrix} \bar{\mathbf{A}}_{ij} & -\bar{\mathbf{B}}_i\mathbf{F}_j \\ \bar{\mathbf{A}}_{ij} & -\bar{\mathbf{B}}_i\mathbf{F}_j \end{bmatrix},$$

$$\bar{\mathbf{G}}_{di}=\begin{bmatrix} \bar{\mathbf{A}}_{di} & \mathbf{0} \\ \bar{\mathbf{A}}_{di} & \mathbf{0} \end{bmatrix},\ \bar{\mathbf{C}}_{ci}=\begin{bmatrix} \mathbf{C}_{ci} & \mathbf{0} \end{bmatrix} \text{ and } \bar{\mathbf{C}}_{cdi}=\begin{bmatrix} \mathbf{C}_{cdi} & \mathbf{0} \end{bmatrix}.$$

In this paper, the passivity theory is employed to discuss the effect of external disturbance on systems. The passivity performance considered in this paper is introduced by the following definition for the augmented system (10).

Definition 1 (Chang et al., 2010)

If there exist constant matrices $\mathbf{S}_1$, $\mathbf{S}_2 \geq \mathbf{0}$ and $\mathbf{S}_3$ for satisfying the following inequality, then the augmented system (10) is called passive with the disturbance $w(k)$ and the output $y_c(k)$ for all terminal time $k_q \geq 0$.

$$E\left\{2\sum_{k=0}^{k_q} y_c^{T}(k)\mathbf{S}_1 w(k)\right\} > E\left\{\sum_{k=0}^{k_q} y_c^{T}(k)\mathbf{S}_2 y_c(k)+\sum_{k=0}^{k_q} w^{T}(k)\mathbf{S}_3 w(k)\right\} \quad (11)$$

#

The passivity theory includes several performance constraints that can be obtained by setting matrices $\mathbf{S}_1$, $\mathbf{S}_2$ and $\mathbf{S}_3$ via the well-known mathematical definition of power supply function. In this paper, the generalization power supply function (11) is proposed to be the constraint index. For illustrating the stability of closed-loop system (10), the following definition is introduced.

Definition 2 (Eli et al., 2005)

For the augmented system (10) without external disturbance input, i.e., $w(k)=0,$, the system is asymptotically mean square stable if $E\{x(k)\}$ and $E\{x^{T}(k)x(k)\}$ are both converged to zero as $k \to \infty$. #

For the discrete-time systems, the discrete Jensen inequality is usually used to develop the less conservative stability criterion. The discrete Jensen inequality introduced by the following lemma is also employed in this paper to derive the less conservative stability conditions.

Lemma 1 (Zhu and Yang, 2008)

For any constant matrix $\mathbf{Q} \in \Re^{n_x \times n_x}$ $\mathbf{Q}=\mathbf{Q}^{T}>0$, integers $\tau_{\min} \leq \tau_{\max}$, vector function $\varpi:\{\tau_{\min},\tau_{\min}+1,...,\tau_{\max}\} \to \Re^{n_x}$ such that the sums in the following are well-defined, then one has

$$-(\tau_{\max}-\tau_{\min}+1)\sum_{k=\tau_{\min}}^{\tau_{\max}} \varpi^{T}(k)\mathbf{Q}\varpi(k) \leq -\left(\sum_{k=\tau_{\min}}^{\tau_{\max}} \varpi(k)\right)^{T} \mathbf{Q}\sum_{k=\tau_{\min}}^{\tau_{\max}} \varpi(k). \quad (12)$$

#

The purpose of this paper is to find the observer-based fuzzy controller (6) such that the closed-loop system (10) is asymptotically mean square stable and satisfies the passivity constraint (11). Applying the above definitions and lemma, the stability analysis and fuzzy controller design of closed-loop system (10) is discussed in the following section.

III. OBSERVER-BASED FUZZY CONTROLLER DESIGN FOR DISCRETE-TIME STOCHASTIC TIME-DELAY T-S FUZZY MODELS

Considering the augmented system (10), the stability conditions are derived in this section. For finding the controller gains and observer gains, an algorithm is developed to solve the proposed non-LMI stability conditions. The stability conditions for the observer-based fuzzy controller are first derived in the following theorem.

Theorem 1

Given performance parameters $\mathbf{S}_1$, $\mathbf{S}_2 \geq 0$, $\mathbf{S}_3$ and time delay constant $\tau > 0$. The augmented system (10) is passive and asymptotically mean square stable if there exist positive definite matrices $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$; any matrices $\mathbf{M}$, $\mathbf{N}$ and $\mathbf{U}$; state feedback gains $\mathbf{F}_i$ and observer gains $\mathbf{L}_i$ to satisfy the following conditions.

$$\bar{\mathbf{\Omega}}_{ij} < 0 \quad \text{for i, j} \leq \text{r} \quad (13)$$

where

$$\bar{\mathbf{\Omega}}_{ij}=\begin{bmatrix} \mathbf{\Omega}_{11} & \mathbf{\Omega}_{12} & \mathbf{\Omega}_{13} & -\mathbf{N}+\mathbf{U}^{T} \\ * & \mathbf{\Omega}_{22} & \mathbf{\Omega}_{23} & -\mathbf{M}-\mathbf{U}^{T} \\ * & * & \mathbf{\Omega}_{33} & \mathbf{0} \\ * & * & * & -\mathbf{U}-\mathbf{U}^{T} \end{bmatrix} \quad (14a)$$

$$\mathbf{\Omega}_{11} = \mathbf{G}_{ij}^{T}\mathbf{P}\mathbf{G}_{ij} + \bar{\mathbf{G}}_{ij}^{T}\mathbf{P}\bar{\mathbf{G}}_{ij} + \left(\mathbf{G}_{ij} - \mathbf{I}\right)^{T}\tau\mathbf{R}\left(\mathbf{G}_{ij} - \mathbf{I}\right) + \bar{\mathbf{G}}_{ij}^{T}\tau\mathbf{R}\bar{\mathbf{G}}_{ij} + \mathbf{Q} - \mathbf{P} - \tau^{-1}\mathbf{R} + \mathbf{N} + \mathbf{N}^{T} + \bar{\mathbf{C}}_{ci}^{T}\mathbf{S}_{2}\bar{\mathbf{C}}_{ci} \tag{14b}$$

$$\mathbf{\Omega}_{12} = \mathbf{G}_{ij}^{T}\mathbf{P}\mathbf{G}_{di} + \bar{\mathbf{G}}_{ij}^{T}\mathbf{P}\bar{\mathbf{G}}_{di} + \left(\mathbf{G}_{ij} - \mathbf{I}\right)^{T}\tau\mathbf{R}\mathbf{G}_{di} + \bar{\mathbf{G}}_{ij}^{T}\tau\mathbf{R}\bar{\mathbf{G}}_{di} + \tau^{-1}\mathbf{R} - \mathbf{N} + \mathbf{M}^{T} + \bar{\mathbf{C}}_{ci}^{T}\mathbf{S}_{2}\bar{\mathbf{C}}_{cdi} \tag{14c}$$

$$\mathbf{\Omega}_{13} = \mathbf{G}_{ij}^{T}\mathbf{P}\mathbf{G}_{Eij} + \left(\mathbf{G}_{ij} - \mathbf{I}\right)^{T}\tau\mathbf{R}\mathbf{G}_{Eij} + \bar{\mathbf{C}}_{ci}^{T}\mathbf{S}_{2}\mathbf{D}_{ci} - \bar{\mathbf{C}}_{ci}^{T}\mathbf{S}_{1} \tag{14d}$$

