



## AN EVENT-TRIGGERED TRANSMISSION POLICY FOR NETWORKED L2-GAIN CONTROL

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# AN EVENT-TRIGGERED TRANSMISSION POLICY FOR NETWORKED $L_2$ -GAIN CONTROL

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Key words: networked control systems,  $L_2$ -gain control, event-triggered transmission, continuous-time systems, linear systems.

## ABSTRACT

This paper considers the design of networked feedback controllers with more efficient allocation of network resources. With a given admissible controller, an  $L_2$ -gain preserving event-triggered transmission policy is proposed for determining whether the currently measured state at the sensor node should be sent to the controller through the network for updating control signal. In this way, not all the measured data are sent and then the network traffic in a networked control system can be reduced. An algorithm is proposed to enlarge the transmission boundary of the event-triggered transmission rule for increasing inter-transmitting times. An illustrative example is given for verifying the benefit of the proposed approach.

## I. INTRODUCTION

In the later decades, modeling, analysis, and synthesis of networked control systems (NCSs) have attracted much attention (Wong and Brockett, 1997, 1999; Hristu and Morgansen, 1999; Walsh et al., 1999, 2002; Hristu, 2000; Walsh and Ye, 2001; Zhang et al., 2001; Tatikonda and Mitter, 2004). For preventing performance degradation causing by the transmission delay, packet dropout, and limited network bandwidth, reducing the network traffic in NCSs is shown to be an efficient way. Therefore, how to design low-network-usage networked feedback controllers to achieve desired performance is an important issue. In NCSs, periodic execution of the control algorithm in general leads to a conservative usage of network resources, since messages are sent through the net-

work at the same rate regardless of the current load in the network and the behavior of the controlled system (Anta and Tabuada, 2009). To relax the periodicity assumption, some researchers have applied event-triggered techniques to design feedback laws of NCSs. By event-triggered policies, data (signal) are sent (updated) only when they are critical for ensuring control performances (Arzen, 1999; Astrom and Bernharsson, 2002; Otanez et al., 2002; Heemels et al., 2007; Tabuada, 2007; Wang and Lemmon, 2008, 2009; Anta and Tabuada, 2010). It has been shown that event-triggered control techniques can significantly reduce the communication between sensors, controllers and actuators (Wang and Lemmon, 2008, 2009).

In this note, a new event-triggered transmission policy is proposed for determining whether the currently measured data at the sensor node is critical for stability and  $L_2$ -gain performance of an NCS. If the currently measured data is not critical, it will not be sent to the controller for saving network usage. In this case, the controller does not update the control signal. If the currently measured data is critical, it will be sent to the controller through the network and the controller updates the control signal (by zero-order holder). In Wang and Lemmon (2008), for the case that the controlled output is the full system state ( $z = x$ ), an event-triggered control scheme (based on Riccati equation) is proposed for guaranteeing finite  $L_2$ -gain stability. The proposed event-triggered transmission policy is derived based on the central controller. In this note, we consider the general case that the controlled output is a linear function of the system state, exogenous input, and control input. In addition, the proposed event-triggered transmission rule, which is derived by Riccati inequality, is applicable for all  $L_2$ -gain rendering controllers but not only for the central controller. In our approach, by the proposed event-triggered transmission policy, the closed-loop system has the same  $L_2$ -gain upper bound as in the point-to-point wiring case. Moreover, an LMI-based algorithm is proposed for seeking a larger transmission boundary of the event-triggered transmission rule. In general, by the proposed algorithm we can find an event-triggered transmission rule that resulting lager inter-transmitting times. The simulation results show that the provided approach can significantly reduce the network usage of NCSs without degrading the  $L_2$ -gain performance.

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## II. PROBLEM FORMULATION

Consider the following system:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_1w(t) + B_2u(t) \\ z(t) &= C_1x(t) + D_{11}w(t) + D_{12}u(t)\end{aligned}\quad (1)$$

where  $x \in R^n$  is the state,  $w \in R^s$  is the disturbance input,  $u \in R^m$  is the control input,  $x \in R^r$  is the controlled output,  $A$ ,  $B_1$ ,  $B_2$ ,  $C_1$ ,  $D_{11}$ , and  $D_{12}$  are constant matrices. Assume that  $(A, B_2)$  is stabilizable and  $(C_1, A)$  is observable. In addition, suppose that the following standard assumption holds.

**Assumption 1:** a)  $D_{11}^T D_{12} = 0$   
b)  $\gamma^2 I - D_{11}^T D_{11} > 0$ . ■

Now, for a given  $\gamma > 0$ , suppose that the following feedback law (by *point-to-point wiring*),

$$u = Fx, \quad (2)$$

be such that the closed-loop system

$$\begin{aligned}\dot{x}(t) &= (A + B_2F)x(t) + B_1w(t) \\ z(t) &= (C_1 + D_{12}F)x(t) + D_{11}w(t)\end{aligned}\quad (3)$$

is internally stable and satisfies the following  $L_2$ -gain requirement: for each  $T > 0$  and every  $w(t) \in L_2[0, T]$ ,

$$\int_0^T z^T(t)z(t)dt \leq \gamma_0^2 \int_0^T w^T(t)w(t)dt, \text{ for some } \gamma_0 < \gamma \quad (4)$$

A matrix  $F$  is called *admissible* if it is such that (3) is internally stable and satisfies (4).

In this note we consider the case that the feedback loop of system (1) is closed through a real-time shared network but not by point-to-point wiring. Suppose that the sensor node keeps measuring the system state, but not all the sampled data are sent to the controller. The transmission at the sensor node is not periodic. Let  $t_i$  ( $i = 1, 2, \dots$ ) be the time that the  $i$ -th transmission occurs at the sensor node. In this case, the controller can get only networked feedback data  $x(t_i)$ ,  $i = 1, 2, \dots$ , and the control signal is updated only at  $t_i$ . That is,

$$u(t) = Fx(t_i) \text{ as } t_i \leq t < t_{i+1}. \quad (5)$$

In general, for reducing the network traffic, inter-transmitting times  $\tau_i \equiv t_{i+1} - t_i$ ,  $i = 1, 2, \dots$ , should be as large as possible. We want to develop a rule to determine whether the currently measured state  $x(t)$  at the sensor node should be sent, through the network, to the controller for updating the control signal to guarantee the  $L_2$ -gain stability of the system. If the

measured data is not critical for  $L_2$ -gain stability, it will not be sent for saving network usage. In this case, the controller does not update the control signal. If the measured data is critical, it will be sent through the network to the controller, and the controller will update the control signal to control the system.

For simplification, this note assumes that the communication in the network is ideal in some fashions: no transmission delay, no packet dropout, and no quantization error.

## III. MAIN RESULTS

This section proposes the main results of this note.

### 1. Event-Triggered Transmission Policies

Consider the system (1). With the event-triggered networked feedback controller (5), the control signal is piecewise constant and the closed-loop system can be described as:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_1w(t) + B_2Fx(t_i), \quad t_i \leq t < t_{i+1}, \\ z(t) &= C_1x(t) + D_{11}w(t) + D_{12}Fx(t_i)\end{aligned}\quad (6)$$

Define the error function  $e(t)$  as

$$e(t) = x(t) - x(t_i), \quad t_i \leq t < t_{i+1}. \quad (7)$$

It is clear that  $e(t_i) = 0$ ,  $i = 1, 2, \dots$ . By (6) and (7) we have

$$\begin{aligned}\dot{x}(t) &= (A + B_2F)x(t) + B_1w(t) - B_2Fe(t), \quad t_i \leq t < t_{i+1}, \\ z(t) &= (C_1 + D_{12}F)x(t) + D_{11}w(t) - D_{12}Fe(t)\end{aligned}\quad (8)$$

For system (1) with the  $L_2$ -gain rendering controller (2), by *Bounded Real Lemma* (Gahinet and Apkarian, 1994), there is a positive definite matrix  $X$  satisfies the Riccati inequality:

$$\begin{aligned}(A + B_2F + B_1(\gamma^2 I - D_{11}^T D_{11})^{-1} D_{11}^T C_1)^T X \\ + X(A + B_2F + B_1(\gamma^2 I - D_{11}^T D_{11})^{-1} D_{11}^T C_1) \\ + (C_1 + D_{12}F)^T (I + D_{11}(\gamma^2 I - D_{11}^T D_{11})^{-1} D_{11}^T) (C_1 + D_{12}F) \\ + XB_1(\gamma^2 I - D_{11}^T D_{11})^{-1} B_1^T X < 0.\end{aligned}\quad (9)$$