$$\mathbf{\Omega}_{22} = \mathbf{G}_{di}^{T}\mathbf{P}\mathbf{G}_{di} + \bar{\mathbf{G}}_{di}^{T}\mathbf{P}\bar{\mathbf{G}}_{di} + \mathbf{G}_{di}^{T}\tau\mathbf{R}\mathbf{G}_{di} + \bar{\mathbf{G}}_{di}^{T}\tau\mathbf{R}\bar{\mathbf{G}}_{di} - \mathbf{Q} - \tau^{-1}\mathbf{R} - \mathbf{M} - \mathbf{M}^{T} + \bar{\mathbf{C}}_{cdi}^{T}\mathbf{S}_{2}\bar{\mathbf{C}}_{cdi} \tag{14e}$$

$$\mathbf{\Omega}_{23} = \mathbf{G}_{di}^{T}\mathbf{P}\mathbf{G}_{Eij} + \mathbf{G}_{di}^{T}\tau\mathbf{R}\mathbf{G}_{Eij} + \bar{\mathbf{C}}_{cdi}^{T}\mathbf{S}_{2}\mathbf{D}_{ci} - \bar{\mathbf{C}}_{cdi}^{T}\mathbf{S}_{1} \tag{14f}$$

$$\mathbf{\Omega}_{33} = \mathbf{G}_{Eij}^{T}\mathbf{P}\mathbf{G}_{Eij} + \mathbf{G}_{Eij}^{T}\tau\mathbf{R}\mathbf{G}_{Eij} + \mathbf{D}_{ci}^{T}\mathbf{S}_{2}\mathbf{D}_{ci} + \mathbf{S}_{3} - \mathbf{D}_{ci}^{T}\mathbf{S}_{1} - \mathbf{S}_{1}\mathbf{D}_{ci} \tag{14g}$$

In symmetric block matrices, the symbol * is used as an ellipsis for terms induced by symmetry.

***Proof*:**

Let us choose a Lyapunov function as

$$V\left(\bar{x}(k)\right) = V_1\left(\bar{x}(k)\right) + V_2\left(\bar{x}(k)\right) + V_3\left(\bar{x}(k)\right) \tag{15}$$

where

$$V_1\left(\bar{x}(k)\right) = \bar{x}(k)^{T}\mathbf{P}\bar{x}(k) \tag{16}$$

$$V_2\left(\bar{x}(k)\right) = \sum_{s=k-\tau}^{k-1}\bar{x}(s)^{T}\mathbf{Q}\bar{x}(s) \tag{17}$$

$$V_3\left(\bar{x}(k)\right) = \sum_{\ell=-\tau}^{-1}\sum_{s=k+\ell}^{k-1}\bar{\eta}(s)^{T}\mathbf{R}\bar{\eta}(s) \tag{18}$$

$$\bar{\eta}(k) = \bar{x}(k+1) - \bar{x}(k) \tag{19}$$

Taking the difference of $V_1\left(\bar{x}(k)\right)$, $V_2\left(\bar{x}(k)\right)$ and $V_3\left(\bar{x}(k)\right)$ along to trajectories of (10) and then taking the mathematical expectation, one has

$$\begin{aligned}
&E\left\{\Delta V_1\left(\bar{x}(k)\right)\right\} \\
&= E\left\{\bar{x}(k+1)^{T}\mathbf{P}\bar{x}(k+1) - \bar{x}(k)^{T}\mathbf{P}\bar{x}(k)\right\} \\
&= E\Bigg\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_i\left(z(k)\right)h_j\left(z(k)\right) \\
&\quad\times\left(\left(\mathbf{G}_{ij}\bar{x}(k) + \mathbf{G}_{Eij}w(k) + \mathbf{G}_{di}\bar{x}(k-\tau)\right)^{T}\right. \\
&\quad\times\mathbf{P}\left(\mathbf{G}_{ij}\bar{x}(k) + \mathbf{G}_{Eij}w(k) + \mathbf{G}_{di}\bar{x}(k-\tau)\right) \\
&\quad+\left(\bar{\mathbf{G}}_{ij}\bar{x}(k) + \bar{\mathbf{G}}_{di}\bar{x}(k-\tau)\right)^{T} \\
&\quad\left.\times\mathbf{P}\left(\bar{\mathbf{G}}_{ij}\bar{x}(k) + \bar{\mathbf{G}}_{di}\bar{x}(k-\tau)\right)\right) - \bar{x}(k)^{T}\mathbf{P}\bar{x}(k)\Bigg\}
\end{aligned} \tag{20}$$

$$E\left\{\Delta V_2\left(\bar{x}(k)\right)\right\} = E\left\{\bar{x}(k)^{T}\mathbf{Q}\bar{x}(k) - \bar{x}(k-\tau)^{T}\mathbf{Q}\bar{x}(k-\tau)\right\} \tag{21}$$

$$E\left\{\Delta V_3\left(\bar{x}(k)\right)\right\} = E\left\{\tau\left(\bar{\eta}(k)^{T}\mathbf{R}\bar{\eta}(k)\right) - \sum_{s=k-\tau}^{k-1}\bar{\eta}(s)^{T}\mathbf{R}\bar{\eta}(s)\right\} \tag{22}$$

Substituting the (10) into the first term of the right-hand side of (22), i.e., $E\left\{\tau\left(\bar{\eta}(k)^{T}\mathbf{R}\bar{\eta}(k)\right)\right\}$, one has

$$\begin{aligned}
&E\left\{\tau\left(\bar{\eta}(k)^{T}\mathbf{R}\bar{\eta}(k)\right)\right\} \\
&= \tau E\Bigg\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_i\left(z(k)\right)h_j\left(z(k)\right) \\
&\quad\times\left(\left(\left(\mathbf{G}_{ij} - \mathbf{I}\right)\bar{x}(k) + \mathbf{G}_{di}\bar{x}(k-\tau) + \mathbf{G}_{Eij}w(k)\right)^{T}\right. \\
&\quad\times\mathbf{R}\left(\left(\mathbf{G}_{ij} - \mathbf{I}\right)\bar{x}(k) + \mathbf{G}_{di}\bar{x}(k-\tau) + \mathbf{G}_{Eij}w(k)\right) \\
&\quad+\left(\bar{\mathbf{G}}_{ij}\bar{x}(k) + \bar{\mathbf{G}}_{di}\bar{x}(k-\tau)\right)^{T} \\
&\quad\left.\times\mathbf{R}\left(\bar{\mathbf{G}}_{ij}\bar{x}(k) + \bar{\mathbf{G}}_{di}\bar{x}(k-\tau)\right)\right)\Bigg\}
\end{aligned} \tag{23}$$

Based on the Lemma 1 and (19), the following inequality can be obtained from the second term of the right-hand side of (22), i.e., $-\sum_{s=k-\tau}^{k-1}\bar{\eta}(s)^{T}\mathbf{R}\bar{\eta}(s)$.

$$\begin{aligned}
&-\sum_{s=k-\tau}^{k-1}\bar{\eta}(s)^{T}\mathbf{R}\bar{\eta}(s) \le -\tau^{-1}\left(\sum_{s=k-\tau}^{k-1}\bar{\eta}(s)\right)^{T}\mathbf{R}\left(\sum_{s=k-\tau}^{k-1}\bar{\eta}(s)\right) \\
&= -\tau^{-1}\left(\bar{x}(k) - \bar{x}(k-\tau)\right)^{T}\mathbf{R}\left(\bar{x}(k) - \bar{x}(k-\tau)\right)
\end{aligned} \tag{24}$$