By Schur complement, it is known that  $X$  also satisfies the matrix inequality:

$$\begin{bmatrix} (A + B_2F)^T X + X(A + B_2F) & XB_1 & (C_1 + D_{12}F)^T \\ B_1^T X & -\gamma^2 I & D_{11}^T \\ C_1 + D_{12}F & D_{11} & -I \end{bmatrix} < 0 \quad (10)$$

Choose  $V(x) = x^T Xx$  as a candidate storage function for the closed-loop NCS (6). If we can show that, along the trajectories of (6) (or (8)),

$$\begin{aligned} \dot{V}(x(t)) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) < 0, \\ \forall x(t) \neq 0 \text{ and } \forall w(t), \end{aligned} \quad (11)$$

then the event-triggered networked feedback closed-loop system (6) is internally stable and satisfies (4).

Let  $X > 0$  satisfy (9) and define

$$\begin{aligned} Y(F, X) \equiv & (A + B_2F + B_1(\gamma^2 I - D_{11}^T D_{11})^{-1} D_{11}^T C_1)^T X \\ & + X(A + B_2F + B_1(\gamma^2 I - D_{11}^T D_{11})^{-1} D_{11}^T C_1) \\ & + (C_1 + D_{12}F)^T (I + D_{11}(\gamma^2 I - D_{11}^T D_{11})^{-1} D_{11}^T) (C_1 + D_{12}F) \\ & + XB_1(\gamma^2 I - D_{11}^T D_{11})^{-1} B_1^T X \end{aligned} \quad (12)$$

Define

$$\begin{aligned} a(F) &= \lambda_{\max}(F^T D_{12}^T D_{12} F) \\ b(F, X) &= \|(XB_2 + C_1^T D_{12})F + F^T D_{12}^T D_{12} F\| \\ c(F, X) &= \lambda_{\max}(Y(F, X)) \end{aligned}$$

By (9), it is clear that  $Y(F, X)$  is negative definite and  $c(F, X) < 0$ . In addition, both  $a(F)$  and  $b(F, X)$  are nonnegative. We have the following result.

**Theorem 1:** Consider system (1). Suppose that  $F$  is such that the point-to-point wiring closed-loop system (3) is internally stable and satisfies (4), and  $X$  is a positive definite solution of (9). The networked closed-loop system (6) is internally stable and satisfies (4), if

$$\|e(t)\| < \eta(F, X) \cdot \|x(t)\|, \quad (13)$$

holds for all  $t \in [t_i, t_{i+1})$  and any  $i = 1, 2, \dots$ , where the *transmission boundary*

$$\eta(F, X) \equiv \begin{cases} \frac{-b(F, X) + \sqrt{b^2(F, X) - a(F)c(F, X)}}{a(F)}, & \text{if } a(F) > 0 \\ -\frac{c(F)}{2b(F, X)}, & \text{if } a(F) = 0 \end{cases}. \quad (14)$$

**Proof:** Along the trajectories of (6) (or (8)) we have

$$\begin{aligned} \dot{V}(x) + z^T z - \gamma^2 w^T w \\ = \dot{x}^T Xx + x^T X\dot{x} + z^T z - \gamma^2 w^T w \\ = ((A + B_2F)x + B_1w - B_2Fe)^T Xx \end{aligned}$$

$$\begin{aligned} + x^T X((A + B_2F)x + B_1w - B_2Fe) \\ + ((C_1 + D_{12}F)x + D_{11}w - D_{12}Fe)^T \\ \cdot ((C_1 + D_{12}F)x + D_{11}w - D_{12}Fe) - \gamma^2 w^T w \\ = x^T ((A + B_2F)^T X + X(A + B_2F) \\ + (C_1 + D_{12}F)^T (C_1 + D_{12}F))x \\ + x^T (XB_1 + C_1^T D_{11})(\gamma^2 I - D_{11}^T D_{11})^{-1} (B_1^T X + D_{11}^T C_1)x \\ - e^T F^T (B_2^T X + D_{12}^T (C_1 + D_{12}F))x \\ - x^T (XB_2 + (C_1 + D_{12}F)^T D_{12})Fe \\ + e^T F^T D_{12}^T D_{12} Fe \\ - (w - w_*(x))^T (\gamma^2 I - D_{11}^T D_{11})(w - w_*(x)) \end{aligned}$$

where

$$w_*(x) = (\gamma^2 I - D_{11}^T D_{11})^{-1} (B_1^T X + D_{11}^T C_1)x.$$

Then,

$$\begin{aligned} \dot{V}(x) + z^T z - \gamma^2 w^T w \\ \leq x^T Yx - 2x^T XB_2Fe - 2x^T (C_1 + D_{12}F)^T D_{12}Fe \\ + e^T F^T D_{12}^T D_{12} Fe \\ \leq \lambda_{\max}(Y) \cdot \|x\|^2 + 2\|x\| \cdot \|XB_2F + (C_1 + D_{12}F)^T D_{12}F\| \cdot \|e\| \\ + \lambda_{\max}(F^T D_{12}^T D_{12} F) \cdot \|e\|^2 \\ \leq c(F, X) \cdot \|x\|^2 + 2b(F, X) \cdot \|x\| \cdot \|e\| + a(F) \cdot \|e\|^2. \end{aligned} \quad (15)$$

Note that if  $a(F) = 0$ , then  $D_{12}F = 0$  and  $b(F, X) = \|XB_2F\| \neq 0$ . Therefore, if (13) holds, we have

$$\begin{aligned} \dot{V}(x(t)) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) < 0, \\ \forall x(t) \neq 0 \text{ and } \forall w(t). \end{aligned} \quad (16)$$

This shows that the networked closed-loop system (6) satisfies (4). To prove the closed-loop stability, letting  $w = 0$  in (16) yields

$$\dot{V}(x(t)) < -z^T(t)z(t) \leq 0, \quad \forall x(t) \neq 0.$$

That is, the closed-loop system is internally stable. ■

Note that if we make an additional standard assumption that  $D_{12}$  has full column rank, then  $a(F) > 0$  for any nonzero  $F$ .

## 2. Synthesis of $L_2$ -gain Rendering Controller

The result in Theorem 1 holds for any (point-to-point wiring)  $L_2$ -gain rendering controller (2). To get a feedback gain  $F$  such that (9) (or (10)) has a positive definite solution  $X$  is not difficult by the linear matrix inequality (LMI) approach.

**Lemma 1** (Gahinet and Apkarian, 1994): Consider system (1). If there exist positive definite matrix  $S \in R^{n \times n}$  and matrix  $M \in R^{m \times n}$  satisfying the following LMI:

$$\begin{bmatrix} AS + SA^T + B_2M + M^T B_2^T & B_1 & SC_1^T + M^T D_{12}^T \\ B_1^T & -\gamma^2 I & D_{11}^T \\ C_1S + D_{12}M & D_{11} & -I \end{bmatrix} < 0, \quad (17)$$

then

$$u = Fx$$

with  $F = MS^{-1}$  is an  $L_2$ -gain rendering feedback law for (1). In addition,  $X = S^{-1}$  satisfies (9) (and then (10)). ■

## 3. An Algorithm for Seeking Larger Transmission Boundary

The result in Theorem 1 holds for any (point-to-point wiring)  $L_2$ -gain rendering controller (2). However, for an admissible  $F$ , there can be infinite solutions to the Riccati inequality (9). To reduce network usage, in general  $\eta(F, X)$  should be as large as possible. On interesting problem is, for a given dismissible  $F$ , how to find a better solution  $X$  to (9) to get a larger  $\eta(F, X)$ . By Theorem 1, it can be seen that, for a given  $F$ , finding a solution  $X$  for (9) to get the maximal  $\eta(F, X)$  is very difficult since it is a nonlinear optimization problem. Here we develop a line search algorithm to find a solution  $X$  for (9) for getting larger  $\eta(F)$ . To this end, for a given admissible feedback gain matrix  $F$ , consider the following LMI:

$$\begin{bmatrix} (A + B_2F)^T X + X(A + B_2F) & XB_1 & (C_1 + D_{12}F)^T \\ B_1^T X & -\gamma^2 I & D_{11}^T \\ C_1 + D_{12}F & D_{11} & -I \end{bmatrix} < -\rho Q \quad (18)$$

where  $Q \in R^{(n+s+r) \times (n+s+r)}$  is a positive semidefinite matrix. For a given  $\rho > 0$ , if there exists a positive definite matrix  $X$  satisfying the inequality (18), it is clear that  $X$  also satisfies inequality (10). Based on this observation, with a given admissible  $F$ , we can vary  $\rho$  and then solve (18) for searching possible larger  $\eta(F, X)$ . The detail algorithm is given below.