According to (23) and (24), one has

$$E\left\{\Delta V_3\left(\bar{x}(k)\right)\right\}$$

$$\leq \tau E\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_i\left(z(k)\right)h_j\left(z(k)\right)\right.$$

$$\times\left(\left(\left(\mathbf{G}_{ij}-\mathbf{I}\right)\bar{x}(k)+\mathbf{G}_{di}\bar{x}(k-\tau)+\mathbf{G}_{Eij}w(k)\right)^{T}\right.$$

$$\times\mathbf{R}\left(\left(\mathbf{G}_{ij}-\mathbf{I}\right)\bar{x}(k)+\mathbf{G}_{di}\bar{x}(k-\tau)+\mathbf{G}_{Eij}w(k)\right)$$

$$\left.\left.+\left(\bar{\mathbf{G}}_{ij}\bar{x}(k)+\bar{\mathbf{G}}_{di}\bar{x}(k-\tau)\right)^{T}\mathbf{R}\left(\bar{\mathbf{G}}_{ij}\bar{x}(k)+\bar{\mathbf{G}}_{di}\bar{x}(k-\tau)\right)\right)\right\}$$

$$-\tau^{-1}\left(\bar{x}(k)-\bar{x}(k-\tau)\right)^{T}\mathbf{R}\left(\bar{x}(k)-\bar{x}(k-\tau)\right) \quad (25)$$

In order to decrease the conservatism of the proposed stability criterion, the following equation is introduced with the concept of free–weighting matrix technique (Souza et al., 2009). For appropriate dimension matrices, the following equation always holds.

$$2\left(\bar{x}(k)^{T}\mathbf{N}+\bar{x}(k-\tau)^{T}\mathbf{M}+\left(\sum_{s=k-\tau}^{k-1}\bar{\eta}(s)\right)^{T}\mathbf{U}\right)$$

$$\times\left(\bar{x}(k)-\bar{x}(k-\tau)-\sum_{s=k-\tau}^{k-1}\bar{\eta}(s)\right)=0 \quad (26)$$

Thus, combining (20), (21), (25) and (26), one has

$$E\left\{\Delta V\left(\bar{x}(k)\right)\right\}=E\left\{\Delta V_1\left(\bar{x}(k)\right)+\Delta V_2\left(\bar{x}(k)\right)+\Delta V_3\left(\bar{x}(k)\right)\right\}$$

$$\leq E\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_i\left(z(k)\right)h_j\left(z(k)\right)\left\{\xi^{T}(k)\bar{\mathbf{\Pi}}_{ij}\xi(k)\right\}\right\}$$

for i, j $\leq$ r (27)

where

$$\xi^{T}(k)=\begin{bmatrix}\bar{x}^{T}(k) & \bar{x}^{T}(k-\tau) & w^{T}(k) & \left(\sum_{\ell=k-\tau}^{k-1}\bar{\eta}(k)\right)^{T}\end{bmatrix}$$

and

$$\bar{\mathbf{\Pi}}_{ij}=\begin{bmatrix}\mathbf{\Pi}_{11} & \mathbf{\Pi}_{12} & \mathbf{\Pi}_{13} & -\mathbf{N}+\mathbf{U}^{T}\\ * & \mathbf{\Pi}_{22} & \mathbf{\Pi}_{23} & -\mathbf{M}-\mathbf{U}^{T}\\ * & * & \mathbf{\Pi}_{33} & \mathbf{0}\\ * & * & * & -\mathbf{U}-\mathbf{U}^{T}\end{bmatrix} \quad (28)$$

$$\mathbf{\Pi}_{11}=\mathbf{G}_{ij}^{T}\mathbf{P}\mathbf{G}_{ij}+\bar{\mathbf{G}}_{ij}^{T}\mathbf{P}\bar{\mathbf{G}}_{ij}+\left(\mathbf{G}_{ij}-\mathbf{I}\right)^{T}\tau\mathbf{R}\left(\mathbf{G}_{ij}-\mathbf{I}\right)+\bar{\mathbf{G}}_{ij}^{T}\tau\mathbf{R}\bar{\mathbf{G}}_{ij}$$

$$+\mathbf{Q}-\mathbf{P}-\tau^{-1}\mathbf{R}+\mathbf{N}+\mathbf{N}^{T} \quad (29a)$$

$$\mathbf{\Pi}_{12}=\mathbf{G}_{ij}^{T}\mathbf{P}\mathbf{G}_{di}+\bar{\mathbf{G}}_{ij}^{T}\mathbf{P}\bar{\mathbf{G}}_{di}+\left(\mathbf{G}_{ij}-\mathbf{I}\right)^{T}\tau\mathbf{R}\mathbf{G}_{di}+\bar{\mathbf{G}}_{ij}^{T}\tau\mathbf{R}\bar{\mathbf{G}}_{di}$$

$$+\tau^{-1}\mathbf{R}-\mathbf{N}+\mathbf{M}^{T} \quad (29b)$$

$$\mathbf{\Pi}_{13}=\mathbf{G}_{ij}^{T}\mathbf{P}\mathbf{G}_{Eij}+\left(\mathbf{G}_{ij}-\mathbf{I}\right)^{T}\tau\mathbf{R}\mathbf{G}_{Eij} \quad (29c)$$

$$\mathbf{\Pi}_{22}=\mathbf{G}_{di}^{T}\mathbf{P}\mathbf{G}_{di}+\bar{\mathbf{G}}_{di}^{T}\mathbf{P}\bar{\mathbf{G}}_{di}+\mathbf{G}_{di}^{T}\tau\mathbf{R}\mathbf{G}_{di}+\bar{\mathbf{G}}_{di}^{T}\tau\mathbf{R}\bar{\mathbf{G}}_{di}$$

$$-\mathbf{Q}-\tau^{-1}\mathbf{R}-\mathbf{M}-\mathbf{M}^{T} \quad (29d)$$

$$\mathbf{\Pi}_{23}=\mathbf{G}_{di}^{T}\mathbf{P}\mathbf{G}_{Eij}+\mathbf{G}_{di}^{T}\tau\mathbf{R}\mathbf{G}_{Eij} \quad (29e)$$

$$\mathbf{\Pi}_{33}=\mathbf{G}_{Eij}^{T}\mathbf{P}\mathbf{G}_{Eij}+\mathbf{G}_{Eij}^{T}\tau\mathbf{R}\mathbf{G}_{Eij} \quad (29f)$$

Let us introduce the following performance function.

$$J\left(\bar{x}(k)\right)=E\left\{\sum_{k=0}^{k_q}\left(y_c^{T}(k)\mathbf{S}_2y_c(k)\right.\right.$$

$$\left.\left.+w^{T}(k)\mathbf{S}_3w(k)-2y_c^{T}(k)\mathbf{S}_1w(k)\right)\right\} \quad (30)$$

For all $w(k)\neq 0$ with zero initial condition, one has

$$J\left(\bar{x}(k)\right)=E\left\{\sum_{k=0}^{k_q}\left\{y_c^{T}(k)\mathbf{S}_2y_c(k)+w^{T}(k)\mathbf{S}_3w(k)\right.\right.$$

$$\left.\left.-2y_c^{T}(k)\mathbf{S}_1w(k)+\Delta V\left(\bar{x}(k)\right)\right\}-V\left(\bar{x}(k_q+1)\right)\right\}$$

$$\leq E\left\{\sum_{k=0}^{k_q}\left(y_c^{T}(k)\mathbf{S}_2y_c(k)+w^{T}(k)\mathbf{S}_3w(k)\right.\right.$$

$$\left.\left.-2y_c^{T}(k)\mathbf{S}_1w(k)+\Delta V\left(\bar{x}(k)\right)\right)\right\} \quad (31)$$

Thus, the following inequality can be obtained by substituting (10b) into (31).

$$J\left(\bar{x}(k)\right)\leq E\sum_{k=0}^{k_q}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_i\left(z(k)\right)h_j\left(z(k)\right)\left\{\xi^{T}(k)\bar{\mathbf{\Omega}}_{ij}\xi(k)\right\}\right\} \quad (32)$$

where

$$\bar{\mathbf{\Omega}}_{ij}=\bar{\mathbf{\Pi}}_{ij}+\mathbf{\Gamma}_i \quad (33)$$

$$\mathbf{\Gamma}_i = \begin{bmatrix} \bar{\mathbf{C}}_{ci}^T\mathbf{S}_2\bar{\mathbf{C}}_{ci} & \bar{\mathbf{C}}_{ci}^T\mathbf{S}_2\bar{\mathbf{C}}_{cdi} & \bar{\mathbf{C}}_{ci}^T\mathbf{S}_2\mathbf{D}_{ci} - \bar{\mathbf{C}}_{ci}^T\mathbf{S}_1 & \mathbf{0} \\ * & \bar{\mathbf{C}}_{cdi}^T\mathbf{S}_2\bar{\mathbf{C}}_{cdi} & \bar{\mathbf{C}}_{cdi}^T\mathbf{S}_2\mathbf{D}_{ci} - \bar{\mathbf{C}}_{cdi}^T\mathbf{S}_1 & \mathbf{0} \\ * & * & \mathbf{D}_{ci}^T\mathbf{S}_2\mathbf{D}_{ci} + \mathbf{S}_3 - \mathbf{D}_{ci}^T\mathbf{S}_1 - \mathbf{S}_1\mathbf{D}_{ci} & \mathbf{0} \\ * & * & * & \mathbf{0} \end{bmatrix} \tag{34}$$

Obviously, if the conditions (13) holds then one has $J(\bar{x}(k)) < 0$. Thus, the following relation can be obtained from (30) according to $J(\bar{x}(k)) < 0$.

$$E\left\{\sum_{k=0}^{k_q}\left(y_c^T(k)\mathbf{S}_2 y_c(k) + w^T(k)\mathbf{S}_3 w(k) - 2y_c^T(k)\mathbf{S}_1 w(k)\right)\right\} < 0 \tag{35a}$$

or

$$E\left\{2\sum_{k=0}^{k_q} y_c^T(k)\mathbf{S}_1 w(k)\right\}$$
$$> E\left\{\sum_{k=0}^{k_q}\left(y_c^T(k)\mathbf{S}_2 y_c(k) + w^T(k)\mathbf{S}_3 w(k)\right)\right\} \tag{35b}$$

Since (35b) is equivalent to (11) of Definition 1, the augmented system (10) is passive for all nonzero external disturbance, i.e., $w(k) \neq 0$. Then, we need to show that the system is asymptotically mean square stable. By omitting the external disturbance, i.e., $w(k) = 0$, if the conditions (13) hold, the following inequality can be obtained from (33).

$$\bar{\mathbf{\Pi}}_{ij} < -\left(\bar{\mathbf{C}}_{ci}\bar{x}(k) + \bar{\mathbf{C}}_{cdi}\bar{x}(k-\tau)\right)^T \times \mathbf{S}_2\left(\bar{\mathbf{C}}_{ci}\bar{x}(k) + \bar{\mathbf{C}}_{cdi}\bar{x}(k-\tau)\right) \tag{36}$$

It can be found that if $\mathbf{S}_2 \geq 0$ holds then $\bar{\mathbf{\Pi}}_{ij} < 0$. From (27), the inequality $E\{\Delta V(x(k))\} < 0$ can be obtained due to $\bar{\mathbf{\Pi}}_{ij} < 0$. In this case, the augmented system (10) is asymptotically mean square stable from Definition 2. The proof of this theorem is completed.

#

Remark 1

In Theorem 1, the Lemma 1 and free-weighting matrix technique are employed to derive stability conditions (13). In Zhu and Yang (2008), the Lemma 1 is applied to derived less conservative stability conditions of nonlinear system with time delay. The reasons for discrete Jensen inequality reducing the conservatism have been illustrated and proven in Zhu and Yang (2008). Besides, the free-weighting matrix technique (Souza et al., 2009) is a famous technique that can also derive the conservative stability conditions of delay systems by adding extra variables without any limitation.

#

The stability conditions stated in (13) developed in Theorem 1 are of the form of Bilinear Matrix Inequality (BMI) that cannot be directly calculated by LMI technique. For this reason, the modified stability conditions are derived from the BMI conditions (13) in the following theorem.

Theorem 2

Given performance parameters $\mathbf{S}_1$, $\mathbf{S}_2 \geq 0$, $\mathbf{S}_3$ and time delay constant $\tau > 0$. The augmented system (10) is passive and asymptotically mean square stable if there exists positive definite matrices $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$; any matrices $\mathbf{M}$, $\mathbf{N}$ and $\mathbf{U}$; state feedback gains $\mathbf{F}_i$ and observer gains $\mathbf{L}_i$ to satisfy the following conditions.

$$\begin{bmatrix} \mathbf{\Theta}_{11} & \mathbf{\Theta}_{12}^T & \mathbf{\Theta}_{13}^T & \mathbf{\Theta}_{14}^T & \mathbf{\Theta}_{15}^T \\ * & -\mathbf{P}^{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -\mathbf{P}^{-1} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\mathbf{R}^{-1} & \mathbf{0} \\ * & * & * & * & -\mathbf{R}^{-1} \end{bmatrix} < 0 \ \text{for i, j} \leq \text{r} \tag{37}$$

where

$$\mathbf{\Theta}_{11} = \begin{bmatrix} \mathbf{\Psi}_{11} + \bar{\mathbf{C}}_{ci}^T\mathbf{S}_2\bar{\mathbf{C}}_{ci} & \tau^{-1}\mathbf{R} - \mathbf{N} + \mathbf{M}^T + \bar{\mathbf{C}}_{ci}^T\mathbf{S}_2\bar{\mathbf{C}}_{cdi} & \bar{\mathbf{C}}_{ci}^T\mathbf{S}_2\mathbf{D}_{ci} - \bar{\mathbf{C}}_{ci}^T\mathbf{S}_1 & -\mathbf{N} + \mathbf{U}^T \\ * & -\mathbf{Q} - \tau^{-1}\mathbf{R} - \mathbf{M} - \mathbf{M}^T + \bar{\mathbf{C}}_{cdi}^T\mathbf{S}_2\bar{\mathbf{C}}_{cdi} & \bar{\mathbf{C}}_{cdi}^T\mathbf{S}_2\mathbf{D}_{ci} - \bar{\mathbf{C}}_{cdi}^T\mathbf{S}_1 & -\mathbf{M} - \mathbf{U}^T \\ * & * & \mathbf{D}_{ci}^T\mathbf{S}_2\mathbf{D}_{ci} + \mathbf{S}_3 - \mathbf{D}_{ci}^T\mathbf{S}_1 - \mathbf{S}_1\mathbf{D}_{ci} & \mathbf{0} \\ * & * & * & -\mathbf{U} - \mathbf{U}^T \end{bmatrix},$$

$$\mathbf{\Theta}_{12} = \begin{bmatrix} \mathbf{G}_{ij} & \mathbf{G}_{di} & \mathbf{G}_{Eij} & \mathbf{0} \end{bmatrix}, \mathbf{\Theta}_{13} = \begin{bmatrix} \bar{\mathbf{G}}_{ij} & \bar{\mathbf{G}}_{di} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\mathbf{\Theta}_{14} = \sqrt{\tau}\begin{bmatrix} \mathbf{G}_{ij} - \mathbf{I} & \mathbf{G}_{di} & \mathbf{G}_{Eij} & \mathbf{0} \end{bmatrix}, \mathbf{\Theta}_{15} = \sqrt{\tau}\mathbf{\Theta}_{13}$$

and

$$\mathbf{\Psi}_{11} = \mathbf{Q} - \mathbf{P} - \tau^{-1}\mathbf{R} + \mathbf{N} + \mathbf{N}^T.$$

***Proof*:**

The stability conditions (13) can be rewritten as follows.