### Algorithm:

1. Let  $\rho(0) = 0$ ,  $\eta(0) = 0$ ,  $\eta_{opt} = 0$ , and  $q = 0$ .
2. Let  $q = q + 1$  and  $\rho(q) = \rho(q - 1) + \Delta\rho$  with a small  $\Delta\rho$ . If  $\rho(q) \geq K$  for a pre-specified  $K > 0$ , go to STEP 5; else solve (18).
3. If there are no positive definite solutions to (20), go to STEP 5. Else, with the obtained solution  $X(q)$ , calculate  $\eta(q)$  by (14).
4. If  $\eta(q) > \eta(q - 1)$ , then  $\eta_{opt} = \eta(q)$  and  $X_{opt} = X(q)$ . Go to STEP 2.  
If  $\eta_{opt} = 0$ , this algorithm is not applicable. Else, the optimal transmission boundary is  $\eta = \eta_{opt}$ , and  $X = X_{opt}$ .

## IV. AN ILLUSTRATIVE EXAMPLE

Consider the following system:

$$\dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t) \quad (19)$$

$$z(t) = C_1x(t) + D_{11}w + D_{12}u(t)$$

with

$$A = \begin{bmatrix} 0 & -1 \\ -0.5 & -0.5 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 \\ -0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix},$$

$$C_1 = [-0.4 \quad 0.3], D_{11} = 0, D_{12} = 0.5.$$

Note that  $D_{12}$  is full column rank. For  $\gamma = 3$ , by solving (17), we have

$$S = \begin{bmatrix} 1.3079 & 0.6712 \\ 0.6712 & 1.3941 \end{bmatrix}$$

and

$$F = MS^{-1} = [-1.5011 \quad 1.7203].$$

With this obtained  $F$ ,  $a(F) \neq 0$ . For verifying the benefits of the proposed event-triggered transmission policy and the algorithm, three cases are considered for comparison.

Case 1 – The algorithm is not used.

Let

$$X = S^{-1} = \begin{bmatrix} 1.0156 & -0.4890 \\ -0.4890 & 0.9527 \end{bmatrix}.$$

In this case, we have

$$Y = \begin{bmatrix} -1.9570 & 1.0143 \\ 1.0143 & -1.9195 \end{bmatrix}$$

and the obtained transmission boundary

$$\eta(F, X) = 0.2371.$$

Case 2 – The algorithm is used with  $Q = \text{diag}\{1, 1, 0, 0\}$ .

By the algorithm we obtain

$$X = \begin{bmatrix} 5.2328 & 1.6338 \\ 1.6338 & 5.9515 \end{bmatrix}$$

$$Y = \begin{bmatrix} -13.5581 & -0.6573 \\ -0.6573 & -11.6116 \end{bmatrix}$$

and the obtained transmission boundary

$$\eta(F, X) = 0.6110.$$

Case 3 – The algorithm is used with  $Q = \text{diag}\{0, 1, 0, 0\}$ .

By the algorithm we have

$$X = \begin{bmatrix} 3.3971 & 1.2302 \\ 1.2302 & 5.2269 \end{bmatrix}$$

$$Y = \begin{bmatrix} -8.2503 & -1.0219 \\ -1.0219 & -10.4115 \end{bmatrix}$$

and the obtained transmission boundary

$$\eta(F, X) = 0.6759.$$

From Theorem 1, the system (21) is internally stable and has  $L_2$  - gain less than 3 if

$$\|e(t)\| = \|x(t) - x(t_i)\| < \eta(F, X)\|x(t)\|,$$

holds for all  $t \in [t_i, t_{i+1})$  and for each  $i = 1, 2, \dots$ . By this condition, let the transmission at the sensor node be triggered by the condition

$$\|e(t)\| = \|x(t) - x(t_i)\| \geq 0.95\eta(F, X)\|x(t)\|.$$

For these three cases, Fig. 1, Fig. 2, and Fig. 3 show the responses of the closed-loop system (initial state is  $[-1 \ 2]^T$ ) with the event-triggered networked controller

$$u(t) = Fx(t_i), \quad t \in [t_i, t_{i+1}), \quad i = 1, 2, \dots, \quad (20)$$

under the influence of the external disturbance

$$w(t) = \sin(x_1(t) + x_2(t)) - e^{-0.1t} \cos(100t).$$

For Case 1, the number of transmission events in the first 5 seconds is 376. The average inter-transmitting time is 0.013