$$\begin{bmatrix} \mathbf{\Psi}_{11} + \bar{\mathbf{C}}_{ci}^{T}\mathbf{S}_2\bar{\mathbf{C}}_{ci} & \tau^{-1}\mathbf{R} - \mathbf{N} + \mathbf{M}^{T} + \bar{\mathbf{C}}_{ci}^{T}\mathbf{S}_2\bar{\mathbf{C}}_{cdi} & \bar{\mathbf{C}}_{ci}^{T}\mathbf{S}_2\mathbf{D}_{ci} - \bar{\mathbf{C}}_{ci}^{T}\mathbf{S}_1 & -\mathbf{N} + \mathbf{U}^{T} \\ * & -\mathbf{Q} - \tau^{-1}\mathbf{R} - \mathbf{M} - \mathbf{M}^{T} + \bar{\mathbf{C}}_{cdi}^{T}\mathbf{S}_2\bar{\mathbf{C}}_{cdi} & \bar{\mathbf{C}}_{cdi}^{T}\mathbf{S}_2\mathbf{D}_{ci} - \bar{\mathbf{C}}_{cdi}^{T}\mathbf{S}_1 & -\mathbf{M} - \mathbf{U}^{T} \\ * & * & \mathbf{D}_{ci}^{T}\mathbf{S}_2\mathbf{D}_{ci} + \mathbf{S}_3 - \mathbf{D}_{ci}^{T}\mathbf{S}_1 - \mathbf{S}_1\mathbf{D}_{ci} & \mathbf{0} \\ * & * & * & -\mathbf{U} - \mathbf{U}^{T} \end{bmatrix}$$

$$+\begin{bmatrix} \mathbf{G}_{ij}^{T} \\ \mathbf{G}_{di}^{T} \\ \mathbf{G}_{Eij}^{T} \\ \mathbf{0} \end{bmatrix} \mathbf{P} \begin{bmatrix} \mathbf{G}_{ij} & \mathbf{G}_{di} & \mathbf{G}_{Eij} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \bar{\mathbf{G}}_{ij}^{T} \\ \bar{\mathbf{G}}_{di}^{T} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \mathbf{P} \begin{bmatrix} \bar{\mathbf{G}}_{ij} & \bar{\mathbf{G}}_{di} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$+\begin{bmatrix} \sqrt{\tau}\mathbf{G}_{ij}^{T} \\ \sqrt{\tau}\mathbf{G}_{di}^{T} \\ \sqrt{\tau}\mathbf{G}_{Eij}^{T} \\ \mathbf{0} \end{bmatrix} \mathbf{R} \begin{bmatrix} \sqrt{\tau}\mathbf{G}_{ij} & \sqrt{\tau}\mathbf{G}_{di} & \sqrt{\tau}\mathbf{G}_{Eij} & \mathbf{0} \end{bmatrix}$$

$$+\begin{bmatrix} \sqrt{\tau}\bar{\mathbf{G}}_{ij}^{T} \\ \sqrt{\tau}\bar{\mathbf{G}}_{di}^{T} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \mathbf{R} \begin{bmatrix} \sqrt{\tau}\bar{\mathbf{G}}_{ij} & \sqrt{\tau}\bar{\mathbf{G}}_{di} & \mathbf{0} & \mathbf{0} \end{bmatrix} < 0 \qquad (38)$$

From the definitions of the matrices below (37), the inequality (38) can also be rewritten in the following form.

$$\mathbf{\Theta}_{11} + \mathbf{\Theta}_{12}^{T}\mathbf{P}\mathbf{\Theta}_{12} + \mathbf{\Theta}_{13}^{T}\mathbf{P}\mathbf{\Theta}_{13} + \mathbf{\Theta}_{14}^{T}\mathbf{R}\mathbf{\Theta}_{14} + \mathbf{\Theta}_{15}^{T}\mathbf{R}\mathbf{\Theta}_{15} < 0 \quad (39)$$

Applying the Schur complement (Boyd, 1994), one can obtain the following inequality from (39).

$$\begin{bmatrix} \mathbf{\Theta}_{11} & \mathbf{\Theta}_{12}^{T} & \mathbf{\Theta}_{13}^{T} & \mathbf{\Theta}_{14}^{T} & \mathbf{\Theta}_{15}^{T} \\ * & -\mathbf{P}^{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -\mathbf{P}^{-1} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\mathbf{R}^{-1} & \mathbf{0} \\ * & * & * & * & -\mathbf{R}^{-1} \end{bmatrix} < 0$$

Obviously, one can find that if the stability conditions (37) of Theorem 2 hold then the stability conditions (13) of Theorem 1 also hold. Thus, if the conditions (37) of Theorem 2 hold then the augmented system (10) is passive and asymptotically mean square stable. The proof of this theorem is complete.

#

In Theorem 2, the conditions of (37) simultaneously includes variables $\mathbf{P}$, $\mathbf{P}^{-1}$, $\mathbf{R}$ and $\mathbf{R}^{-1}$ such that the condition (37) is still not a LMI problem. For applying the LMI technique, let us introduce two new variables, i.e, $\mathbf{X}$ and $\mathbf{T}$, such that

$$\mathbf{PX} = \mathbf{I} \text{ and } \mathbf{RT} = \mathbf{I} \qquad (40)$$

Hence, (37) becomes

$$\begin{bmatrix} \mathbf{\Theta}_{11} & \mathbf{\Theta}_{12}^{T} & \mathbf{\Theta}_{13}^{T} & \mathbf{\Theta}_{14}^{T} & \mathbf{\Theta}_{15}^{T} \\ * & -\mathbf{X} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -\mathbf{X} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\mathbf{T} & \mathbf{0} \\ * & * & * & * & -\mathbf{T} \end{bmatrix} < 0 \text{ for } i, j \le r \qquad (41)$$

Based on the cone complementarity technique (Ghaoui et al., 1997), the following nonlinear minimization problem can be proposed by replacing $\mathbf{P}^{-1}$ and $\mathbf{R}^{-1}$ as $\mathbf{X}$ and $\mathbf{T}$ in (37).

Minimize $\text{trace}(\mathbf{PX})$ and $\text{trace}(\mathbf{RT})$

$$\text{Subject to (41), } \begin{bmatrix} \mathbf{P} & \mathbf{I} \\ \mathbf{I} & \mathbf{X} \end{bmatrix} \ge 0 \text{ and } \begin{bmatrix} \mathbf{R} & \mathbf{I} \\ \mathbf{I} & \mathbf{T} \end{bmatrix} \ge 0 \qquad (42)$$

Although the above minimization problem gives sub-optimal solutions for (41), it is much easier to find feasible solutions than the original non-convex conditions (13). Based on the nonlinear minimization problem stated in (42), the feasible solutions of stability condition (37) can be solved by using the following algorithm.

Algorithm 1

Step 1. Choose performance matrices $\mathbf{S}_1$, $\mathbf{S}_2 \ge 0$ and $\mathbf{S}_3$. Find the feasible solutions $\mathbf{P}^{t\text{-}1}$, $\mathbf{R}^{t\text{-}1}$, $\mathbf{X}^{t\text{-}1}$ and $\mathbf{T}^{t\text{-}1}$ for satisfying the (41). Set t =1 and the obtained variables to be initial variables set $\{\mathbf{P}^{0}, \mathbf{R}^{0}, \mathbf{X}^{0}, \mathbf{T}^{0}\}$.

Step 2. Solve the parameters $\{\mathbf{P}^{t}, \mathbf{R}^{t}, \mathbf{X}^{t}, \mathbf{T}^{t}\}$ from the following LMI problem with obtained variables $\{\mathbf{P}^{t\text{-}1}, \mathbf{R}^{t\text{-}1}, \mathbf{X}^{t\text{-}1}, \mathbf{T}^{t\text{-}1}\}$.

Minimize

$\text{trace}\{\mathbf{P}^{t}\mathbf{X}^{t\text{-}1} + \mathbf{X}^{t}\mathbf{P}^{t\text{-}1}\}$ and $\text{trace}\{\mathbf{R}^{t}\mathbf{T}^{t\text{-}1} + \mathbf{T}^{t}\mathbf{R}^{t\text{-}1}\}$.