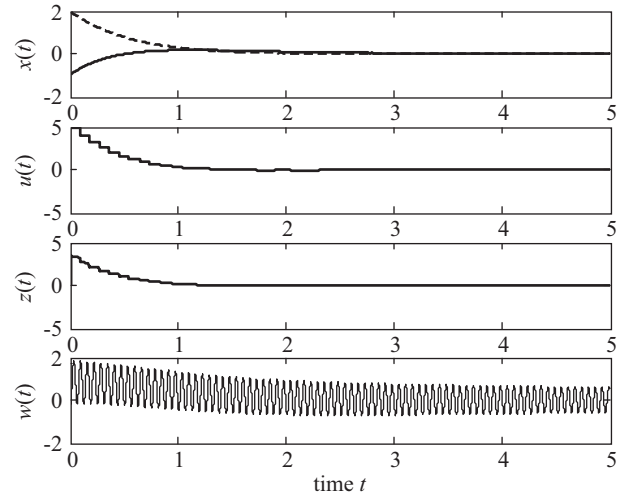


Fig. 1. Responses of the closed-loop system for Case 1.

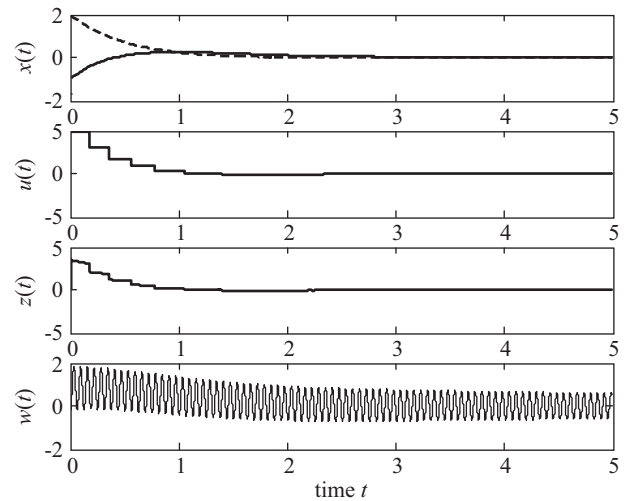


Fig. 2. Responses of the closed-loop system for Case 2.

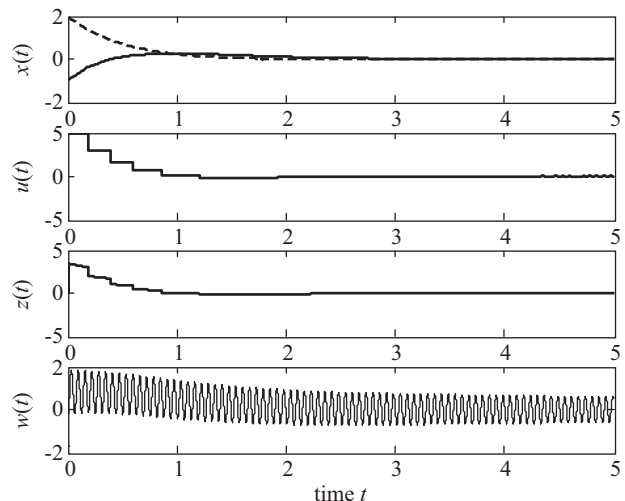


Fig. 3. Responses of the closed-loop system for Case 3.

sec. For Case 2, the number of transmission events in the first 5 seconds is 111. The average inter-transmitting time is 0.045 sec. For Case 3, the number of transmission events in the first 5 seconds is 104. The average inter-transmitting time is 0.48 sec. This shows that the proposed algorithm can significantly reduce the network usage in NCSs.

## V. CONCLUSIONS

In this paper, we have developed a new approach for designing stabilizing and  $L_2$ -gain rendering networked feedback controllers with low transmission rate. We have derived a state-dependent event-triggered transmission policy for NCSs to reduce network usage. By the simulation results we can see that the proposed method can significantly reduce network traffics in NCSs.

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