Subject to (41), $\begin{bmatrix} \mathbf{P}^{t} & \mathbf{I} \\ \mathbf{I} & \mathbf{X}^{t} \end{bmatrix} \ge 0$ and $\begin{bmatrix} \mathbf{R}^{t} & \mathbf{I} \\ \mathbf{I} & \mathbf{T}^{t} \end{bmatrix} \ge 0$.

Step 3. Substitute the obtained solutions from Step 2 into (37). If the inequality (37) is satisfied, then stop and quit the algorithm. Otherwise, Set t = t + 1 and go to Step 2.

In the following section, a numerical example for the control of nonlinear pendulum system with time delay is provided to illustrate the application of proposed observer-based fuzzy controller design method.

IV. NUMERICAL EXAMPLE FOR THE CONTROL OF NONLINEAR PENDULUM SYSTEMS

According to the design method developed in this paper, the observer-based fuzzy controller for nonlinear pendulum system with time delay is designed in this section. Referring to Feng (2006), the dynamic equation of continuous-time nonlinear time-delay pendulum system is described as follows.

$$\dot{x}_1(t) = \alpha x_2(t) + (1-\alpha)x_2(t-\tau)$$

$$\dot{x}_2(t) = -\frac{g\sin x_1(t) + (2b/lm)\left(\alpha x_2(t) + (1-\alpha)x_2(t-\tau)\right)}{(4/3)l - aml\cos^2 x_1(t)}$$
$$+\frac{(aml/2)\left(\alpha x_2(t) + (1-\alpha)x_2(t-\tau)\right)^2 \sin(2x_1(t))}{(4/3)l - aml\cos^2 x_1(t)}$$
$$+w(t)$$

$$y(t) = \sin x_1(t) + \alpha x_2(t) + (1-\alpha)x_2(t-\tau) + w(\mathrm{t}) \qquad (43)$$

where $x_1(t)$ denotes the angle of the pendulum from the vertical, $x_2(t)$ is the angular velocity and w(t) is a zero-mean white noise with variance 1. Besides, g is the gravity constant, m is the mass of the pendulum, M is the mass of the cart, $a = 1/(m + M)$, $2l$ is the length of the pendulum and b is the damping coefficient of the pendulum around the pivot. In this example, the pendulum parameters are chosen as $g = 9.8$ m/s^2, $m = 2$ kg, $M = 8$ kg, $l = 0.5$ m, $b = 0.7$ Nm/s and the retarded coefficient is given as $\alpha = 0.9$. In order to consider the stochastic behaviors, the multiplicative noise term is added in the system. By linearizing the plant around the operation points, i.e., $x_{op1} = (0, 0)$, $x_{op2} = (\pm\pi/3, 0)$ and $x_{op3} = (\pm\pi/2, 0)$, and discretizing it with the sampling period T = 0.1 s, then the discrete-time T-S fuzzy model with multiplicative noise can be obtained as follows.

Rule i : IF $x_1(\mathrm{k})$ is about M_{i1} THEN

$$x(k+1) = \mathbf{A}_{\mathrm{i}}x(k) + \mathbf{B}_{\mathrm{i}}u(k) + \mathbf{E}_{\mathrm{i}}w(k) + \mathbf{A}_{\mathrm{di}}x(k-\tau)$$
$$+\left(\bar{\mathbf{A}}_{\mathrm{i}}x(k) + \bar{\mathbf{B}}_{\mathrm{i}}u(k) + \bar{\mathbf{A}}_{\mathrm{di}}x(k-\tau)\right)\beta(k) \qquad (44a)$$

$$y_{\mathrm{m}}(k) = \mathbf{C}_{\mathrm{mi}}x(k) + \mathbf{C}_{\mathrm{mdi}}x(k-\tau) + \mathbf{D}_{\mathrm{mi}}w(k) \qquad (44b)$$

$$y_{\mathrm{c}}(k) = \mathbf{C}_{\mathrm{ci}}x(k) + \mathbf{C}_{\mathrm{cdi}}x(k-\tau) + \mathbf{D}_{\mathrm{ci}}w(k) \qquad (44c)$$

for i = 1, 2, 3

where

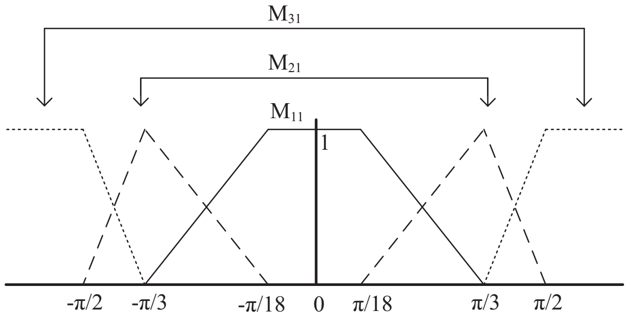

Fig. 1. Membership functions of x_1(k).

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0.045 \\ -0.8558 & 0.7894 \end{bmatrix},\ \mathbf{A}_2 = \begin{bmatrix} 1 & 0.045 \\ -0.6315 & 0.8018 \end{bmatrix},$$

$$\mathbf{A}_3 = \begin{bmatrix} 1 & 0.045 \\ -0.4679 & 0.8055 \end{bmatrix},\ \mathbf{B} = \mathbf{B}_{\mathrm{i}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$\mathbf{A}_{\mathrm{d1}} = \begin{bmatrix} 0 & 0.005 \\ 0 & 0.0877 \end{bmatrix},\ \mathbf{A}_{\mathrm{d2}} = \begin{bmatrix} 0 & 0.005 \\ 0 & 0.0891 \end{bmatrix},$$

$$\mathbf{A}_{\mathrm{d3}} = \begin{bmatrix} 0 & 0.005 \\ 0 & 0.0895 \end{bmatrix},\ \mathbf{E} = \mathbf{E}_{\mathrm{i}} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},$$

$$\mathbf{C}_{\mathrm{m1}} = \mathbf{C}_{\mathrm{c1}} = \begin{bmatrix} 0.9949 & 1 \end{bmatrix},\ \mathbf{C}_{\mathrm{m2}} = \mathbf{C}_{\mathrm{c2}} = \begin{bmatrix} 0.8270 & 1 \end{bmatrix},$$

$$\mathbf{C}_{\mathrm{m3}} = \mathbf{C}_{\mathrm{c3}} = \begin{bmatrix} 0.6366 & 1 \end{bmatrix},\ \mathbf{C}_{\mathrm{cd}} = \mathbf{C}_{\mathrm{cdi}} = \begin{bmatrix} 0 & 0.1 \end{bmatrix},$$

$$\mathbf{C}_{\mathrm{md}} = \mathbf{C}_{\mathrm{mdi}} = \begin{bmatrix} 0 & 0.1 \end{bmatrix},\ \mathbf{D}_{\mathrm{mi}} = \mathbf{D}_{\mathrm{ci}} = \mathbf{D} = 1,\ \bar{\mathbf{A}}_1 = 0.01\mathbf{A}_1,$$

$$\bar{\mathbf{A}}_2 = 0.02\mathbf{A}_2,\ \bar{\mathbf{A}}_3 = 0.03\mathbf{A}_3,\ \bar{\mathbf{A}}_{\mathrm{d1}} = 0.01\mathbf{A}_{\mathrm{d1}},\ \bar{\mathbf{A}}_{\mathrm{d2}} = 0.02\mathbf{A}_{\mathrm{d2}},$$

$$\bar{\mathbf{A}}_{\mathrm{d3}} = 0.03\mathbf{A}_{\mathrm{d3}},\ \bar{\mathbf{B}}_1 = 0.01\mathbf{B}_1,\ \bar{\mathbf{B}}_2 = 0.02\mathbf{B}_2 \text{ and } \bar{\mathbf{B}}_3 = 0.03\mathbf{B}_3.$$

The membership functions of $x_1(\mathrm{k})$ are shown in Fig. 1.

Before starting the calculation process, let us set the parameters as $\mathbf{S}_1 \triangleq 1.5$, $\mathbf{S}_2 \triangleq 0.1$ and $\mathbf{S}_3 \triangleq 0.8$ and $\tau = 2$ for designing observer-based fuzzy controller. Using the LMI Toolbox of MATLAB, the following state feedback gains and observer gains can be obtained via Theorem 2 and Algorithm 1.

$$\mathbf{F}_1 = \begin{bmatrix} -2.7498 & -0.8487 \end{bmatrix},\ \mathbf{F}_2 = \begin{bmatrix} -2.9325 & -0.8534 \end{bmatrix},$$

$$\mathbf{F}_3 = \begin{bmatrix} -2.9253 & -0.8221 \end{bmatrix},\ \mathbf{L}_1 = \begin{bmatrix} 0.2305 \\ 101.1767 \end{bmatrix} \times 10^{-3},$$

$$\mathbf{L}_2 = \begin{bmatrix} 0.2762 \\ 101.1041 \end{bmatrix} \times 10^{-3},\ \mathbf{L}_3 = \begin{bmatrix} 0.2905 \\ 101.0155 \end{bmatrix} \times 10^{-3},$$

$$\mathbf{P}=\begin{bmatrix} 2.2914 & 0.3841 & 0.0000 & 0.0000 \\ 0.3841 & 0.1038 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 72.9209 & 13.5689 \\ 0.0000 & 0.0000 & 13.5689 & 5.6213 \end{bmatrix}\times 10^{3},$$

$$\mathbf{Q}=\begin{bmatrix} 0.7972 & 0.1954 & 0.0000 & 0.0000 \\ 0.1954 & 0.0581 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 6.6579 & 1.5105 \\ 0.0000 & 0.0000 & 1.5105 & 0.8153 \end{bmatrix}\times 10^{3},$$

$$\mathbf{R}=\begin{bmatrix} 8.3269 & -8.7031 & 0.0000 & 0.0000 \\ -8.7031 & 11.9025 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.6983 & -0.1113 \\ 0.0000 & 0.0000 & -0.1113 & 0.6728 \end{bmatrix}\times 10^{-6},$$

$$\mathbf{M}=\begin{bmatrix} -57.9341 & -18.3108 & -2.1159 & -0.7236 \\ -11.2313 & -4.3747 & -8.8851 & -3.3178 \\ -1.2456 & -0.3977 & -366.5764 & -26.1862 \\ -0.4059 & -0.1618 & -322.5181 & -165.8799 \end{bmatrix},$$

$$\mathbf{N}=\begin{bmatrix} 0.0764 & 0.0242 & -0.0129 & -0.0044 \\ 0.0144 & 0.0056 & -0.0097 & -0.0031 \\ -0.0129 & -0.0035 & 1.3118 & 0.6437 \\ -0.0036 & -0.0012 & 0.3719 & 0.2344 \end{bmatrix}\times 10^{3} \text{ and}$$

$$\mathbf{U}=\begin{bmatrix} 0.1872 & 0.0426 & -0.0021 & -0.0007 \\ 0.0513 & 0.0136 & -0.0005 & -0.0003 \\ -0.0012 & -0.0004 & 1.7713 & 0.5836 \\ -0.0004 & 0.00005 & 0.6076 & 0.3192 \end{bmatrix}\times 10^{3}.$$

Based on the above feedback gains, the observer-based fuzzy controller can be obtained by PDC concept such as

$$u(k)=\sum_{i=1}^{r} h_i\left(z(k)\right)\left(\mathbf{F}_i\hat{x}(k)\right) \tag{45}$$

With the designed fuzzy controller (45), the state responses of the augmented system are shown in Figs. 2 and 3 with the initial conditions $x(0)=[\pi/4\ \ 0]^{\mathrm{T}}$ and $\hat{x}(0)=[\pi/6\ \ 0]^{\mathrm{T}}$. From the simulated responses, the following ratio can be introduced to check the passivity constraint (11).

$$\frac{E\left\{\sum_{k=0}^{k_q} 0.1 y_c^{\mathrm{T}}(k) y_c(k)+0.8 w^{T}(k) w(k)\right\}}{E\left\{\sum_{k=0}^{k_q} 3 y_c^{\mathrm{T}}(k) w(k)\right\}}=0.3156 \tag{46}$$

It is easy to find that the value of (46) is smaller than one;

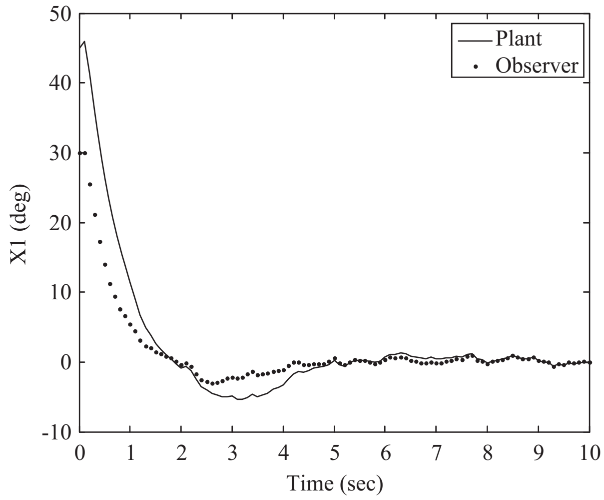

Fig. 2. Responses of x_1(k) and $\hat{x}_1$(k).

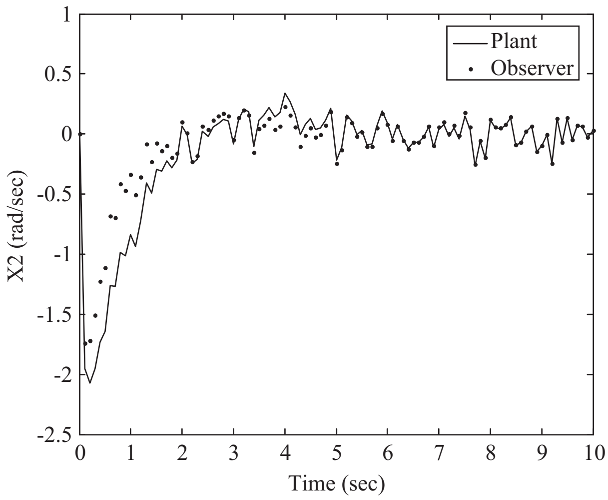

Fig. 3. Responses of x_2(k) and $\hat{x}_2$(k).

hence, the passivity constraint (11) with $\mathbf{S}_1 \triangleq 1.5$, $\mathbf{S}_2 \triangleq 0.1$ and $\mathbf{S}_3 \triangleq 0.8$ can be satisfied. From Figs. 2 and 3 and (46), one can find that the system (43) driven by fuzzy controller (45) is passive and asymptotically mean square stable.

V. CONCLUSIONS

In this paper, the observer-based fuzzy controller design method was considered for discrete-time nonlinear stochastic systems with time delay. And, the stochastic T-S fuzzy model was proposed to represent the nonlinear stochastic systems via T-S fuzzy model approach. For discussing the stability and stabilization problems, the sufficient conditions have been derived via the Lyapunov stability criterion and the passivity theorem. Besides, the discrete Jensen inequality and the free-weighting matrix technique were employed to reduce the conservatism of the proposed stability criterion. In order to calculate the non-LMI problems, an algorithm was proposed in this paper by using cone complementarity technique. Via the proposed design methods, the observer-based fuzzy controller can be designed for stabilizing the discrete-time stochastic T-S fuzzy model with time delay.

ACKNOWLEDGMENTS

The authors would like to express their sincere gratitude to anonymous reviewers who gave us some constructive comments, criticisms and suggestions.

REFERENCES

Boyd, S., L. E. Ghaoui, E. Feron and V. Balakrishnan (1994). Linear Matrix Inequalities in System and Control Theory. SIAM, Philadelphia, PA.

Chang, W. J. and W. Chang (2005). Synthesis of nonlinear discrete control systems via time-delay affine Takagi-Sugeno fuzzy models. ISA Transactions 44, 243-257.

Chang, W. J. and W. Chang (2006). Discrete fuzzy control of time-delay affine Takagi-Sugeno fuzzy models with H_∞ constraint. IEE Proceedings, Part D, Control Theory and Applications 153, 745-752.

Chang, W. J., C. C. Ku and P. H. Huang (2010). Robust fuzzy control of uncertain stochastic time-delay Takagi-Sugeno fuzzy models for achieving passivity. Fuzzy Sets and Systems 161, 2012-2032.

Chang, W. J., C. C. Ku and P. H. Huang (2011). Robust fuzzy control via observer feedback for passive stochastic fuzzy systems with time-delay and multiplicative noise. International Journal of Innovative Computing, Information and Control 7, 345-364.

Chang, W. J. C. C. Ku and C. H. Chang (2012). PDC and Non-PDC fuzzy control with relaxed stability conditions for continuous-time multiplicative noised fuzzy systems. Journal of the Franklin Institute - Engineering and Applied Mathematics 349, 2664-2686.

Chang, W. J., C. T. Yang and P. H. Chen (2013). Robust fuzzy congestion control of TCP/AQM router via perturbed Takagi-Sugeno fuzzy models. International Journal of Fuzzy Systems 15, 203-213.

Chang, W. J. and B. J. Huang (2014). Robust fuzzy control subject to state variance and passivity constraints for perturbed nonlinear systems with multiplicative noises. ISA Transactions 53, 1787-1795.

Chen, B. S. and Y. T. Chang (2009). Fuzzy state-space modeling and robust observer-based control design for nonlinear partial differential systems. IEEE Transactions on Fuzzy Systems 17, 1025-1043.

Choi, H. H. (2007). LMI-based nonlinear fuzzy observer-controller design for uncertain MIMO nonlinear systems. IEEE Transactions on Fuzzy Systems 15, 956-971.

Dong, J. and G. H. Yang (2009). H_∞ controller synthesis via switched PDC scheme for discrete-time T-S fuzzy systems. IEEE Transactions on Fuzzy Systems 17, 544-555.

Eli, G., S. Uri and Y. Isaac (2005). H_∞ Control and Estimation of State-Multiplicative Linear Systems. Springer, London.

Feng, G. (2006). Nonsynchronized state estimation of discrete time piecewise linear systems. IEEE Transactions on Signal Processing 54, 295-303.

Gassara, H., A. E. Hajjaji and M. Chaabane (2010). Observer-based robust H_∞ reliable control for uncertain T-S fuzzy systems with state time delay. IEEE Transactions on Fuzzy Systems 18, 1027-1040.

Ghaoui, L. E., F. Oustry and M. A. Rami (1997). A cone complementarity linearization algorithm for static output-feedback and related problems. IEEE Transactions on Automatic Control 42, 1171-1176.

Hsiao, F. H., C. W. Chen, Y. W. Liang, S. D. Xu and W. L. Chaing (2005a). T-S fuzzy controllers for nonlinear interconnected systems with multiple time-delays. IEEE Transactions on Circuits and Systems I: Regular Papers 52, 1883-1893.

Hsiao, F. H., J. D. Hwang, C. W. Chen and Z. R. Tsai (2005b). Robust stabilization of nonlinear multiple time-delay large-scale systems via decentralized fuzzy control. IEEE Transactions on Fuzzy Systems 13, 152-163.

Hwang, C. L. and H. Y. Lin (2004). A fuzzy decentralized variable structure tracking control with optimal and improved robustness design and applications. IEEE Transactions on Fuzzy Systems 12, 615-630.

Kim, S. H. and P. G. Park (2009). Observer-based relaxed H_∞ control for fuzzy systems using a multiple Lyapunov function. IEEE Transactions on Fuzzy Systems 17, 477-484.

Ku, C. C., P. H. Huang and W. J. Chang (2010). Passive fuzzy controller design for nonlinear systems with multiplicative noises. Journal of the Franklin Institute: Engineering and Applied Mathematics 347, 732-750.

Liang, J., Z. Wang and X. Liu (2010). On passivity and passification of stochastic fuzzy systems with delays: the discrete-time case. IEEE Transactions on Systems, Man, and Cybernetic, Part B: Cybernetics 40, 964-969.

Lin, C., Q. G. Wang, T. H. Lee and Y. He (2008). Design of observer-based H_∞ control for fuzzy time-delay systems. IEEE Transactions on Fuzzy Systems 16, 534-543.

Lin, W. W., W. J. Wang and S. H. Yang (2007). A novel stabilization criterion for large-scale T-S fuzzy systems. IEEE Transactions on Systems, Man, and Cybernetic, Part B: Cybernetics 37, 1074-1079.

Lozano, R., B. Brogliato, O. Egeland and B. Maschke (2001). Dissipative Systems Analysis and Control Theory and Application. Springer, London.

Ma, S. and E. K. Boukas (2009). Stability and robust stabilisation for uncertain discrete stochastic hybrid singular systems with time delay. Proceedings of IET Control Theory and Applications 3, 1217-1225.

Nachidi, M., A. Benzaouia, F. Tadeo and M. A. Rami (2008). LMI-based approach for output-feedback stabilization for discrete-time Takagi-Sugeno systems. IEEE Transactions on Fuzzy Systems 16, 1188-1196.

Souza, F. O., L. A. Mozelli and R. M. Palhares (2009). On stability and stabilization of T-S fuzzy time-delayed systems. IEEE Transactions on Fuzzy Systems 17, 1450-1455.

Tanaka, K. and H. O. Wang (2001). Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach. John Wiley & Sons., New York.

Tseng, C. S. (2008). A novel approach to H_∞ decentralized fuzzy-observer-based fuzzy control design for nonlinear interconnected systems. IEEE Transactions on Fuzzy Systems 16, 1337-1350.

Tseng, C. S. and B. S. Chen (2009). Robust fuzzy observer-based fuzzy control design for nonlinear discrete-time systems with persistent bounded disturbances. IEEE Transactions on Fuzzy Systems 17, 711-723.

Tseng, C. S. and B. S. Chen (2001). H_∞ -decentralized fuzzy model reference tracking control design for nonlinear interconnected systems. IEEE Transactions on Fuzzy Systems 9, 795-809.

Wang, W. J. and W. W. Lin (2005). Decentralized PDC for large-scale T-S fuzzy systems. IEEE Transactions on Fuzzy Systems 13, 779-786.

Wang, Y., D. Cheng, C. Li and Y. Ge (2003). Dissipative Hamiltonian realization and energy-based L_2-disturbance attenuation control of multimachine power systems. IEEE Transactions on Automatic Control 48, 1428-1433.

Wen, Y. and X. Ren (2010). Observer-based fuzzy adaptive control for nonlinear time-varying delay systems with unknown control direction. Proceedings of IET Control Theory and Applications 4, 2757-2769.

Yang, K. J., K. S. Hong and F. Matsuno (2005). Energy-based control of axially translating beams: varying tension, varying speed, and disturbance adaptation. IEEE Transactions on Control Systems Technology 13, 1045-1054.

Zhu, X. L. and G. H. Yang (2008). Jensen inequality approach to stability analysis of discrete-time system with time-varying delay. Proceedings of American Control Conference, Seattle, WA, USA, 1644-1649.